F1

THREE PHASE AC SUPPLIES

Double Subscript Notation

AC voltages are written using double subscript notation and as complex numbers in polar form.

Voltage E_{AB} = voltage at point A with reference to point B.
Voltage E_{BA} = voltage at point B with reference to point A.

The second mentioned letter is always the reference point.

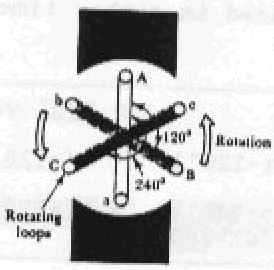
Example: $E_{AB} = 200\angle 0^\circ$ volts
$E_{BA} = 200\angle \pm 180^\circ$ volts (opposite to E_{AB})

Generation of a Three Phase Set of EMFs

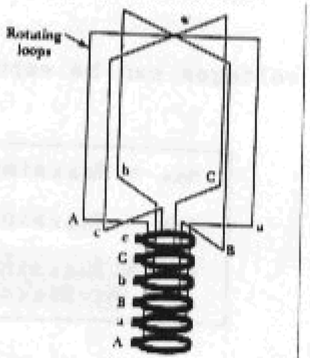
A machine with a single rotating coil in a magnetic field generates one cycle of sinusoidal emf for each revolution of the coil. This is called a single phase emf.

If three coils are spaced equally, 120° apart, around the machine, then the generated emfs in each coil will pass zero and reach peaks at different times. The three emfs generated are three single phase emfs.

Refer to FIG 1 which shows three coils displaced by 120° from each other, rotating between two magnetic poles, and the electrical connections to the coils through sliprings and brushes.



(a) Three-phase generation



(b) Output connections of generator

FIG 1

The coils are labelled A, B and C, and the emfs will reach peaks in the order A, B, C, if the coil system is rotated anticlockwise.

The coils are electrically separate and the emfs measured across the ends of each coil are identified as e_{Aa}, e_{Bb} and e_{Cc}, in accordance with double subscript notation.

The three separate generator winding voltages are equal in magnitude but displaced from each other in phase by ±120°

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FIG 2 below shows the waveforms of the three separate <u>phase</u> voltages in each coil, e_{Aa}, e_{Bb} and e_{Cc}.

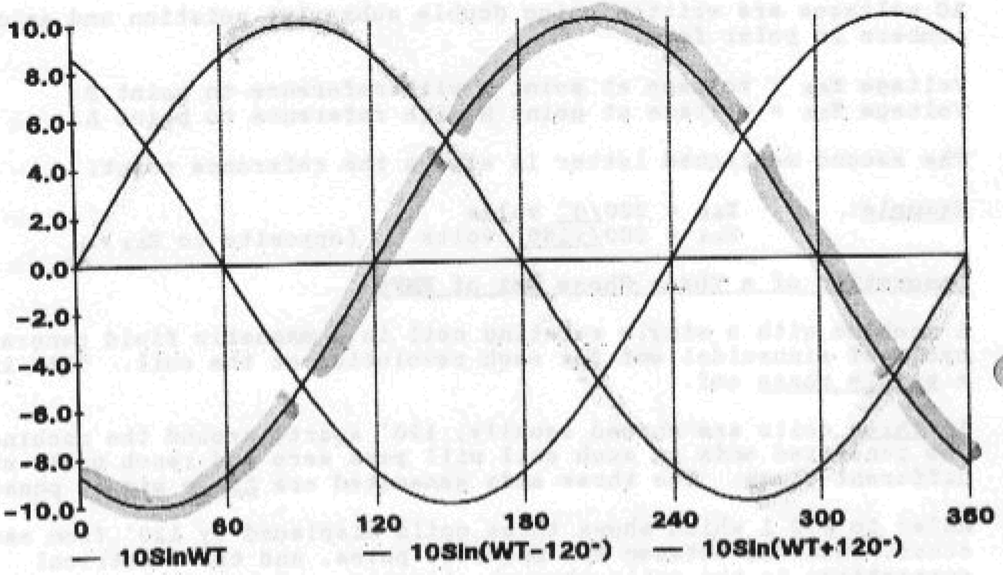

FIG 2

The three voltages can be expressed in either time domain or complex form.

$e_{Aa} = E_{MAX}\sin wt$	$= E\angle 0^\circ$ volts
$e_{Bb} = E_{MAX}\sin(wt-120^\circ)$	$= E\angle -120^\circ$ volts
$e_{Cc} = E_{MAX}\sin(wt-240^\circ)$ or $E_{MAX}\sin(wt+120^\circ)$	$= E\angle +120^\circ$ volts

Refer to FIG 3 which shows the phase relationships of ±120° and the sequence A, B and C of the three emfs on a phasor diagram

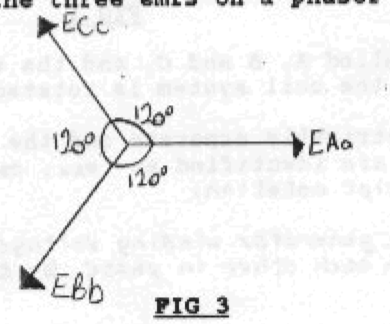

FIG 3

Three Phase Notation

The three phase windings in a generator can be labelled in any way, however, standard identification has been adopted.

Phases in a three phase supply system can be identified as:

a) A, B, C or
b) 1, 2, 3 or
c) Red, Yellow, Blue (R, Y, B) old system or
d) Red, White, Blue (R, W, B) new international standard.

The use of colours to identify equipment and wiring is an industrial standard.

R-W-B is the more recent standard adopted, however, R-Y-B labelled equipment is still quite common in industrial installations.

Phase Sequence (Phase Rotation)

Phase sequence or phase rotation is defined as the order in which the phase voltages reach their peak values.

If the coils on the machine in FIG 1 are rotated anticlockwise, the phase rotation is in the order A-B-C.
This is called "**Positive Phase Sequence**"

If the coils on the machine in FIG 1 are rotated clockwise, the phase rotation is in the order A-C-B.
This is called "**Negative Phase Sequence**"

Effect of Incorrect Phase Sequence on Rotating Machinery

The direction of rotation of a three phase motor, will depend on the phase sequence of the supply voltage.

If the phase sequence is **reversed**, then the machine will rotate in the **reverse** direction.

Connection of Three Phase Generators

The three windings of the machine could be kept electrically separate, and supple separate single phase loads.

This would require six separate wires to the ends of each of the three windings, in which case the connections would be as shown in FIG 4.

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This would require six separate wires to the ends of each of the three windings, in which case the connections would be as shown in FIG 4.

Voltages in a Balanced Star Connected Generator

Line voltages ($E_{line-line}$) are measured between any two line terminals.

There are three line voltages in the circuit of FIG 5., E_{AB}, E_{BC}, E_{CA}.

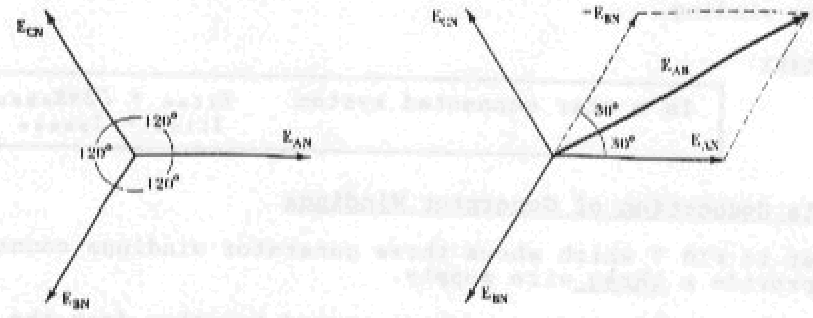
Phase Voltages ($E_{line-neutral}$) are measured across any winding or between any line terminal and the neutral terminal.

There are three phase voltages in the circuit of FIG 5., E_{AN}, E_{BN}, E_{CN}.

The three generator winding voltages (phase voltages), are equal in magnitude (balanced) but displaced from each other in phase, by ±120°.

Each line voltage is the vector sum of two phase voltages.

Refer to the phasor diagrams in FIG 6. which show three phase voltages and the vector addition of the phase voltages to give the three line voltages.



(a) Relationship of phase voltages

(b) Line voltage E_{AB} is the phasor difference $E_{AN} - E_{BN}$

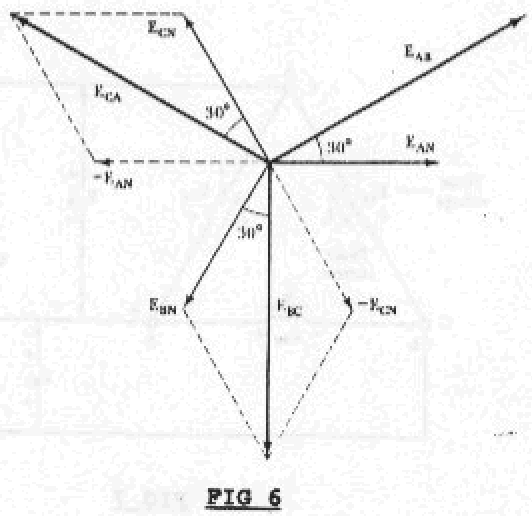


FIG 6

Line Voltage $E_{AB} = E_{AN} + E_{NB}$
Line Voltage $E_{BC} = E_{BN} + E_{NC}$
Line Voltage $E_{CA} = E_{CN} + E_{NA}$

Notes: To carry out these vector additions, one of the phase voltage must be reversed.
Line voltages are all equal in magnitude.
Line voltages are $\sqrt{3}$ times the phase voltages.
Line voltages lead the phase voltages by 30°.

Currents in a Balanced Star Connected Generator

From the circuit diagram in FIG 5 it can be seen that the current flowing in a line conductor is the same as the current flowing in a phase winding.

Summary:

In a star connected system	$E_{line} = \sqrt{3} \times E_{phase}$ $I_{line} = I_{phase}$

Delta Connection of Generator Windings

Refer to FIG 7 which shows three generator windings connected in delta to provide a three wire supply.

The ends of the windings are connected together from the start of one winding to the finish of the next winding to form a delta.

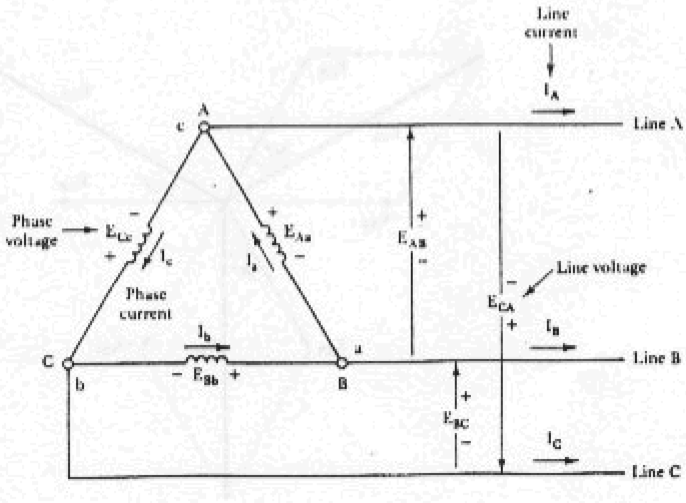


FIG 7

Voltages in a Balanced Delta Connected Generator

Line voltages ($E_{line-line}$) are measured between any two line terminals.

There are three line voltages in the circuit of FIG 7., E_{AB}, E_{BC}, E_{CA}.

Phase Voltages (E_{phase}) are measured across any winding and are the same as the line voltages in the delta circuit.

There are three phase voltages in the circuit of FIG 7., E_{AB}, E_{BC}, E_{CA}.

Refer to the phasor diagram in FIG 8. which shows the phase voltages and line voltages in a delta system.

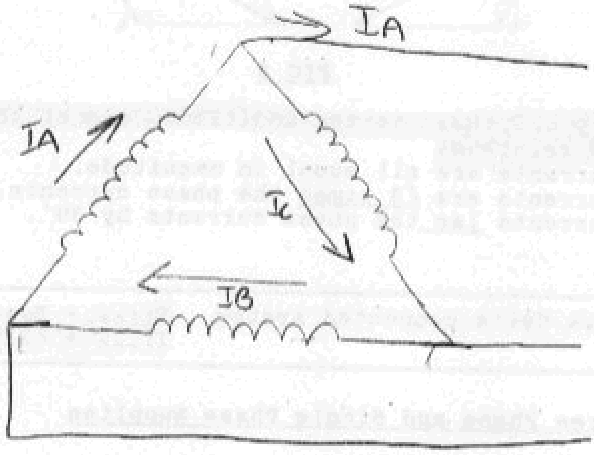


FIG 8

Currents in a Balanced Delta Connected Generator

From the circuit diagram in FIG 7 it can be seen that the current flowing in a line conductor is vector sum of two currents flowing in the phase windings.

Applying Kirchhoff's Current Law at each line terminal:

Line Current $I_A = I_A - I_C$

Line Current $I_B = I_B - I_A$

Line Current $I_C = I_C - I_B$

Refer to the phasor diagram in FIG 9. which shows the vector addition of the phase currents to give the three line currents.

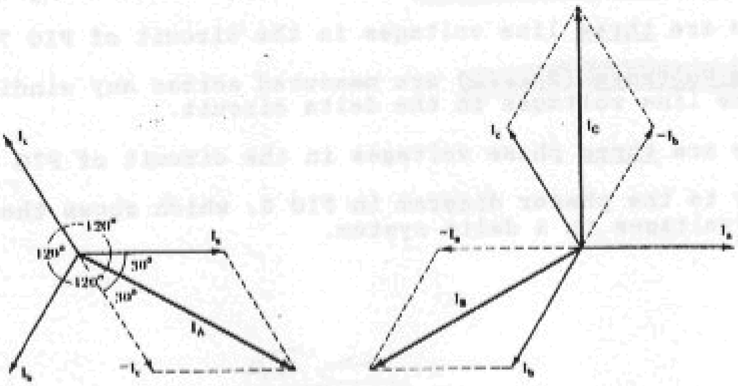


FIG 9

Notes: To carry out these vector additions, one of the phase currents must be reversed.
Line currents are all equal in magnitude.
Line currents are $\sqrt{3}$ times the phase currents.
Line currents lag the phase currents by 30°.

Summary:

In a delta connected system	$E_{line} = E_{phase}$ $I_{line} = \sqrt{3} I_{phase}$

Comparison of Three Phase and Single Phase Supplies

Power Supplied:
Power delivered to a load from a single phase supply is in two pulses per cycle of the supply voltage.
Power delivered to a load by a three phase supply, comes from each phase in turn, which results in a smoother application of power to the load. (3 phase motors run smoother than 1 phase motors and are physically smaller for the same power rating)

Voltages:
A three phase supply allows flexibility of two different voltage levels and a choice of 1, 2 or 3 phase supply.

Currents:
As power is supplied from three sources, the current necessary to deliver the same power as a single phase supply, is less.
This means that the conductor size can be smaller.
Smaller current is an advantage for motor starting.

Number of Conductors:
Three separate single phase supplies would require six conductors joining the supply to the load.
A three phase supply requires only three or four conductors of smaller cross sectional area.

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CONNECTION OF BALANCED THREE PHASE LOADS

Three phase supplies can be either three wire or four wire.

Three phase loads however, can be connected to the supply in a number of ways.

Three phase loads can be either **Star** or **Delta** connected, and may be either **balanced** or **unbalanced**.

A **balanced** three phase load is defined as having **equal** impedances in all phases.

An **unbalanced** three phase load is defined as having **unequal** impedances in all phases.

Four Wire Supply with a Balanced Star Connected Load

Refer to FIG 1 which shows a star connected load connected to a four wire three phase supply.

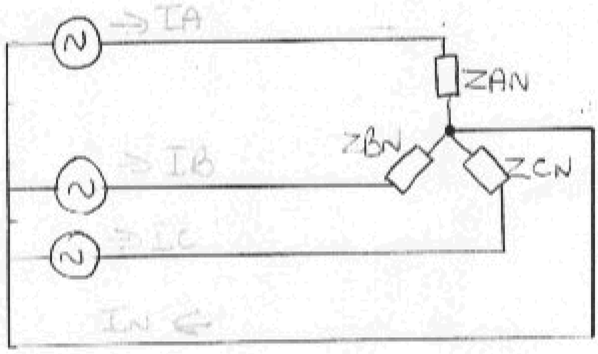

FIG 1

Notes: The impedances in the three phases are equal in magnitude and angle.

Line current I_{line} = Phase current I_{phase}

All line currents are equal.

Phase voltage $E_{phase} = E_{line}/\sqrt{3}$

The connection of the neutral (fourth wire), ensures that all phase voltages are equal at the load.

Neutral current is equal to the vector sum of the line currents.

$$I_{NEUTRAL} = I_A + I_B + I_C = 0 \text{ (for a balanced load)}$$

Example: A balanced three phase star connected load, consists of three 100Ω resistors connected via a four wire balanced supply system, to an alternator, having a line-neutral voltage of 100V rms.

Assume voltage E_{AN} as reference quantity.

Calculate:

a) the line voltages in polar form,
b) the line currents in polar form,
c) the neutral current in polar form

Solution:

Refer to the circuit diagram in FIG 2.

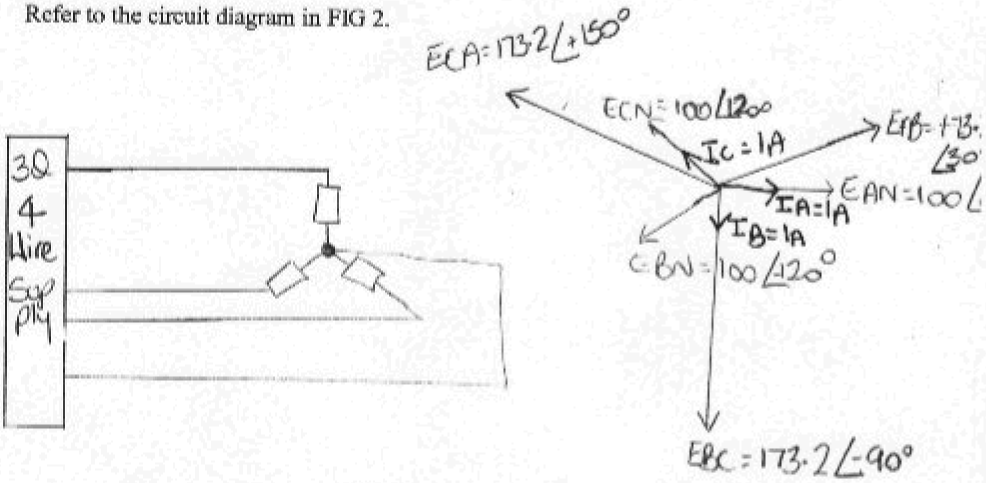

FIG 2

a) Since the supply is balanced,

$E_{AN} = 100\angle 0°$ volts
$E_{BN} = 100\angle -120°$ volts
$E_{CN} = 100\angle +120°$ volts

$Z = R + jx$
$= 100 + j0$
$= 100\angle 0°\ \Omega$

All Line voltages = $\sqrt{3} \times E_{phase}$ = $\sqrt{3} \times 100$ = 173.2V rms.

$E_{AB} = 173.2\angle +30°$ volts
$E_{BC} = 173.2\angle -90°$ volts
$E_{CA} = 173.2\angle +150°$ volts

b) Line current $I_A = \dfrac{E_{AN}}{Z_{AN}} = \dfrac{100\angle 0°}{100\angle 0°} = 1\angle 0°$ amps rms

Line current $I_B = \dfrac{E_{BN}}{Z_{BN}} = \dfrac{100\angle -120°}{100\angle 0°} = 1\angle -120°$ amps rms

Line current $I_C = \frac{E_{CN}}{Z_{CN}} = \frac{100/+120°}{100/0°} = 1/+120°$ amps rms

All line currents are equal in magnitude and displaced by 120°.

c) $I_{NEUTRAL} = I_A + I_B + I_C$
$= 1/0° + 1/-120° + 1/+120°$
$= (1 + j0) + (-0.5 - j0.866) + (-0.5 + j0.866)$
$= 0$

Balanced Delta Connected Loads

Refer to FIG 3 which shows a balanced delta load connected to a three wire balanced supply.

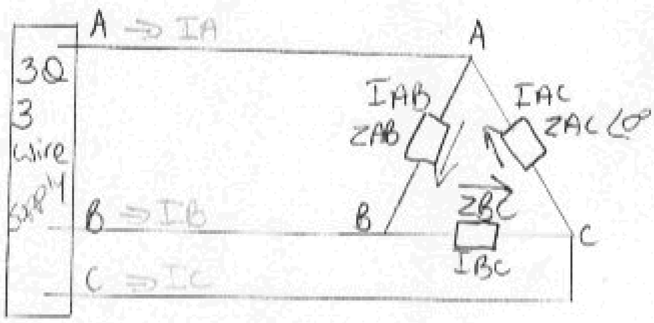

FIG 3

Notes: All phase currents are equal and symmetrical in phase.
All line currents are equal and symmetrical in phase.
Line currents are equal to √3 times the phase currents.
The vector sum of the three line currents is zero.

Phase current $I_{AB} = \frac{E_{AB}}{Z_{AB}}$

Phase current $I_{BC} = \frac{E_{BC}}{Z_{BC}}$

Phase current $I_{CA} = \frac{E_{CA}}{Z_{CA}}$

Line current $I_A = I_{AB} - I_{CA} = \sqrt{3} \times I_{PHASE}$

Line current I_B = $I_{BC} - I_{AB}$ = $\sqrt{3} x I_{PHASE}$

Line current I_C = $I_{CA} - I_{BC}$ = $\sqrt{3} x I_{PHASE}$

Example: A balanced delta load of R = 50Ω per phase is connected across a three phase balanced supply of 400V rms line-line.
Using voltage E_{AB} as reference, calculate:

a) all phase currents in polar form
b) all line currents in polar form.
Draw the complete phasor diagram.

Solution:

Refer to the circuit diagram in FIG 4.

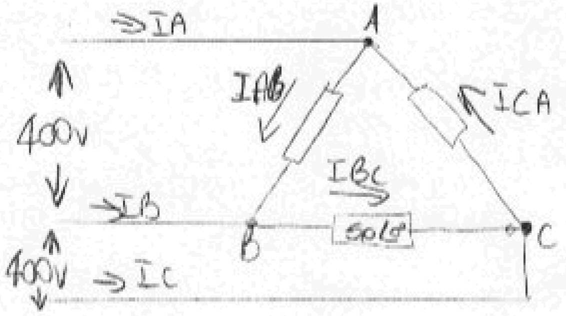

FIG 4

a) Phase current $I_{AB} = \frac{E_{AB}}{Z_{AB}} = \frac{400\angle 0°}{50\angle 0°} = 8\angle 0°$ amps rms

Phase current $I_{BC} = \frac{E_{BC}}{Z_{BC}} = \frac{400\angle -120°}{50\angle 0°} = 8\angle -120°$ amps rms

Phase current $I_{CA} = \frac{E_{CA}}{Z_{CA}} = \frac{400\angle +120°}{50\angle 0°} = 8\angle +120°$ amps rms

b) Line Current I_A = $I_{AB} - I_{CA}$
= $8\angle 0° - 8\angle +120°$

= (8 + j0) - (-4 + j6.9)
= 12 - j6.9
= 13.8/-30° amps rms

Line Current I_B = $I_{BC} - I_{AB}$
= 8/-120° - 8/0°
= (-4 - j6.9) - (8 + j0)
= -12 - j6.9
= 13.8/-150° amps rms

Line Current I_C = $I_{CA} - I_{BC}$
= 8/+120° - 8/-120°
= (-4 + j6.9) - (-4 - j6.9)
= 0 + j13.8
= 13.8/+90° amps rms

The sum of the line currents is:

$(I_A + I_B + I_C)$ = (12 - j6.9) + (-12 - j6.9) + (0 + j13.8)
= 0

Notes: With a balanced load, we can calculate one line current and then displace the other two by ±120° since they are equal and symmetrical.
The sum of the three line currents in a three wire system is always equal to zero, since there is no path for unbalance current to flow.

Refer to FIG 5 which is the complete phasor diagram.

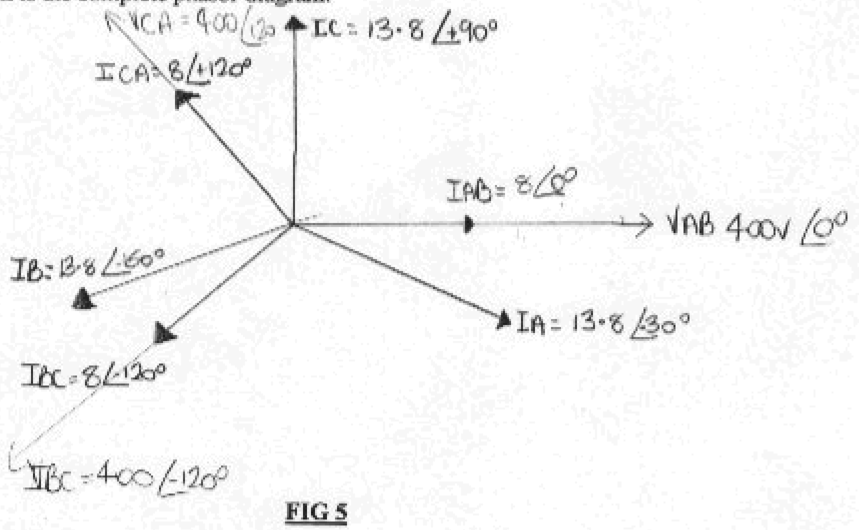

FIG 5

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DELTA-STAR AND STAR-DELTA CONVERSIONS

Star Connected Impedances

Three impedances are connected in the "Star" or "Wye" connection when they are connected as shown in FIG 1.

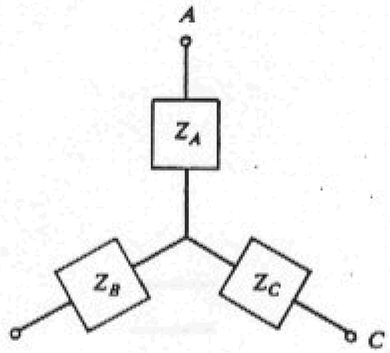

FIG 1

This connection is used in three phase circuits where the three phase supply is available either as a three or four wire system.

Impedances may also be connected in this configuration in either DC or single phase AC circuits that may need to be simplified and solved.

Delta Connected Impedances

Three impedances are connected in the "Delta" connection when they are connected as shown in FIG 2.

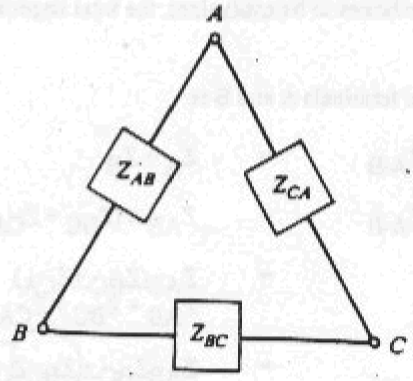

FIG 2

This connection is used in three phase circuits where the three phase supply is available as a three wire system.

Impedances may also be connected in this configuration in either DC or single phase AC circuits that may need to be simplified and solved.

Delta-Star Transformation

A delta connected set of impedances can be replaced by an equivalent star connected set of impedances that will appear to be the same impedance between each two line terminals.

Refer to FIG 3 which shows a star connected set of impedances Z_A, Z_B and Z_C, and also a delta connected set of impedances Z_{AB}, Z_{BC} and Z_{CA}.

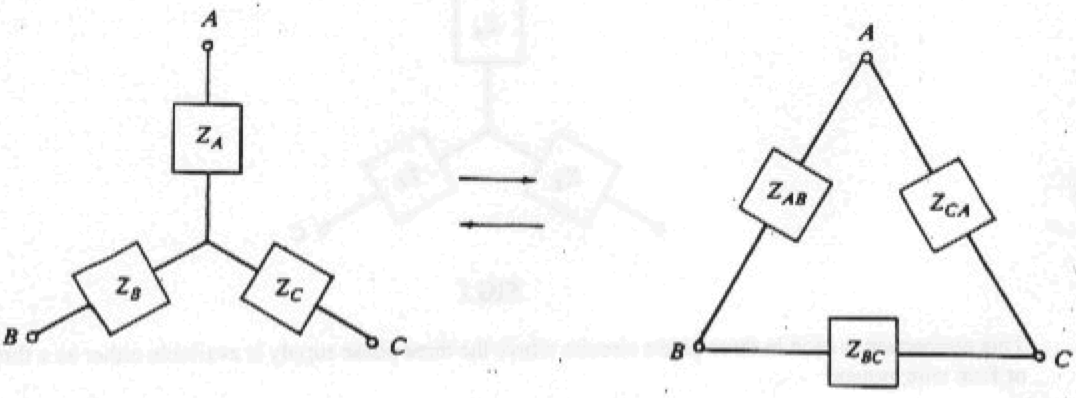

FIG 3

For the two sets of impedances to be equivalent, the total impedance between any two line terminals must be the same.

Impedance between line terminals A and B is:

In star connection: $Z_{A\text{-}B} = Z_A + Z_B$

In delta connection: $Z_{A\text{-}B} = Z_{AB} \,//\, (Z_{BC} + Z_{CA})$

$$= \frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} = \frac{\text{product}}{\text{sum}}$$

$$= \frac{Z_{AB}Z_{BC} + Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Correct Z_{AB}

Equate the star and delta impedances:

$$Z_A + Z_B = \frac{Z_{AB}Z_{BC} + Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}} \qquad \text{Equation 1.}$$

Fig

CONNECTION OF BALANCED THREE PHASE LOADS

Three phase supplies can be either three wire or four wire.

Three phase loads however, can be connected to the supply in a number of ways.

Three phase loads can be either **Star** or **Delta** connected, and may be either **balanced** or **unbalanced**.

A **balanced** three phase load is defined as having **equal** impedances in all phases.

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Four Wire Supply with a Balanced Star Connected Load

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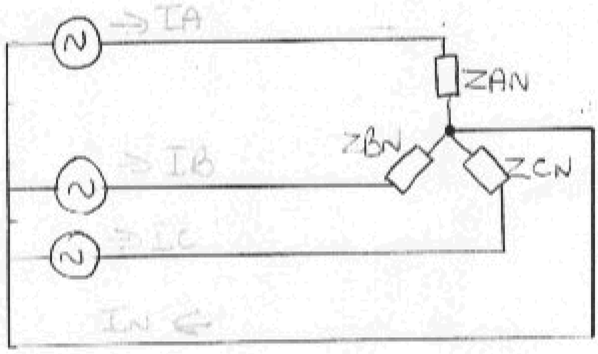

FIG 1

Notes: The impedances in the three phases are equal in magnitude and angle.

Line current I_{line} = Phase current I_{phase}

All line currents are equal.

Phase voltage $E_{phase} = E_{line}/\sqrt{3}$

The connection of the neutral (fourth wire), ensures that all phase voltages are equal at the load.

Neutral current is equal to the vector sum of the line currents.

$$I_{NEUTRAL} = I_A + I_B + I_C = 0 \text{ (for a balanced load)}$$

Example: A balanced three phase star connected load, consists of three 100Ω resistors connected via a four wire balanced supply system, to an alternator, having a line-neutral voltage of 100V rms.

Assume voltage E_{AN} as reference quantity.

Calculate:

a) the line voltages in polar form,

b) the line currents in polar form,

c) the neutral current in polar form

Solution:

Refer to the circuit diagram in FIG 2.

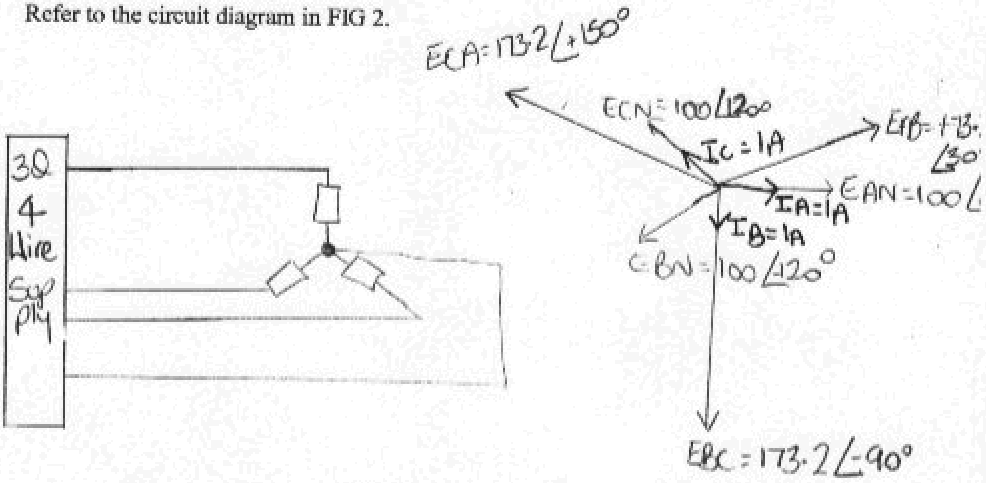

FIG 2

a) Since the supply is balanced,

$E_{AN} = 100\angle 0°$ volts

$E_{BN} = 100\angle -120°$ volts

$E_{CN} = 100\angle +120°$ volts

$Z = R + jX_C$
$= 100 + j0\,\Omega$
$= 100\angle 0°\,\Omega$

All Line voltages = $\sqrt{3} \times E_{phase}$ = $\sqrt{3} \times 100$ = 173.2V rms.

$E_{AB} = 173.2\angle +30°$ volts

$E_{BC} = 173.2\angle -90°$ volts

$E_{CA} = 173.2\angle +150°$ volts

b) Line current $I_A = \dfrac{E_{AN}}{Z_{AN}} = \dfrac{100\angle 0°}{100\angle 0°} = 1\angle 0°$ amps rms

Line current $I_B = \dfrac{E_{BN}}{Z_{BN}} = \dfrac{100\angle -120°}{100\angle 0°} = 1\angle -120°$ amps rms

Line current $I_C = \dfrac{E_{CN}}{Z_{CN}} = \dfrac{100\angle{+120°}}{100\angle{0°}} = 1\angle{+120°}$ amps rms

All line currents are equal in magnitude and displaced by 120°.

c) $I_{NEUTRAL} = I_A + I_B + I_C$
$= 1\angle{0°} + 1\angle{-120°} + 1\angle{+120°}$
$= (1 + j0) + (-0.5 - j0.866) + (-0.5 + j0.866)$
$= 0$

Balanced Delta Connected Loads

Refer to FIG 3 which shows a balanced delta load connected to a three wire balanced supply.

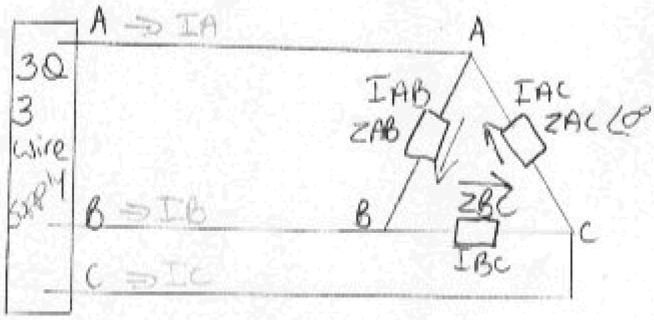

FIG 3

Notes:
All phase currents are equal and symmetrical in phase.
All line currents are equal and symmetrical in phase.
Line currents are equal to √3 times the phase currents.
The vector sum of the three line currents is zero.

Phase current $I_{AB} = \dfrac{E_{AB}}{Z_{AB}}$

Phase current $I_{BC} = \dfrac{E_{BC}}{Z_{BC}}$

Phase current $I_{CA} = \dfrac{E_{CA}}{Z_{CA}}$

Line current $I_A = I_{AB} - I_{CA} = \sqrt{3} \times I_{PHASE}$

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Line current I_B = $I_{BC} - I_{AB}$ = $\sqrt{3} x I_{PHASE}$

Line current I_C = $I_{CA} - I_{BC}$ = $\sqrt{3} x I_{PHASE}$

Example: A balanced delta load of R = 50Ω per phase is connected across a three phase balanced supply of 400V rms line-line.
Using voltage E_{AB} as reference, calculate:

a) all phase currents in polar form
b) all line currents in polar form.
Draw the complete phasor diagram.

Solution:

Refer to the circuit diagram in FIG 4.

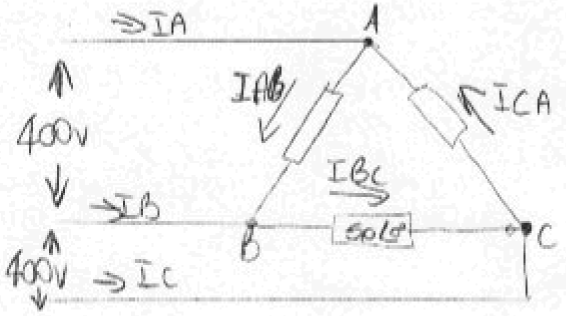

FIG 4

a) Phase current $I_{AB} = \frac{E_{AB}}{Z_{AB}} = \frac{400\angle 0°}{50\angle 0°} = 8\angle 0°$ amps rms

Phase current $I_{BC} = \frac{E_{BC}}{Z_{BC}} = \frac{400\angle -120°}{50\angle 0°} = 8\angle -120°$ amps rms

Phase current $I_{CA} = \frac{E_{CA}}{Z_{CA}} = \frac{400\angle +120°}{50\angle 0°} = 8\angle +120°$ amps rms

b) Line Current I_A = $I_{AB} - I_{CA}$
= $8\angle 0° - 8\angle +120°$

	=	(8 + j0) - (-4 + j6.9)
	=	12 - j6.9
	=	13.8/-30° amps rms
Line Current I_B	=	$I_{BC} - I_{AB}$
	=	8/-120° - 8/0°
	=	(-4 - j6.9) - (8 + j0)
	=	-12 - j6.9
	=	13.8/-150° amps rms
Line Current I_C	=	$I_{CA} - I_{BC}$
	=	8/+120° - 8/-120°
	=	(-4 + j6.9) - (-4 - j6.9)
	=	0 + j13.8
	=	13.8/+90° amps rms

The sum of the line currents is:

$(I_A + I_B + I_C)$ = (12 - j6.9) + (-12 - j6.9) + (0 + j13.8)
= 0

Notes: With a balanced load, we can calculate one line current and then displace the other two by ±120° since they are equal and symmetrical.
The sum of the three line currents in a three wire system is always equal to zero, since there is no path for unbalance current to flow.

Refer to FIG 5 which is the complete phasor diagram.

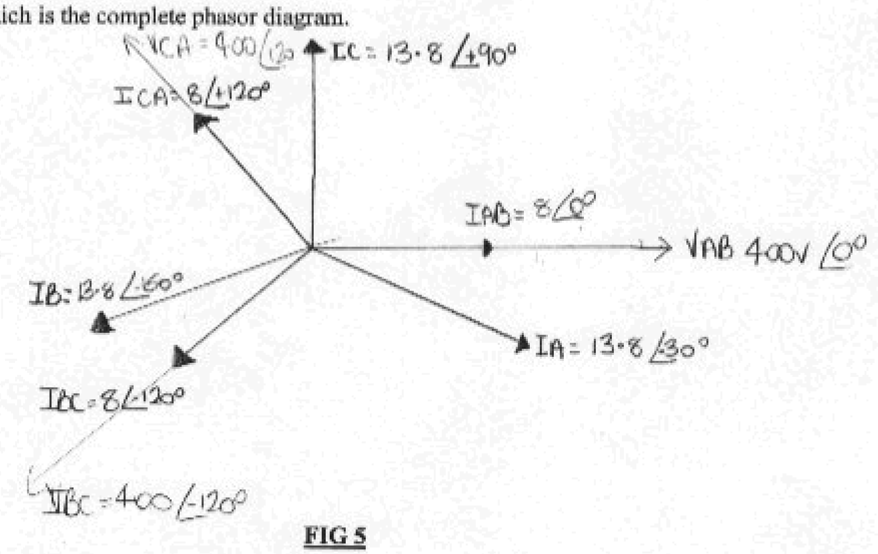

FIG 5

POWER AND ENERGY IN AC CIRCUITS

Definition of Power

Power is the rate of doing work. (Power = work/time)

The SI unit of "real or true" power is the "**WATT**"

Revision of Real Power in DC circuits

In a purely resistive dc circuit, real power in watts, can be calculated by any of the following expressions:

$P = EI$ $\quad$ $P = I^2R$ $\quad$ $P = E^2/R$

The power is called "real" power because the energy is consumed by the resistor and converted to another form, such as light, heat or mechanical energy, and cannot return to the source.

Remember, "real" power in watts can only be consumed by resistors or the resistive part of a circuit.

Energy stored in Magnetic or Electric Fields

In dc circuit transient analysis, using R-L or R-C circuits, it is revealed that some energy is consumed by the resistor, and some energy is supplied from the source, to establish a magnetic field in an inductor, or an electric field in a capacitor during the "charging" transient time.

This field energy is not consumed and lost, but is stored in the field until released by providing a "discharge" path for current to flow.

When the discharge current flows through a discharge resistor, during the "discharge" transient time, the energy is then consumed by the resistor, converted to heat and lost.

The energy does not become "real" power in watts until consumed by a resistor or the resistive part of a circuit.

Power in AC Circuits

When a sinusoidal voltage is applied to a circuit, the current will also be sinusoidal.

There will be a phase relationship between current and voltage depending on the circuit components (R, L, C or combinations of these).

Summary of Phase Relationships

Purely resistive circuit:	E and I in phase
Purely capacitive circuit:	I leads E by 90°
Purely inductive circuit:	I lags E by 90°
R-C circuit:	I leads E by an angle between 0° and 90°
R-L circuit:	I lags E by an angle between 0° and 90°
R-L-C circuit:	I may lead or lag E by an angle between 0° and 90°depending on overall impedance.

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Summary: The voltage and current waveforms are in phase (purely resistive circuit).
The power waveform is twice the frequency of the supply voltage, indicating that there are two pulses of power for every cycle of the supply voltage.
The power waveform is **always** positive, indicating that power always flows from the supply to the resistor but never back the other way.
The **average** value of power supplied to the resistor, is the average value of the power waveform, and is equal to the constant component.

Average Power $p_{ave} = \frac{1}{2}E_{MAX}I_{MAX} = E_{RMS}I_{RMS}$

Power in a Purely Inductive AC Circuit

In a purely inductive ac circuit:

If $e = E_{MAX}\sin\omega t$

then $i = I_{MAX}\sin(\omega t - 90°) = -I_{MAX}\cos\omega t$ (lagging e by 90°)

then instantaneous power $p = E_{MAX}\sin\omega t \times -I_{MAX}\cos\omega t$

$p = -E_{MAX}I_{MAX}\sin\omega t\cos\omega t$

Using the trigonometrical identity $2\sin\omega t\cos\omega t = \sin 2\omega t$ becomes: $\sin\omega t\cos\omega t = \frac{1}{2}\sin 2\omega t$

$p = -E_{MAX}I_{MAX}(\frac{1}{2}\sin 2\omega t) = -\frac{1}{2}E_{MAX}I_{MAX}\sin 2\omega t$

Power is a double frequency sine wave with maximum value of $\frac{1}{2}E_{MAX}I_{MAX}$.

FIG 2 shows the voltage, current and resulting power waveform for the purely inductive circuit.

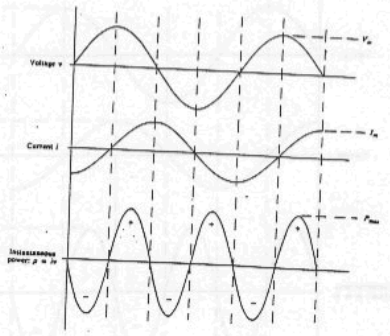

FIG 2

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Summary: The current waveform lags the voltage waveform by 90° (purely inductive circuit). The power waveform is twice the frequency of the supply voltage, indicating that there are two pulses of power for every cycle of the supply voltage.
The power waveform has equal positive and negative half cycles, indicating that power flows from the supply to the inductor and then back the other way.
This energy between the source and the inductor, is used to produce the magnetic field surrounding the inductor and when the field collapses, the energy is returned to the source.
The **average** value of "real" power supplied to the inductor, is the average value of the power waveform, and is equal to **zero**.
The pure inductor consumes no "real" power in watts since there is no resistive component in the circuit.
The power that flows back and forth is called "**reactive**" power or "**volt-amperes reactive**" (**VARS**), sometimes called "inductive" or "lagging" VARS.

Power in a Purely Capacitive AC Circuit

In a purely capacitive ac circuit:

If e $= E_{MAX}\sin\omega t$
then i $= I_{MAX}\sin(\omega t + 90°) = I_{MAX}\cos\omega t$ (leading e by 90°)

then instantaneous power p $= E_{MAX}\sin\omega t \times I_{MAX}\cos\omega t = E_{MAX}I_{MAX}\sin\omega t\cos\omega t$

becomes: p $= E_{MAX}I_{MAX}(\frac{1}{2}\sin 2\omega t) = \frac{1}{2}E_{MAX}I_{MAX}\sin 2\omega t$

Power is a double frequency sine wave with maximum value of $\frac{1}{2}E_{MAX}I_{MAX}$.

FIG 3 shows the voltage, current and resulting power waveform for the purely capacitive circuit.

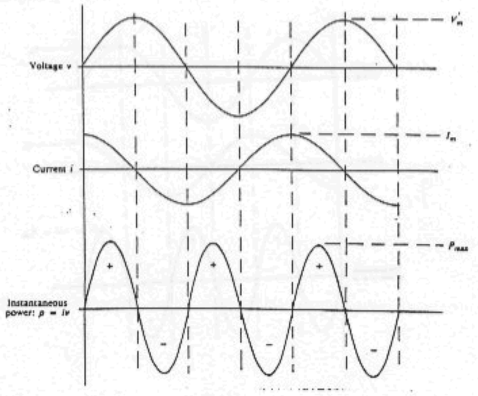

FIG 3

Summary: The current waveform leads the voltage waveform by 90° (purely capacitive circuit).
The power waveform is <u>twice</u> the frequency of the supply voltage, indicating that there are two pulses of power for every cycle of the supply voltage.
The power waveform has equal positive and negative half cycles, indicating that power flows from the supply to the capacitor and then back the other way.
This energy between the source and the capacitor, is used to produce the electric field in the capacitor and when the field collapses, the energy is returned to the source.
The **average** value of "real" power supplied to the capacitor, is the average value of the power waveform, and is **zero**.
The pure capacitor consumes no "real" power in watts since there is no resistive component in the circuit.
The power that flows back and forth is called "**reactive**" power or "**volt-amperes reactive**" (**VARS**), sometimes called "capacitive" or "leading" VARS.

Inductive and Capacitive VARS

Only inductors and capacitors can generate or consume <u>reactive power</u> in vars and <u>do not</u> consume real power in watts.

When comparing the power waveforms in FIG 2 and FIG 3, it can be seen, that when the inductor requires energy for its magnetic field, the capacitor is giving back its energy and when the capacitor requires energy for its electric field, the inductor is giving back its energy.

This means that the inductor satisfies the needs of the capacitor and vice versa for their <u>reactive power</u> requirements, and so inductance tends to cancel the effect of capacitance and vice versa in a circuit.

This effect has already been noted in the study of series and parallel R-L-C circuits.

Power in a Series R-L AC Circuit

In a series R-L ac circuit:

If $e = E_{MAX}\sin\omega t$

then $i = I_{MAX}\sin(\omega t - \theta°)$ (lagging e by θ°)

then instantaneous power $p = ei = E_{MAX}I_{MAX}\sin\omega t\sin(\omega t - \theta°)$

using trig identity $\sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A + B)$

becomes: $p = \frac{1}{2}E_{MAX}I_{MAX}[\cos\theta° - \cos(2\omega t - \theta°)]$

expanding brackets gives:

$$p = \frac{1}{2}E_{MAX}I_{MAX}\cos\theta° - \frac{1}{2}E_{MAX}I_{MAX}\cos(2\omega t - \theta°)$$

$$= E_{RMS}I_{RMS}\cos\theta° - E_{RMS}I_{RMS}\cos(2\omega t - \theta°)$$

p = constant - double frequency cosine wave shifted by -θ°

FIG 4 shows the voltage, current and resulting power waveform for the series R-L circuit.

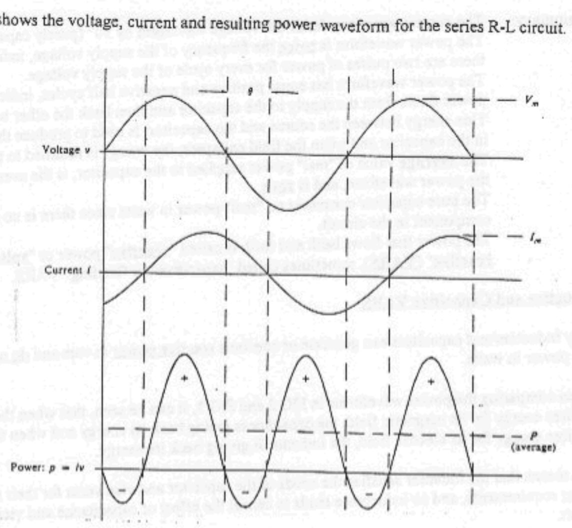

FIG 4

Summary: The current waveform lags the voltage waveform by θ° (inductive circuit).
The power waveform is twice the frequency of the supply voltage, indicating that there are two pulses of power for every cycle of the supply voltage.
The power waveform has a larger positive than negative value, indicating that more power flows from the supply to the circuit than flows back to the supply.
This energy between the source and the circuit, is used to supply **real** power to the resistor and **reactive** power to produce the magnetic field in the inductor, and when the field collapses, the reactive power is returned to the source.
The **average** value of "real" power supplied to the R-L circuit, is the average value of the power waveform.

$$\text{Real Power in Watts} = E_{RMS} I_{RMS} \cos\theta^\circ$$

where θ° is the angle of lag between the supply voltage and the circuit current and is also the impedance angle of the circuit.
The inductor consumes no "real" power in watts.
The negative part of the power waveform, represents the inductive VARS moving back and forth between the source and the inductor.

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Power in a Series R-C AC Circuit

In a series R-C ac circuit:

If $e = E_{MAX}\sin\omega t$

then $i = I_{MAX}\sin(\omega t + \theta°)$ (leading e by θ°)

then instantaneous power $p = ei = E_{MAX}I_{MAX}\sin\omega t\sin(\omega t + \theta°)$

using trig identity $\sin A\sin B = ½(\cos(A-B) - \cos(A + B))$

becomes: $p = ½E_{MAX}I_{MAX}[\cos\theta° - \cos(2\omega t + \theta°)]$

expanding brackets gives:

$$p = ½E_{MAX}I_{MAX}\cos\theta° - ½E_{MAX}I_{MAX}\cos(2\omega t + \theta°)$$

$$= E_{RMS}I_{RMS}\cos\theta° - E_{RMS}I_{RMS}\cos(2\omega t + \theta°)$$

p = constant - double frequency cosine wave shifted by +θ°

FIG 5 shows the voltage, current and resulting power waveform for the series R-C circuit.

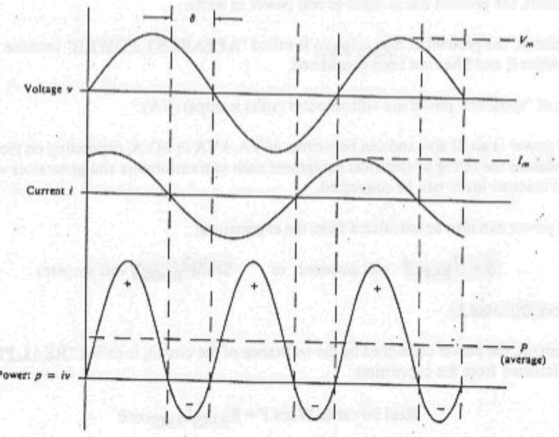

FIG 5

Summary: The current waveform leads the voltage waveform by θ° (capacitive circuit).
The power waveform is twice the frequency of the supply voltage, indicating that there are two pulses of power for every cycle of the supply voltage.
The power waveform has a larger positive than negative value, indicating that more power flows from the supply to the circuit than flows back to the supply.
This energy between the source and the circuit, is used to supply **real** power to the resistor and **reactive** power to produce the electric field in the capacitor, and when the field collapses, the reactive power is returned to the source.
The **average** value of "real" power supplied to the R-C circuit, is the average value of the power waveform.

$$\text{Real Power in Watts} = E_{RMS}I_{RMS}\cos\theta°$$

where θ° is the angle of lead between the supply voltage and the circuit current and is also the impedance angle of the circuit.
The capacitor consumes no "real" power in watts.
The negative part of the power waveform, represents the capacitive VARS moving back and forth between the source and the capacitor.

Apparent Power, Real Power and Reactive Power in AC Circuits

Apparent Power (Symbol S)

In a dc circuit, the product ExI is equal to real power in watts.

In an ac circuit, the product of $E_{RMS}I_{RMS}$ is called "**APPARENT POWER**" because the phase angle between E and I has not been considered.

The units of "apparent" power are volt amperes (volts x amps) (VA).

Apparent power is an SI unit and can be written as VA, kVA or MVA, depending on the size, and is used to measure the rating of electrical equipment such as transformers and generators where it is likely that reactive loads will be connected.

Apparent power can also be calculated from the expressions:

$S = I^2_{RMS}Z$ volt amperes or $S = V^2_{RMS}/Z$ volt amperes

Real Power (Symbol P)

In an ac circuit, the power consumed by the resistance of the circuit, is called "**REAL POWER**" and can be calculated from the expression:

$$\text{Real Power in Watts } P = E_{RMS}I_{RMS}\cos\theta$$

where θ° is the angle of lead or lag between the supply voltage and the circuit current and is also the impedance angle of the circuit.

Real power can also be calculated from the expressions:

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$P = I^2_{RMS}R$ watts or $P = E^2_{RMS}/R$ watts

where I_{RMS} is the current passing through the resistor and E_{RMS} is the voltage across the resistor.

Real power is an SI unit and can be written as μW, mW, kW, MW etc depending on the size.

Reactive Power (Symbol Q)

In an ac circuit, the temporary energy requirement of reactive elements such as inductors or capacitors is called "**REACTIVE POWER**" and can be calculated from the expression:

$$\text{Reactive Power in Vars } Q = E_{RMS}I_{RMS}\sin\theta$$

where θ° is the angle of lead or lag between the supply voltage and the circuit current and is also the impedance angle of the circuit.

Reactive power can also be calculated from the expressions:

$Q = I^2_{RMS}X$ vars or $Q = E^2_{RMS}/X$ vars

where I_{RMS} is the current passing through the reactance and E_{RMS} is the voltage across the reactance and X can be either $+jX_L$ or $-jX_C$.

There is a convention to identify the types of reactive power.
Inductive (lagging) Vars are consumed by inductors (+ sign).
Capacitive (leading) Vars are generated by capacitors (- sign).

Reactive power is an SI unit and can be written as kVar, Mvar etc depending on the size.

The Power Triangle

The Power Triangle shows the relationship between apparent, real and reactive power in an ac circuit.

FIG 6 shows the power triangles for capacitive and inductive circuits.

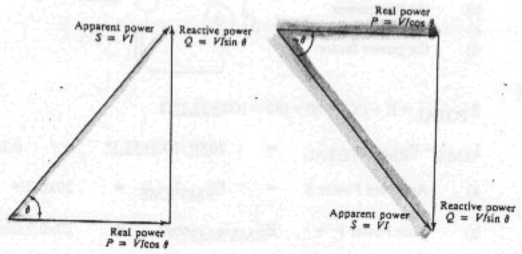


FIG 6

The angle θ° in the triangle, is the angle of lead or lag between the supply voltage and the circuit current and is also the impedance angle of the circuit.

This means that the shape of the power triangle and the impedance triangle are the same.

The sides of the triangle represent the apparent power S, the real power P, and the reactive power Q in the ac circuit.

In a purely resistive circuit, $\theta = 0°$ and so real power P = apparent power S and reactive power Q is zero.

In a purely inductive or purely capacitive circuit, $\theta = 90°$ and so reactive power Q = apparent power S and real power P is zero.

Power Factor

The ratio of real power/apparent power in an ac circuit is called "**POWER FACTOR**".

Power Factor is the factor used to multiply the apparent power to obtain the real power.

Power Factor (pf) = real power/apparent power

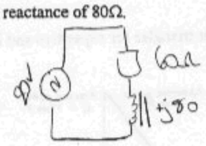
$= E_{RMS}I_{RMS}\cos\theta / E_{RMS}I_{RMS}$

$= \cos\theta$

Note: When writing a value for power factor, it must be specified whether the circuit is inductive or capacitive.
This is done by adding the word "leading" for capacitive circuit, or "lagging" for inductive circuit after the numerical value of power factor.

Example: A circuit consists of a 20 volt rms source connected to a series combination of a 60Ω resistor and an inductor with a reactance of 80Ω.
Calculate for the circuit:

a) the apparent power
b) the true power
c) the reactive power
d) the power factor.



Solution:

$Z_{TOTAL} = R + jX_L = 60 + j80 = 100\angle{+53.1°}\Omega$

$I_{RMS} = E_{RMS}/Z_{TOTAL} = 20\angle{0°}/100\angle{+53.1°} = 0.2\angle{-53.1°}$ amps

a) Apparent Power S $= E_{RMS}I_{RMS} = 20 \times 0.2 = 4$VA

b) True Power P $= E_{RMS}I_{RMS}\cos\theta = 20 \times 0.2 \times \cos 53.1°$

$= 2.4$ Watts

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OR $= I^2_{RMS}R = (0.2)^2 x60 = 2.4Watts$

c) Reactive Power Q $= E_{RMS}I_{RMS}\sin\theta = 20x0.2x\sin 53.1° = 3.2Vars$

OR $= I^2_{RMS}X_L = (0.2)^2 x80 = 3.2Vars$

d) Power Factor $= \cos\theta = \cos 53.1° = 0.6$ lagging (inductive)

Calculation of Power using Complex Numbers

Voltages and currents are written in complex form (rectangular or polar) to make calculations easier.

Power can also be written in complex form because it is the product of a voltage and a current.

Apparent Power (S) = Real Power (P) ± j Reactive Power (Q) volt amperes

$$S = P \pm jQ \text{ volt amperes}$$

Note: Reactive power component can have either a positive or negative sign in front of "j" term, depending on whether the reactive power (vars) are inductive or capacitive.

Convention: Inductive vars have a **positive** "j" term.

Complex power can be calculated from the equation:

$$S = E_{RMS}I_{RMS}^* \text{ volt amperes}$$

where I_{RMS}^* = conjugate of I_{RMS}

Note on Conjugate of a Complex Number

If $I = a + jb = I\angle{+\theta°}$ then $I^* = a - jb = I\angle{-\theta°}$ (opposite sign for "j")

Example: In an AC circuit, the supply voltage $E_{RMS} = 240\angle{0°}$ volts and the resulting current is $I_{RMS} = 10\angle{-10°}$ amps.
Calculate for the circuit:

a) apparent power S in polar form,
b) real power P in watts,
c) reactive power Q in vars (state whether inductive or capacitive),
d) power factor (state whether leading or lagging).

Solution:

The first observation that can be made from the given information, is that the circuit is inductive, since current lags the applied voltage (by 10°).

a) Complex Power S $= E_{RMS}I_{RMS}^*$

$= 240\angle{0°} \times 10\angle{+10°}$ VA

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$= 2400\angle{+10^\circ}$

$= 2363 + j416.7$ VA

b) Real power in watts $= P = 2363$ watts

c) Reactive power in vars $= Q = 416.7$ vars (inductive)

d) Power Factor pf $= \cos\theta = \cos 10^\circ = 0.98$ (lagging)

Example: An impedance of $Z = 30 + j40\ \Omega$ is connected to a 250 volt rms supply. Calculate:

a) apparent power in VA
b) real power in WATTS
c) reactive power in VARS (state whether inductive or capacitive)
d) circuit power factor (state whether leading or lagging).

Solution:

a) Total Circuit Impedance $Z = 30 + j40 = 50\angle{53.1^\circ}\ \Omega$ (inductive)

Now Current $I_{RMS} = E_{RMS}/Z$

$= 250\angle{0^\circ}/50\angle{53.1^\circ}$

$= 5\angle{-53.1^\circ}$ amps

Apparent Power S $= P + jQ$ VA

$= E_{RMS}I_{RMS}^*$

$= 250\angle{0^\circ} \times 5\angle{+53.1^\circ}$

$= 1250\angle{+53.1^\circ}$ VA or (1.25kVA)

b) Apparent Power S $= 1250\angle{+53.1^\circ}$ VA

$= 750 + j1000$ VA

$= P + jQ$

So Real Power = P = 750 watts or 0.75kW

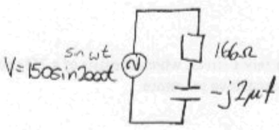
c) Reactive Power = Q = 1000VARS or 1kVAR (inductive)

d) Power Factor pf $= \cos\theta = \cos 53.1^\circ = 0.6$ lagging

Example: A voltage $v = 150\sin 2000t$ is applied to a resistance of 166Ω in series with a 2μF capacitor.
Calculate:
a) apparent power in VA
b) real power in WATTS
c) reactive power in VARS (state whether inductive or capacitive)
d) circuit power factor (state whether leading or lagging).
e) draw the power triangle.

Solution:

First draw the circuit diagram.



Capacitive reactance X_C = $1/\omega C$ = $1/(2000 \times 2 \times 10^{-6})$ = $-j250\Omega$

Total Impedance Z_{TOTAL} = $R - jX_C$

= $166 - j250$

= $300\angle{-56.4°}\,\Omega$

E_{RMS} = $E_{MAX} \times 0.707$

= 150×0.707

= $106\angle{0°}$ volts

I_{RMS} = E_{RMS}/Z_{TOTAL}

= $106\angle{0°}/300\angle{-56.4°}$

= $0.35\angle{+56.4°}$ A

a) Apparent Power S = $P + jQ$

= $E_{RMS} I_{RMS}^{*}$

= $106\angle{0°} \times 0.35\angle{-56.4°}$ VA

= $37.45\angle{-56.4°}$ VA

= $20.7 - j31.2$ VA

b) Real Power P = 20.7 watts

c) Reactive Power Q = 31.2 Vars (capacitive)

d) Power Factor = $\cos\theta$ = $\cos 56.4°$ = 0.55 leading

e) Power triangle

Example: A $50V_{RMS}$ voltage supplies a series circuit which consists of a 3Ω resistor, a 6Ω inductive reactance and a 2Ω capacitive reactance.
Calculate:
a) apparent power in VA
b) real power in WATTS
c) reactive power in VARS (state whether inductive or capacitive)
d) circuit power factor (state whether leading or lagging).

Solution:
Draw the circuit diagram.

Total Impedance Z_{TOTAL} = $R + j(X_L - X_C)$
= $3 + j(6 - 2)$
= $3 + j4$
= $5\angle{+53.1°}\,\Omega$

Total Current I_{RMS} = E_{RMS}/Z_{TOTAL}
= $50\angle{0°}/5\angle{+53.1°}$
= $10\angle{-53.1°}$ A

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a) Apparent Power S $= E_{RMS}I_{RMS}^*$

$= 50\angle 0^\circ \times 10\angle +53.1^\circ$

$= 500\angle +53.1^\circ$ VA

$= 300 + j400$ VA

b) Real Power P $=$ 300 watts

c) Reactive Power Q $=$ 400 vars

d) Power Factor $= \cos\theta = \cos 53.1^\circ = 0.6$ lagging

Power in Parallel Circuits

Total power consumed by parallel circuits can be determined by calculating power in each branch and then adding the power values together.

Example: A two branch parallel circuit is connected to a supply voltage of $E = 20\angle 60^\circ V_{RMS}$. The impedance of branch 1 is $Z_1 = 4\angle 30^\circ \Omega$ and impedance of branch 2 is $Z_2 = 5\angle 60^\circ \Omega$.

Determine:

a) power triangle for Z_1
b) power triangle for Z_2
c) total circuit power triangle
d) overall circuit power factor.

Solution:

Draw circuit diagram.

a) **Branch 1:**

Current $I_1 = E/Z_1$

$= 20\angle 60^\circ / 4\angle 30^\circ$ A

$= 5\angle 30^\circ$ A

Apparent Power $S_1 = E_{RMS}I_{1RMS}^*$

$= 20\angle 60^\circ \times 5\angle -30^\circ$ VA

= $100\angle 30°$

= 86.6 + j50 VA

Draw power triangle for branch 1.

b) **Branch 2:**

Current I_2 = E/Z_2

= $20\angle 60° / 5\angle 60°$ A

= $4\angle 0°$ A

Apparent Power S_2 = $E_{RMS} I_{2RMS}^*$

= $20\angle 60° \times 4\angle 0°$ VA

= $80\angle 60°$

= 40 + j69.2 VA

Draw power triangle for branch 2.

c) Total Apparent Power S_T = $S_1 + S_2$

= (86.6 + j50) + (40 + j69.2) VA

= 126.6 + j119.2 VA

= $173.9\angle 43.3°$ VA

Draw power triangle for total circuit.

d) Overall power factor = $\cos\theta$ = $\cos 43.3°$ = 0.7 lagging.

Note: Total circuit values could have been determined:

a) by adding the two power triangles together graphically or

b) by calculating the total circuit impedance first and then calculating power for total circuit.

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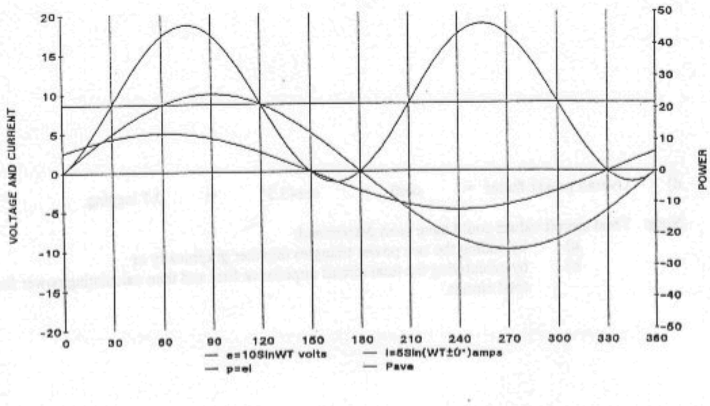
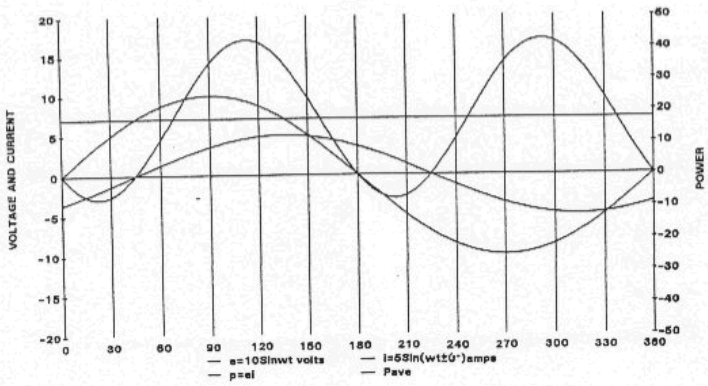
POWER IN AC CIRCUIT IMPEDANCE ANGLE θ= -30



POWER IN AC CIRCUIT IMPEDANCE ANGLE 45



Power in a Purely Resistive AC Circuit

In a dc circuit, $P_{DC} = EI$ (real power in watts)

In an ac circuit, $p_{AC} = ei$ (instantaneous values)

In a purely resistive ac circuit:

If $e = E_{MAX}\sin\omega t$
then $i = I_{MAX}\sin\omega t$ (in phase with e)

then instantaneous power $p = E_{MAX}\sin\omega t \times I_{MAX}\sin\omega t$

$$p = E_{MAX}I_{MAX}\sin^2\omega t$$

Using the trigonometrical identity $\sin^2\omega t = (½ - ½\cos2\omega t)$ to simplify:

$$p = E_{MAX}I_{MAX}(½ - ½\cos2\omega t)$$

expanding brackets gives $p = ½E_{MAX}I_{MAX} - ½E_{MAX}I_{MAX}\cos2\omega t$

P = a constant + a double frequency cosine wave

FIG 1 shows the voltage, current and resulting power waveform for the purely resistive circuit.

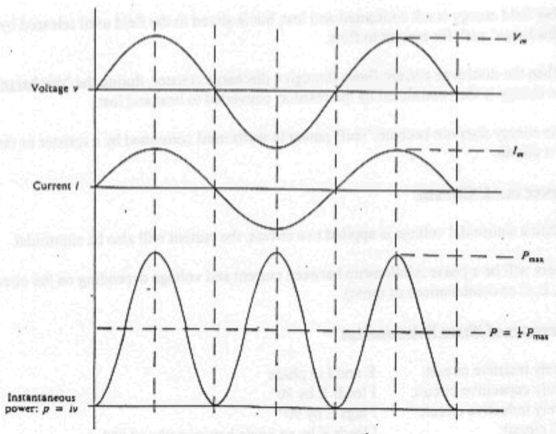


FIG 1

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WATTMETERS

ELECTRODYNAMIC INSTRUMENTS (Dynamometer)

The electrodynamic instrument is similar to a PMMC instrument except that the permanent magnet is replaced by two field coils. If a current is passed through these coils, it will produce a magnetic field.

FIG 1 shows the basic construction of a dynamometer instrument.

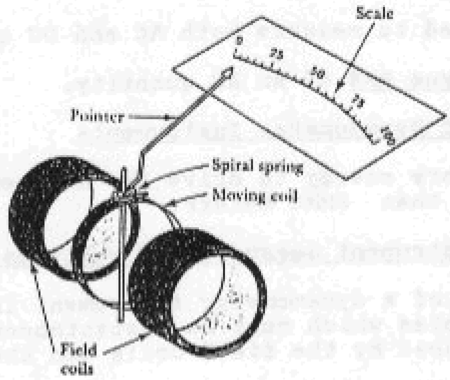

FIG 1

In a PMMC meter, the deflection is proportional to current in the moving coil and the field strength.

In a dynamometer instrument, the deflection is proportional to the current in the moving coil and the current in the field coils.

Deflection is proportional to $I_{FIELD} \times I_{MOVING\ COIL}$

When the instrument is connected in a DC circuit as shown in FIG 2, it will measure circuit current and voltage and deflect proportional to power (ExI) in watts, consumed by the load.

The instrument is now called a dynamometer wattmeter.

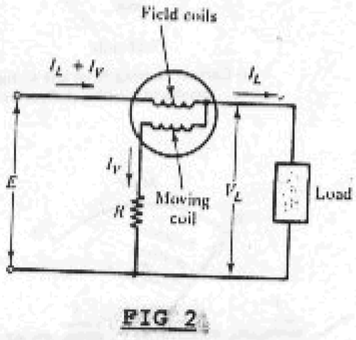

FIG 2

Notes: The moving coil is measuring voltage, and the resistor R connected in series with the coil is a multiplier resistor, similar to that used with voltmeters to extend their range. There are polarity markings on both coils to ensure that correct readings are obtained.

The dynamometer instrument can also be used to measure voltage alone or current alone by connection of field coils and moving coil in series (voltmeter), or field coils and moving coil in parallel (ammeter).

Advantages of Dynamometer Instruments

1. Can be used to measure both AC and DC quantities.

2. Measure true RMS of an AC quantity.

Disadvantage of Dynamometer Instruments

1. Require more energy to drive the movement, and are therefore less sensitive than PMMC meters.

Dynamometer Instrument measuring AC voltages and currents

The deflection of a dynamometer instrument is caused by the repulsion of two magnetic poles which must be instantaneously the same. These two poles are produced by the field coils and the moving coil.

Refer to FIG 3.

If we apply alternating quantities to the coils, and the coils are in series (for a voltmeter or ammeter), then the magnetic poles will all change polarity when the measured quantity reverses, thus maintaining an upscale deflection.

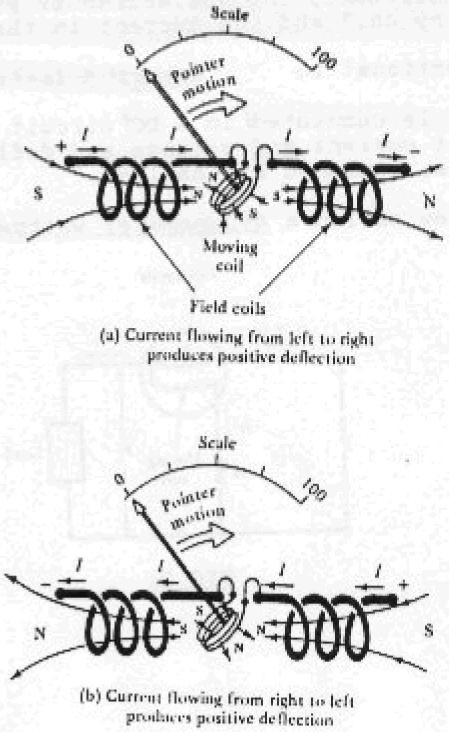

FIG 3

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Dynamometer Instrument measuring AC Power

When the dynamometer instrument is connected as a wattmeter, the deflection is caused by the voltage and in phase component of current because these components will produce instantaneous like poles.

Deflection is proportional to ExIcosθ = True power in WATTS

However, the connections are important, because the reversal of one coil is equivalent to a phase reversal of current or voltage and will cause the meter to read in error.

Fig 4 shows the connection of a wattmeter and correct polarity markings to ensure accurate reading.

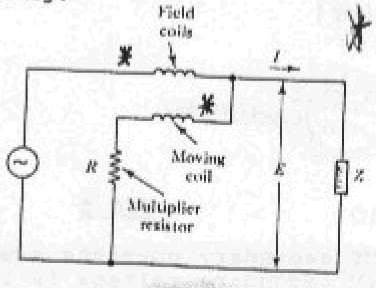

FIG 4

Precautions to be taken when using Wattmeters.

When wattmeters are measuring power at low power factors, the current and voltage applied to the meter may still be high.

Unlike a voltmeter or ammeter, where an overscale reading is obvious, the wattmeter may be reading a small deflection on scale but have excessive current in the current coil, or excessive voltage on the voltage coil which will burn out the meter.

1. Never exceed current or voltage ratings of coils.
2. Always connect an ammeter in series with the current coil, to monitor the current applied to the wattmeter.
3. Always connect an voltmeter in parallel with the voltage coil, to monitor the voltage applied to the wattmeter.
4. Always observe correct polarity markings and use correct voltage and current ranges.

Using Wattmeters in high voltage and high current circuits

Where the currents and voltages to be measured by a wattmeter exceed the maximum ratings of the wattmeter coils, the quantities must be stepped down to a safe value before being applied to the wattmeter.

The step-down is done through instrument transformers. A current transformer (CT) steps down current. A voltage transformer (VT) steps down voltage.

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Refer to FIG 5 which shows a wattmeter connected to a circuit through a CT with ratio 100/1 amps and a VT with a ratio 3300/110 volts.

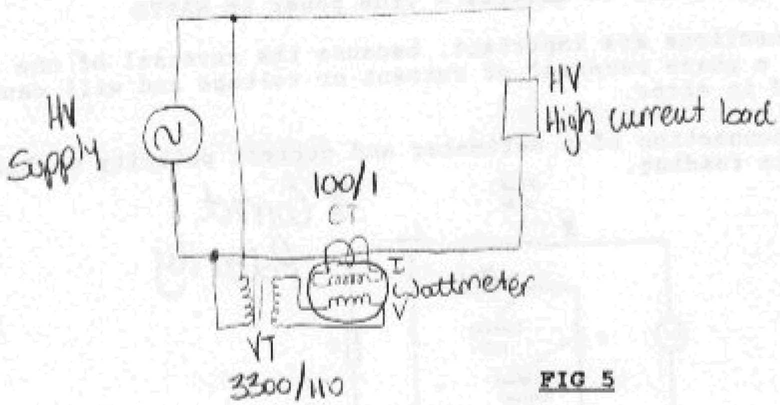

FIG 5

Notes: Standard CT secondary currents are 1 amp and 5 amps. Standard VT secondary voltage is 110V or 110 and 63.5V for three phase VTs.

Calculation of Wattmeter Reading

To ensure accurate readings and measurements when using wattmeters, the following points should be observed:

a) correct polarity connections,
b) which voltage range is selected,
c) which current range is selected,
d) power factor (pf) of wattmeter (see note below),
e) CT ratio if used,
f) VT ratio if used,
g) total number of divisions on scale,
h) the reading on scale.

Note: There are high and low power factor wattmeters available. The low pf meter is more accurate when used to measure in low pf circuits which are highly inductive or capacitive.

Full Scale Deflection of Wattmeter

The power required for full scale deflection on a wattmeter can be determined from:

Full Scale Deflection in Watts = Vrange x Irange x Meter power Factor

Wattmeter Constant

Meter Constant = CT ratio x VT ratio

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Example: A unity power factor (pf = 1) wattmeter is scaled 0-100 and reads 65 on scale.
The meter is used to measure power in a high voltage circuit, by connecting it via a CT with ratio of 150/1 amp and a VT with ratio 660/110 volts.
The 1A current range and the 110V voltage range are used on the meter.
Calculate the true power consumed by the circuit under test.

Solution:

Full Scale Deflection in Watts = Vrange x Irange x Meter power Factor
= 110 x 1 x 1
= 110 watts

Reading on scale = 65 divisions

Wattmeter reading = 110 x (65/100) 65% reading scale / total scale
= 71.5 watts

Actual power in circuit = Meter constant x Wattmeter reading
= CTratio x VTratio x 71.5 watts
= (150/1) x (660/110) x 71.5
= 64.35kW

Calculation of Power Factor from Wattmeter/Voltmeter Readings

If a wattmeter, voltmeter and ammeter are measuring quantities in an AC circuit, the readings can be used to determine the circuit power factor.

True Power P = $E_{RMS} I_{RMS} \cos\theta$

Power Factor $\cos\theta$ = P/(EI)
= wattmeter/(voltmeter x ammeter)

Example: A wattmeter measures AC power in a load as 100 watts, an ammeter measures circuit current as 1.5A and a voltmeter measures circuit voltage as 100V.
Calculate:
a) power factor of the load
b) phase angle between circuit current and voltage.

Solution:

a) Power P = ERMSIRMScosθ
Power factor cosθ = P/(ExI)
= 100/(100 x 1.5)
= 0.667 (leading or lagging ??)

b) phase angle θ = $\cos^{-1}$)0.667
= 48.2° (leading or lagging ??)

Note: From the information given, we do not know whether the circuit is inductive or capacitive.

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THREE PHASE POWER

The total power consumed by a three phase load is:

P_{TOTAL} = Sum of individual phase powers.

= $P_{Aphase} + P_{Bphase} + P_{Cphase}$

Single phase power = $E \times I \times \cos\theta$

where
E = applied rms voltage (load voltage)
I = rms load current
θ = angle of phase difference between E and I.

The power consumed by individual phases of a three phase load are:

$P_{phase} = E_{phase} \times I_{phase} \times \cos\theta$

Total Power in Balanced Three Phase Loads

The power in each phase of a balanced load will be the same.

P_{TOTAL} = 3x(Power in one phase)

= $3 \times E_{phase} I_{phase} \cos\theta$

but in a three phase system:

Balanced Star connected load: $E_{phase} = \frac{E_{line}}{\sqrt{3}}$ and $I_{phase} = I_{line}$

Replacing the phase values with line values in the equation above:

$$P_{TOTAL} = \sqrt{3} E_{line} I_{line} \cos\theta \quad \text{watts}$$

Balanced Delta connected load: $E_{phase} = E_{line}$ and $I_{phase} = \frac{I_{line}}{\sqrt{3}}$

$$P_{TOTAL} = \sqrt{3} E_{line} I_{line} \cos\theta \quad \text{watts}$$

This is the same equation for both balanced star and delta connected loads.

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Apparent and Reactive Power in Balanced Loads

Equations for apparent and reactive power, are derived from the single phase equations.

$$\text{Three phase Apparent Power} = \sqrt{3}E_{line}I_{line} \quad \text{VA}$$

$$\text{Three phase Reactive Power} = \sqrt{3}E_{line}I_{line}\sin\theta \text{ Vars}$$

Example:

Calculate the apparent power, real power and reactive power consumed by a balanced three phase delta connected load with an impedance of $10 + j0\Omega$ in each phase and supplied by a 200Vrms line-line source.

Solution:

Current in each phase $= \dfrac{E_{phase}}{Z_{phase}}$

$= \dfrac{200}{10}$

$I_{phase} = 20A$

Line current $= \sqrt{3} \times I_{phase}$ (in a balanced load)

$= \sqrt{3} \times 20$

$I_{line} = 34.6A$

Apparent Power $= \sqrt{3}E_{line}I_{line}$ VA

$= \sqrt{3} \times 200 \times 34.6$

≈ 12000 VA (12kVA)

Real Power $= \sqrt{3}E_{line}I_{line}\cos\theta$ Watts

$= \sqrt{3} \times 200 \times 34.6 \times \cos 0°$

≈ 12000 W (12kW)

Since load is purely resistive there will be no vars generated or consumed in the load.

Reactive Power = 0 Vars

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Power in Unbalanced Three Phase Loads

Total power in an unbalanced three phase load can be determined by calculating each individual phase power, and adding the three phase powers to obtain total three phase power.

Example:

An unbalanced three phase star connected load has impedances of $Z_A = 10 + j0\Omega$, $Z_B = 3 + j4\Omega$ and $Z_C = 0 - j5\Omega$.
The load is supplied by a three phase, four wire source with line-line voltages of 346Vrms.
Calculate the total apparent power, real power and reactive power consumed by the load.

Solution:

$$E_{phase} = \frac{E_{line}}{\sqrt{3}} = \frac{346}{\sqrt{3}} = 200\text{Vrms}$$

Assume E_{AN} is reference quantity ($E_{AN} = 200\angle 0°$V)

Phase A

Line Current $I_A = \frac{E_{AN}}{Z_A}$ Arms

$= \frac{200\angle 0°}{10\angle 0°}$

$= 20\angle 0°$ Arms

Complex Apparent Power $S_A = ExI^*$ VA

$= 200\angle 0° \times 20\angle 0°$ VA

$= 4000\angle 0°$ VA

$= 4000 + j0$ VA

Real power = 4000 W

Reactive power = 0 Vars

Phase B

Line Current $I_B = \frac{E_{BN}}{Z_B}$ Arms

$= \frac{200\angle -120°}{5\angle 53.1°}$

$= 40\angle -173.1°$ Arms

Complex Apparent Power $S_B = ExI^*$ VA

$= 200\angle -120° \times 40\angle +173.1°$ VA

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= $8000\underline{/+53.1°}$ VA

= $4800 + j6400$ VA

Real power = 4800 W

Reactive power = 6400 Vars (inductive)

Phase C

Line Current I_C = $\frac{E_{CN}}{Z_C}$ Arms

= $\frac{200\underline{/+120°}}{5\underline{/-90°}}$

= $40\underline{/+210°}$ Arms

Complex Apparent Power S_C = ExI^* VA

= $200\underline{/+120°}x40\underline{/-210°}$ VA

= $8000\underline{/-90°}$ VA

= $0 - j8000$ VA

Real power = 0 W

Reactive power = 8000 Vars (capacitive)

Total Three Phase Apparent Power = $S_A + S_B + S_C$ VA

= $(4000 + j0) + (4800 + j6400) + (0 - j8000)$

= $8800 - j1600$ VA

= $8944\underline{/-10.3°}$ VA (8.94kVA)

Total Three Phase Real Power = $P_A + P_B + P_C$ Watts

= $4000 + 4800 + 0$

= 8800W (8.8kW)

Total Three Phase Reactive Power = $Q_A + Q_B + Q_C$ Vars

= $0 + j6400 - j8000$

= 1600Vars (capacitive) (1.6kVar)

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Measurement of Three Phase Power using Wattmeters

Using One Wattmeter

Power in balanced three phase loads can be measured using only one wattmeter.

Refer to FIG 1 which shows one wattmeter connected to measure power in a balanced three phase star connected load.

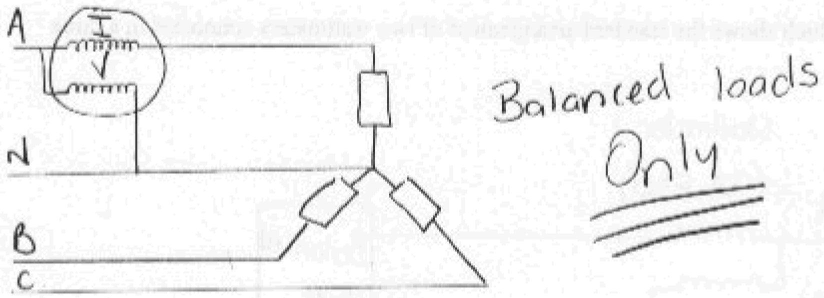

FIG 1

Notes: The wattmeter current coil is a measuring line current, and the voltage coil is measuring a line-neutral voltage.
The wattmeter reads $E_{phase}I_{phase}\cos\theta$ which is the real power in one phase only.
Total three phase power is 3x(Wattmeter reading)

Refer to FIG 2 which shows one wattmeter connected to measure power in a balanced three phase delta connected load.

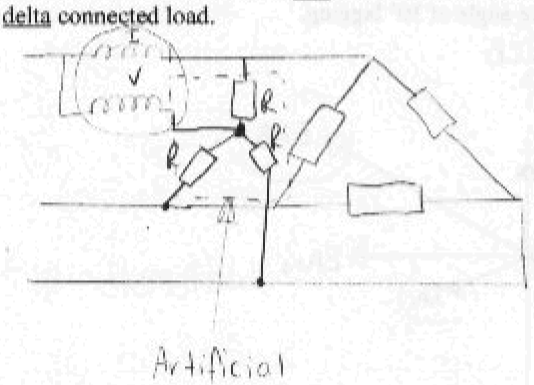

FIG 2

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Notes: The three resistors produce an artificial neutral point.
The wattmeter current coil is measuring a line current, and the voltage coil is measuring a line-neutral voltage.
The wattmeter reads $E_{phase}I_{line}\cos\theta$ which is equivalent to √3 times the real power in one phase only.
Total three phase power is √3x(Wattmeter reading)

Two Wattmeter Method of Measuring Three Phase Power

Refer to FIG 3 which shows the standard arrangement of two wattmeters connected in a three phase circuit.

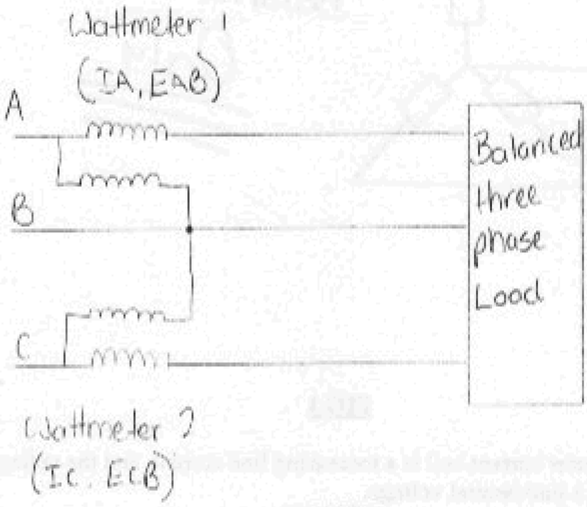

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FIG 3

Example:
Refer to the phasor diagram of FIG 4 which shows the conditions in a balanced three phase load, having an impedance angle of 30° lagging.

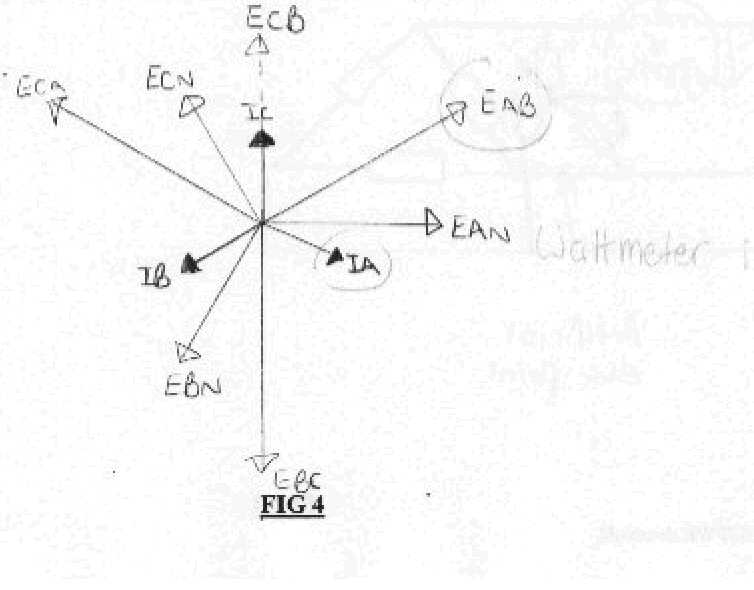

FIG 4

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Notes:

Wattmeter 1	measures	$E_{AB}I_A\cos\varphi_1$
where $\varphi_1 = (30 + \theta)°$	and θ = angle of lag of I_A wrt E_{AN}.	
Wattmeter 2	measures	$E_{CB}I_C\cos\varphi_2$
where $\varphi_2 = (30 - \theta)°$	and θ = angle of lag of I_C wrt E_{CN}.	

In this example, $\theta = 30°$, and $\varphi_1 = 60°$ and $\varphi_2 = 0°$.

Sum of Wattmeter Readings:

Reading on W_1 $= E_{line}I_{line}\cos(30 + \theta)$

$= E_{line}I_{line}(\cos30\cos\theta - \sin30\sin\theta)$

Reading on W_2 $= E_{line}I_{line}\cos(30 - \theta)$

$= E_{line}I_{line}(\cos30\cos\theta + \sin30\sin\theta)$

$W_1 + W_2$ $= E_{line}I_{line}(\cos30\cos\theta - \sin30\sin\theta) + E_{line}I_{line}(\cos30\cos\theta + \sin30\sin\theta)$

$= E_{line}I_{line}(2\cos30\cos\theta)$

but $\cos30° = \sqrt{3}/2$

$$W_1 + W_2 = \sqrt{3}E_{line}I_{line}\cos\theta \text{ (three phase power in balanced load)}$$

Therefore the sum of the two wattmeter readings is equal to the total three phase power.

Notes: As the phase angle of the line currents changes, the readings on the two wattmeters will also change.
If the load power factor angle is greater than 60° lag, then the reading on W_1 will become negative, since the angle between E_{AB} and I_A is >90°.
If the load power factor angle is greater than 60° lead, then the reading on W_2 will become negative, since the angle between E_{CB} and I_C is >90°.
If the load power factor angle is equal to 60° lag, then the reading on $W_1 = 0$.
If the load power factor angle is equal to 60° lead, then the reading on $W_2 = 0$.
If the load power factor angle is equal to 0° (purely resistive load), then $W_1 = W_2$.

It can be shown that the sum of the two wattmeter readings will be equal to the total three phase power consumed by either star or delta connected three phase loads whether they are balanced or unbalanced.

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Difference of Two Wattmeter Readings in a Balanced Load ($W_2 - W_1$)

$$W_2 - W_1 = E_{line}I_{line}(\cos30\cos\theta + \sin30\sin\theta) - E_{line}I_{line}(\cos30\cos\theta - \sin30\sin\theta)$$

$$= 2E_{line}I_{line}\sin30\sin\theta$$

$$= E_{line}I_{line}\sin\theta$$

If $W_2 - W_1$ is multiplied by $\sqrt{3}$:

$\sqrt{3}(W_2 - W_1) = \sqrt{3}E_{line}I_{line}\sin\theta$ (Three phase reactive power)

Calculation of Power Factor from Wattmeter Readings

Note: Method only valid for balanced loads.

Since Watts $= W_1 + W_2$

and Vars $= \sqrt{3}(W_2 - W_1)$

Then $\tan\theta = \dfrac{\text{Vars}}{\text{Watts}}$

$$= \frac{\sqrt{3}(W_2 - W_1)}{(W_1 + W_2)}$$

Power Factor $= \cos\theta$.

Note: The solution to this calculation does not indicate whether the load power factor is leading or lagging.
Inspection of the sign and relative values of W_1 and W_2 may show whether the load is leading or lagging.

Effect of Reverse Phase Sequence on Wattmeter Readings

If the phase sequence of the supply voltages is reversed, then the two wattmeter readings W_1 and W_2 will change, but the total apparent power, real power and reactive power in the load will not change.

Example: Refer to FIG 5 which shows an unbalanced delta connected load supplied by a three phase 200Vrms line-line set of voltages, with phase sequence A-B-C.
Calculate:

a) all line currents in polar form,

b) the readings on two wattmeters connected in standard way (W_1 reads I_A and E_{AB}, W_2 reads I_C and E_{CB}),

c) total real power consumed by the load using the wattmeter readings,

d) check the answer in c) using another method,

e) total reactive power in the load.

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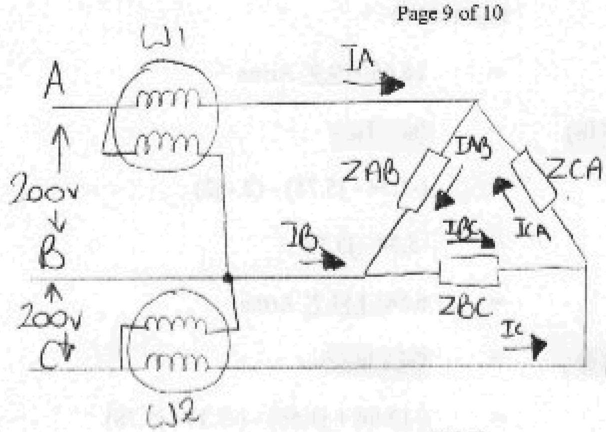
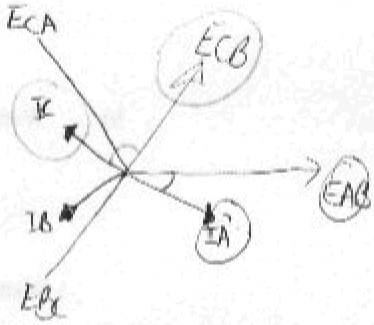


W2 = |ECB| x |IC| x cos 77.6°

FIG 5

Solution:

a) Use voltage E_{AB} as reference



Phase current $I_{AB} = \frac{E_{AB}}{Z_{AB}}$

$= \frac{200\angle 0^\circ}{70.7\angle 45^\circ}$

$= 2.83\angle -45^\circ$ Arms

$= 2 - j2$ Arms

Phase current $I_{BC} = \frac{E_{BC}}{Z_{BC}}$

Wattmeter 1 = |EAB||IA| cos 19.9°

$= \frac{200\angle -120^\circ}{30\angle 0^\circ}$

$= 6.67\angle -120^\circ$ Arms

$= -3.34 - j5.78$ Arms

Phase current $I_{CA} = \frac{E_{CA}}{Z_{CA}}$

$= \frac{200\angle +120^\circ}{14.14\angle -45^\circ}$

$= 14.14\angle 165^\circ$ Arms

$= -13.66 + j3.66$ Arms

Line Current $I_A = I_{AB} - I_{CA}$

$= (2 - j2) - (-13.66 + j3.66)$

$= 15.66 - j5.66$ Arms

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	=	$16.65\underline{/-19.9°}$ Arms
Line Current I_B	=	$I_{BC} - I_{AB}$
	=	$(-3.34 - j5.78) - (2 - j2)$
	=	$-5.34 - j3.78$
	=	$6.54\underline{/-144.7°}$ Arms
Line Current I_C	=	$I_{CA} - I_{BC}$
	=	$(-13.66 + j3.66) - (-3.34 - j5.78)$
	=	$-10.32 + j9.44$
	=	$14\underline{/137.6°}$ Arms
b) Wattmeter 1 reads	=	$E_{AB} x I_A x \cos\varphi_1$
	=	$200\underline{/0°} x 16.65\underline{/-19.9°} x \cos 19.9°$
	=	+3131 Watts
Wattmeter 2 reads	=	$E_{CB} x I_C x \cos\varphi_2$
	=	$200\underline{/60°} x 14\underline{/137.6°} x \cos 77.6°$
	=	+601 Watts
c) Total Real Power	=	$W_1 + W_2$
	=	3131 + 601
	=	3732 Watts
d) Total Real power	=	$I_{AB}^2 R_{AB} + I_{BC}^2 R_{BC} + I_{CA}^2 R_{CA}$
	=	$2.83^2 x50 + 6.67^2 x30 + 14.14^2 x10$
	=	400 + 1335 + 2000
	=	3735 Watts check OK
e) Total Reactive Power	=	$I_{AB}^2 X_{AB} + I_{BC}^2 X_{BC} + I_{CA}^2 X_{CA}$
	=	$2.83^2 x j50 + 6.67^2 x0 + 14.14^2 x{-j10}$
	=	400 + 0 - 2000
	=	-1600 Vars (capacitive)

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POWER FACTOR CORRECTION

Most industrial loads consist of inductive loads (motors, induction heating etc), which result in lagging supply currents and lagging power factors.

The supply current must transmit both real power (WATTS) and reactive power (VARS) for the inductive load.

However, electrical energy is sold as kilowatt-hours (real power component).

To make the power supply system more efficient, we would like to supply only the real power in watts.

Why??
Refer to the phasor diagram in FIG 1, showing the phase relationship between voltage and current in an inductive load.

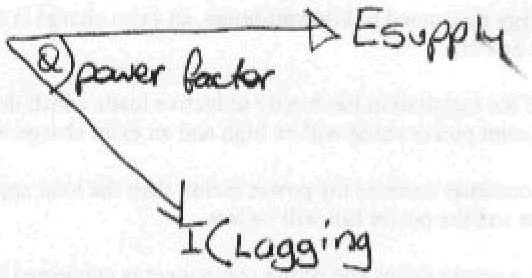


FIG 1

A lagging inductive load will require current I to transmit the WATTS and the VARS to the load.

However, only the in-phase component $I\cos\theta$ is required to transmit the WATTS component, and $I\cos\theta$ is less than the value of I.

If we could avoid having to supply the VARS from the source, then the total load current would be less.

Power losses occur in the power supply lines and equipment caused by I^2R.

The power system losses would be lower if we reduced the level of current flowing through the supply system and this would increase the efficiency of power transmission.

Power factor is cosine of the angle θ between V and I, so if θ could be reduced to 0°, then the power factor would be cos0° = 1 (UNITY), which is the maximum value.

Power factor correction is the connection of a suitable reactance in parallel with a reactive load, so that the reactive power (VARS) requirement of the load will be satisfied locally and a minimum of reactive power (VARS) will have to be supplied by the source.

Normally power factor correction is carried out on inductive loads by connecting suitable capacitors in parallel with inductive loads.

However, it is possible to correct power factor of a highly capacitive load by connecting in parallel, a suitable inductor.

Beside static capacitors, **synchronous motors** are very useful for power factor correction, as their excitation can be varied so that they can be made to look either inductive or capacitive.

The power factor correction is best applied at the location of the load.

Industrial consumers are encouraged to correct the power factor of their installation themselves or else pay a penalty with their power charges.

Penalties for having Low (Poor) Power Factor

Power supply authorities charge industrial consumers two rates for power consumption.

In addition to energy consumed in kilowatt-hours, an extra charge is made for maximum demand in kVA (apparent power).

This means that if the installation has highly inductive loads which draw excessive reactive power then the total apparent power value will be high and an extra charge will be made.

However, if the consumer corrects the power factor, then the total apparent power taken from the supply will be less and the power bill will be less.

FIG 2 shows how a power factor correcting component is connected in parallel with the load.

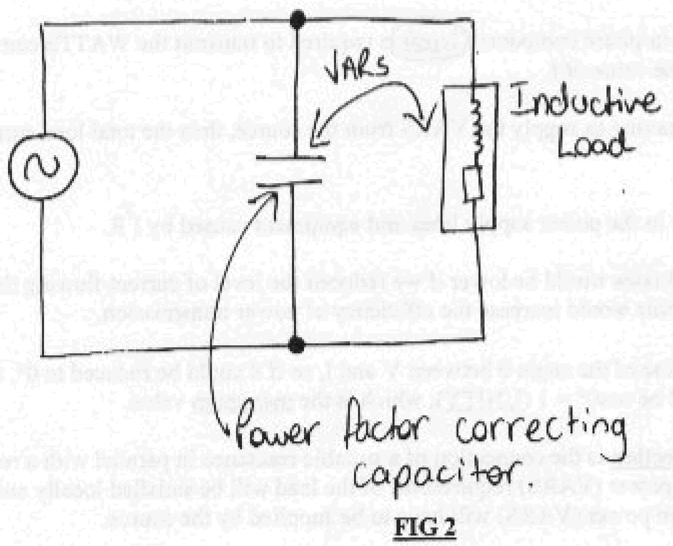


FIG 2

F76

Notes: When the power factor correcting component is added to the circuit, the real power (WATTS) supplied by the source will remain the same, but the reactive power (VARS) supplied by the source will be reduced to the required value for the new value of power factor.

Power factor is not always corrected exactly to unity, as this will only satisfy one value of load impedance and usually loads are varying continually as plant and equipment is switched on and off, so power factor is usually corrected to just slightly lagging. (0.95-1)

Example: A load of 5kVA at 0.6 power factor lagging, is connected to a 250V rms 50Hz supply.
Calculate the value of the additional component that must be connected in parallel with the load, to increase the total power factor to unity.

Solution:

Firstly draw a circuit diagram as shown in FIG 3.

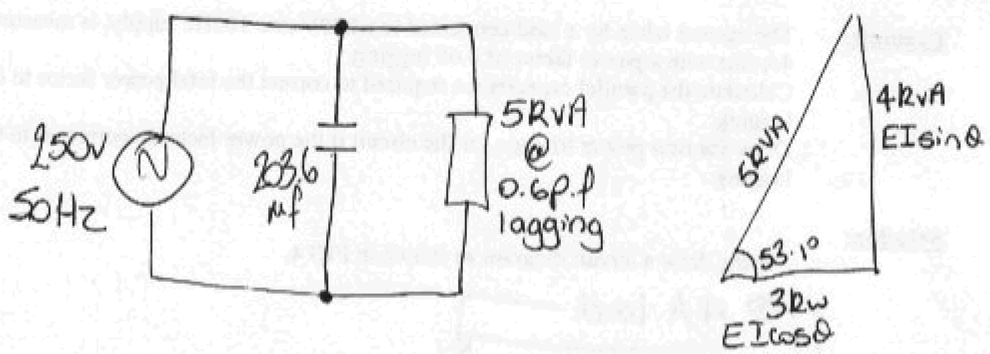


FIG 3

As the load is inductive, a capacitor must be used as a power factor correction component.

We must first solve the power triangle for the load.

Apparent power ExI = 5kVA.

Power factor cosθ = 0.6

$\theta = \cos^{-1} 0.6 = 53.1°$ lagging

Real power watts = EIcosθ = $5 \times 10^{-3} \times 0.6$ = 3kW

Xc Reactive Power = EIsinθ = $5 \times 10^{-3} \times 0.8$ = 4kVAR (inductive)

To correct the power factor to unity ($\theta = 0°$), we must connect a capacitor in parallel with the load so that it will supply 4kVAR of capacitive reactive power.

Reactive power from Capacitor $\quad Q_C = E^2/X_C$

so Capacitive Reactance $\quad X_C = E^2/Q_C$

$= (250 \times 250)/(4 \times 10^3)$

$= 15.63\Omega$

Capacitance C $= 1/(\omega X_C)$

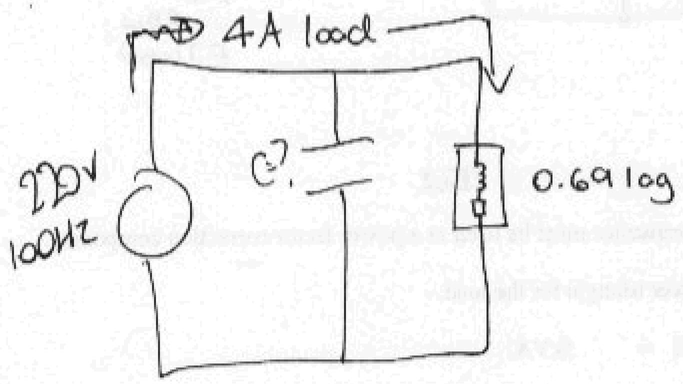
$= 1/(2\pi \times 50 \times 15.63)$

$= 203.6\mu F$

Example: The current taken by a load connected to a 220V rms 100Hz supply, is measured as 4A rms with a power factor of 0.69 lagging.
Calculate the parallel capacitance required to correct the total power factor to 0.97 lagging.
Draw the new power triangle for the circuit if the power factor is corrected to 0.97 lagging.

Solution:

Firstly, draw a circuit diagram as shown in FIG 4.



Real Power P $= E_{RMS} I_{RMS} \cos\theta$ watts

$= 220 \times 4 \times 0.69$

$= 607.2$ watts

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Apparent Power S = $E_{RMS}I_{RMS}$ VA

= 220x4

= 880 VA

Reactive Power Q = $E_{RMS}I_{RMS}\sin\theta$ VARS

= 220x4x0.72

= 636.9 VARS

Draw power triangle as shown in FIG 5.

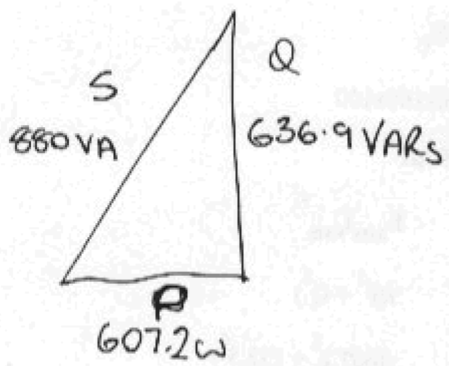

FIG 5

When the power factor is corrected, the real power in WATTS will remain the same (607.2 watts) but the reactive power (VARS) will be reduced.

If the power factor is corrected to 0.97 lagging then:

θ = $\cos^{-1} 0.97$

= 14.1° lagging

Tan θ = VARS/WATTS

VARS = WATTS x Tan14.1°

= 607.2 x 0.251

= 152.5 VARS

The old requirement for vars was 636.9 VARS and the new requirement for VARS is 152.5 VARS.

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The parallel connected capacitor must produce:

Q_{CAP} = 636.9 - 152.5 VARS

= 484.4 VARS

$Q_C = E_{RMS}^2/X_C$

$X_C = E_{RMS}^2/Q_C$

= 220x220/484.4

= 100Ω

Capacitance C = $1/\omega X_C$

= $1/2\pi$x100x100

= 15.92μF

Apparent Power S = $E_{RMS}I_{RMS}^*$

Also Apparent Power S = $\sqrt{(P^2 + Q^2)}$

= $\sqrt{(607.2^2 + 152.5^2)}$

= 626.1/14.1° VA

Overall power triangle is as shown in FIG 6.

$I = \frac{VA}{V}$

$= \frac{626.1}{220}$

= 2.85A

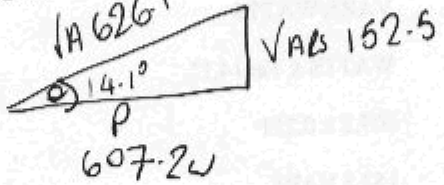

FIG 6

HIGH VOLTAGE TRANSMISSION LINE LOSSES

If the electrical parameters (resistance, inductance, capacitance) of a transmission line are known, then the performance of the line under various operating conditions can be determined.

To determine such things as receiving end voltage under load or no load, voltage drop along the line or line charging current, a Transmission Line Equivalent Circuit must be drawn.

Short Line Equivalent Circuit

Any line less than 80-100kM in length, is usually described as a "Short" line.

The capacitive effect of the short line can be neglected except when the line is unloaded.

Refer to FIG 1 which shows the single phase equivalent circuit of a short line with a source voltage and load impedance connected.

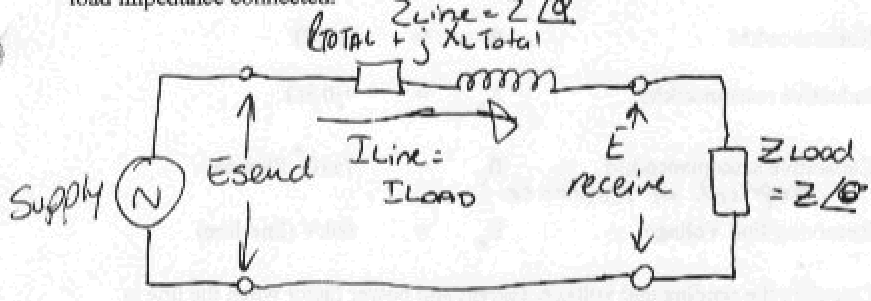

FIG 1

The equivalent circuit is a single phase representation, and all calculations use phase values.

Refer to FIG 2 which shows the phasor diagram for the equivalent circuit of FIG 1.

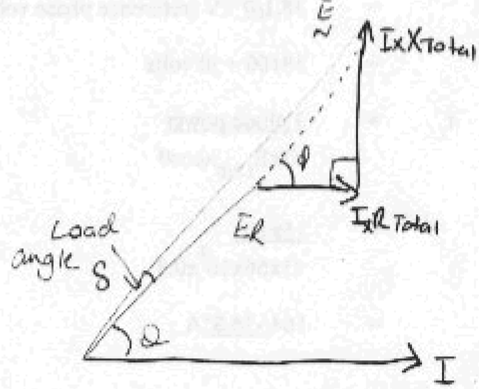

FIG 2

Quantities shown in the phasor diagram are:

I	=	load current
E_R	=	load or receiving end voltage
E_S	=	supply or sending end voltage
IR	=	voltage drop across line resistance R
IX	=	voltage drop across line inductive reactance X_L
IZ	=	voltage drop across line impedance
θ	=	load power factor angle
φ	=	line impedance angle
δ	=	"load" angle (phase shift along the line)

Depending on the magnitude of the voltage drop along the line, there will be a difference between E_S and E_R in both magnitude and angle.

Example: A transmission line is 100kM long and has the following parameters.

Resistance/kM	R	=	0.25Ω
Inductive reactance/kM	X_L	=	+j0.8Ω
Capacitive susceptance/kM	B_C	=	$7x10^{-6}$ Siemen
Receiving End Voltage	E_R	=	66kV (line-line)

Not used for this example — recprical of reactance

Calculate the sending end voltage, current and power factor when the line is delivering 15MW at a power factor of 0.8pf lagging.

Solution: Perform calculations on a per phase basis, and neglect the effect of line capacitance.

Load Voltage $E_R = \frac{66}{\sqrt{3}}$ kV

$= 38.1\angle 0°$kV (reference phase voltage)

$= 38100 + j0$ volts

Line Current $I_L = \frac{\text{3 phase power}}{\sqrt{3} x E_{LINE} x \cos\theta}$

$= \frac{15x10^6}{\sqrt{3}x66x10^3x0.8}$

$= 164\angle -36.8°$A

Line Impedance Z_L = 100(0.25 + j0.8)

= 25 + j80Ω

= 83.8/72.6°Ω

Draw single phase equivalent circuit as shown in FIG 3.

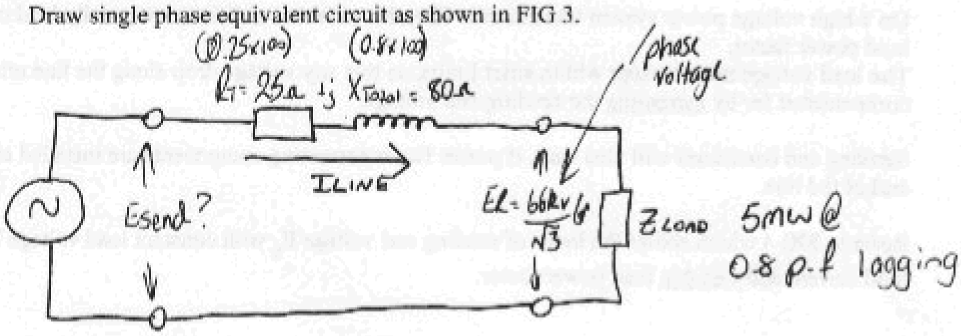

FIG 3

Voltage Drop along the line = $I_L x Z_L$

= 164/-36.8° x 83.8/72.6° volts

= 13743/35.8° volts

= 11146 + j8039 volts

Sending End Voltage E_S = $E_R + E_{LINE}$

= (38100 + j0) + (11146 + j8039)

= 49426 + j8039

= 49898/9.3° volts

Sending End power factor = cos (36.8° + 9.3°)

= cos 46.1° lag

= 0.69 lagging

Notes: There is a phase shift of 9.3° between E_S and E_R along the line.
The sending end power factor is worse than the load power factor due to the line impedance.

Locus Diagram of E_S for the Short Line

A locus diagram shows how a phasor quantity of voltage or current will vary when other conditions in the circuit are changed.

On a high voltage power system transmission line, the variable conditions are usually load current and load power factor.
The load voltage must be kept within strict limits, so that any voltage drop along the line must be compensated for by increasing the sending end voltage.

Sending end conditions will also vary, if power factor correcting components are installed at the load end of the line.

Refer to FIG 4 which shows the locus of sending end voltage E_S with constant load voltage E_R, constant load current and variable load power factor.

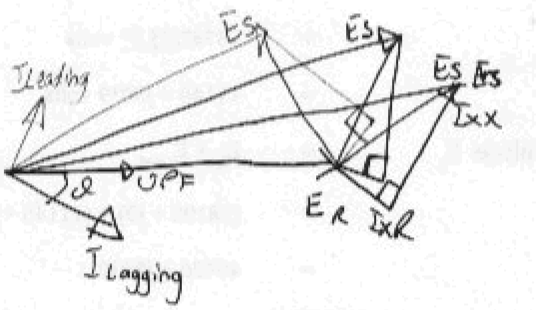

FIG 4

Note: As load power factor changes, sending end voltage ES will change, and so too will the load angle δ.

Long Line Equivalent Circuit

For long line analysis, the capacitance of the line must be included in calculations.

The line capacitance is connected **across** the line.

There are several ways that the equivalent circuit may be drawn.

a) **Nominal π Method** half of the total line capacitance at each end of the line, with total line impedance between,

b) **Nominal T Method** half of the total line impedance at each end with all the line capacitance in between,

c) **Lumped Capacitance Method** lump the total line capacitance at the load end of the line,

Nominal π Method

Refer to FIG 5 which shows the equivalent circuit where line capacitance is divided between the load and the supply end of the line.

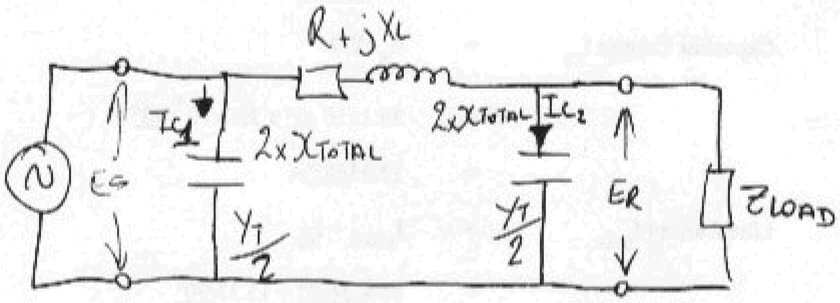

FIG 5

Note: The total capacitive susceptance Y_{TOTAL} is split into two components ($Y_T/2$), one connected at each end of the line.

Re-calculating the previous example using the Nominal π Method.

Draw the Nominal π equivalent circuit as shown in FIG 6.

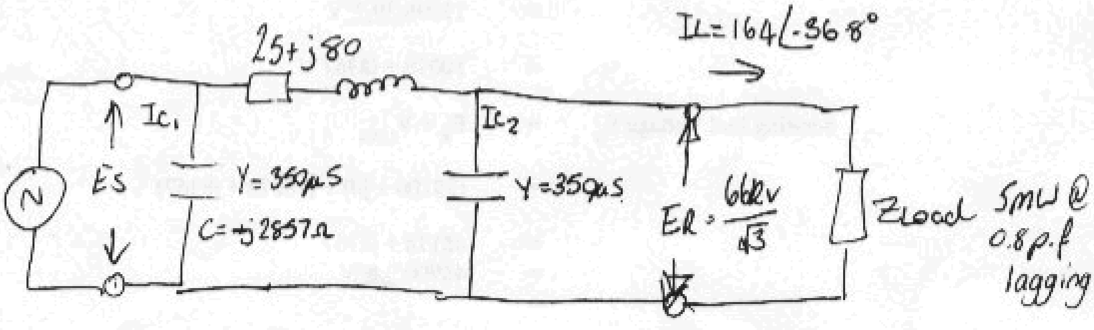

FIG 6

Load Voltage E_R	=	$\frac{66}{\sqrt{3}}$ kV
	=	$38.1\angle 0°$ kV (reference phase voltage)
	=	$38100 + j0$ volts
Load Current I_{LOAD}	=	$\frac{\text{3 phase power}}{\sqrt{3} x E_{LINE} x \cos\theta}$
	=	$\frac{15x10^6}{\sqrt{3}x66x10^3 x0.8}$
	=	$164\angle -36.8°$ A
Capacitor Current I_{C2}	=	$E_R x Y/2$
	=	$38.1x10^3\angle 0° \text{ x } 350x10^{-6}\angle 90°$
	=	$13.33\angle 90°$ A
Line Current I_{LINE}	=	$I_{LOAD} + I_{C2}$
	=	$164\angle -36.8° + 13.33\angle 90°$
	=	$(131.3 - j98.2) + (0 + j13.33)$
	=	$131.3 - j84.9$
	=	$156.3\angle -32.9°$ A
Voltage Drop along line V_{LINE}	=	$I_{LINE} x Z_{LINE}$
	=	$156.3\angle -32.9° \text{ x } 83.8\angle 72.6°$
	=	$13098\angle 39.7°$ V
	=	$10078 + j8367$ V
Sending End Voltage E_S	=	$E_R + V_{LINE}$
	=	$(38100 + j0) + (10078 + j8367)$
	=	$48178 + j8367$
	=	$48900\angle 9.8°$ V

Capacitor Current I_{C1} = $E_S x Y/2$

= $48900\underline{/9.8°} \times 350x10^{-6} \underline{/90°}$

= $17.12\underline{/99.8°}$

= $-2.9 + j16.87A$

Sending End Current I_S = $I_{C1} + I_{LINE}$

= $(-2.9 + j16.87) + (131.3 - j84.9)$

= $128.4 - j68$

= $145.3\underline{/-27.9°}A$

Sending End Power Factor = $\cos(27.9 + 9.8)$

= $\cos 37.7°$

= 0.791 lagging

Notes: These results are more accurate than the first method used, where line capacitance was not considered.
The sending end current is <u>less</u> than the load current due to the small power factor correction effect of the line capacitance.

Nominal T Method

Refer to FIG 7 which shows the equivalent circuit where line impedance is divided between the load and the supply end of the line, and the line capacitance is lumped between the two impedances.

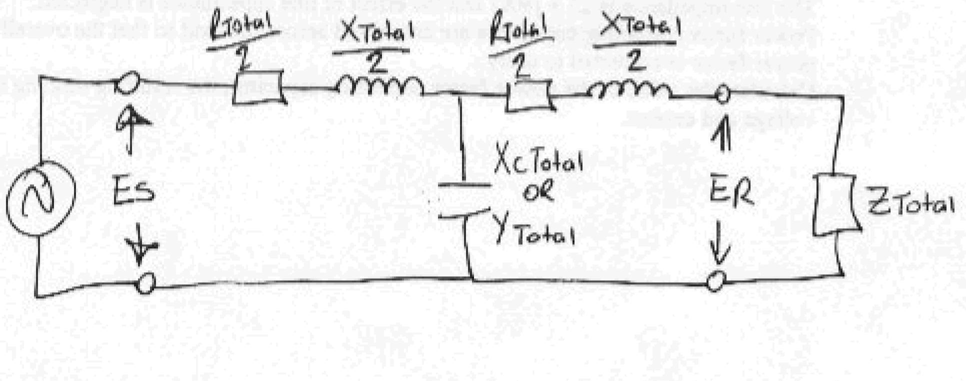

FIG 7

Note: The total line impedance Z_{TOTAL} is split into two components ($Z_T/2$), one connected at each end of the line.

In a similar way to the Nominal π method, line current and sending end voltage can be calculated.

Lumped Capacitance Method

Refer to FIG 8 which shows the equivalent circuit where the line capacitance is lumped at the receiving end of the line.

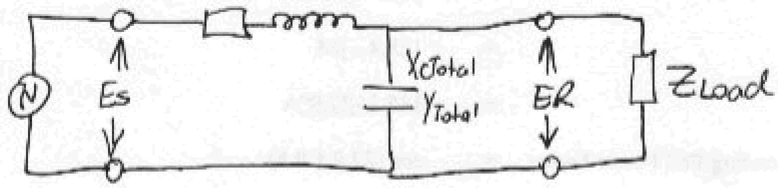

FIG 8

In a similar way to the Nominal π method and the Nominal T method, line current and sending end voltage can be calculated.

Effect of Power Factor Correction on Line Conditions

If a power factor correction capacitor is connected across the load, the load VARS will be supplied locally and will not need to be transmitted along the line from the source.

Example: A transmission line is delivering 15MW at a power factor of 0.8pf lagging and line voltage of 66kV.
The line impedance is $25 + j80\Omega$ and the effect of line capacitance is neglected.
Power factor correcting capacitors are connected across the load so that the overall power factor is corrected to unity.
Calculate the value of the power factor correcting capacitor, the resulting sending end voltage and current.

Solution:

Draw simplified single phase equivalent circuit as shown in FIG 9.

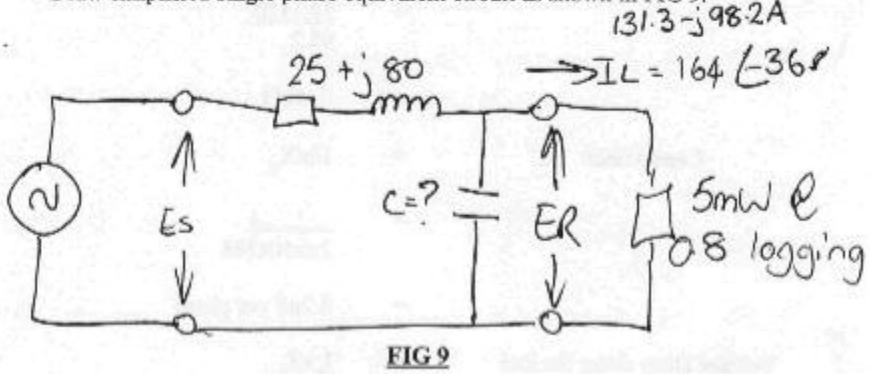

FIG 9

Load Voltage E_R = $\frac{66}{\sqrt{3}}$ kV

= $38.1\underline{/0°}$ kV (reference phase voltage)

= $38100 + j0$ volts

Line Impedance Z_L = $25 + j80\Omega$

= $83.8\underline{/72.6°}\Omega$

Load Current I_{LOAD} = $\frac{\text{3 phase power}}{\sqrt{3} \times E_{LINE} \times \cos\theta}$

= $\frac{15\times10^6}{\sqrt{3}\times66\times10^3\times0.8}$

= $164\underline{/-36.8°}$ A

= $131.3 - j98.2$ A

To correct the power factor of the load to unity, the power factor correcting capacitor must supply $(0 + j98.2\text{A})$ of capacitive current

Line Current I_{LINE} (after correction) = $131.3 + j0$ A

= $131.3\underline{/0°}$ A

Reactance of Capacitor X_C = E_R/I_C

= $\frac{38.1 \times 10^3}{98.2}$

= $-j388\Omega$

Capacitance C = $1/\omega X_C$

= $\frac{1}{2\pi \times 50 \times 388}$

= $8.2\mu F$ per phase

Voltage Drop along the line = $I_L x Z_L$

= $131.3\angle 0^\circ \times 83.8\angle 72.6^\circ$ volts

= $11003\angle 72.6^\circ$ volts

= $3290 + j10500$ volts

Sending End Voltage E_S = $E_R + E_{LINE}$

= $(38100 + j0) + (3290 + j10500)$

= $41390 + j10500$

= $42701\angle 14.2^\circ$ volts

Sending End power factor = $\cos(0^\circ + 14.2^\circ)$

= $\cos 14.2^\circ$ lag

= 0.97 lagging

Notes: There is a phase shift of 14.2° between E_S and E_R along the line.
The sending end power factor is better than the load power factor due to the power factor correction.
The line current has been decreased by power factor correction.

Voltage Rise Along an Unloaded Transmission Line

An energised but unloaded transmission line draws capacitive "charging" current through its line capacitance, and a long line may draw several hundred amps of charging current.

The leading capacitive currents passing through the series inductive reactance of the line, may cause a voltage rise along the line, so that the receiving end voltage is larger than the sending end voltage.

This "negative voltage regulation" is known as the "Ferranti Effect", and in some cases the receiving end voltage may exceed the sending end voltage by as much as 50%.

The overvoltage is undesirable, as it places excessive stress on the insulation of HV equipment.

The Ferranti Effect is also seen on lightly loaded transmission lines and cables, whose capacitance is even greater than that of overhead lines.

The overvoltage effect is minimised by connecting in circuit, shunt reactors, which consume some of the excessive capacitive VARS generated by the line or cable.

Example: A 132kV 50Hz three phase transmission line, is energised at the sending end but is not loaded, has phase impedance of $10 + j120\Omega$ and the line charging current is 90A. Calculate the receiving end voltage if the sending end voltage is fixed at 132kV.

Solution:

Use the simplified single phase equivalent circuit where the capacitance is lumped at the receiving end of the line.
Neglect the line resistance because it is less than 0.1 of the value of X_L.
Draw the equivalent circuit as shown in FIG 10.

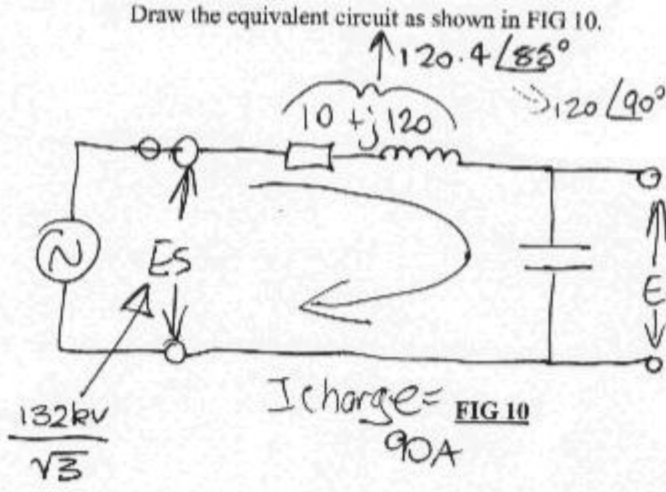

FIG 10

Line Voltage Drop	=	$I_L x X_L$
	=	(0 + j90)(0 + j120)
V_{LINE}	=	10800/180°V
Receiving End Voltage E_R	=	E_{SEND} - V_{LINE}
	=	76100/0° - 10800/180°
	=	86700/0°V

Thus there has been a voltage rise along the unloaded line, so that the receiving end voltage is ≈ 14% higher than the sending end voltage.

95X Fg2

SYMMETRICAL COMPONENTS

Balanced Three Phase System

Voltages: All voltages (line and phase) are equal and are displaced by 120°.

Currents: All line currents are equal and displaced by 120°.

Unbalanced (Assymetrical) Three Phase System

A three Phase system is unbalanced when the three phase currents and/or voltages are not equal in magnitude or not displaced by 120°.

Unbalance can occur due to:

a) unbalanced loading of phases,

b) short circuit (fault) conditions,

c) partial short circuit (fault) conditions, not a zero ohm fault. e.g. arching faults have some form of resistance.

d) open circuit phase or windings.

FIG 1 shows examples of unbalanced three phase voltages.

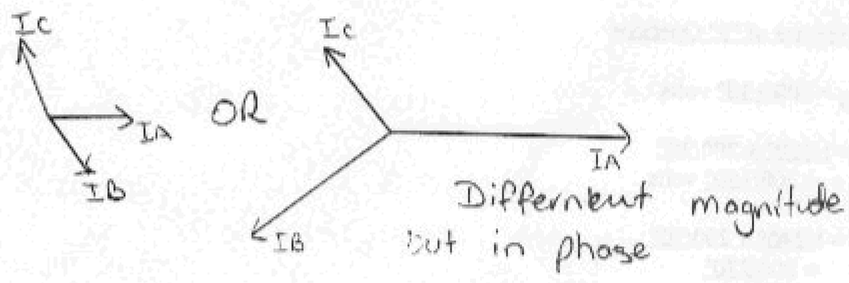

FIG 1

Vector Operators

The "j" Operator

The "j" operator is a vector operator which rotates a vector (phasor) by 90° anti-clockwise, and is equal to $1\underline{/90^\circ}$ (-1).
It is used to represent complex numbers and each successive multiplication by "j" will rotate the vector by 90° anti-clockwise.

$j = 1\underline{/90^\circ}$ $j^2 = 1\underline{/180^\circ}$ $j^3 = 1\underline{/270^\circ}$ $j^4 = 1\underline{/360^\circ} = 1\underline{/0^\circ}$

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The "a" Operator

The "a" operator is a vector operator which rotates a vector (phasor) by 120° anti-clockwise, and is equal to $1\angle 120°$.
It is used specifically for manipulating three phase vectors.

FIG 2 shows 1, a and a^2 as a symmetrical balanced set of vectors

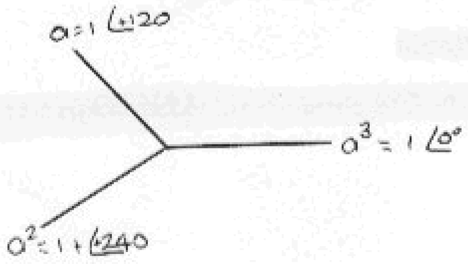

FIG 2

If the three vectors are added together:

$$
\begin{aligned}
1 + a + a^2 &= 1\angle 0° + 1\angle +120° + 1\angle +240° \\
&= (1 + j0) + (-0.5 + j0.866) + (-0.5 - j0.866) \\
&= 0
\end{aligned}
$$

Therefore the sum of a symmetrical balanced set of vectors is zero.

Examples of the use of "a" Operator

If $E_{AB} = 200\angle +30°$ volts

then $aE_{AB} = 1\angle 120° \times 200\angle 30°$
$= 200\angle 150°$ volts

then $a^2E_{AB} = 1\angle 240° \times 200\angle 30°$
$= 200\angle 270°$
OR $= 200\angle -90°$ volts

Thus voltages or current phasors can be manipulated by "a" or "a^2".

The Theorem of Symmetrical Components

In a three phase system, any set of unbalanced phasors (voltage or current) can be represented by the sum of two or three sets of balanced phasors (Superposition Theorem) called Symmetrical Components.

The three possible sets of balanced or symmetrical components are known as:

a) the positive sequence set (subscript 1) .

b) the negative sequence set (subscript 2)

c) the zero sequence set (subscript 0).

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Note: A <u>zero</u> sequence set of phasors has no sequence and all three phasors are equal and in phase.

FIG 3 shows a zero sequence set of phasors.

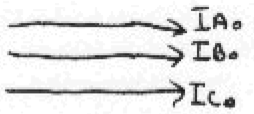


FIG 3

Representing the unbalanced phasors as a sum of Symmetrical Components

Each phasor in the original unbalanced set can be represented as follows:

$$I_A = I_{A1} + I_{A2} + I_{A0}$$
$$I_B = I_{B1} + I_{B2} + I_{B0}$$
$$I_C = I_{C1} + I_{C2} + I_{C0}$$

Depending on the type of unbalance, not all of the components will exist.

Possible Combinations of Components

The possible combinations of symmetrical components that represent an unbalanced set of phasors are:

a) positive and negative sequence only, which exist when sum of the original three phasors is equal to zero. ($I_A + I_B + I_C = 0$)

b) positive, negative and zero sequence, which exist when sum of the original three phasors is not equal to zero. ($I_A + I_B + I_C$ not $= 0$)

Rules for Separating Components

The unbalanced phasors must be manipulated in the following way to determine the value of each of the symmetrical components.

Positive Sequence Components

The positive sequence component of A phase is:

$$I_{A1} = 1/3(I_A + aI_B + a^2I_C)$$

The balanced positive sequence set of components can now be drawn by positioning I_{A1} and drawing I_{B1} and I_{C1} at ±120° from I_{A1} in positive sequence.

FIG 4 shows a positive sequence set of components.

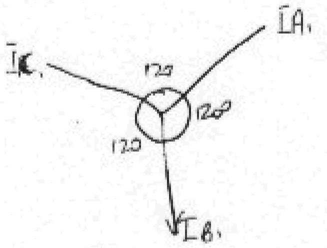


FIG 4

Negative Sequence Components

The negative sequence component of A phase is:

$$I_{A2} = 1/3(I_A + a^2 I_B + aI_C)$$

The balanced negative sequence set of components can now be drawn by positioning I_{A2} and drawing I_{B2} I_{C2} at ±120° from I_{A2} in negative sequence.

FIG 5 shows a negative sequence set of components.

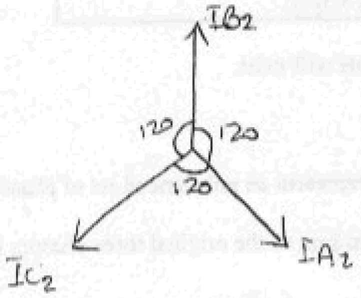


FIG 5

Zero Sequence Components

The zero sequence component of A phase is:

$$I_{A0} = 1/3(I_A + I_B + I_C)$$

The balanced zero sequence set of components can now be drawn by positioning I_{A0} and drawing I_{B0} and I_{C0} in phase with I_{A0}.

FIG 6 shows a zero sequence set of components.

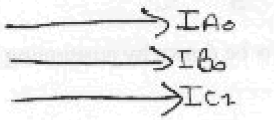


FIG 6

F96

Notes: The neutral current in a four wire unbalanced three phase system is equal to the sum of the zero sequenc components.
Earth fault currents consist of zero sequence components.

Example: Resolve the following unbalanced currents into their symmetrical components.

$I_A = 100\angle 0°$ amps
$I_B = 100\angle 180°$ amps
$I_C = 0$

Solution:

Zero sequence components

$I_{A0} = 1/3(I_A + I_B + I_C)$
$= 1/3(100\angle 0° + 100\angle 180° + 0)$
$= 1/3((100 + j0) + (-100 + j0))$
$= 0$

There is no zero sequence component.

Positive sequence Components

$I_{A1} = 1/3(I_A + aI_B + a^2I_C)$
$= 1/3(100\angle 0° + (1\angle 120° \times 100\angle 180°) + (1\angle 240° \times 0))$
$= 1/3(100 + j0 + 100\angle 300° + 0)$
$= 1/3((100 + j0) + (50 - j86.6))$
$= 1/3(150 - j86.6)$
$= 1/3(173.2\angle -30°)$
$= 57.7\angle -30°$ amps

$I_{B1} = 57.7\angle -150°$ amps

$I_{C1} = 57.7\angle +90°$ amps

FIG 7 shows the phasor diagram of the positive sequence components.

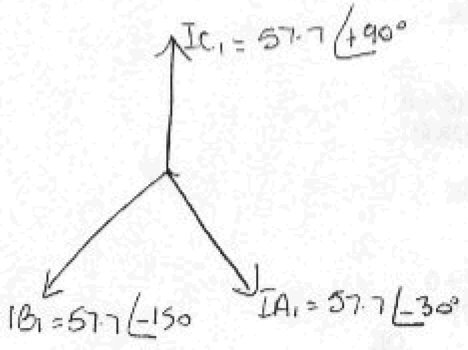


FIG 7

Negative sequence Components

$I_{A2} = 1/3(I_A + a^2 I_B + aI_C)$
$= 1/3(100\underline{/0^\circ} + (1\underline{/240^\circ}\times 100\underline{/180^\circ}) + (1\underline{/120^\circ}\times 0))$
$= 1/3(100 + j0 + 100\underline{/420^\circ} + 0)$
$= 1/3((100 + j0) + (50 + j86.6))$
$= 1/3(150 + j86.6)$
$= 1/3(173.2\underline{/30^\circ})$
$= 57.7\underline{/30^\circ}$ amps

$I_{B2} = 57.7\underline{/+150^\circ}$ amps

$I_{C2} = 57.7\underline{/-90^\circ}$ amps

FIG 8 shows the phasor diagram of the negative sequence components.

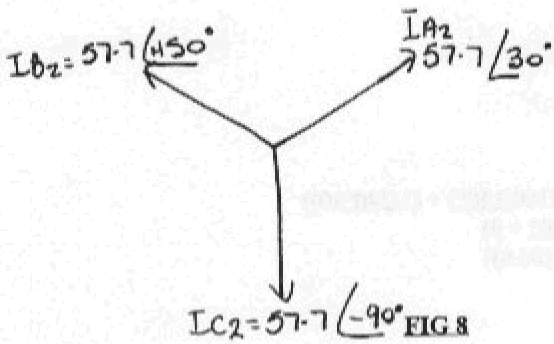


FIG 8

Check sum of components.

$I_A = I_{A1} + I_{A2} + I_{A0}$
$= 57.7\underline{/-30^\circ} + 57.7\underline{/+30^\circ} + 0$
$= (50 - j28.8) + (50 + j28.8)$
$= 100 + j0$
$= 100\underline{/0^\circ}$ amps OK.

$I_B = I_{B1} + I_{B2} + I_{B0}$
$= 57.7\underline{/-150^\circ} + 57.7\underline{/+150^\circ} + 0$
$= (-50 - j28.8) + (-50 + j28.8)$
$= -100 + j0$
$= 100\underline{/180^\circ}$ amps OK

$I_C = I_{C1} + I_{C2} + I_{C0}$
$= 57.7\underline{/+90^\circ} + 57.7\underline{/-90^\circ} + 0$
$= (0 + j57.7) + (0 - j57.7)$
$= 0$ amps OK

Notes: These unbalanced currents have only positive and negative sequence components.
They are the currents that would represent a phase-phase fault on a three phase system.
The current in the third unfaulted phase is zero or negligibly small.
There are no zero sequence components as the fault does not involve earth.

F92

FIG 9 is a phasor diagram showing the positive and negative sequence components added to give the original currents.

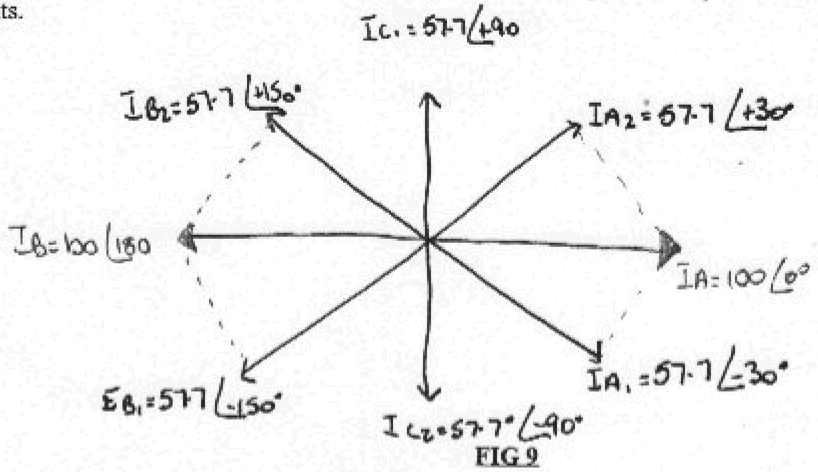


FIG 9

F73-1

DISTRIBUTION OF FAULT CURRENTS THROUGH POWER SYSTEMS

Fault currents can be calculated at points in the power system other than at the fault.

The impedance diagrams must be used to determine the distribution of current through each branch of the network.

Example:

Refer to Tutorial 5.

Depending on the type of fault, there may exist the following components of fault current:

Transmission line:
positive, negative and zero sequence.

Transformer secondary windings (132kV earthed star):
positive, negative and zero sequence.

Transformer primary windings (33kV unearthed star):
positive and negative sequence only.

Transformer tertiary windings (11kV delta):
zero sequence only.

Generator windings (33kV unearthed star):
positive and negative sequence only.

Consider the distribution of current through the transformer for the single phase to earth fault on A phase.

Refer to FIG 1 which shows the phase sequence components that exist in each set of windings for the single phase to earth fault.

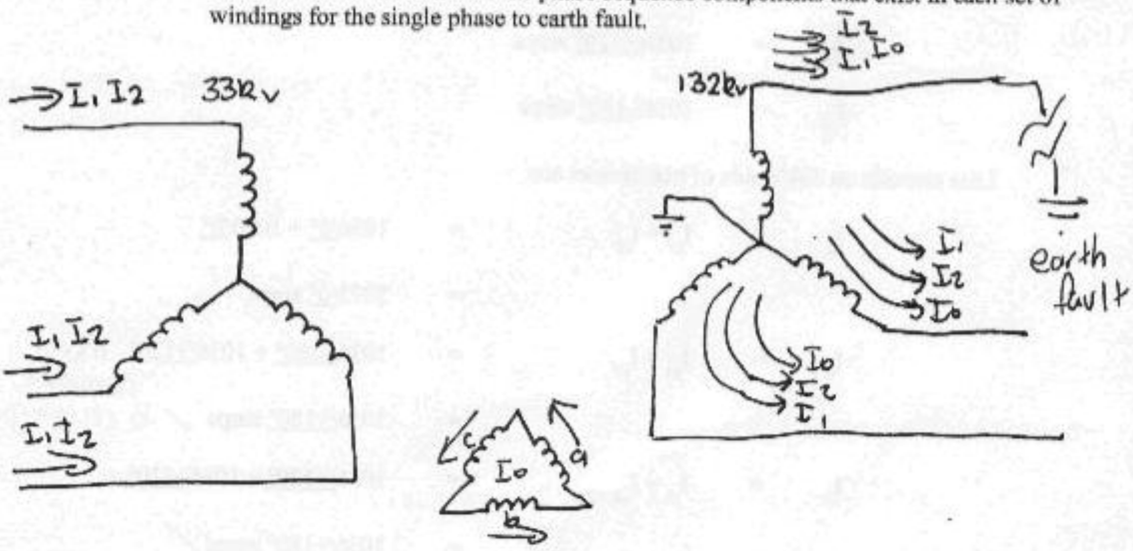


FIG 1

773-2

The ampere turns of MMF in the transformer core must balance.

Use the previously calculated fault currents on the 132kV transmission line which will be the same in the 132kV windings of the transformer.

On 132kV side of transformer: $I_A = 777\underline{/0^\circ}$ amps
$I_B = 0$
$I_C = 0.$

$$I_{A1} = I_{A2} = I_{A0} = 259\underline{/0^\circ} \text{ amps}$$

The positive and negative sequence components will be balanced by currents flowing in the 33kV primary windings.

The zero sequence components will be balanced by currents flowing around the 11kV delta tertiary windings.

The ratio of transformation between primary and secondary is 132/33 = 4

Positive sequence components in 33kV windings are:

$$I_{a1} = \frac{259 \times 132 /0^\circ}{33} = 1036\underline{/0^\circ} \text{ amps}$$

$$I_{b1} = 1036\underline{/-120^\circ} \text{ amps}$$

$$I_{c1} = 1036\underline{/+120^\circ} \text{ amps}$$

Negative sequence components in 33kV windings are:

$$I_{a2} = \frac{259 \times 132 /0^\circ}{33} = 1036\underline{/0^\circ} \text{ amps}$$

$$I_{b1} = 1036\underline{/+120^\circ} \text{ amps}$$

$$I_{c1} = 1036\underline{/-120^\circ} \text{ amps}$$

Line currents on 33kV side of transformer are:

$$I_a = I_{a1} + I_{a2} = 1036\underline{/0^\circ} + 1036\underline{/0^\circ}$$

$$= 2072\underline{/0^\circ} \text{ amps}$$

$$I_b = I_{b1} + I_{b2} = 1036\underline{/-120^\circ} + 1036\underline{/+120^\circ}$$ must convert to rectangular

$$= 1036\underline{/+180^\circ} \text{ amps}$$ ✓

$$I_c = I_{c1} + I_{c2} = 1036\underline{/+120^\circ} + 1036\underline{/-120^\circ}$$

$$= 1036\underline{/+180^\circ} \text{ amps}$$ ✓

F13-9

Ratio of transformation between secondary and tertiary windings $(132/\sqrt{3})/11$.

Zero sequence currents flowing around the 11kV delta winding are:

$$I_{a0} = I_{b0} = I_{c0} = \frac{I_{A0} \times 132 \times 1\angle 0^\circ}{\sqrt{3} \times 11}$$

$$= \frac{250 \times 132 \times 1\angle 0^\circ}{\sqrt{3} \times 11}$$

$$= 1795\angle 0^\circ \text{ amps.}$$

Refer to FIG 2 which shows the calculated current values flowing in the transformer windings.

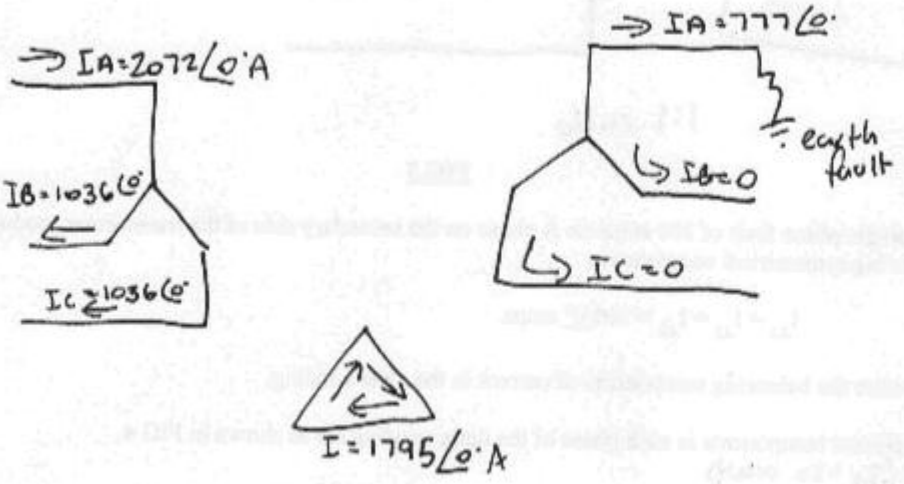


FIG 2

Note: There are no positive or negative phase sequence components in the delta windings because the winding has no external connections.

F 13-4

Fault Current Distribution through Star Delta Transformers

Refer to FIG 3 which shows a Delta/Star transformer with turns ratio 1:1 and a single phase to earth fault on the star connected side.

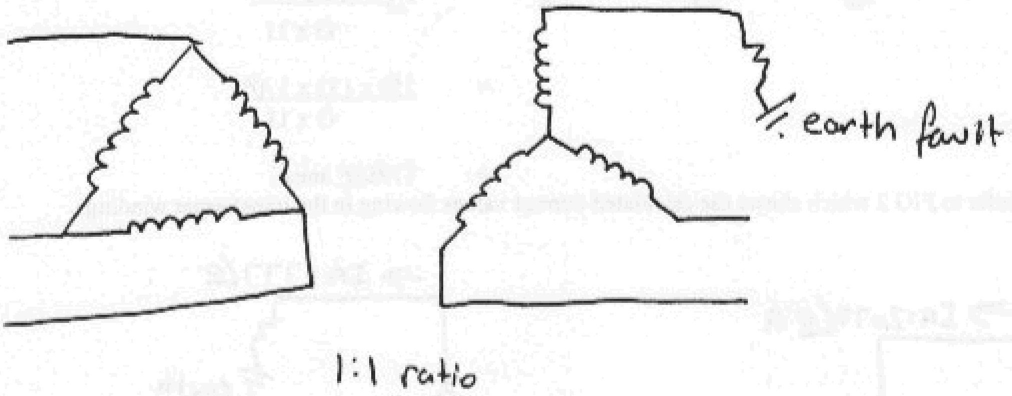


FIG 3

The single phase fault of 300 amps on A phase on the secondary side of the transformer produces the following symmetrical components.

$$I_{A1} = I_{A2} = I_{A0} = 100\underline{/0^\circ} \text{ amps.}$$

Calculate the balancing components of current in the delta winding.

The current components in each phase of the delta winding are as shown in FIG 4.

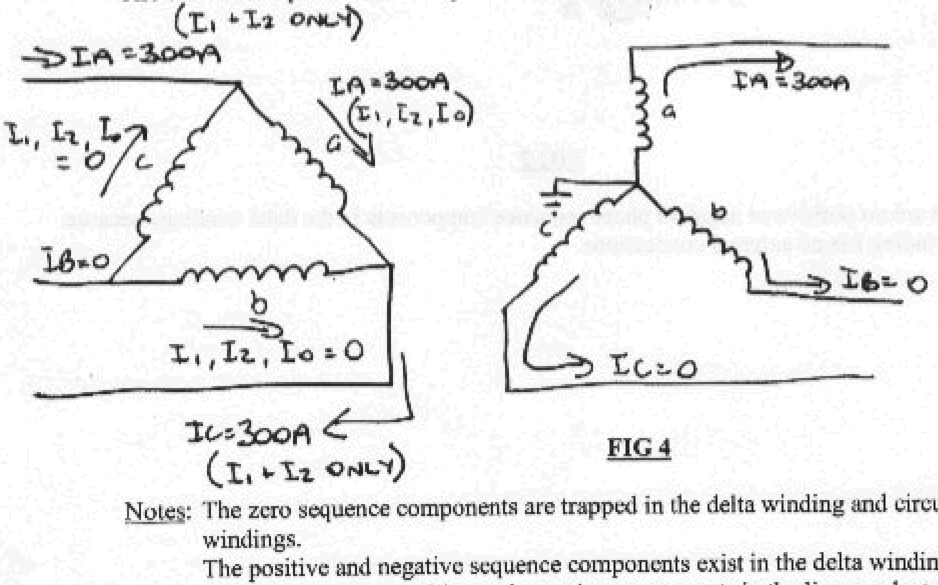


FIG 4

Notes: The zero sequence components are trapped in the delta winding and circulate in the windings.
The positive and negative sequence components exist in the delta windings and add vectorially to give positive and negative components in the line conductors.

PHASE SEQUENCE IMPEDANCE DIAGRAMS FOR POWER SYSTEMS

There are two classes of faults on power systems:

a) Balanced — three phase short circuit

b) Unbalanced — one phase to earth
phase to phase
phase to phase to earth.

Balanced Faults

A short circuit across three phases produces symmetrical and balanced three phase currents and voltages in the fault.

The currents and voltages are positive sequence components only. No negative or zero sequence components exist.

Unbalanced Faults

Unbalanced faults involving earth will produce positive, negative and zero sequence components.
Unbalanced faults not involving earth will produce only positive and negative sequence components.

To calculate the effects of faults, the impedance of the faulted system must be known.

Impedance of System Components

All high voltage equipment such as Generators, Transformers and Transmission Lines will have an impedance value for every phase sequence component.

Z_0 = zero sequence impedance

Z_1 = positive sequence component

Z_2 = negative sequence component

These values may be different or the same, depending on the type of equipment and how it is connected.

We must draw an impedance diagram for each component before solving for fault currents.

Impedance of Static Devices

Static devices are such things as Transformers, Overhead lines, Cables and Reactors but not rotating machines such as Generators.

Generally $Z_1 = Z_2$ for static devices.

However, Z_0 will depend on whether there is a path for zero sequence currents and is determined by how the device is connected (earthed or not).

Two Winding Transformer Equivalent Circuit

FIG 1 shows the equivalent circuit of a two winding transformer.

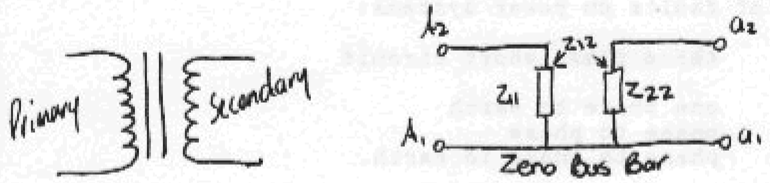


FIG 1

Z_{11} and Z_{22} are self impedances of the two windings.

Z_{12} is the mutual impedance between the two windings.

The equivalent circuit in FIG 1 can be re-drawn as shown in FIG 2.

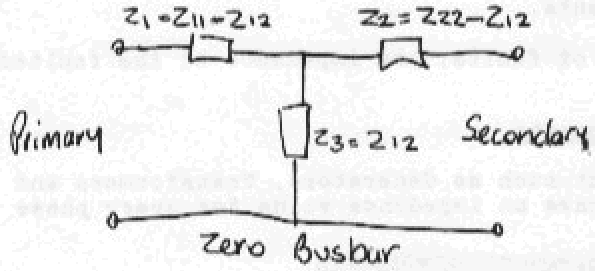


FIG 2

Z_1 and Z_2 are leakage impedances.

Z_3 is the mutual impedance (very large compared to Z_1 and Z_2) and can be ignored.

Positive Sequence Equivalent Circuit of Two Winding Transformer

Refer to FIG 3 which is the simplified circuit of FIG 2 and represents the Positive Sequence Equivalent Circuit.

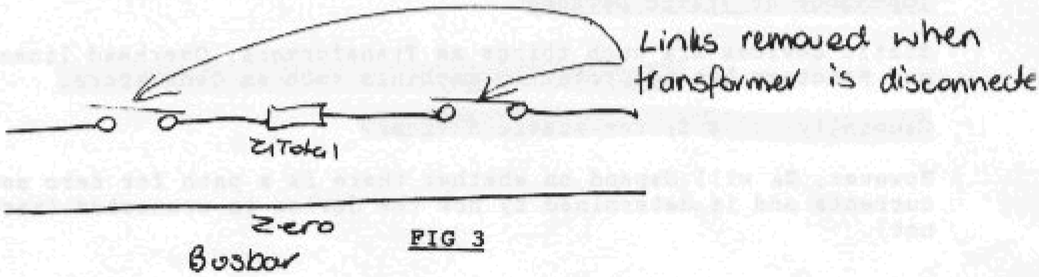


FIG 3

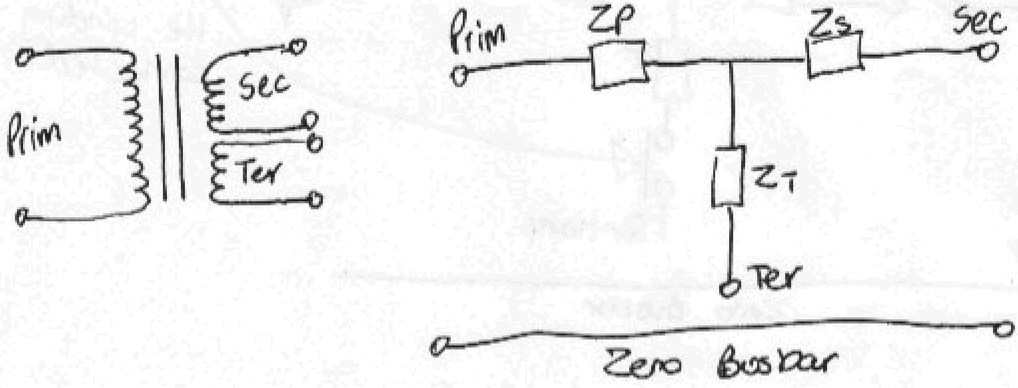
Z1 diagram

Negative Sequence Equivalent Circuit of Two Winding Transformer

Since $Z_1 = Z_2$ in static devices, then FIG 3 also represents the Negative Sequence Equivalent circuit of the two winding transformer.

Three Winding Transformer Equivalent Circuit

Refer to FIG 4 which shows the equivalent circuit of a three winding transformer.



Impedance diagram

FIG 4

The total impedance between each pair of terminals is determined as follows:

$$Z_{PS} = Z_P + Z_S$$

$$Z_{ST} = Z_S + Z_T$$

$$Z_{PT} = Z_P + Z_T$$

Similarly, the individual impedances of the windings can be represented by:

$$Z_P = \tfrac{1}{2}(Z_{PS} + Z_{PT} - Z_{TS})$$

$$Z_S = \tfrac{1}{2}(Z_{PS} + Z_{TS} - Z_{PT})$$

$$Z_T = \tfrac{1}{2}(Z_{PT} + Z_{TS} - Z_{PS})$$

Positive Sequence Equivalent Circuit of Three Winding Transformer

Refer to FIG 5 which is the circuit of FIG 4 re-drawn and includes links in each circuit which will be closed if the particular winding is connected.

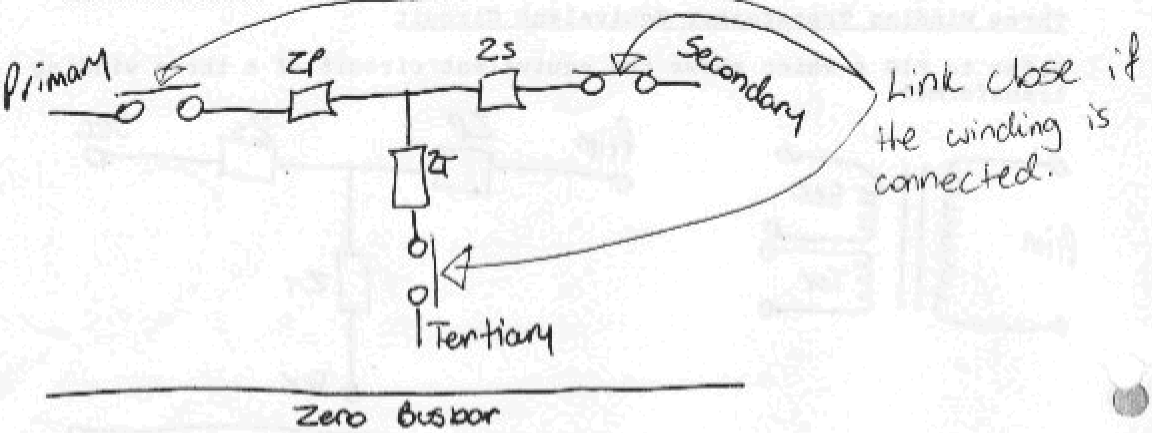


FIG 5

Negative Sequence Equivalent Circuit of Three Winding Transformer

Since $Z_1 = Z_2$ in static devices, then FIG 5 also represents the Negative Sequence Equivalent circuit of the three winding transformer.

Generator Equivalent Circuits

In rotating machinery, Positive sequence impedance Z_1 is not equal to Negative sequence impedance Z_2, due to the effects of Negative Sequence components on the rotor of the machine.

Refer to FIG 6 which is the equivalent circuit of a generator.

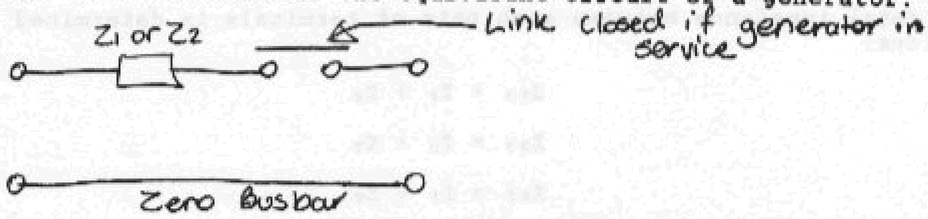


FIG 6

The circuit is the same for both positive and negative sequence, except that the impedance value will be different.

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Zero Sequence Equivalent Circuits of Transformers

Refer to FIG 7 which shows the equivalent zero sequence circuits for various transformer connections.

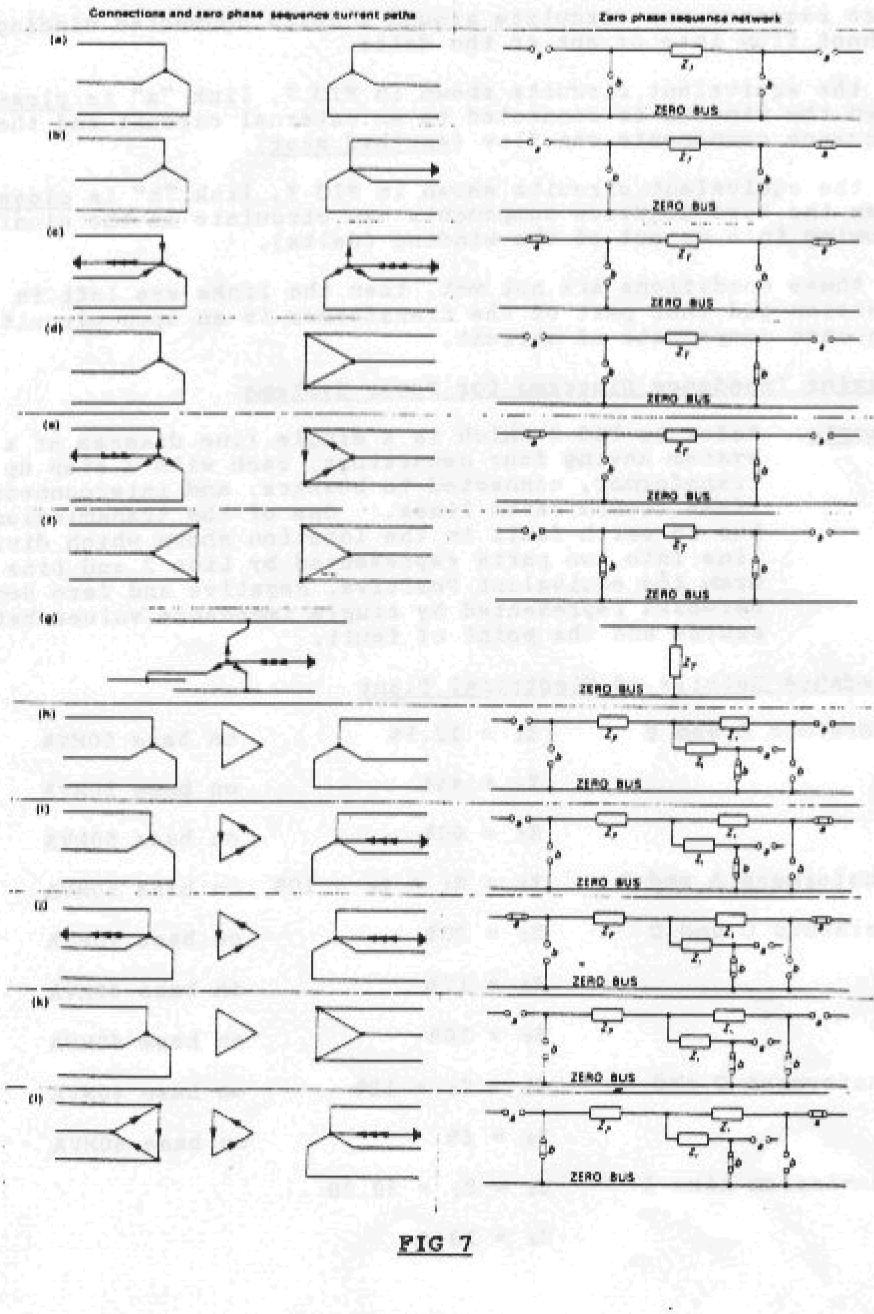

FIG 7

Zero sequence components of current can flow into and out of a transformer, when the transformer forms part of a closed loop to the unidirectional currents.

Zero sequence components can flow in a star connected winding which has an earthed star point and is connected to an earthed source.

Zero sequence can circulate around a delta connected winding but cannot flow into or out of the delta.

In the equivalent circuits shown in FIG 7, link "a" is closed only when the winding is connected to an external circuit and the zero sequence components can flow (earthed star).

In the equivalent circuits shown in FIG 7, link "b" is closed only when the zero sequence components can circulate in the winding without flowing into or out of the winding (delta).

If these conditions are not met, then the links are left in the open position and that part of the transformer is an open circuit to zero sequence components of current.

Drawing Impedance Diagrams for Power Systems

Example: Refer to FIG 8 which is a single line diagram of a power system having four generators, each with a step up transformer, connected to busbars, and interconnected by two 132kV transmission lines. One of the transmission lines has an earth fault in the location shown which divides the line into two parts represented by Line 2 and Line 3. Draw the equivalent Positive, Negative and Zero Sequence Networks represented by single impedance values between the source and the point of fault.

Impedance Details of Electrical Plant

Generators A and B	Z_1 = 32.5%	on base 50MVA
	Z_2 = 45%	on base 50MVA
	Z_0 = 60%	on base 50MVA
Transformers A and B	Z_1 = Z_2 = Z_0 = 10%	on base 50MVA
Generators C and D	Z_1 = 20%	on base 40MVA
	Z_2 = 12%	on base 40MVA
	Z_0 = 10%	on base 40MVA
Transformers C and D	Z_1 = Z_2 = 12%	on base 40MVA
	Z_0 = 6%	on base 40MVA
Transmission Line 1	Z_1 = Z_2 = 38.3Ω	
	Z_0 = 134Ω	

Fig8

Transmission Line 2 $Z_1 = Z_2 = 10.5\Omega$

$Z_0 = 36.6\Omega$

Transmission Line 3 $Z_1 = Z_2 = 27.8\Omega$

$Z_0 = 97.4\Omega$

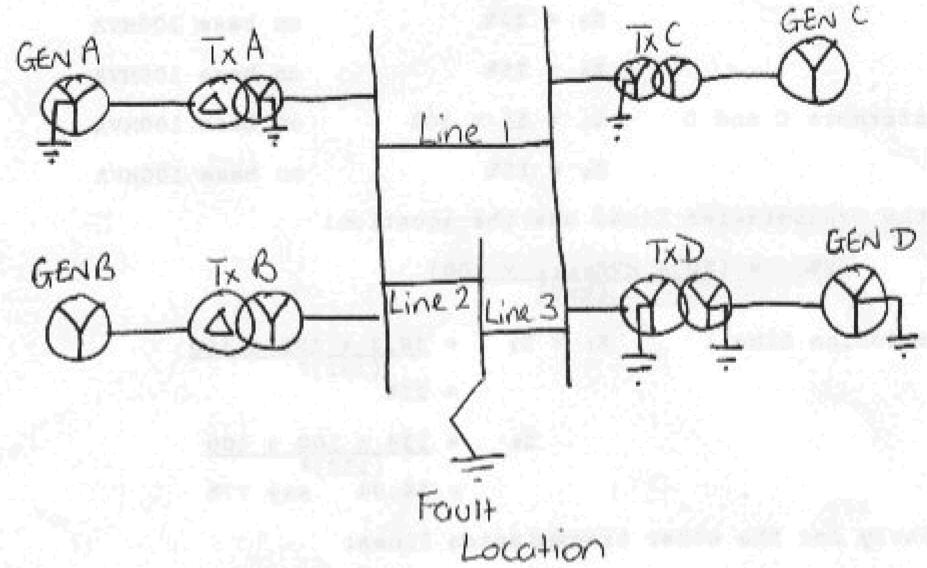

FIG 8

Solution:

Convert all impedance values to a common base (say 100MVA).

For the Generators and Transformers use the equation:

$$Z\% \text{ (new MVA base)} = Z\% \text{ (on old MVA base)} \times \frac{\text{new base MVA}}{\text{old base MVA}}$$

Generators A and B $Z_1 = \frac{32.5 \times 100}{50}$

$= 65\%$ on base 100MVA

Similarly all other Z% values can be converted.

	Z_2 = 90%	on base 100MVA
	Z_0 = 120%	on base 100MVA
Transformers A and B	Z_1 = Z_2 = Z_0 = 20%	on base 100MVA
Generators C and D	Z_1 = 50%	on base 100MVA
	Z_2 = 30%	on base 100MVA
	Z_0 = 25%	on base 100MVA
Transformers C and D	Z_1 = Z_2 = 30%	on base 100MVA
	Z_0 = 15%	on base 100MVA

For the Transmission Lines use the equation:

$$Z\% = \frac{(Z\Omega \times MVA_{BASE} \times 100)}{(kV)^2}$$

Transmission Line 1

$$Z_1 = Z_2 = \frac{38.3 \times 100 \times 100}{(132)^2} = 22\%$$

$$Z_0 = \frac{134 \times 100 \times 100}{(132)^2} = 76.9\% \quad \text{say } 77\%$$

Similarly for the other transmission lines:

Transmission Line 2	Z_1 = Z_2	= 6%
	Z_0	= 21%
Transmission Line 3	Z_1 = Z_2	= 16%
	Z_0	= 56%.

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Negative Sequence Impedance Diagram

Draw the Negative Sequence Diagram for the System as shown in FIG 13.

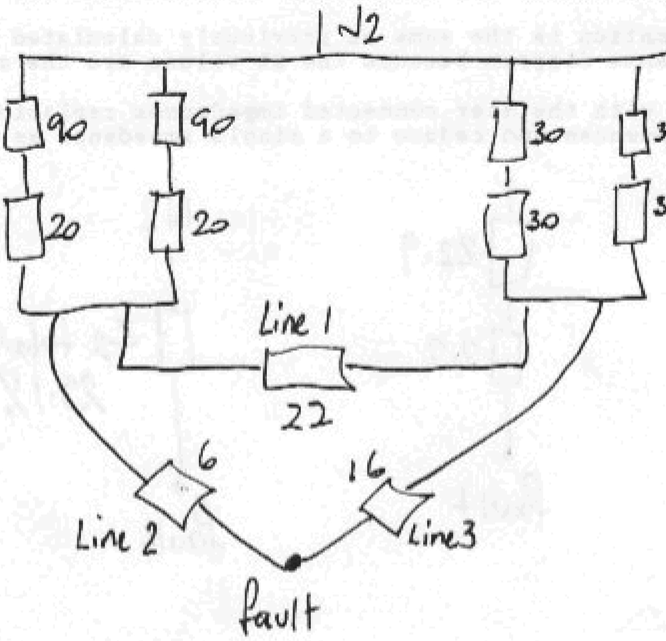

FIG 13

Note that the diagram does not include a voltage source.

The network shown in FIG 13 can be simplified by combining series and parallel impedances as shown in FIG 14.

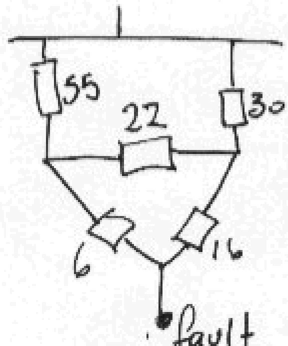

FIG 14

To further simplify the network, a star-delta transformation must be carried out on the delta connected impedances of 22%, 6% and 16% in FIG 14.

This transformation is the same as previously calculated for the positive sequence diagram because the Z% values are the same.

Redraw FIG 14 with the star connected impedances replacing the delta connected impedances and reduce to a single impedance as shown in FIG 15.

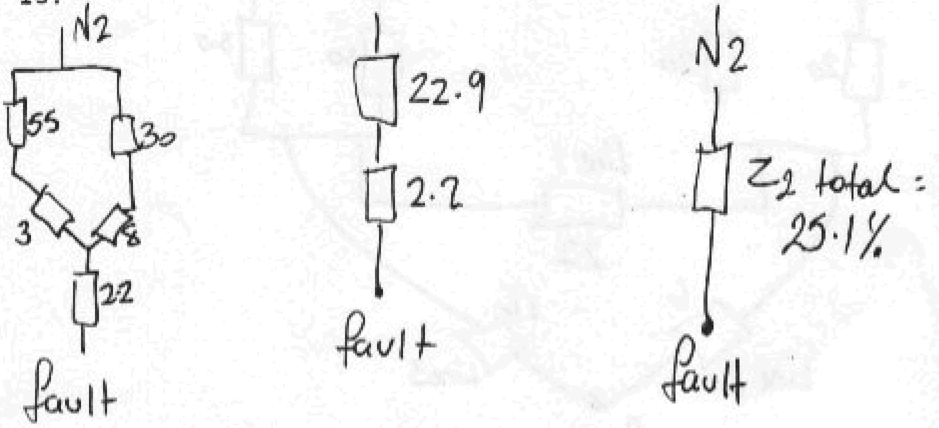

FIG 15

The total Negative sequence impedance Z_2 is 25.1%.

Positive Sequence Impedance Diagram

Draw the Positive Sequence Diagram for the System as shown in FIG 9.

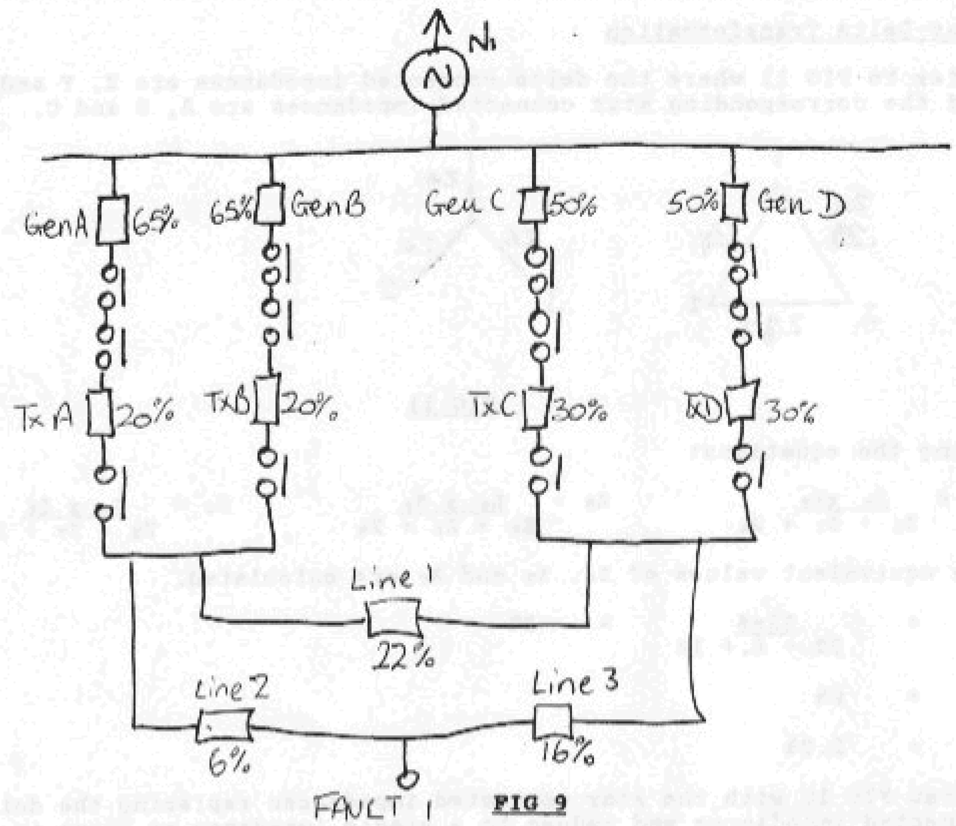

FIG 9

Note that the diagram includes a positive sequence voltage source which is the system voltage supply.

The network shown in FIG 9 can be simplified by combining series and parallel impedances as shown in FIG 10.

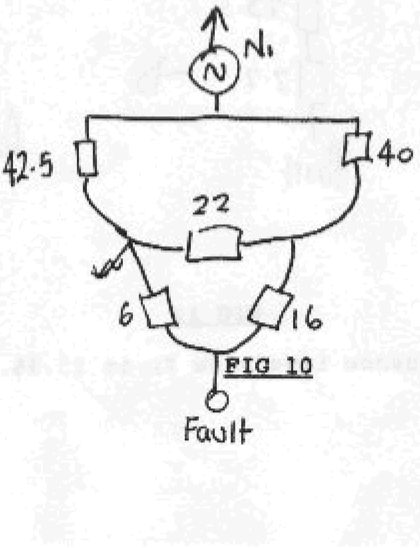

FIG 10

 F11P

To further simplify the network, a star-delta transformation must be carried out on the delta connected impedances of 22%, 6% and 16% in FIG 10.

Star-Delta Transformation

Refer to FIG 11 where the delta connected impedances are X, Y and Z and the corresponding star connected impedances are A, B and C.

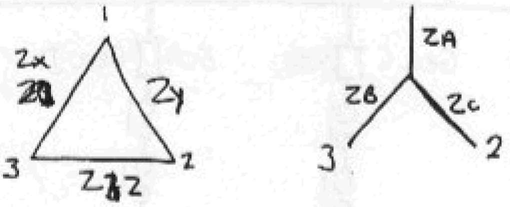

FIG 11

Using the equations:

$$Z_A = \frac{Z_X \times Z_Y}{Z_X + Z_Y + Z_Z} \qquad Z_B = \frac{Z_Y \times Z_Z}{Z_X + Z_Y + Z_Z} \qquad Z_C = \frac{Z_X \times Z_Z}{Z_X + Z_Y + Z_Z}$$

The equivalent values of Z_A, Z_B and Z_C are calculated.

$$Z_A = \frac{22 \times 6}{22 + 6 + 16} = 3\%$$

Z_B = 8%

Z_C = 2.2%

Redraw FIG 10 with the star connected impedances replacing the delta connected impedances and reduce to a single impedance as shown in FIG 12.

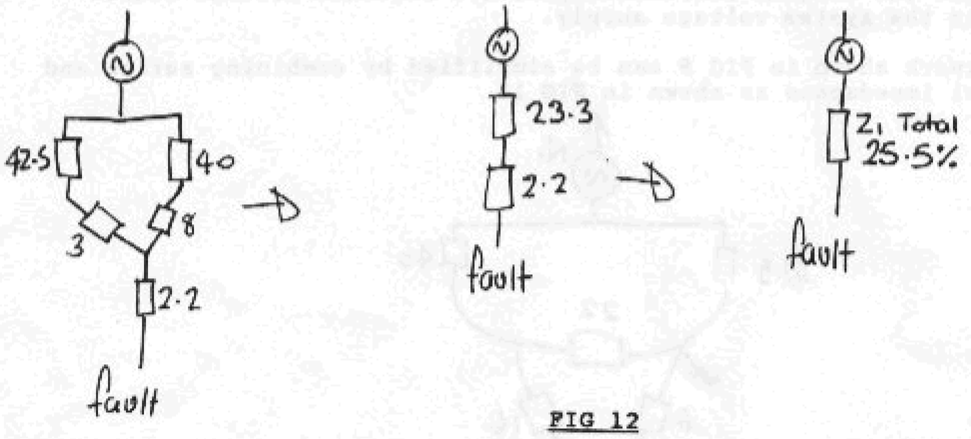

FIG 12

The total Positive sequence impedance Z_1 is 25.5%.

E111

<u>Zero Sequence Impedance Diagram</u>

Draw the Zero Sequence Diagram for the System as shown in FIG 16.

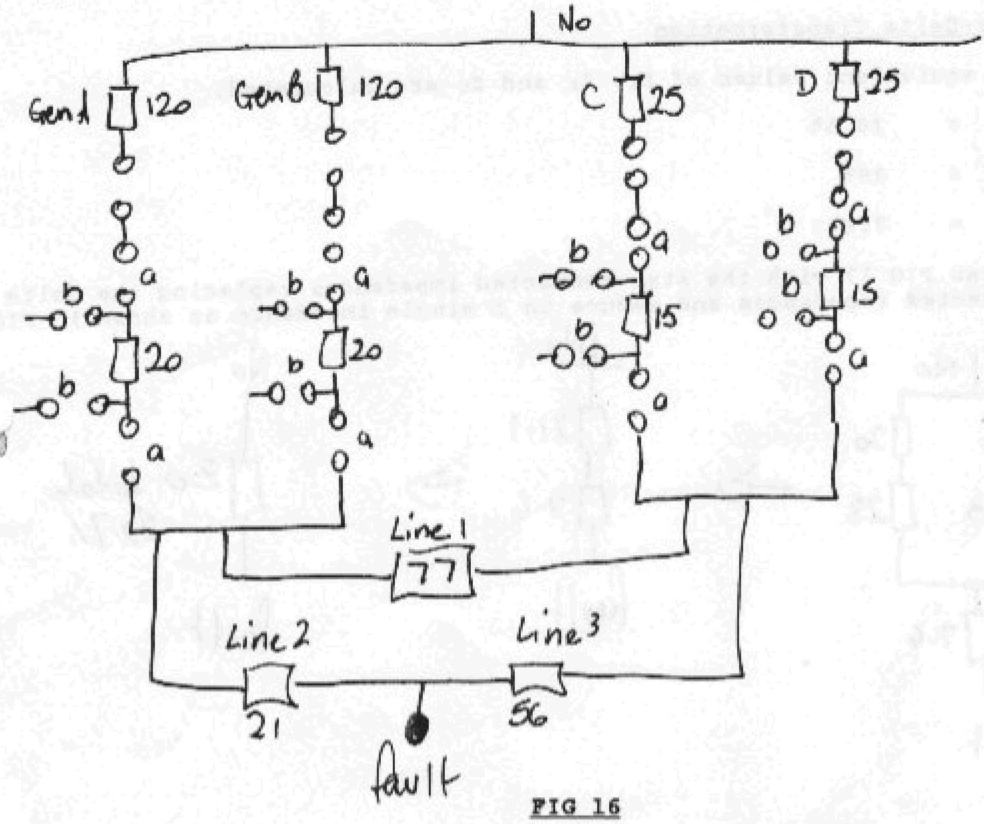

FIG 16

Note that the diagram does not include voltage source.

The network shown in FIG 16 can be simplified by combining series and parallel impedances as shown in FIG 17.

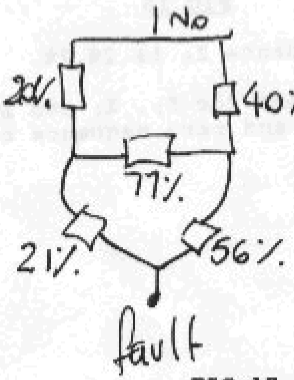

FIG 17

To further simplify the network, a star-delta transformation must be carried out on the delta connected impedances of 77%, 21% and 56% in FIG 17.

Star-Delta Transformation

The equivalent values of Z_A, Z_B and Z_C are calculated.

Z_A = 10.5%

Z_B = 28%

Z_C = 7.6%

Redraw FIG 17 with the star connected impedances replacing the delta connected impedances and reduce to a single impedance as shown in FIG 18.

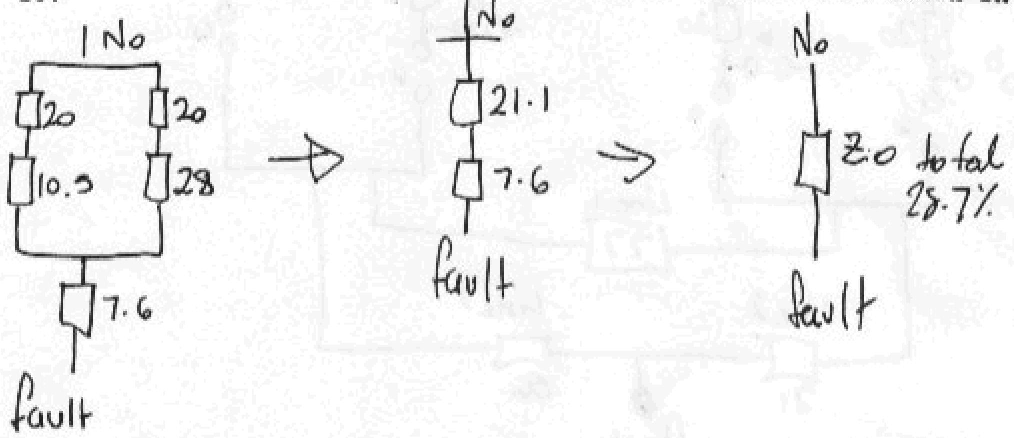

FIG 18

The total Zero sequence impedance Z_0 is 28.7%.

The equivalent impedance values for Z_1, Z_2 and Z_0 can now be used to calculate positive, negative and zero sequence components of the fault current.

PHASE SEQUENCE DETECTORS

Detection of Zero Sequence Currents/Voltages

Zero sequence currents flow in the neutral wire when unbalanced loads are connected to a four wire system.
Zero sequence currents also flow in a three wire system under earth fault conditions. The conditions are:

a) a system connection to earth at two or more points,
b) a potential difference between the points resulting in a current flow.

Residual Current Detection

Refer to FIG 1 which shows three current transformers connected in star, supplying an ammeter or relay.

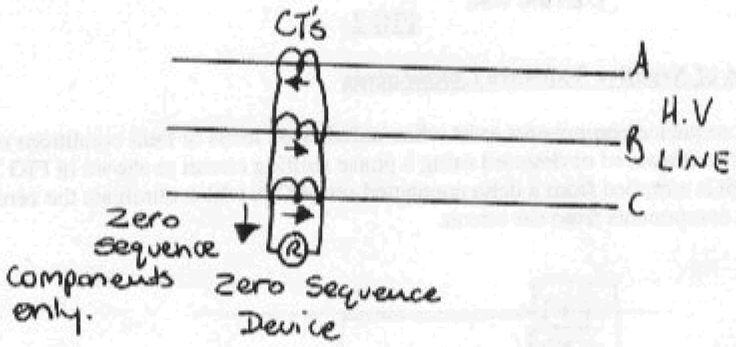

FIG 1

The ammeter or relay measures the vector sum of the three line currents, which is proportional to the zero sequence current since:

$$I_0 = 1/3(I_A + I_B + I_C)$$

Residual Voltage Detection

Refer to FIG 2 which shows a three phase voltage transformer with the secondary winding connected in open delta and supplying a voltmeter or relay.
The voltmeter or relay measures the vector sum of the secondary voltages which is proportional to the zero sequence component.

$$E_0 = E_A + E_B + E_C$$

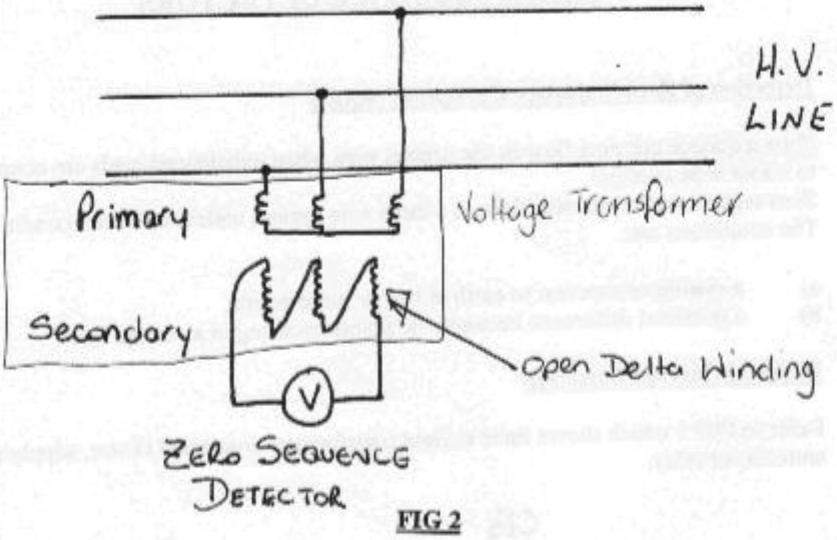

FIG 2

Detection of Negative Sequence Components

Negative sequence components exist when unbalanced loads or fault conditions occur. They can be measured or detected using a phase shifting circuit as shown in FIG 3. The circuit is supplied from a delta connected set of CTs which eliminate the zero sequence components from the circuit.

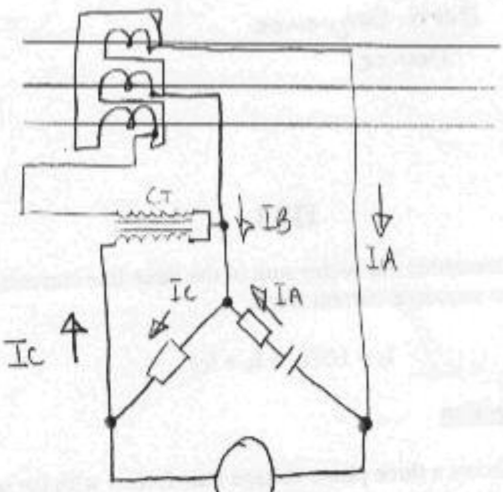

FIG 3

FAULT CALCULATIONS ON POWER SYSTEMS

F11s

There are two classes of faults on power systems:

a)	Balanced	three phase short circuit
b)	Unbalanced	one phase to earth phase to phase phase to phase to earth.

Each of these fault conditions can be described by voltage and current equations.

Single Phase to Earth Fault

Consider a single phase to earth fault on "A" phase as shown in FIG 1.

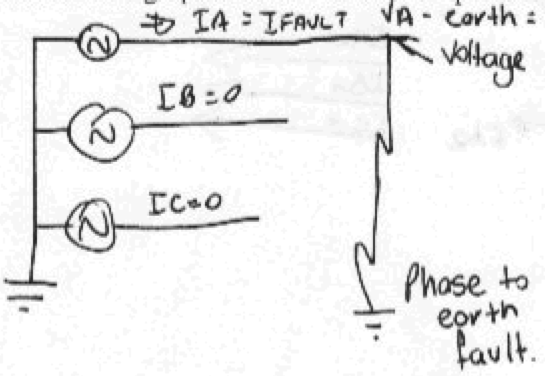

FIG 1

Assumptions: a) All load currents are zero.

b) Fault is through zero impedance.

Current Equations for the circuit are:

$I_A = I_{FAULT}$
$I_B = 0$
$I_C = 0$

Voltage Equations for the circuit are:

$V_{A\text{-}EARTH} = 0$ (short circuit to earth)
V_B = Normal voltage
V_C = Normal voltage.

The phasor diagram for the system line currents is shown in FIG 2.

FIG 2

A single phase to earth fault current will contain positive, negative and zero sequence components.

Using symmetrical component analysis, the unbalanced current phasor shown in FIG 2 can be replaced by the positive, negative and zero sequence components shown in FIG 3.

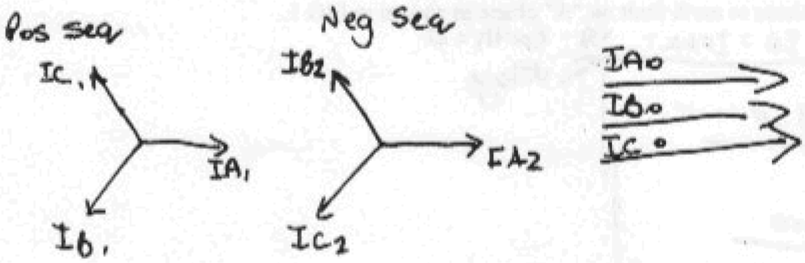

FIG 3

Note: $I_0 = I_1 = I_2 = I_A/3$ (all components are in phase).

$$I_{FAULT} = I_1 + I_2 + I_0 = 3_{I1}$$

F14

Connection of Phase Sequence Impedance Diagrams

The single phase to earth fault contains positive, negative and zero sequence components of current.

This means that to determine the total fault impedance between the source and the fault, the positive, negative and zero sequence impedance diagrams for the system must all be connected together as shown in FIG 4.

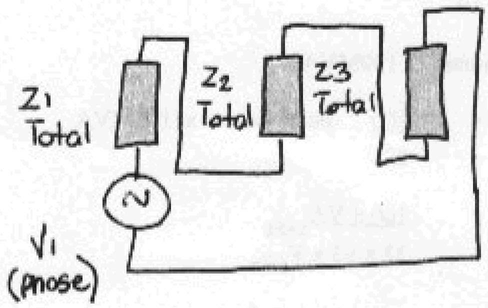

FIG 4

Note: As all sequence components are equal and in phase, the sequence impedances Z_1, Z_2 and Z_0 are connected in series.

The voltage equation for this circuit is:

$$V = I_1Z_1 + I_2Z_2 + I_0Z_0$$

$$= I_1(Z_1 + Z_2 + Z_0) = I_1 \times Z_{TOTAL}$$

where V is the phase to neutral value.

Calculation of Fault Current using Sequence Impedance Diagram

$$I_1 = I_2 = I_0 = \frac{100 \times \text{Current at Base MVA}}{Z\%_{TOTAL}}$$

$$I_{FAULT} = I_1 + I_2 + I_0 = 3_{I1}$$

$$3QVA = \sqrt{3}\ EL \times IL$$

$$I = \frac{3QVA}{\sqrt{3}EL}$$

Example:

A 132kV power system has a positive sequence impedance Z_1 of 6%, a negative sequence impedance Z_2 of 7% and a zero sequence impedance Z_0 of 10%, all calculated on a base of 100MVA.
Calculate the current flowing into a single phase to earth fault at 132kV on "A" phase.

$$Z\%_{TOTAL} = Z_1 + Z_2 + Z_0$$

$$= 6 + 7 + 10$$

$$= 23\% \text{ on base of 100MVA.}$$

$$I_{A1} = I_{A2} = I_{A0} = \frac{100}{23} \text{ pu of current at 100MVA}$$

$$= \frac{100 \times VA_{BASE}}{23 \times \sqrt{3} \times V_{LINE}}$$

$$= \frac{100 \times 100 \times 10^6}{23 \times \sqrt{3} \times 132 \times 10^3}$$

$$= 1902 \text{ amps}$$

$$\text{Fault Current } I_A = I_{A1} + I_{A2} + I_{A0}$$

$$= 3 \times I_{A1}$$

$$= 5706 \text{ amps}$$

Phase to Phase Fault

Consider a phase to phase fault on "A to B" phases as shown in FIG 5.

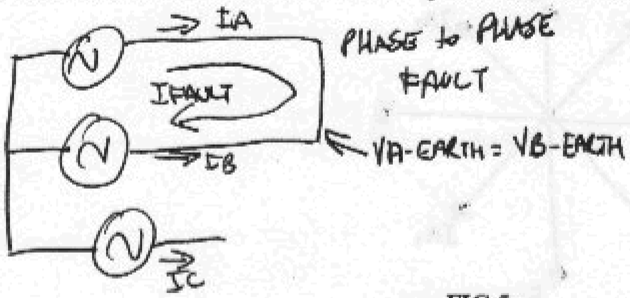

FIG 5

Assumptions: a) All load currents are zero.

b) Fault is through zero impedance.

Current Equations for the circuit are:

$I_C = 0$
$I_A = -I_B$

Voltage Equation for the circuit is:

$V_A = V_B$ (short circuit between A and B)

The phasor diagram for the system line currents is shown in FIG 6.

FIG 6

A phase to phase fault current will contain only positive and negative sequence components.

Using symmetrical component analysis, the unbalanced current phasor shown in FIG 6 can be replaced by the positive and negative sequence components shown in FIG 7.

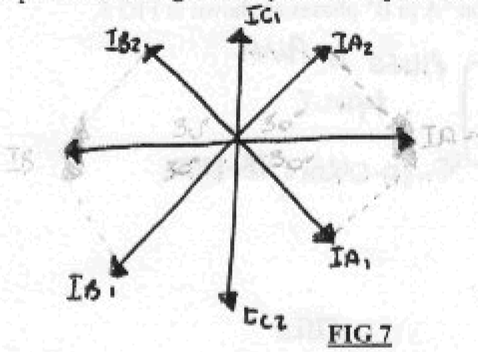

FIG 7

Notes: $I_C = I_{C1} + I_{C2} = 0$

$I_{C1} = - I_{C2}$

$$I_{FAULT} = \sqrt{3} \times I_1 \angle +30°$$

Connection of Phase Sequence Impedance Diagrams

The phase to phase fault contains only positive and negative sequence components of current.

This means that to determine the total fault impedance between the source and the fault, the positive and negative sequence impedance diagrams for the system must be connected together as shown in FIG 8.

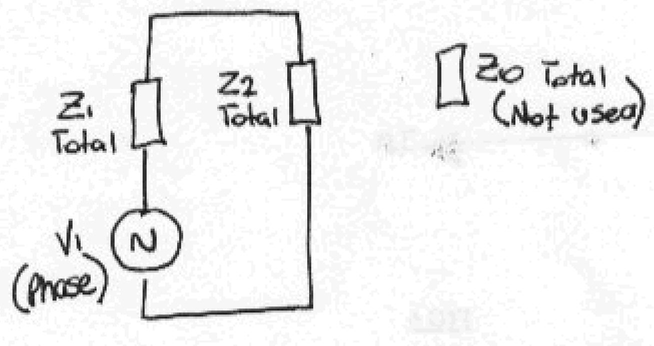

FIG 8

The voltage equation for this circuit is:

F(2) F(2)

$$V = I_1(Z_1 + Z_2) \quad \text{or}$$

$$= -I_2(Z_1 + Z_2)$$

where V is the phase to neutral value.

Example:

A power system has a positive sequence impedance Z_1 of 10% and a negative sequence impedance Z_2 of 12% calculated on a base of 100MVA. Calculate the current flowing into a phase to phase fault (A to B) at 66kV.

$$Z\%_{TOTAL} = Z_1 + Z_2$$

$$= 10 + 12$$

$$= 22\% \text{ on base of 100MVA.}$$

$$I_1 = -I_2 = \frac{100 \text{ pu of Current at 100MVA}}{Z\%_{TOTAL}}$$

$$= \frac{100 \times VA_{BASE}}{22 \times \sqrt{3} \times V_{LINE}}$$

$$= \frac{100 \times 100 \times 10^6}{22 \times \sqrt{3} \times 66 \times 10^3}$$

$$= 3976 \text{ amps}$$

$$\text{Fault Current } \sqrt{3}I_1 = \sqrt{3} \times 3976$$

$$= 6886 \text{ amps}$$

$$I_A = I_B = 6886 \text{ amps}$$

F122

Phase to Phase to Earth Fault

Consider a phase to phase to earth fault on (A to B to E) as shown in FIG 9.

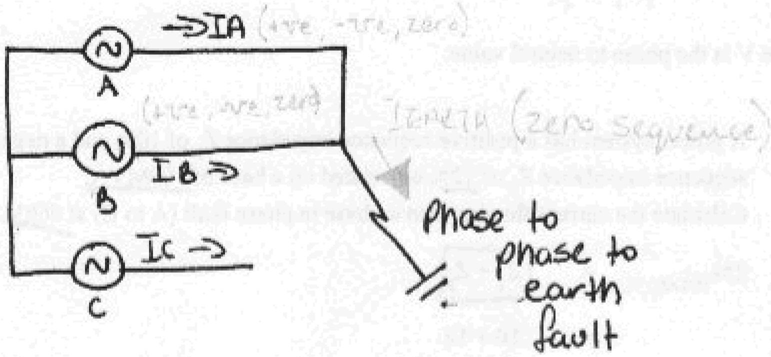

FIG 9

Assumptions: a) All load currents are zero.

b) Fault is through zero impedance.

Current Equation for the circuit is:

$$I_C = 0$$

Voltage Equations for the circuit are:

$V_A = 0$
$V_B = 0$ (short circuit between A and B)

All symmetrical components exist since earth fault conditions are present.

$$I_{A0} = I_{B0} = I_{C0} = 1/3(I_A + I_B + I_C)$$

$$I_C = I_{C1} + I_{C2} + I_{C0} = 0$$

$$I_{C1} = -(I_{C2} + I_{C0})$$

From voltage equations:

$$V_{C1} = 1/3(V_C + aV_A + a^2V_B)$$

$$= 1/3(V_C + 0 + 0)$$

$$= 1/3V_C$$

Similarly $V_{C2} = 1/3V_C$