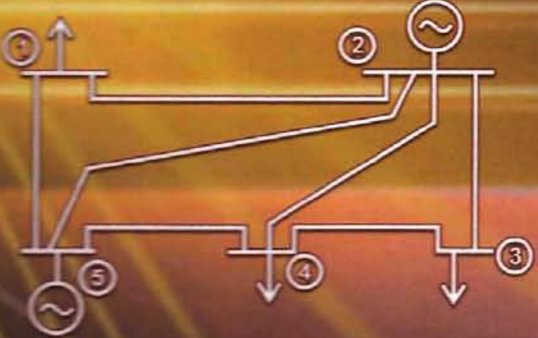
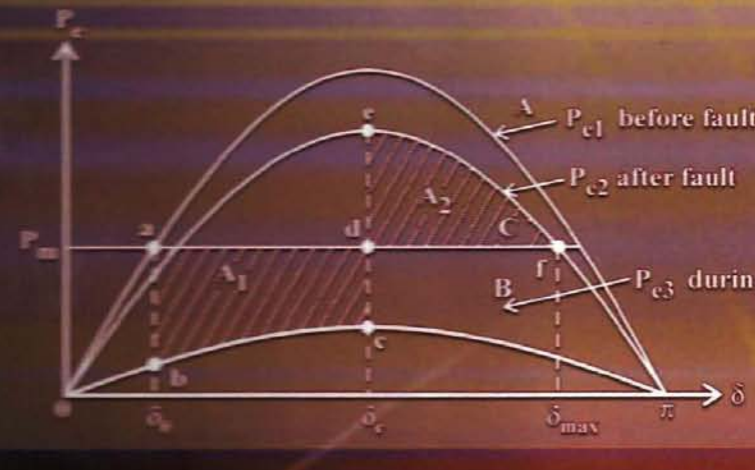


POWER SYSTEM ANALYSIS



P.S.R Murty

BS Publications

Power System Analysis

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Power System Analysis

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Preface

Power System analysis is a pre-requisite course for electrical power engineering students.

In Chapter 1, introductory concepts about a Power system, network models, faults and analysis, the primitive network and stability are presented.

Chapter 2 deals with the graph theory that is relevant to various incidence matrices required for network modelling are explained.

Chapter 3 explains the various incidence matrices and network matrices.

Chapter 4 discusses, step-by-step method of building of network matrices.

Chapter 5 deals with power flow studies. Both Gauss-Seidel method and Newton-Raphson methods are explained. In Newton-Raphson method both the Cartesion coordinates method and polar coordinates methods are discussed.

In chapter 6 short circuit analysis is explained Per unit quantity and percentage values are defined. Analysis for symmetrical faults is discussed. The utility of reactors for bus bar and generator protection is also explained.

Unbalanced fault analysis is presented in chapter 7. Use of symmetrical components and network connections are explained.

Chapter 8 deals with the power system stability problem. Steady state stability, transient stability and dynamic stability are discussed.

It is earnestly hoped that this book will meet the requirements of students in the subject power system analysis.

- Author

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Acknowledgment

My sincere thanks go to Mr. Nikhil Shah, Proprietor of BS Publications for his constant encouragement to me to write and complete this book on 'Power System Analysis'.

My thanks also go to Mr. M.V.L. Narasimha Rao, my well wisher for taking great pains in transferring script material and proof material between my residence and the press with a great smiley face, day and night over the last few months.

I thank Mrs. Swarupa for MATLAB assistance.

-Author

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3 Incidence Matrices

4 Building of Network Matrices

5 Power Flow Studies

7 Unbalanced Fault Analysis

8 Power System Stability

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1 INTRODUCTION

Power is an essential pre-requisite for the progress of any country. The modern power system has features unique to it self. It is the largest man made system in existence and is the most complex system. The power demand is more than doubling every decade.

Planning, operation and control of interconnected power system poses a variety of challenging problems, the solution of which requires extensive application of mathematical methods from various branches.

Thomas Alva Edison was the first to conceive an electric power station and operate it in Newyork in 1882. Since then, power generation originally confined to steam engines expanded using (steam turbines) hydro electric turbines, nuclear reactors and others.

The inter connection of the various generating stations to load centers through EHV and UHV transmission lines necessitated analytical methods for analysing various situations that arise in operation and control of the system.

Power system analysis is the subject in the branch of electrical power engineering which deals with the determination of voltages at various buses and the currents that flow in the transmission lines operating at different voltage levels.

1.1 The Electrical Power System

The electrical power system is a complex network consisting of generators, loads, transmission lines, transformers, buses, circuit breakers etc. For the analysis of a power system in operation a suitable model is needed. This model basically depends upon the type of problem on hand. Accordingly it may be algebraic equations, differential equations, transfer functions etc. The power system is never in steady state as the loads keep changing continuously.

However, it is possible to conceive a quasistatic state during which period the loads could be considered constant. This period could be 15 to 30 minutes. In this state power flow equations are non-linear due to the presence of product terms of variables and trigonometric terms. The solution techniques involves numerical (iterative) methods for solving non-linear algebraic equations. Newton-Raphson method is the most commonly used mathematical technique. The analysis of the system for small load variations, wherein speed or frequency and voltage control may be required to maintain the standard values, transfer function and state variable models are better suited to implement proportional, derivative and integral controllers or optimal controllers using Kalman's feed back coefficients. For transient stability studies involving sudden changes in load or circuit condition due to faults, differential equations describing energy balance over a few half-cycles of time period are required. For studying the steady state performance a number of matrix models are needed.

Consider the power System shown in Fig. 1.1. The equivalent circuit for the power system can be represented as in Fig. 1.2. For study of fault currents the equivalent circuit in Fig. 1.2 can be reduced to Fig. 1.3 upto the load terminals neglecting the shunt capacitances of the transmission line and magnetizing reactances of the transformers.

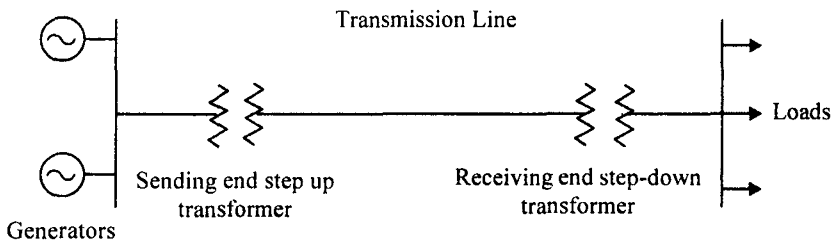


Fig. 1.1

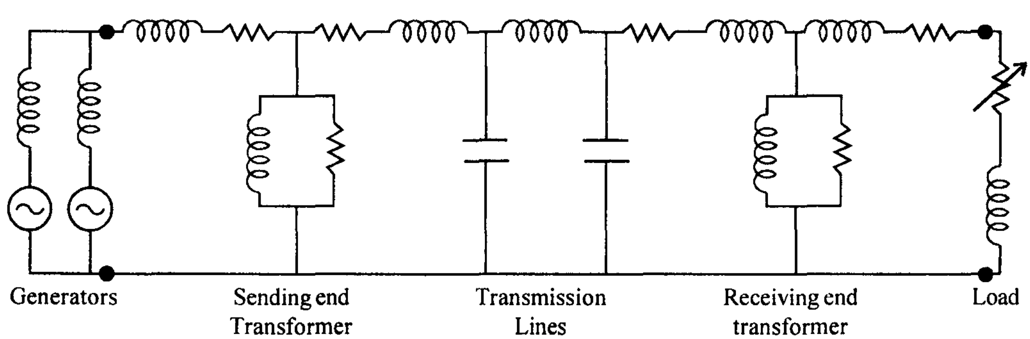


Fig. 1.2

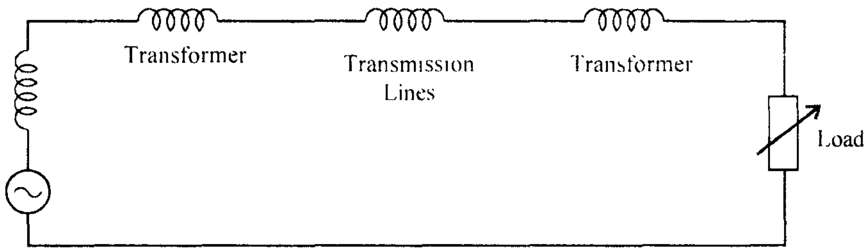


Fig. 1.3

While the reactances of transformers and lines which are static do not change under varying conditions of operation, the machine reactances may change and assume different values for different situations. Also, composite loads containing 3-phase motors, 1-phase motors, d-c motors, rectifiers, lighting loads, heaters, welding transformers etc., may have very different models depending upon the composition of its constituents.

The control of a turbo generator set to suit to the varying load requirement requires a model. For small variations, a linearized model is convenient to study. Such a model can be obtained using transfer function concept and control can be achieved through classical or modern control theory. This requires modeling of speed governor, turbo generator and power system itself as all these constitute the components of a feedback loop for control. The ultimate objective of power system control is to maintain continuous supply of power with acceptable quality. Quality is defined in terms of voltage and frequency.

1.2 Network Models

Electrical power network consists of large number of transmission lines interconnected in a fashion that is dictated by the development of load centers. This interconnected network configuration expands continuously. A systematic procedure is needed to build a model that can be constantly up-graded with increasing interconnections.

Network solutions can be carried out using Ohm's law and Kirchoff's laws.

Either $e = Z \cdot i$

or $i = Y \cdot e$

model can be used for steady state network solution. Thus, it is required to develop both Z-bus and Y-bus models for the network. To build such a model, graph theory and incidence matrices will be quite convenient.

1.3 Faults and Analysis

Study of the network performance under fault conditions requires analysis of a generally balanced network as an unbalanced network. Under balanced operation, all the three-phase voltages are equal in magnitude and displaced from each other mutually by 120° (elec.). It may be noted that unbalanced transmission line configuration is balanced in operation by transposition, balancing the electrical characteristics.

Under fault conditions, the three-phase voltages may not be equal in magnitude and the phase angles too may differ widely from 120° (elec.) even if the transmission and distribution networks are balanced. The situation changes into a case of unbalanced excitation.

Network solution under these conditions can be obtained by using transformed variables through different component systems involving the concept of power invariance.

In this course all these aspects will be dealt with in modeling so that at an advanced level, analyzing and developing of suitable control strategies could be easily understood using these models wherever necessary.

1.4 The Primitive Network

Network components are represented either by their impedance parameters or admittance parameters. Fig (1.4) represents the impedance form, the variables are currents and voltages. Every power system element can be described by a primitive network. A primitive network is a set of unconnected elements.

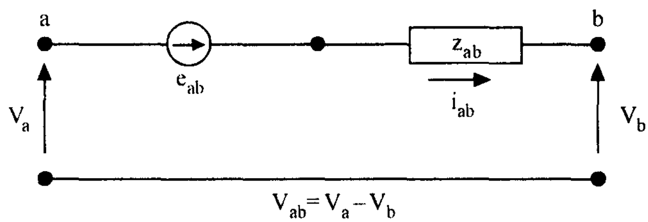


Fig. 1.4

a and b are the terminals of a network element a-b. V_a and V_b are voltages at a and b.

V_{ab} is the voltage across the network element a - b.

e_{ab} is the source voltage in series with the network element a - b

z_{ab} is the self impedance of network element a - b.

j_{ab} is the current through the network element a - b.

From the Fig.(1.4) we have the relation

$$V_{ab} + e_{ab} = z_{ab}\, i_{ab} \qquad .....(1.1)$$

In the admittance form the network element may be represented as in Fig. (1.5).

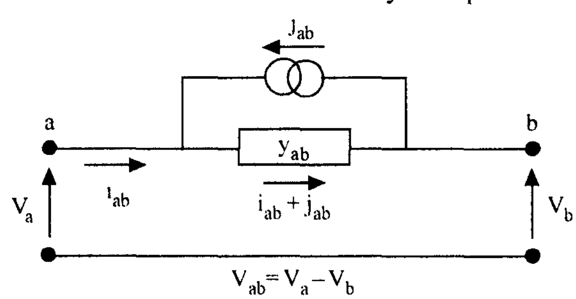


Fig. 1.5

y_{ab} is the self admittance of the network element a-b

j_{ab} is the source current in parallel with the network element a-b

From Fig.(1.5) we have the relation

$$i_{ab} + j_{ab} = y_{ab} v_{ab} \quad .....(1.2)$$

The series voltage in the impedance form and the parallel source current in the admittance form are related by the equation.

$$-j_{ab} = y_{ab} e_{ab} \quad .....(1.3)$$

A set of unconnected elements that are depicted in Fig.(1.4) or (1.5) constitute a primitive network. The performance equations for the primitive networks may be either in the form

$$\underline{e} + \underline{v} = [z]\, \underline{i} \quad .....(1.4)$$

or in the form

$$\underline{i} + \underline{j} = [y]\, \underline{v} \quad .....(1.5)$$

In eqs.(1.4) and (1.5) the matrices [z] or [y] contain the self impedances or self admittances denoted by $z_{ab,\, ab}$ or $y_{ab,ab}$. The off-diagonal elements may in a similar way contain the mutual impedances or mutual admittances denoted by $z_{ab,\, cd}$ or $y_{ab,\, cd}$ where ab and cd are two different elements having mutual coupling. If there is no mutual coupling, then the matrices [z] and [y] are diagonal matrices. While in general [y] matrix can be obtained by inverting the [z] matrix, when there is no mutual coupling, elements of [y] matrix are obtained by taking reciprocals of the elements of [z] matrix.

1.5 Power System Stability

Power system stability is a word used in connection with alternating current power systems denoting a condition where in, the various alternators in the system remain in synchronous with each other. Study of this aspect is very important, as otherwise, due to a variety of changes, such as, sudden load loss or increment, faults on lines, short circuits at different locations, circuit opening and reswitching etc., occuring in the system continuously some where or other may create blackouts.

Study of simple power systems with single machine or a group of machines represented by a single machine, connected to infinite bus gives an insight into the stability problem.

At a first level, study of these topics is very important for electrical power engineering students.

2 GRAPH THEORY

2.1 Introduction

Graph theory has many applications in several fields such as engineering, physical, social and biological sciences, linguistics etc. Any physical situation that involves discrete objects with interrelationships can be represented by a graph. In ***Electrical Engineering*** Graph Theory is used to predict the behaviour of the network in analysis. However, for smaller networks node or mesh analysis is more convenient than the use of graph theory. It may be mentioned that Kirchoff was the first to develop theory of trees for applications to electrical network. The advent of high speed digital computers has made it possible to use graph theory advantageously for larger network analysis. In this chapter a brief account of graphs theory is given that is relevant to power transmission networks and their analysis.

2.2 Definitions

Element of a Graph : Each network element is replaced by a line segment or an arc while constructing a graph for a network. Each line segment or arc is called an *element*. Each potential source is replaced by a short circuit. Each current source is replaced by an open circuit.

Node or Vertex : The terminal of an element is called a *node* or a *vertex*.

Edge : An element of a graph is called an *edge*.

Degree : The number of edges connected to a vertex or node is called its *degree*.

Graph : An element is said to be incident on a node, if the node is a terminal of the element. Nodes can be incident to one or more elements. The network can thus be represented by an interconnection of elements. The actual interconnections of the elements gives a graph.

Rank : The *rank of a graph* is $n - 1$ where n is the number of nodes in the graph.

Sub Graph : Any subset of elements of the graph is called a *subgraph* A subgraph is said to be proper if it consists of strictly less than all the elements and nodes of the graph.

Path : A path is defined as a subgraph of connected elements such that not more than two elements are connected to any one node. If there is a path between every pair of nodes then the graph is said to be connected. Alternatively, a graph is said to be connected if there exists at least one path between every pair of nodes.

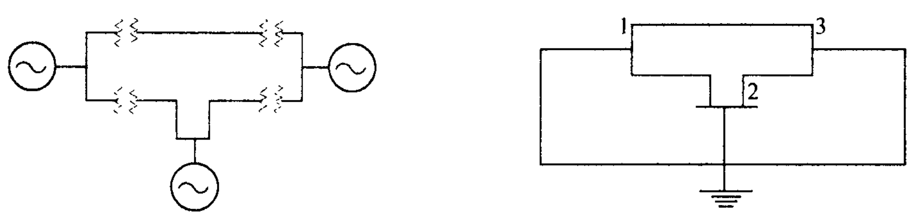
Planar Graph : A graph is said to be planar, if it can be drawn without-out cross over of edges. Otherwise it is called non-planar (Fig. 2.1).

(a) (b)

Fig. 2.1 (a) Planar Graph (b) Non-Planar Graph.

Closed Path or Loop : The set of elements traversed starting from one node and returning to the same node form a closed path or loop.

Oriented Graph : An oriented graph is a graph with direction marked for each element Fig. 2.2(a) shows the single line diagram of a simple power network consisting of generating stations, transmission lines and loads. Fig. 2.2(b) shows the positive sequence network of the system in Fig. 2.2(a). The oriented connected graph is shown in Fig. 2.3 for the same system.



(a) (b)

Fig. 2.2 (a) Power system single-line diagram (b) Positive sequence network diagram

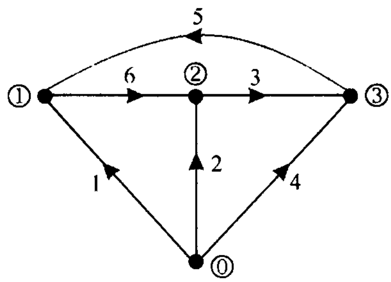


Fig. 2.3 Oriented connected graph.

2.3 Tree and Co-Tree

Tree : A tree is an oriented connected subgraph of an oriented connected graph containing all the nodes of the graph, but, containing no loops. A tree has (n-1) branches where n is the number of nodes of graph G. The branches of a tree are called twigs. The remaining branches of the graph are called links or chords.

Co-tree : The links form a subgraph, not necessarily connected called co-tree. Co-tree is the complement of tree. There is a co-tree for every tree.

For a connected graph and subgraph:

1. There exists only one path between any pair of nodes on a tree
2. every connected graph has at least one tree
3. every tree has two terminal nodes and
4. the rank of a tree is n-1 and is equal to the rank of the graph.

The number of nodes and the number of branches in a tree are related by

$$b = n - 1 \qquad .....(2.1)$$

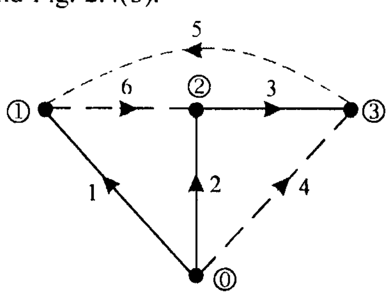
If e is the total number of elements then the number of links l of a connected graph with branches b is given by

$$l = e - b \qquad .....(2.2)$$

Hence, from eq. (2.1), it can be written that

$$l = e - n + 1 \qquad .....(2.3)$$

A tree and the corresponding co - tree of the graph for the system shown in Fig. 2.3 are indicated in Fig. 2.4(a) and Fig. 2.4(b).



n = number of nodes = 4

e = number of elements = 6

b = n – 1 = 4 – 1 = 3

l = e – n + 1 = 6 – 4 + 1 = 3

Fig. 2.4 (a) Tree for the system in Fig. 2.3.

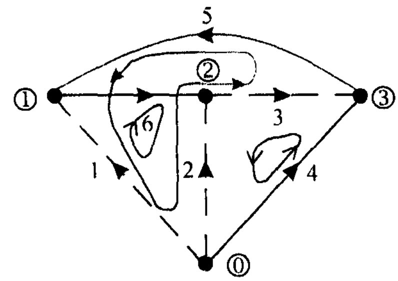


Fig. 2.4 (b) Co-tree for the system in Fig. 2.3.

2.4 Basic Loops

A loop is obtained whenever a link is added to a tree, which is a closed path. As an example to the tree in Fig. 2.4(a) if the link 6 is added, a loop containing the elements 1-2-6 is obtained. Loops which contain only one link are called *independent loops* or *basic loops.*

It can be observed that the number of basic loops is equal to the number of links given by equation (2.2) or (2.3). Fig. 2.5 shows the basic loops for the tree in Fig. 2.4(a).

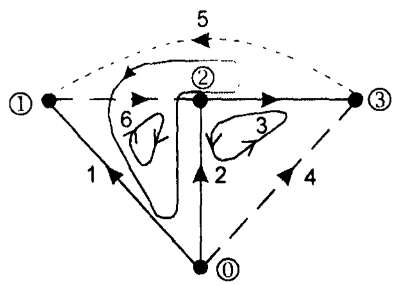


Fig. 2.5 Basic loops for the tree in Fig. 2.4(a).

2.5 Cut-Set

A Cut set is a minimal set of branches K of a connected graph G, such that the removal of all K branches divides the graph into two parts. It is also true that the removal of K branches reduces the rank of G by one, provided no proper subset of this set reduces the rank of G by one when it is removed from G.

Consider the graph in Fig. 2.6(a).

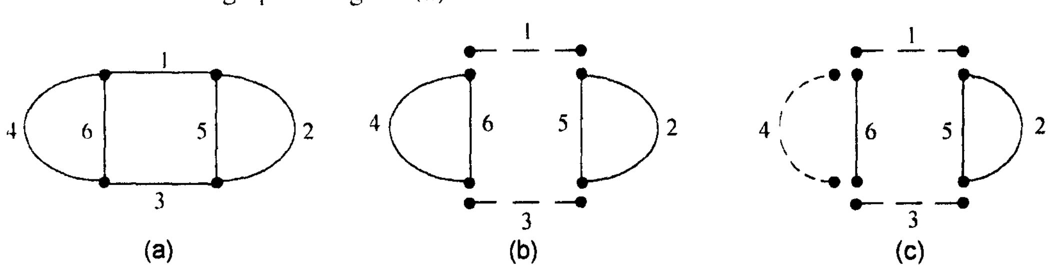


Fig. 2.6

The rank of the graph = (no. of nodes n – 1) = 4 – 1 = 3. If branches 1 and 3 are removed two sub graphs are obtained as in Fig. 2.6(b). Thus 1 and 3 may be a cut-set. Also, if branches 1, 4 and 3 are removed the graph is divided into two sub graphs as shown in Fig. 2.6(c) Branches 1, 4, 3 may also be a cut-set. In both the above cases the rank both of the sub graphs is 1 + 1 = 2. It can be noted that (1, 3) set is a sub-set of (1, 4, 3) set. The cut set is a minimal set of branches of the graph, removal of which cuts the graph into two parts. It separates nodes of the graphs into two graphs. Each group is in one of the two sub graphs.

2.6 Basic Cut-Sets

If each cut-set contains only one branch, then these independent cut-sets are called basic cut-sets. In order to understand basic cut-sets select a tree. Consider a twig b_k of the tree. If the twig is removed the tree is separated into two parts. All the links which go from one part of this disconnected tree to the other, together with the twig b_k constitutes a cut-set called basic cut-set. The orientation of the basic cut-set is chosen as to coincide with that of the branch of the tree defining the cut-set. Each basic cut-set contains at least one branch with respect to which the tree is defined which is not contained in the other basic cut-set. For this reason, the n -1 basic cut-sets of a tree are linearly independent.

Now consider the tree in Fig. 2.4(a).

Consider node (1) and branch or twig 1. Cut-set A contains the branch 1 and links 5 and 6 and is oriented in the same way as branch 1. In a similar way C cut-set cuts the branch 3 and links 4 and 5 and is oriented in the same direction as branch 3. Finally cut-set B cutting branch 2 and also links 4, 6 and 5 is oriented as branch 2 and the cutsets are shown in Fig. 2.7.

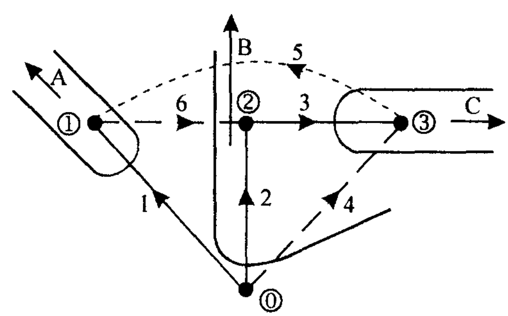


Fig. 2.7 Cut-set for the tree in Fig. 2.4(a).

Worked Examples

2.1 For the network shown in figure below, draw the graph and mark a tree. How many trees will this graph have? Mark the basic cutsets and basic loops.

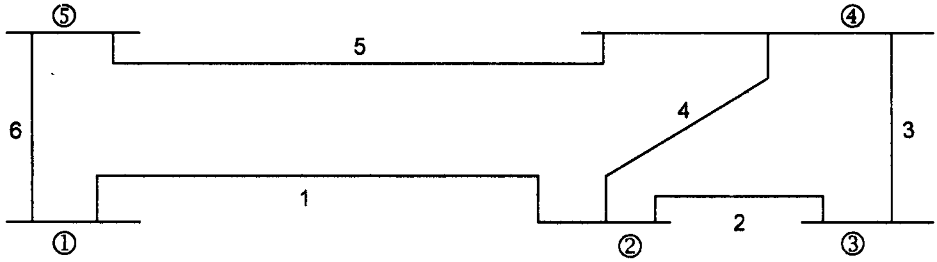


Fig. E.2.1

Solution :

Assume that bus (1) is the reference bus

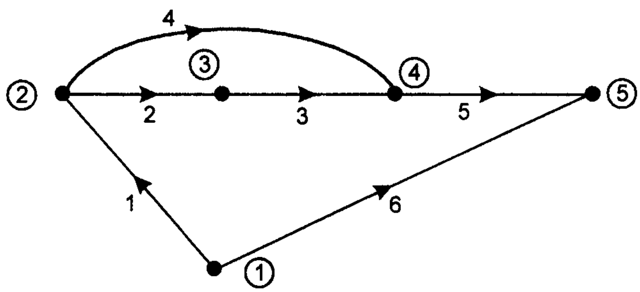


Fig. E.2.2

Number of nodes n = 5

Number of elements e = 6

The graph can be redrawn as,

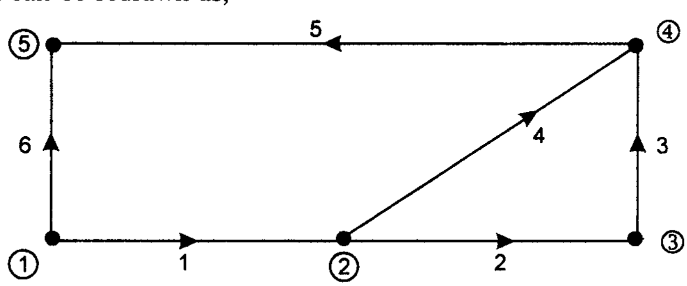


Fig. E.2.3

Tree : A connected subgraph containing all nodes of a graph, but no closed path is called a *tree*.

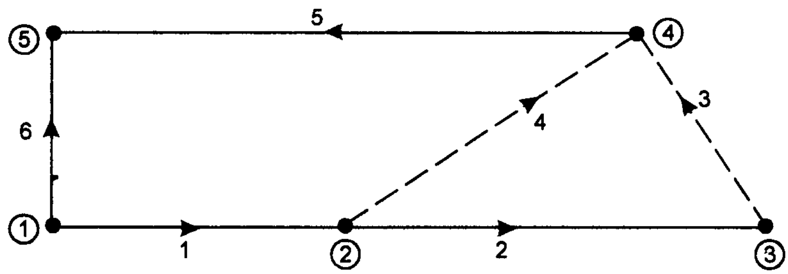


Fig. E.2.4

Number of branches n-1 = 5–1 = 4

Number of links = e-b = 6-4 = 2

(*Note :* Number of links = Number of co-trees).

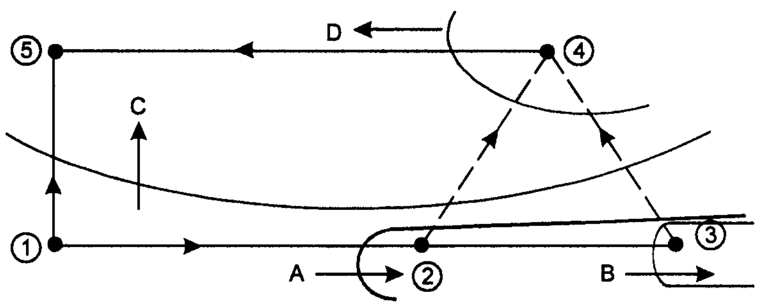


Fig. E.2.5

The number of basic cutsets = no. of branches = 4; the cutsets A,B,C,D, are shown in figure.

2.2 Show the basic loops and basic cutsets for the graph shown below and verify any relations that exist between them.

(Take 1-2-3-4 as tree 1).

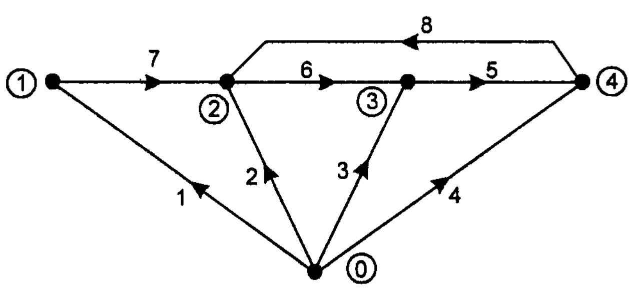


Fig. E.2.6

Solution :

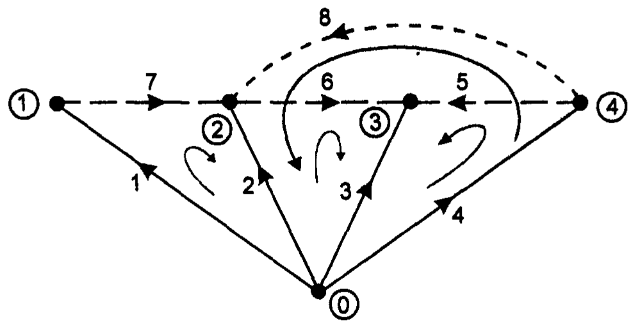


Fig. E.2.7 Tree and Co-tree for the graph.

If a link is added to the tree a loop is formed, loops that contain only one link are called *basic loops*.

Branches, $b = n - 1 = 5 - 1 = 4$

$l = e - b = 8 - 4 = 4$

The four loops are shown in Fig.

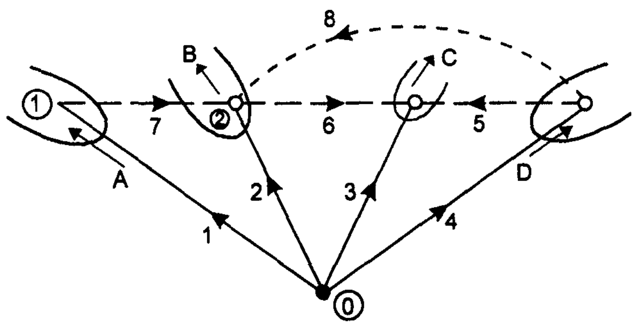


Fig. E.2.8 Basic cut sets A, B, C, D.

The number of basic cuts (4) = number of branches b(4).

2.3 For the graph given in figure below, draw the tree and the corresponding co-tree. Choose a tree of your choice and hence write the cut-set schedule.

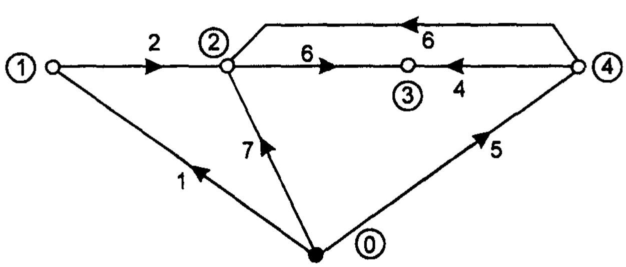


Fig. E.2.9 Oriented connected graph.

Solution :

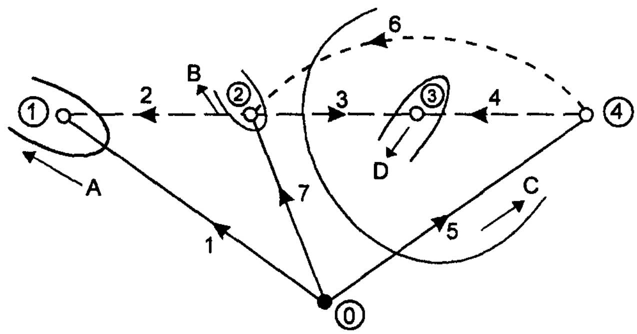


Fig. E.2.10 Basic cut sets A, B, C, D.

The f-cut set schedule (fundamental or basic)

A: 1,2
B : 2,7,3, 6
C : 6,3,5
D: 3,4

2.4 For the power systems shown in figure draw the graph, a tree and its co-tree.

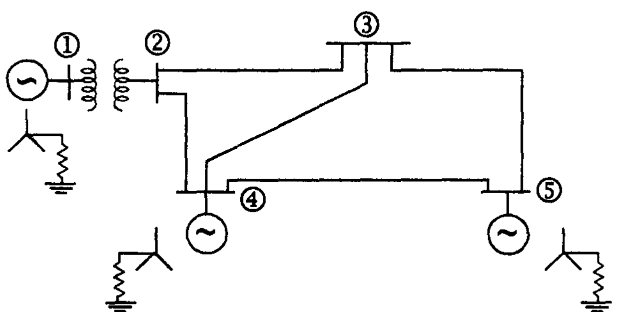


Fig. E.2.11

Solution :

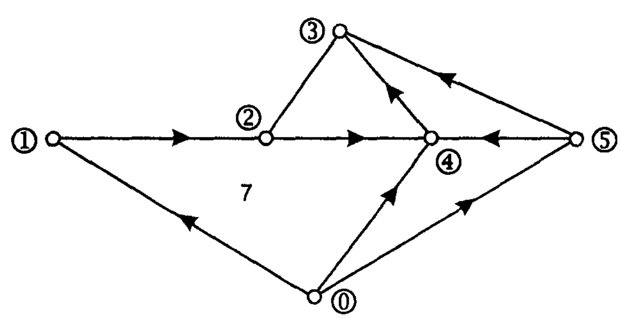


Fig. E.2.12

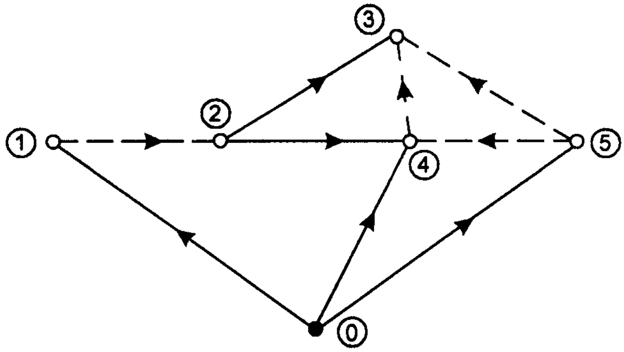


Fig. E.2.13 Tree and Co-tree 2.4.

Problems

P 2.1 Draw the graph for the network shown. Draw a tree and co-tree for the graph.

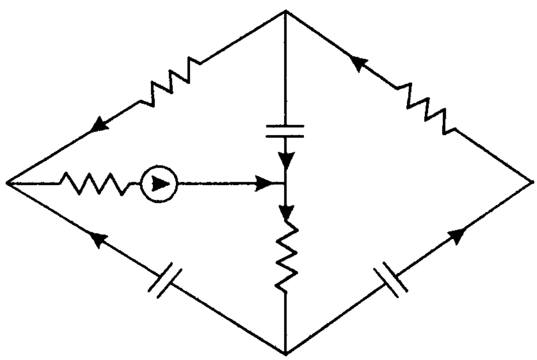


Fig. P.2.1

P 2.2 Draw the graph for the circuit shown.

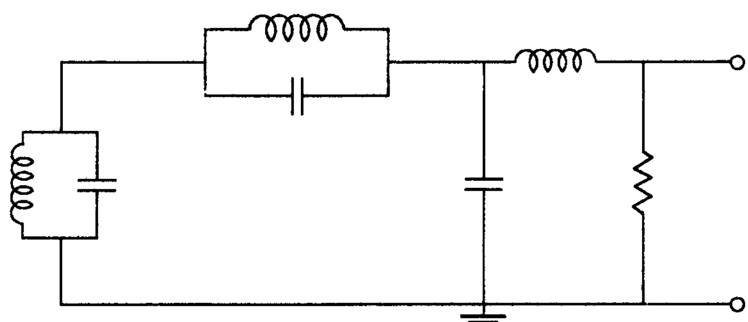


Fig. P.2.2

P 2.3 Draw the graph for the network shown.

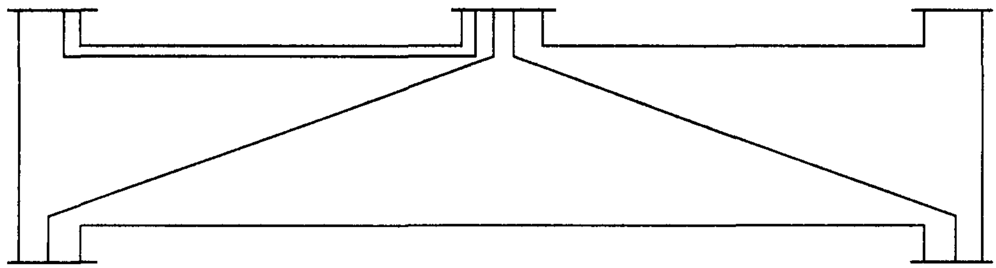


Fig. P.2.3

Mark basic cutsets, basic loops and open loops.

Questions

2.1 Explain the following terms :
   (i) Basic loops
   (ii) Cut set
   (iii) Basic cut sets

2.2 Explain the relationship between the basic loops and links; basic cut-sets and the number of branches.

2.3 Define the following terms with suitable example :
   (i) Tree
   (ii) Branches
   (iii) Links
   (iv) Co-Tree
   (v) Basic loop

2.4 Write down the relations between the number of nodes, number of branches, number of links and number of elements.

2.5 Define the following terms.
   (i) Graph
   (ii) Node
   (iii) Rank of a graph
   (iv) Path

3 INCIDENCE MATRICES

There are several incidence matrices that are important in developing the various networks matrices such as bus impedance matrix, branch admittance matrix etc., using singular or non singular transformation.

These various incidence matrices are basically derived from the connectivity or incidence of an element to a node, path, cutset or loop.

Incidence Matrices

The following incidence matrices are of interest in power network analysis.

(a) Element-node incidence matrix

(b) Bus incidence matrix

(c) Branch path incidence matrix

(d) Basic cut-set incidence matrix

(e) Basic loop incidence matrix

Of these, the bus incidence matrices is the most important one :

3.1 Element Node Incidence Matrix

Element node incidence matrix $\bar{A}$ shows the incidence of elements to nodes in the connected graph. The incidence or connectivity is indicated by the operator as follows :

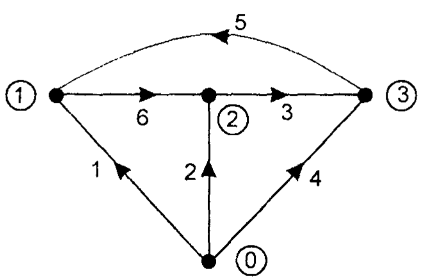
$\alpha_{pq} = 1$ if the p^{th} element is incident to and directed away from the q the node.

$\alpha_{pq} = -1$ if the p^{th} element is incident to and directed towards the q the node.

$\alpha_{pq} = 0$ if the p^{th} element is not incident to the q^{th} node.

The element-node incidence matrix will have the dimension exn where '*e*' is the number of elements and n is the number of nodes in the graph. It is denoted by $\bar{A}$.

The element node incidence matrix for the graph of Fig. 2.3 is shown in Fig. 3.1.



$\bar{A}$ =

e \ n	(0)	(1)	(2)	(3)
1	1	–1		
2	1		–1	
3			1	–1
4	1			–1
5		–1		1
6		1	–1	

Fig. 3.1 Element-node incidence-matrix for the graph of Fig. (2.3).

It is seen from the elements of the matrix that

$$\sum_{q=0}^{3} \alpha_{pq} = 0; \; p = 1, 2, \ldots\ldots 6 \qquad \ldots\ldots(3.1)$$

It can be inferred that the columns of $\bar{A}$ are linearly independent. The rank of $\bar{A}$ is less than n the number of nodes in the graph.

3.2 Bus Incidence Matrix

The network in Fig. 2.2(b) contains a reference reflected in Fig. 2.3 as a reference node. In fact any node of the connected graph can be selected as the reference node. The matrix obtained by deleting the column corresponding to the reference node in the element node incidence matrix $\bar{A}$ is called *bus incidence matrix A*. Thus, the dimension of this matrix is ex (n – 1) and the rank will therefore be, n – 1 = b, where b is the number of branches in the graph. Deleting the column corresponding to node (0) from Fig. 3.1 the bus-incidence matrix for the system in Fig. 2.2(a) is obtained. This is shown in Fig. 3.2.

A =

bus \ e	(1)	(2)	(3)
1	–1		
2		–1	
3		+1	–1
4			–1
5	–1		+1
6	1	–1	

Fig. 3.2 Bus Incidence Matrix for graph in (2.3).

If the rows are arranged in the order of a specific tree, the matrix A can be partitioned into two submatrices A_b of the dimension bx (n – 1) and A_l of dimension lx (n – 1). The rows of A_b correspond to branches and the rows of A_l correspond to links. This is shown in (Fig. 3.3) for the matrix in (Fig. 3.2).

A =

e \ bus	(1)	(2)	(3)
1	–1		
2		–1	
3		1	–1
4			–1
5	–1		1
6	1	–1	

A =

e \ bus	(1)	(2)	(3)	
1				
2		A_b		Branches
3				
4				
5		A_l		Links
6				

Fig. 3.3 Partitioning of matrix A.

3.3 Branch - Path Incidence Matrix K

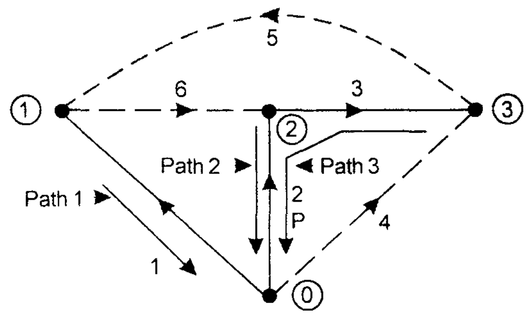
Branch path incidence matrix, as the name itself suggests, shows the incidence of branches to paths in a tree. The elements of this matrix are indicated by the operators as follows :

$K_{pq} = 1$ If the pth branch is in the path from qth bus to reference and oriented in the same direction.

$K_{pq} = -1$ If the pth branch is in the path from qth bus to reference and oriented in the opposite direction.

$K_{pq} = 0$ If the pth branch is not in the path from the qth bus to reference.

For the system in Fig. 2.4(a), the branch-path incidence matrix K is shown in Fig. 3.4. Node (0) is assumed as reference.



path / b	(1)	(2)	(3)
1	−1		
2		−1	−1
3		0	−1

—— tree — — — cotree

Fig. 3.4 Branch-Path Incidence Matrix for network

While the branch path incidence matrix relates branches to paths, the sub matrix A_b of Fig. 3.3 gives the connectivity between branches and buses. Thus, the paths and buses can be related by $A_b K^t = U$ where U is a unit matrix.

Hence $$K^t = A_b^{-1} \qquad .....(3.2)$$

3.4 Basic Cut-Set Incidence Matrix

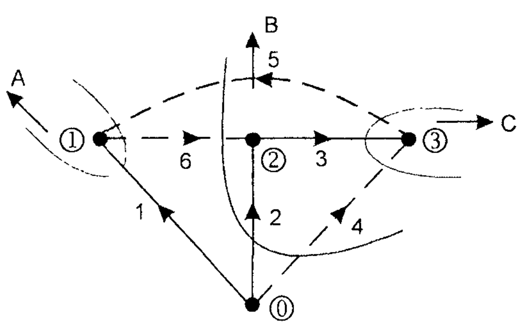
This matrix depicts the connectivity of elements to basic cut-sets of the connected graph. The elements of the matrix are indicated by the operator as follows :

$\beta_{pq} = 1$ if the pth element is incident to and oriented in the same direction as the qth basic cut-set.

$\beta_{pq} = -1$ if the pth element is incident to and oriented in the opposite direction as the qth basic cut-set.

$\beta_{pq} = 0$ if the pth element is not incident to the qth basic cut-set.

The basic cut-set incidence matrix has the dimension e × b. For the graph in Fig. 2.3(a) the basic cut-set incidence matrix B is obtained as in Fig.



B =

b / e	Basic Cut-sets A	B	C
1	1		
2		1	
3			1
4		1	1
5	1	−1	−1
6	−1	1	

Fig. 3.5 Basic Cut-set incidence matrix for the graph in 3.5(a) drawn and shown.

It is possible to partition the basic cut-set incidence matrix B into two submatrices U_B and U_l corresponding to branches and links respectively. For the example on hand, the partitioned matrix is shown in (Fig. 3.6).

B =

e \ b	Basic Cut-sets A	B	C
1	1		
2		1	
3			1
4		1	1
5	1	–1	–1
6	–1	1	0

	Basic Cut-sets
Branches	U_b
Links	B_l

Fig. 3.5

The identity matrix U_b shows the one-to-one correspondence between branches and basic cut-sets.

It may be recalled that the incidence of links to buses is shown by submatrix A_l and the incidence of branches to buses by A_b. There is a one-to-one correspondence between branches and basic cut-sets. Since the incidence of links to buses is given by

$$B_l\, A_b = A_l \qquad \text{.....(3.3)}$$

Therefore $$B_l = A_l\, A_b^{-1} \qquad \text{.....(3.4)}$$

However from equation (3.2) $K^t = A_b^{-1}$

Substituting this result in equation (3.1)

$$B_l = A_l\, K^t \qquad \text{.....(3.5)}$$

This is illustrated in Fig. (3.6).

$$\begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Fig. 3.6 Illustration of equation $A_l\, K^t = B_l$.

3.5 Basic Loop Incidence Matrix

In section 2.3 basic loops are defined and in Fig. 3.7 basic loops for the sample system under discussion are shown. Basic Loop incidence matrix C shows the incidence of the elements of the connected graph to the basic loops. The incidence of the elements is indicated by the operator as follows :

$\gamma_{pq} = 1$ if the pth element is incident to and oriented in the same direction as the qth basic loop.

$\gamma_{pq} = -1$ if the pth element is incident to and oriented in the opposite direction as the qth basic loop.

$\gamma_{pq} = 0$ if the pth element is not incident to the qth loop.

The basic loop incidence matrix has the dimension e × l and the matrix is shown in Fig. 3.8.

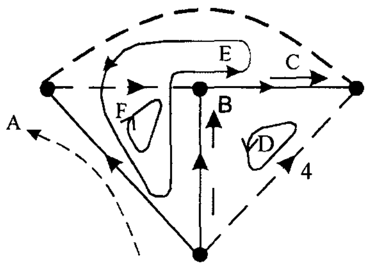


Fig. 3.7 Basic loops (D, E, F) and open loops (A, B, C).

C =

e \ l	D	E	F
1		–1	1
2	–1	1	–1
3	–1	1	
4	1		
5		1	
6		0	1

Fig. 3.8 Basic loop incidence matrix for Fig.3.7.

It is possible to partition the basic loop incidence matrix as in Fig. 3.9.

C =

e \ b	Basic loops		
1			1
2	–1		–1
3	–1	1	
4	1		
5		1	
6		1	

e \ l	Basic loops
Branches	C_b
Links	U_l

Fig. 3.9 Partitioning of basic loop incidence matrix.

The unit matrix U_l shown the one-to-one correspondence of links to basic loops.

3.6 Network Performance Equations

The power system network consists of components such as generators, transformers, transmission lines, circuit breakers, capacitor banks etc., which are all connected together to perform specific function. Some are in series and some are in shunt connection.

Whatever may be their actual configuration, network analysis is performed either by nodal or by loop method. In case of power system, generally, each node is also a bus. Thus, in the bus frame of reference the performance of the power network is described by (n-1) independent nodal equations, where n is the total number of nodes. In the impedance form the performance equation, following Ohm's law will be

$$\bar{V} = [Z_{BUS}]\,\bar{I}_{BUS} \qquad .....(3.6)$$

where, $\bar{V}_{BUS}$ = Vector of bus voltages measured with respect to a reference bus.

$\bar{I}_{BUS}$ = Vector of impressed bus currents.

$[Z_{BUS}]$ = Bus impedance matrix

The elements of bus impedance matrix are open circuit driving point and transfer impedances.

Consider a 3-bus or 3-node system. Then

$$[Z_{BUS}] = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array}\begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \end{array}$$

The impedance elements on the principal diagonal are called driving point impedances of the buses and the off-diagonal elements are called transfer impedances of the buses. In the admittance frame of reference

$$\bar{I}_{BUS} = [Y_{BUS}].\bar{V}_{BUS} \qquad .....(3.7)$$

where $[Y_{BUS}]$ = bus admittance matrix whose elements are short circuit driving point and transfer admittances.

By definition $[Y_{BUS}] = [Z_{BUS}]^{-1}$ .....(3.8)

In a similar way, we can obtain the performance equations in the branch frame of reference. If b is the number of branches, then b independent branch equation of the form

$$\bar{V}_{BR} = [Z_{BR}].\bar{I}_{BR} \qquad .....(3.9)$$

describe network performance. In the admittance form

$$\bar{I}_{BR} = [Y_{BR}]\bar{V}_{BR} \qquad .....(3.10)$$

where

$\bar{I}_{BR}$ = Vector of currents through branches.

$\bar{V}_{BR}$ = Vector of voltages across the branches.

$[Y_{BR}]$ = Branch admittance matrix whose elements are short circuit driving point and transfer admittances of the branches of the network.

$[Z_{BR}]$ = Branch impedance matrix whose elements are open circuit driving point and transfer impedances of the branches of the network.

Like wise, in the loop frame of reference, the performance equation can be described by l independent loop equations where l is the number of links or basic loops. In the impedance from

$$\bar{V}_{LOOP} = [Z_{LOOP}].\bar{I}_{LOOP} \quad .....(3.11)$$

and in the admittance form

$$\bar{I}_{LOOP} = [Y_{LOOP}].\bar{V}_{LOOP} \quad .....(3.12)$$

where, $\bar{V}_{LOOP}$ = Vector of basic loop voltages

$\bar{I}_{LOOP}$ = Vector of basic loop currents

$[Z_{LOOP}]$ = Loop impedance matrix

$[Y_{LOOP}]$ = Loop admittance matrix

3.7 Network Matrices

It is indicated in Chapter - 1 that network solution can be carried out using Ohm's Law and Kirchoff's Law. The impedance model given by

$$e = Z . i$$

or the admittance model

$$i = Y. e$$

can be used depending upon the situation or the type of problem encountered. In network analysis students of electrical engineering are familiar with nodal analysis and mesh analysis using Kirchoff's laws. In most of the power network solutions, the bus impedance or bus admittance are used. Thus it is necessary to derive equations that relate these various models.

Network matrices can be formed by two methods:

Viz. (a) Singular transformation and

(b) Direct method

Singular Transformations

The network matrices that are used commonly in power system analysis that can be obtained by singular transformation are :

(i) Bus admittance matrix
(ii) Bus impedance matrix
(iii) Branch admittance matrix
(iv) Branch impedance matrix
(v) Loop impedance matrix
(vi) Loop admittance matrix

3.8 Bus Admittance Matrix and Bus Impedance Matrix

The bus admittance matrix Y_{BUS} can be obtained by determining the relation between the variables and parameters of the primitive network described *in section (2.1)* to bus quantities of the network using bus incidence matrix. Consider eqn. (1.5).

$$\bar{i} + \bar{j} = [y]\bar{\upsilon}$$

Pre multiplying by $[A^t]$, the transpose of the bus incidence matrix

$$[A^t]\,\bar{i} + [A^t]\,\bar{j} = A^t\,[y]\bar{\upsilon} \quad .....(3.13)$$

Matrix A shows the connections of elements to buses. $[A^t]$ i thus is a vector, wherein, each element is the algebraic sum of the currents that terminate at any of the buses. Following Kirchoff's current law, the algebraic sum of currents at any node or bus must be zero. Hence

$$[A^t]\,\bar{i} = 0 \quad .....(3.14)$$

Again $[A^t]\,\bar{j}$ term indicates the algebraic sum of source currents at each of the buses and must equal the vector of impressed bus currents. Hence,

$$\bar{I}_{BUS} = [A^t]\,\bar{j} \quad .....(3.15)$$

Substituting eqs. (3.14) and (3.15) into (3.13)

$$\bar{I}_{BUS} = [A^t][y]\bar{\upsilon} \quad .....(3.16)$$

In the bus frame, power in the network is given by

$$[\bar{I}_{BUS}^{*}]^t\,\bar{V}_{BUS} = P_{BUS} \quad .....(3.17)$$

Power in the primitive network is given by

$$\left(\bar{j}^{*}\right)^t\bar{\upsilon} = P \quad .....(3.18)$$

Power must be invariant, for transformation of variables to be invariant. That is to say, that the bus frame of referee corresponds to the given primitive network in performance. Power consumed in both the circuits is the same.

Therefore $$[\bar{I}_{BUS}^{*}]\bar{V}_{BUS} = [\bar{j}^{*}]\bar{\upsilon} \qquad .....(3.19)$$

Conjugate transpose of eqn. (3.15) gives

$$[\bar{I}_{BUS}^{*}]^{t} = [j^{*}]^{t}A^{*} \qquad .....(3.20)$$

However, as A is real matrix $A = A^{*}$

$$[\bar{I}_{BUS}^{*}]^{t} = (j^{*})^{t}[A] \qquad .....(3.21)$$

Substituting (3.21) into (3.19)

$$(j^{*})^{t}[A] \cdot \bar{V}_{BUS} = (j^{*})^{t}\bar{\upsilon} \qquad .....(3.22)$$

i.e., $$[A]\,\bar{V}_{BUS} = \bar{\upsilon} \qquad .....(3.23)$$

Substituting eqn. (3.22) into (3.16)

$$\bar{I}_{BUS} = [A^{t}]\,[y]\,[A]\,\bar{V}_{BUS} \qquad .....(3.24)$$

From eqn. 3.7

$$\bar{I}_{BUS} = [\bar{Y}_{BUS}]\,\bar{V}_{BUS} \qquad .....(3.25)$$

Hence

$$[Y_{BUS}] = [A^{t}]\,[y]\,[A] \qquad .....(3.26)$$

Once $[Y_{BUS}]$ is evaluated from the above transformation, (Z_{BUS}) can be determined from the relation.

$$Z_{BUS} = Y_{BUS}^{-1} = \left\{[A^{t}][y][A]\right\}^{-1} \qquad .....(3.27)$$

3.9 Branch Admittance and Branch Impedance Matrices

In order to obtain the branch admittance matrix Y_{BR}, the basic cut-set incidence matrix [B], is used. The variables and parameters of primitive network are related to the variables and parameters of the branch admittance network.

For the primitive network

$$\bar{i} + \bar{j} = [y]\bar{\upsilon} \qquad .....(3.28)$$

Premultiplying by B^{t}

$$[B]^{t}\,i + [B]^{t}\,j = [B]^{t}\,[y]\,\bar{\upsilon} \qquad .....(3.29)$$

It is clear that the matrix [B] shows the incidence of elements to basic cut-sets.

Each element of the vector $[B^t]\ \bar{i}$ is the algebraic sum of the currents through the elements that are connected to a basic cut-set. Every cut-set divides the network into two connected sub networks. Thus each element of the vector $[B^t]\ \bar{i}$ represents the algebraic sum of the currents entering a sub network which must be zero by Kirchoff's law.

Hence, $$[B^t]\ \bar{i} = 0 \qquad .....(3.30)$$

$[B^t]\ \bar{j}$ is a vector in which each element is the algebraic sum of the source currents of the elements incident to the basic cut-set and represents the total source current in parallel with a branch.

$$[B^t]\ \bar{j} = \bar{I}_{BR} \qquad .....(3.31)$$

therefore, $$\bar{I}_{BR} = [B^t][y]\bar{\upsilon} \qquad .....(3.32)$$

For power invariance.

$$\bar{I}_{BR}^{*\,t} . \bar{V}_{BR} = \bar{j}^{*t}\,\bar{\upsilon} \qquad .....(3.33)$$

conjugate transpose of eqn. (3.31) gives $j^{*\,t}\,[B]^{*} = \bar{I}_{BR}^{*t}$. Substituting this in the previous eqn. (3.32)

$$(j)^{*t}\,[B]^{*}\,\bar{V}_{BR} = (j^{*})^{t}\,\bar{\upsilon}$$

As [B] is a real matrix $[B]^* = [B]$ $\qquad .....(3.34)$

Hence, $$(J^{*})^{t}\,[B]\ \bar{V}_{BR} = (j^{*})^{t}\,\bar{\upsilon} \qquad .....(3.35)$$

(i.e.) $$\bar{\upsilon} = [B]\bar{V}_{BR} \qquad .....(3.36)$$

Substituting eqs. (3.35) into (3.32)

$$\bar{I}_{BR} = [B]^{t}[y][B]\bar{V}_{BR} \qquad .....(3.37)$$

However, the branch voltages and currents are related by

$$\bar{I}_{BR} = [Y_{BR}].\bar{V}_{BR} \qquad .....(3.38)$$

comparing (3.37) and (3.38)

$$[Y_{BR}] = [B]^{t}\,[y]\,[B] \qquad .....(3.39)$$

Since, the basic cut-set matrix [B] is a singular matrix the transformation $[Y_{BR}]$ is a singular transformation of [y]. The branch impedance matrix, then, is given by

$$[Z]_{BR} = [Y_{BR}]^{-1}$$

$$[Z]_{BR} = [Y]_{BR}^{-1} = \{[B^{t}][y][B]\}^{-1} \qquad .....(3.40)$$

3.10 Loop Impedance and Loop Admittance Matrices

The loop impedance matrix is designated by $[Z_{Loop}]$. The basic loop incidence matrix [C] is used to obtain $[Z_{Loop}]$ in terms of the elements of the primitive network.

The performance equation of the primitive network is

$$\bar{\upsilon} + \bar{e} = [Z]\,\bar{i} \qquad .....(3.41)$$

Premultiplying by $[C^t]$

$$[C]^t[\bar{\upsilon}] + [C]^t\bar{e} = [C]^t[z]\bar{i} \qquad .....(3.42)$$

As the matrix [C] shows the incidence of elements to basic loops, $[C^t]\,\bar{\upsilon}$ yields the algebraic sum of the voltages around each basic loop.

By Kirchoff's voltage law, the algebraic sum of the voltages around a loop is zero. Hence, $[C^t]\,\bar{\upsilon} = 0$. Also $[C^t]\,\bar{e}$ gives the algebraic sum of source voltages around each basic loop; so that,

$$\bar{V}_{LOOP} = [C^t]\bar{e} \qquad .....(3.43)$$

From power invariance condition for both the loop and primitive networks.

$$\left(\bar{I}_{LOOP}{}^*\right)^t \cdot \bar{V}_{LOOP} = (\bar{i}^*)^t\bar{e} \qquad .....(3.44)$$

for all values of $\bar{e}$.

Substituting $\bar{V}_{LOOP}$ from eqn. (3.43)

$$\left(\bar{I}_{LOOP}{}^*\right)^t [C^t]\bar{e} = [\bar{i}^*]^t\bar{e} \qquad .....(3.45)$$

Therefore,

$$\bar{i} = [C^*]^t\,\bar{I}_{LOOP} \qquad .....(3.46)$$

However, as [C] is a real matrix $[C] = [C^*]$

Hence, $\qquad \bar{i} = [C]\bar{I}_{LOOP} \qquad .....(3.47)$

From eqns. (3.43), (3.45) & (3.47)

$$\bar{V}_{LOOP} = [C^t][z][C]\bar{I}_{LOOP} \qquad .....(3.48)$$

However, for the loop frame of reference the performance equation from eqn. (3.11) is

$$\bar{V}_{LOOP} = [Z_{LOOP}]\bar{I}_{LOOP} \qquad .....(3.49)$$

comparing (3.48) & (3.49) equation

$$[Z_{LOOP}] = [C^t] [z] [C] \quad .....(3.50)$$

[C] being a singular matrix the transformation eqn. (3.50) is a singular transformation of [z].

The loop admittance matrix is obtained from

$$[Y_{LOOP}] = [Z^{-1}_{LOOP}] = \{[C]^t [z] [C]\}^{-1} \quad .....(3.51)$$

Summar.y of Singular Transformations
$[z]^{-1} = [y]$
$[A^t] [y] [A] = [Y_{BUS}]$;
$[Y_{BUS}]^{-1} = [Z_{BUS}]$
$[B^t] [y] [B] = [Y_{BR}]$;
$[Y_{BR}]^{-1} = [Z_{BR}]$
$[C^t] [Z] [C] = [Z_{LOOP}]$;
$[Z_{LOOP}]^{-1} = [Y_{LOOP}]$

3.11 Bus Admittance Matrix by Direct Inspection

Bus admittance matrix can be obtained for any network, if there are no mutual impedances between elements, by direct inspection of the network. This is explained by taking an example.

Consider the three bus power system as shown in Fig.

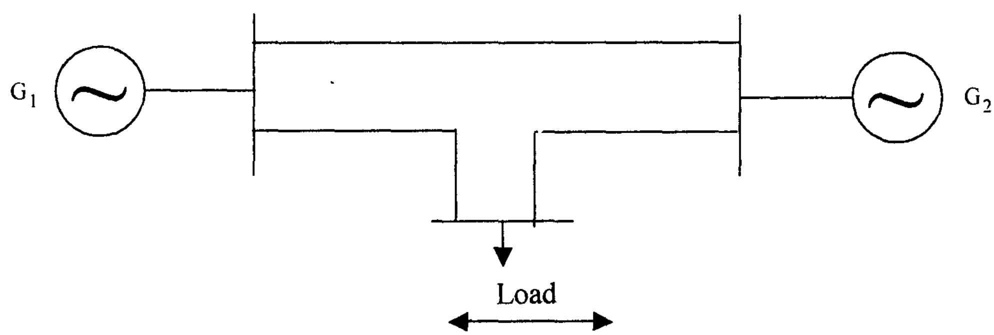


Fig. 3.10

The equivalent circuit is shown in Fig. below. The generator is represented by a voltage source in series with the impedance. The three transmission lines are replaced by their "π equivalents".

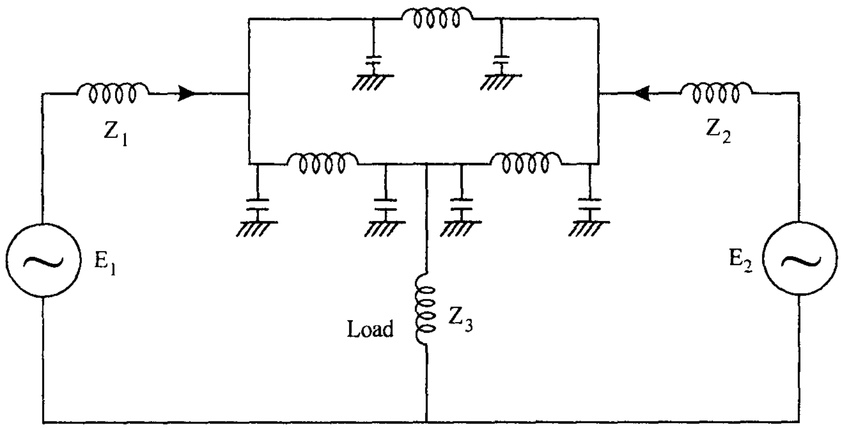


Fig. 3.11

The equivalent circuit is further simplified as in the following figure combining the shunt admittance wherever feasible.

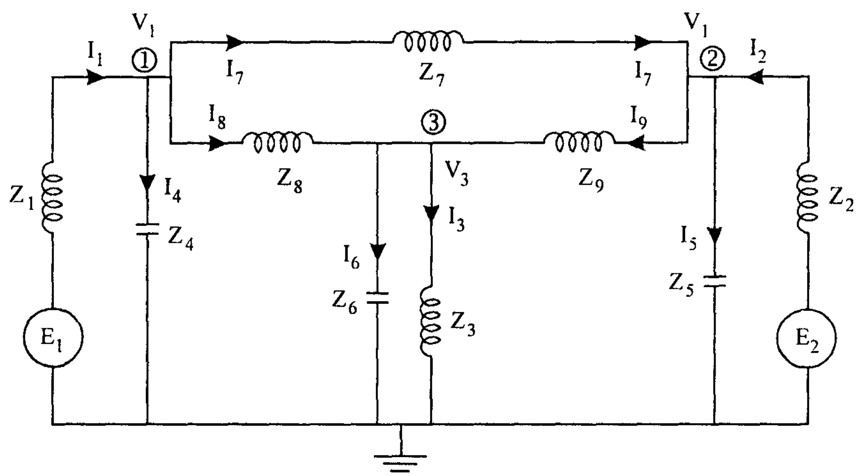


Fig. 3.12

The three nodes are at voltage V_1, V_2 and V_3 respectively above the ground. The Kirchoff's nodal current equations are written as follows :

At Node 1 :

$$I_1 = I_7 + I_8 + I_4$$

$$I_1 = (V_1 - V_2)\,Y_7 + (V_1 - V_3)\,Y_8 + V_1\,Y_4 \qquad ....(3.52)$$

At Node 2 :

$$I_2 = I_5 + I_9 - I_7$$

$$= V_2\,Y_5 + (V_2 - V_3)\,Y_9 - (V_1 - V_2)\,Y_7 \qquad .....(3.53)$$

At Node 3 :

$$I_3 = I_8 + I_9 - I_6$$

$$= (V_1 - V_3)\,Y_8 + (V_2 - V_3)\,Y_9 - V_3\,Y_6 \qquad .....(3.54)$$

Re arranging the terms the equations will become

$$I_1 = V_1\,(Y_4 + Y_7 + Y_8) - V_2\,Y_7 - V_3\,Y_8 \qquad .....(3.55(a))$$

$$I_2 = -V_1\,Y_7 + V_2\,(Y_5 + Y_7 + Y_9) - V_3\,Y_9 \qquad .....(3.55(b))$$

$$I_3 = V_1\,Y_8 + V_2\,Y_9 - V_3\,(Y_8 + Y_9 + Y_6) \qquad .....(3.55(c))$$

The last of the above equations may be rewritten as

$$-I_3 = -V_1\,Y_8 - V_2\,Y_9 + V_3\,(Y_6 + Y_8 + Y_9) \qquad .....(3.56)$$

Thus we get the matrix relationship from the above

$$\begin{bmatrix} I_1 \\ I_2 \\ -I_3 \end{bmatrix} = \begin{bmatrix} (Y_4 + Y_7 + Y_8) & -Y_7 & -Y_8 \\ -Y_7 & (Y_5 + Y_7 + Y_9) & -Y_9 \\ -Y_8 & -Y_9 & (Y_6 + Y_8 + Y_9) \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad .....(3.57)$$

It may be recognized that the diagonal terms in the admittance matrix at each of the nodes are the sum of the admittances of the branches incident to the node. The off – diagonal terms are the negative of these admittances branch – wise incident on the node. Thus, the diagonal element is the negative sum of the off – diagonal elements. The matrix can be written easily by direct inspection of the network.

The diagonal elements are denoted by

$$\left.\begin{aligned} Y_{11} &= Y_4 + Y_7 + Y_8 \\ Y_{22} &= Y_5 + Y_7 + Y_9 \\ Y_{33} &= Y_6 + Y_8 + Y_9 \end{aligned}\right\} \text{ and} \qquad .....(3.58)$$

They are called self admittances of the nodes or driving point admittances. The off-diagonal elements are denoted by

$$\left.\begin{aligned} Y_{12} &= -Y_7 \\ Y_{13} &= -Y_8 \\ Y_{21} &= -Y_7 \\ Y_{23} &= -Y_8 \\ Y_{31} &= -Y_8 \\ Y_{32} &= -Y_9 \end{aligned}\right\} \qquad .....(3.59)$$

using **double suffix** denoting the nodes across which the admittances exist. They are called mutual admittances or transfer admittances. Thus the relation in eqn. 3.91 can be rewritten as

$$\begin{bmatrix} I_1 \\ I_2 \\ -I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad .....(3.60)$$

$$\bar{I}_{BUS} = [Y_{BUS}] \bullet \bar{V}_{BUS} \qquad .....(3.61)$$

In power systems each node is called a bus. Thus, if there are n independent buses, the general expression for the source current towards the node i is given by

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j; \quad i \neq j \qquad .....(3.62)$$

Worked Examples

E 3.1 For the network shown in figure form the bus incidence matrix, A, branch path incidence matrix K and loop incidence matrix C.

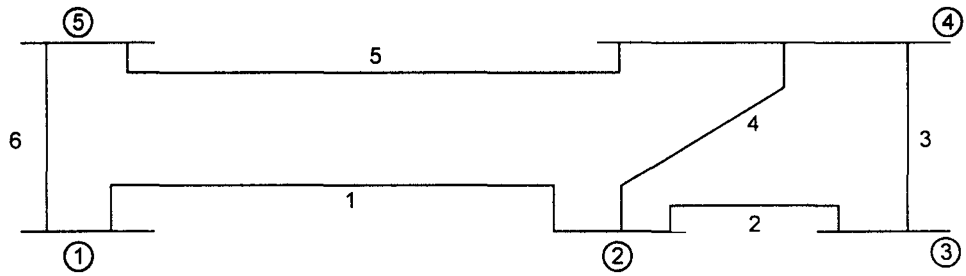


Fig. E.3.1

Solution :

For the tree and co-tree chosen for the graph shown below, the basic cutsets are marked. Bus (1) is taken as reference.

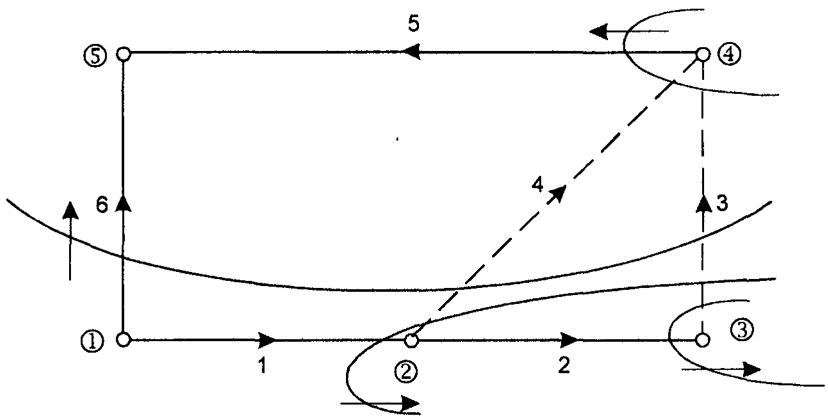


Fig. E.3.2

The basic loops are shown in the following figure.

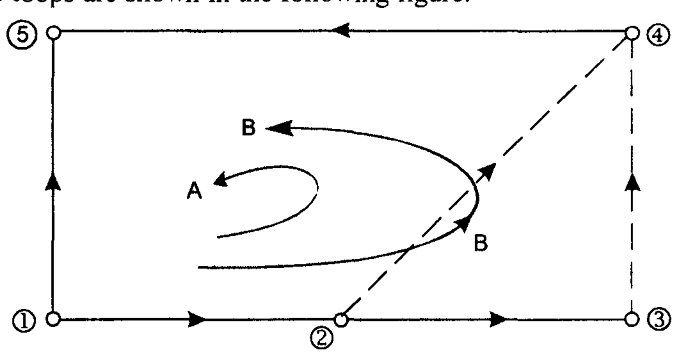


Fig. E.3.3

(i) Bus incidence matrix

Number of buses = number of nodes

A =

e \ Bus	(2)	(3)	(4)	(5)
1	–1	0	0	0
2	1	–1	0	0
3	0	1	–1	0
4	1	0	–1	0
5	0	0	1	–1
6	0	0	0	–1

Fig. E.3.4

A =

	e \ Bus	(2)	(3)	(4)	(5)
Branches	1	–1	0	0	0
	2	1	–1	0	0
	5	0	1	1	–1
	6	0	0	0	–1
Link	3	0	1	–1	1
	4	1	0	–1	0

=

e \ Bus	Buses
B branches	A_b
L links	A_l

Fig. E.3.5

(ii) Branch path incidence matrix (K) :

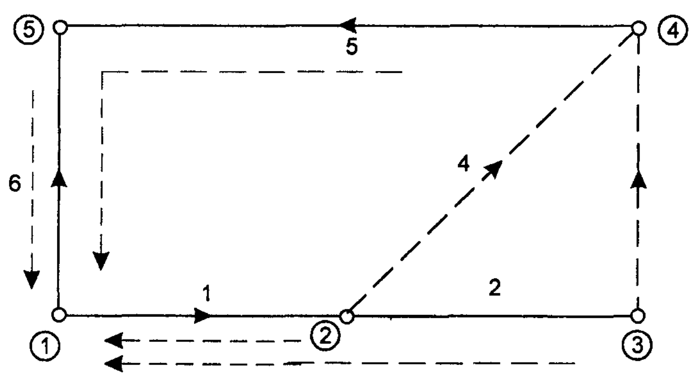


Fig. E 3.6 Branches and the paths.

K =

e \ Path	(2)	(3)	(4)	(5)
1	–1	–1		
2		–1	0	
5			1	
6			–1	–1

Fig. E.3.7 Branches and the paths.

(iii) Basic loop incidence matrix C :

C =

e \ l	A	B
1	1	1
2	0	1
3	0	1
4	1	0
5	1	1
6	–1	–1

C =

e \ l	A	B
1	1	1
2	0	1
5	1	1
6	–1	–1
3	0	1
4	1	0

$$= \begin{bmatrix} C_b \\ C_l \end{bmatrix}$$

Fig. E 3.8 Branches and the paths.

E 3.2 Form the Y_{BUS} by using singular transformation for the network shown in Fig. including the generator buses.

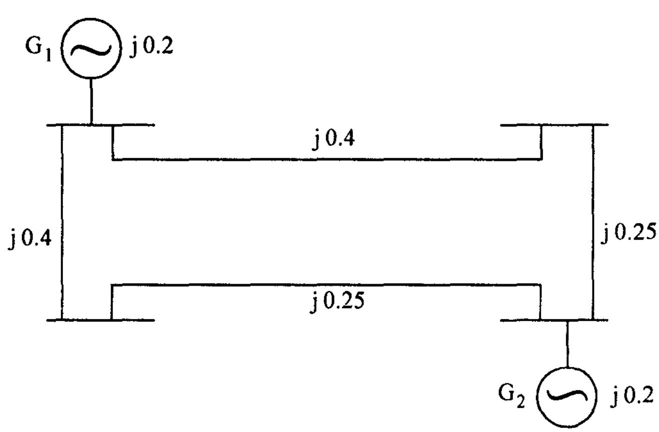


Fig. E.3.9

Solution :

The given network is represented in admittance form

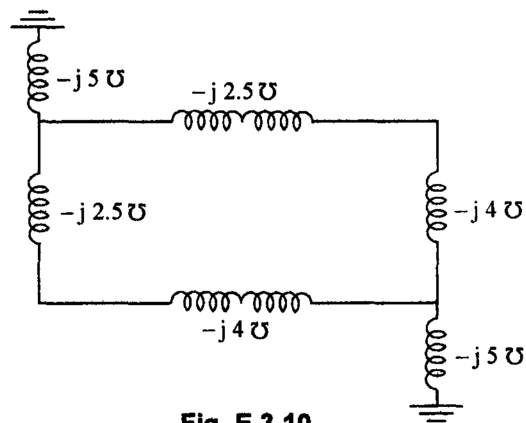


Fig. E.3.10

The oriented graph is shown in Fig. below

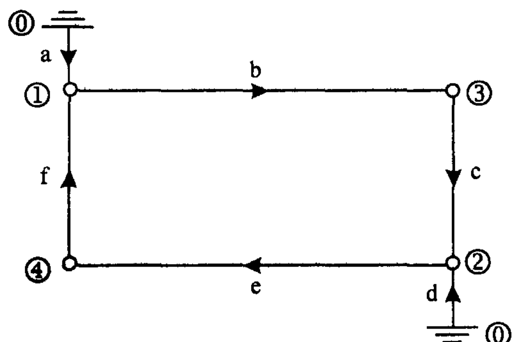


Fig. E.3.11

The above graph can be converted into the following form for convenience

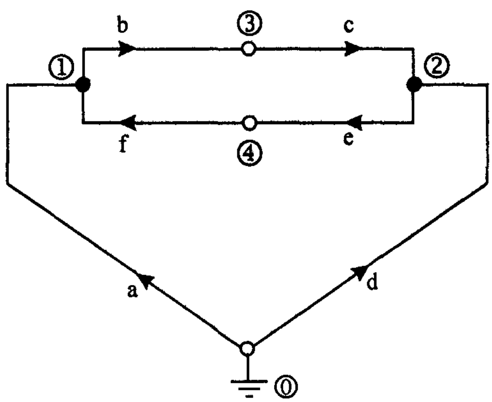


Fig. E.3.12

the element node incidence matrix is given by

$$\hat{A} = \begin{array}{c|ccccc} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ \hline a & +1 & -1 & 0 & 0 & 0 \\ b & 0 & +1 & 0 & -1 & 0 \\ c & 0 & 0 & -1 & +1 & 0 \\ d & +1 & 0 & -1 & 0 & 0 \\ e & 0 & 0 & +1 & 0 & -1 \\ f & 0 & -1 & 0 & 0 & +1 \end{array}$$

Bus incidence matrix is obtained by deleting the column corresponding to the reference bus.

$$A = \begin{array}{c|cccc} e\backslash b & 1 & 2 & 3 & 4 \\ \hline a & -1 & 0 & 0 & 0 \\ b & +1 & 0 & -1 & 0 \\ c & 0 & -1 & +1 & 0 \\ d & 0 & -1 & 0 & 0 \\ e & 0 & +1 & 0 & -1 \\ f & -1 & 0 & 0 & +1 \end{array}$$

$$A^t = \begin{array}{c|cccccc} b\backslash e & a & b & c & d & e & f \\ \hline 1 & -1 & 1 & 0 & 0 & 0 & -1 \\ 2 & 0 & 0 & -1 & -1 & 1 & 0 \\ 3 & 0 & -1 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}$$

The bus admittance matrix

$$Y_{BUS} = [A]^t\,[y]\,[A]$$

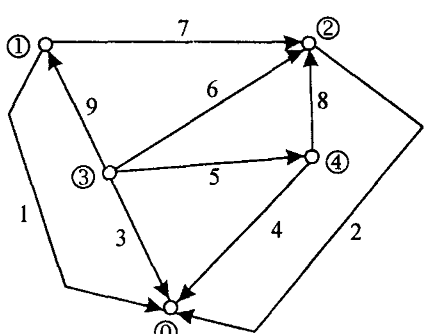
$$[y]\,[A] = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} y_a & 0 & 0 & 0 & 0 & 0 \\ 0 & y_b & 0 & 0 & 0 & 0 \\ 0 & 0 & y_c & 0 & 0 & 0 \\ 0 & 0 & 0 & y_d & 0 & 0 \\ 0 & 0 & 0 & 0 & y_e & 0 \\ 0 & 0 & 0 & 0 & 0 & y_f \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$[y][A] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ -2.5 & 0 & 2.5 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & -4 & 0 & 4 \\ 2.5 & 0 & 0 & -2.5 \end{bmatrix}$$

$$Y_{BUS} = [A]^t\,[y]\,[A] = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 2 & 0 & - & 0 \\ -2.5 & 0 & 2.5 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & -4 & 0 & 4 \\ 2.5 & 0 & 0 & -2.5 \end{bmatrix}$$

whence,
$$Y_{BUS} = \begin{bmatrix} -10 & 0 & 2.5 & 2.5 \\ 0 & -13 & 4 & 4 \\ 2.5 & 4 & -6.5 & 0 \\ 2.5 & 4 & 0 & -6.5 \end{bmatrix}$$

E 3.3 Find the Y_{BUS} using singular transformation for the system shown in Fig. E.3.5.



$Y = \text{dia}\,[\,Y_{10} Y_{20} Y_{30} Y_{40} Y_{34} Y_{12} Y_{23} Y_{24} Y_{13}]$

Fig. E.3.13

Solution : The graph may be redrawn for convenient as follows

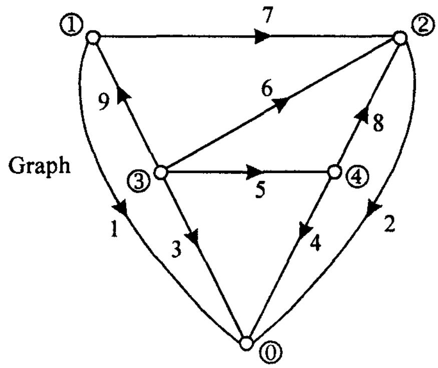


Fig. E.3.14

A tree and a co-tree are identified as shown below.

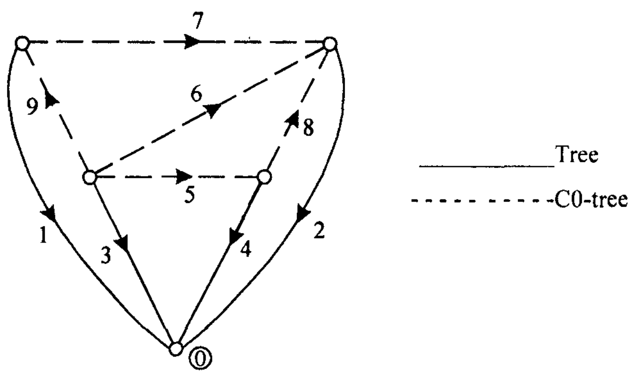


Fig. E.3.15

The element mode incidence matrix $\hat{A}$ is given by

$\hat{A} =$

	(0)	(1)	(2)	(3)	(4)
1	−1	1	0	0	0
2	−1	0	1	0	0
3	−1	0	0	1	0
4	−1	0	0	0	1
5	0	0	0	1	−1
6	0	0	−1	1	0
7	0	1	−1	0	0
8	0	0	−1	0	1
9	0	−1	0	1	0

The bus incidence matrix is obtained by deleting the first column taking (0) node as reference.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}\begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \end{array} = \begin{bmatrix} A_b \\ \hline A_l \end{bmatrix} = \begin{bmatrix} U \\ \hline A_l \end{bmatrix}$$

Given
$$[y] = \begin{bmatrix} y_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{30} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{40} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & y_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y3 \end{bmatrix}$$

$$[Y_{Bus}] = A^t [y] A$$

$$[y]\,[A] = \begin{bmatrix} y_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{30} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{40} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & y_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & y_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} y_{10} & 0 & 0 & 0 \\ 0 & y_{20} & 0 & 0 \\ 0 & 0 & y_{30} & 0 \\ 0 & 0 & 0 & y_{40} \\ 0 & 0 & y_{34} & -y_{34} \\ 0 & -y_{23} & y_{23} & 0 \\ y_{12} & -y_{12} & 0 & 0 \\ 0 & -y_{24} & 0 & y_{24} \\ -y_{13} & 0 & y_{13} & 0 \end{bmatrix}$$

$$[A_t]\,[y]\,[A] = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{bmatrix} y_{10} & 0 & 0 & 0 \\ 0 & y_{20} & 0 & 0 \\ 0 & 0 & y_{30} & 0 \\ 0 & 0 & 0 & y_{40} \\ 0 & 0 & y_{34} & -y_{34} \\ 0 & -y_{23} & y_{23} & 0 \\ y_{12} & -y_{12} & 0 & 0 \\ 0 & -y_{24} & 0 & y_{24} \\ -y_{13} & 0 & y_{13} & 0 \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} (y_{10} + y_{12} + y_{13}) & -y_{12} & -y13 & 0 \\ -y_{12} & (y_{20} + y_{12} + y_{23} + y_{24}) & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & (y_{30} + y_{13} + y_{23} + y_{34}) & -y_{34} \\ 0 & -y_{24} & -y_{34} & (y_{40} + y_{34} + y_{24}) \end{bmatrix}$$

E 3.4 Derive an expression for Z_{loop} for the oriented graph shown below

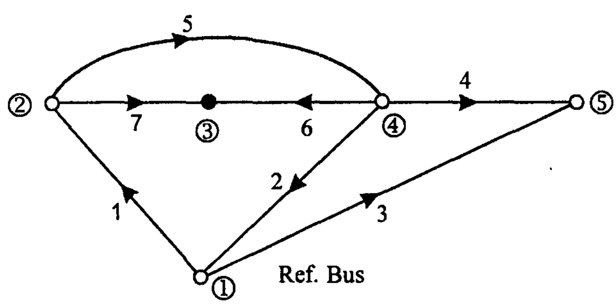


Fig. E.3.16

Solution :

Consider the tree and co-tree identified in the Fig. shown

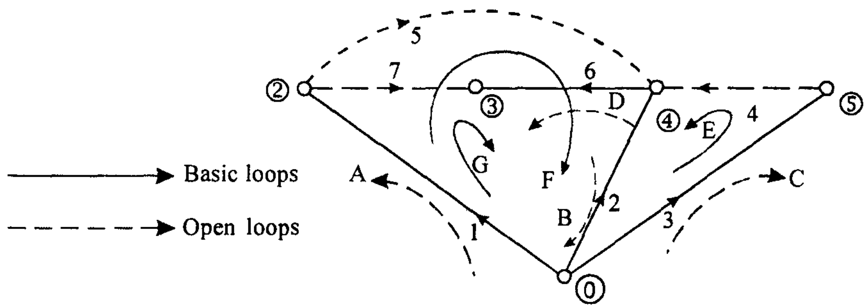


Fig. E.3.17

The augmented loop incidence matrix $\hat{C}$ is obtained as shown from the Fig.

$\hat{C}$ =

e \ l	A	B	C	D	E	F	G
1	1				0	1	1
2		1			1	1	1
3			1		1	0	0
4				1	0	0	–1
5					1		
6						1	
7							1

$$= \left[\begin{array}{c|c} U_b & C_b \\ \hline 0 & U_l \end{array}\right]$$

The basic loop incidence matrix

C =

e \ l	E	F	G
1	0	1	1
2	1	1	1
3	1	0	0
6	0	0	–1
4	1		
5		1	
7			1

$$= \left[\frac{C_b}{1}\right]$$

$$Z_{loop} = [Ct]\,[Z]\,[C]$$

$$= \left[C_b^t [U_l]\right] \begin{bmatrix} Z_{bb} & Z_{bl} \\ Z_{lb} & Z_{ll} \end{bmatrix} \begin{bmatrix} C_b \\ U_l \end{bmatrix}$$

$$[Z_{loop}] = C_b^t [Z_{bb}] C_b + [Z_{bb}] C_b + C_b^t [Z_{bl}] + [Z_{ll}]$$

Note : It is not necessary to form the augmented loop incidence matrix for this problem only loop incidence matrix suffices].

E 3.5 For the system shown in figure obtain Y_{BUS} by inspection method. Take bus (1)as reference. The impedance marked are in p.u.

Solution :

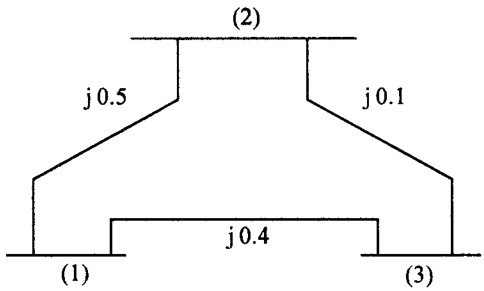


Fig. E.3.18

$$Y_{BUS} = \begin{array}{c} (2) \\ (3) \end{array}\begin{bmatrix} \dfrac{1}{j0.5}+\dfrac{1}{j0.1} & -\dfrac{1}{j0.1} \\ -\dfrac{1}{j0.1} & \dfrac{1}{j0.1}+\dfrac{1}{j0.4} \end{bmatrix} = \begin{array}{c} \\ (2) \\ (3) \end{array}\begin{array}{c} \begin{array}{cc} (2) & (3) \end{array} \\ \begin{bmatrix} -j12 & +j10 \\ j10 & -j12.5 \end{bmatrix} \end{array}$$

E 3.6 Consider the linear graph shown below which represents a 4 bus transmission system with all the shunt admittance lumped together. Each line has a series impedance of (0.02 + j 0.08) and half line charging admittance of j0.02. Compute the Y_{BUS} by singular transformation. Compute the Y_{BUS} also by inspection.

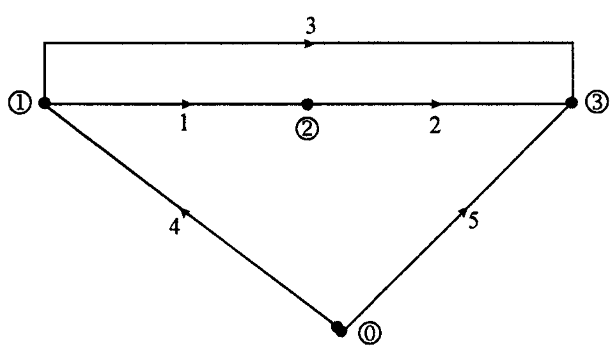


Fig. E.3.19

Solution :

The half – line charging admittances are all connected to ground. Taking this ground as reference and eliminating it. The bus incidence matrix is given by

$$A = \begin{array}{c|c|c|c|c|}
 & (0) & (1) & (2) & (3) \\ \hline
1 & 0 & 1 & -1 & 0 \\ \hline
2 & 0 & 0 & 1 & -1 \\ \hline
3 & 0 & 1 & 0 & -1 \\ \hline
4 & 1 & -1 & 0 & 0 \\ \hline
5 & 1 & 0 & 0 & -1 \\ \hline
\end{array} \; a$$

incidence matrix is given by; the transport of the bus

$$A^t = \begin{array}{c|c|c|c|c|c|}
 & 1 & 2 & 3 & 4 & 5 \\ \hline
0 & 0 & 0 & 0 & 1 & 1 \\ \hline
1 & 1 & 0 & 1 & -1 & 0 \\ \hline
2 & -1 & 1 & 0 & 0 & 0 \\ \hline
3 & 0 & -1 & -1 & 0 & -1 \\ \hline
\end{array}$$

The primitive admittance matrix [y] is shown below.

$$[y] = \begin{bmatrix} y_1 & 0 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 & 0 \\ 0 & 0 & y_3 & 0 & 0 \\ 0 & 0 & 0 & y_4 & 0 \\ 0 & 0 & 0 & 0 & y_5 \end{bmatrix}$$

The admittance of all the branches are the same

i.e $$y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{Z} = \frac{1}{0.02 + j0.08}$$

$$[y] = \begin{bmatrix} 2.94 - j11.75 & 0 & 0 & 0 & 0 \\ 0 & 2.94 - j11.75 & 0 & 0 & 0 \\ 0 & 0 & 2.94 - j11.75 & 0 & 0 \\ 0 & 0 & 0 & 2.94 - j11.75 & 0 \\ 0 & 0 & 0 & 0 & 2.94 - j11.75 \end{bmatrix}$$

[y] [A] =

$$\begin{bmatrix} 2.94-j11.75 & 0 & 0 & 0 & 0 \\ 0 & 2.94-j11.75 & 0 & 0 & 0 \\ 0 & 0 & 2.94-j11.75 & 0 & 0 \\ 0 & 0 & 0 & 2.94-j11.75 & 0 \\ 0 & 0 & 0 & 0 & 2.94-j11.75 \end{bmatrix} \bullet \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+j0 & 2.94-j11.75 & -2.94+j11.75 & 0+j0 \\ 0+j0 & 0+j0 & 2.94-j11.75 & -2.94+j11.75 \\ 0+j0 & 2.94-j11.75 & 0+j0 & -294+j11.75 \\ 2.94-j11.75 & -2.94+j11.75 & 0+j0 & 0+j0 \\ 2.94-j11.75 & 0+j0 & 0+j0 & -2.94+j11.75 \end{bmatrix}$$

$Y_{BUS} = [A^t] [y] [A]$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 \end{bmatrix} \bullet \begin{bmatrix} 0+j0 & 2.94-j11.75 & -2.94+j11.75 & 0+j0 \\ 0+j0 & 0+j0 & 2.94-j11.75 & -2.94+j11.75 \\ 0+j0 & 2.94-j11.75 & 0+j0 & -294+j11.75 \\ 2.94-j11.75 & -2.94+j11.75 & 0+j0 & 0+j0 \\ 2.94-j11.75 & 0+j0 & 0+j0 & -2.94+j11.75 \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} 5.88-j23.5 & -2.94+j11.75 & 0+j0 & -2.94+j11.75 \\ -2.94+j11.75 & 8.82-j35.25 & -2.94+j11.75 & -2.94+j11.75 \\ 0+j0 & -2.94+j11.75 & 5.88-j23.5 & -2.94+j11.75 \\ -2.94+j11.75 & -2.94+j11.75 & -2.94+j11.75 & 8.82-j35.7 \end{bmatrix}$$

Solution by inspection including line charging admittances:

$$y_{00} = y_{01} + y_{03} + y_{0\,½} + y_{0\,3/2}$$

$$y_{00} = [2.94 - j11.75 + j0.02 + j0.02 + 2.94 - j11.75]$$

$$y_{00} = [5.88 - j\,23.46]$$

$$y_{00} = y_{22} = 5.88 - j23.46$$

$$y_{11} = y_{10} + y_{12} + y_{13} + y_{10/2} + y_{12/2} + y_{13/2}$$

$$y_{33} = [y_{30} + y_{31} + y_{32} + y_{30/2} + y_{31/2} + y_{32/2}]$$

$$y_{11} = y_{33} = 3\ (2.94 - j\ 11.75) + 3\ (j0.02)$$
$$= 8.82 - j\ 35.25 + j\ 0.006$$
$$y_{11} = y_{33} = [8.82 - j\ 35.19]$$

the off diagonal elements are

$$y_{01} = y_{10} = -y_{01} = -2.94 + j\ 11.75$$
$$y_{12} = y_{21} = (-y_{02}) = 0$$
$$y_{03} = y_{30} = (-y_{03}) = -2.94 + j\ 11.75$$
$$y_{2b} = y_{32} = (-y_{23}) = (-2.94 + j\ 11.75)$$
$$y_{31} = y_{13} = (-y_{13}) = (-2.94 + j\ 11.75)$$

$$Y_{BUS} = \begin{bmatrix} 5.88 - j23.46 & -2.94 + j11.75 & 0 & -2.94 + j11.75 \\ -2.94 + j11.75 & 8.82 - j35.19 & -2.94 + j11.75 & -2.94 + j11.75 \\ 0 & -2.94 + j11.75 & 5.88 - j23.46 & -2.94 + j11.75 \\ -2.94 + j11.75 & -2.94 + j11.75 & -2.94 + j11.75 & 8.82 - j35.19 \end{bmatrix}$$

The slight changes in the imaginary part of the diagnal elements are due to the line charging capacitances which are not neglected here.

E 3.7 A power system consists of 4 buses. Generators are connected at buses 1 and 3 reactances of which are j0.2 and j0.1 respectively. The transmission lines are connected between buses 1-2, 1-4, 2-3 and 3-4 and have reactances j0.25, j0.5, j0.4 and j 0.1 respectively. Find the bus admittance matrix (i) by direct inspection (ii) using bus incidence matrix and admittance matrix.

Solution :

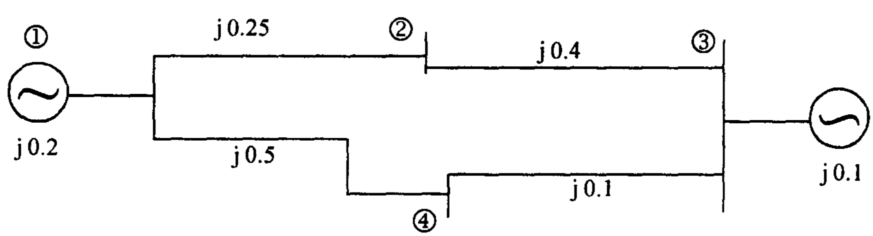


Fig. E.3.20

Taking bus (1) as reference the graph is drawn as shown in Fig.

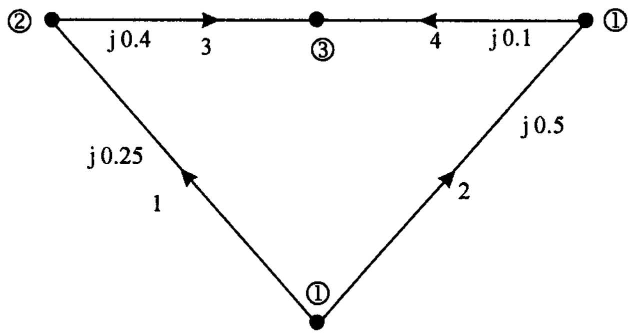


Fig. E.3.21

Only the network reactances are considered. Generator reactances are not considered.

By direct inspection :

$$Y_{BUS} = \begin{array}{c|c|c|c|c} & (1) & (2) & (3) & (4) \\ \hline 1 & \frac{1}{j0.25}+\frac{1}{j0.5} & -\frac{1}{j0.25} & 0 & -\frac{1}{j0.5} \\ \hline 2 & -\frac{1}{j0.25} & \frac{1}{j0.4}+\frac{1}{j0.25} & -\frac{1}{j0.4} & 0 \\ \hline 3 & 0 & -\frac{1}{j0.4} & \frac{1}{j0.4}+\frac{1}{j0.1} & -\frac{1}{j0.1} \\ \hline 4 & -\frac{1}{j0.5} & 0 & -\frac{1}{j0.1} & \frac{1}{j0.1}+\frac{1}{j0.5} \end{array}$$

This reduces to

$$Y_{BUS} = \begin{array}{c|c|c|c|c} & (1) & (2) & (3) & (4) \\ \hline 1 & -j6.0 & j4.0 & 0 & j2.0 \\ \hline 2 & j4.0 & -j6.5 & j2.5 & 0 \\ \hline 3 & 0 & j2.5 & -j12.5 & j10 \\ \hline 4 & +j2 & 0 & j10 & -j12 \end{array}$$

Deleting the reference bus (1)

$$Y_{BUS} = \begin{array}{c|c|c|c} & (2) & (3) & (4) \\ \hline (2) & -j6.5 & j2.5 & 0 \\ \hline (3) & j2.5 & -j12.5 & j10 \\ \hline (4) & 0 & j10 & -j12.0 \end{array}$$

By singular transformation

The primitive impedance matrix

$$[z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.25 & 0 & 0 & 0 \\ 0 & j0.5 & 0 & 0 \\ 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & j0.1 \end{bmatrix} \end{array}$$

The primitive admittance matrix is obtained by taking the reciprocals of z elements since there are no matrices.

$$[y] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -j4 & & & \\ & j2 & & \\ & & -j2.5 & \\ & & & -j10 \end{bmatrix} \end{array}$$

The bus incidence matrix is from the graph

A =	(2)	(3)	(4)
1	−1	0	0
2	0	0	−1
3	+1	−1	0
4	0	−1	+1

and $$y.A = \begin{bmatrix} j4 & 0 & 0 \\ 0 & 0 & j2 \\ -j2.5 & j2.5 & 0 \\ 0 & j10 & -j10 \end{bmatrix}$$

$$A^t y A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} j4 & 0 & 0 \\ 0 & 0 & j2 \\ -j2.5 & j2.5 & 0 \\ 0 & j10 & -j10 \end{bmatrix} = \begin{bmatrix} -j6.5 & j2.5 & 0 \\ j2.5 & -j12.5 & j10 \\ 0 & j10 & -j12.0 \end{bmatrix}$$

E 3.8 For the system shown in figure for m Y_{BUS}.

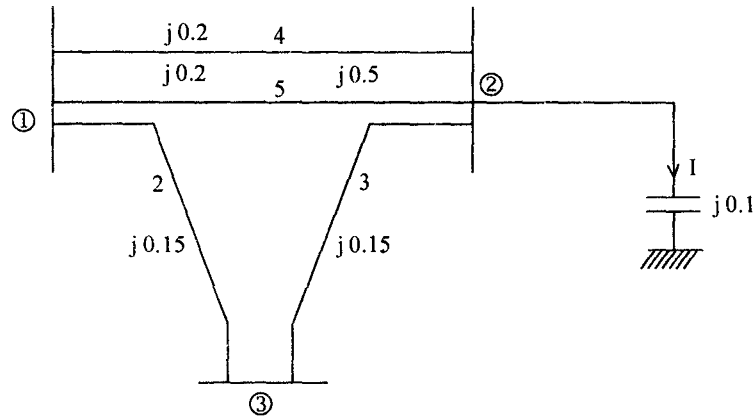


Fig. E.3.22

Solution :

Solution is obtained using singular transformation

The primitive admittance matrix is obtained by inverting the primitive impedance as

$$[y] = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 6.66 & 0 & 0 & 0 \\ 0 & 0 & 6.66 & 0 & 0 \\ 0 & 0 & 0 & -0.952 & 2.381 \\ 0 & 0 & 0 & 2.831 & -0.952 \end{bmatrix} \end{array}$$

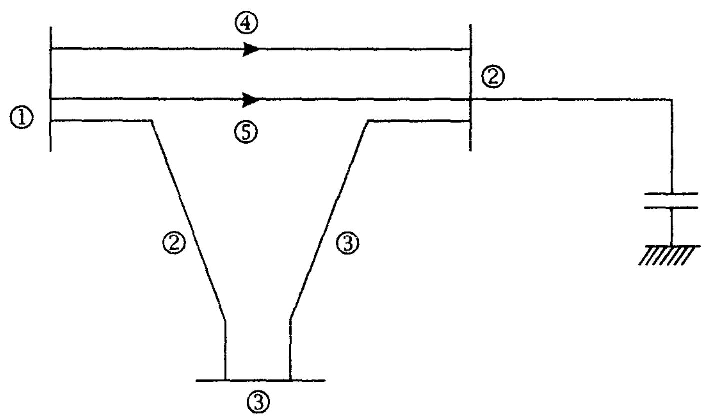


Fig. E.3.23

From the graph shown in figure the element-node incidence matrix is given by

$$\bar{A} = \begin{array}{c|cccc} e \backslash node & 0 & 1 & 2 & 3 \\ \hline 1 & -1 & 0 & +1 & 0 \\ 2 & 0 & +1 & -1 & 0 \\ 3 & 0 & +1 & -1 & 0 \\ 4 & 0 & -1 & 0 & +1 \\ 5 & 0 & 0 & -1 & +1 \end{array}$$

Taking bus zero as reference and eliminating its column the bus incidence matrix A is given by

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} 0 & +1 & 0 \\ +1 & -1 & 0 \\ +1 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & -1 & +1 \end{bmatrix} \end{array}$$

$$[y] = \begin{matrix} 1 \\ 2 \end{matrix} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 6.66 & 0 & 0 & 0 \\ 0 & 0 & 6.66 & 0 & 0 \\ 0 & 0 & 0 & -0.952 & 2.381 \\ 0 & 0 & 0 & 2.831 & -0.952 \end{bmatrix} \bullet \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

The bus admittance matrix Y_{BUS} is obtained from

$$Y_{BUS} = A^t \, y \, A$$

$$= \begin{bmatrix} 0 & +1 & +1 & -1 & 0 \\ +1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & +1 & +1 \end{bmatrix} \bullet \begin{bmatrix} 0 & 10 & 0 \\ -6.66 & 0 & 6.66 \\ 0 & -6.66 & 6.66 \\ 1.42 & -1.42 & 0 \\ 1.42 & -1.42 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9.518 & -2.858 & -6.66 \\ -2.8580 & 19.51 & -6.66 \\ -6.66 & -6.66 & 13.32 \end{bmatrix}$$

Problems

P 3.1 Determine Z_{LOOP} for the following network using basic loop incidence matrix.

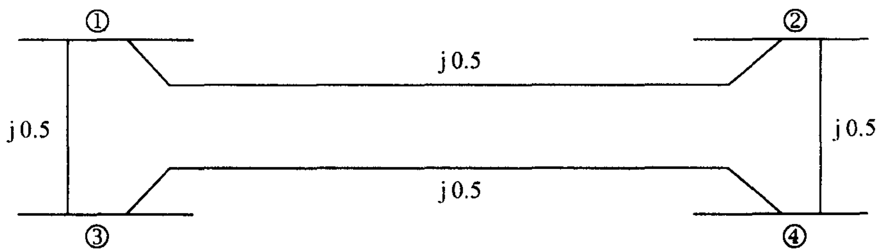


Fig. P.3.1

P 3.2 Compute the bus admittance matrix for the power shown in figure by (i) direct inspection method and (ii) by using singular transformation.

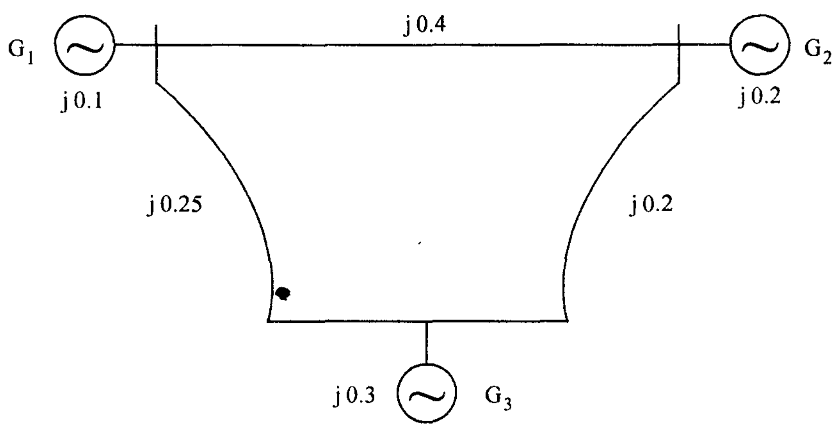


Fig. P.3.2

Questions

3.1 Derive the bus admittance matrix by singular transformation

3.2 Prove that $Z_{BUS} = K^t Z_{BR} K$

3.3 Explain how do you form Y_{BUS} by direct inspection with a suitable example.

3.4 Derive the expression for bus admittance matrix Y_{BUS} in terms of primitive admittance matrix and bus incidence matrix.

3.5 Derive the expression for the loop impedance matrix Z_{LOOP} using singular transformation in terms of primitive impedance matrix Z and the basic loop incidence matrix C.

3.6 Show that $Z_{LOOP} = C^t [z] C$

3.7 Show that $Y_{BR} = B^t [y] B$ where [y] is the primitive admittance matrix and B is the basic cut set matrix

3.8 Prove that $Z_{BR} = A_B Z_{BUS} A_B^T$ with usual notation

3.9 Prove that $Y_{BR} = K Y_{BUS} K^t$ with usual notation

4 BUILDING OF NETWORK MATRICES

Introduction

In Chapter 4 methods for obtaining the various network matrices are presented. These methods basically depend upon incidence matrices. A, B, C, K and $\tilde{B}, \tilde{C}$ for singular and non-singular transformation respectively. Thus, the procedure for obtaining Y or Z matrices in any frame of reference requires matrix transformations involving inversions and multiplications. This could be a very laborious and time consuming process for large systems involving hundreds of nodes. It is possible to build the Z_{bus} by using an algorithm where in systematically element by element is considered for addition and build the complete network directly from the element parameters. Such an algorithm would be very convenient for various manipulations that may be needed while the system is in operation such as addition of lines, removal of lines and change in parameters.

The basic equation that governs the performance of a network is

$$\bar{V}_{BUS} = [Z_{BUS}].\bar{I}_{BUS}$$

4.1 Partial Network

In order to build the network element by element, a partial network is considered. At the beginning to start with, the building up of the network and its Z_{BUS} or Y_{BUS} model a single element 1 is considered. Further, this element having two terminals connected to two nodes say (1) and (2) will have one of the terminals as reference or ground. Thus if node (1) is the reference then the element will have its own self impedance as Z_{BUS}. When we connect any other element 2 to this element 1, then it may be either a branch or a link. The branch is connected in series with the existing node either (1) or (2) giving rise to a third node (3). On the contrary, a link is connected across the terminals (1) and (2) parallel to element 1. This is shown in (Fig. 4.1).

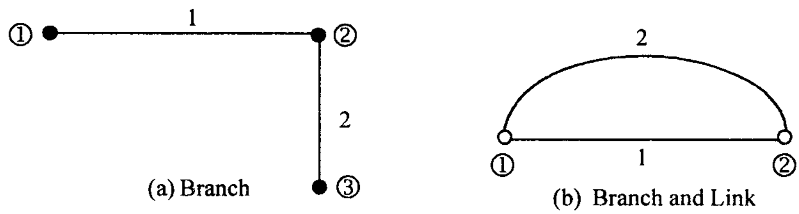


Fig. 4.1

In this case no new bus is formed. The element 1 with nodes (1) and (2) is called the partial network that is already existing before the branch or link 2 is connected to the element. We shall use the notation (a) and (b) for the nodes of the element added either as a branch or as a link. The terminals of the already existing network will be called (x) and (y). Thus, as element by element is added to an existing network, the network already in existence is called the partial network, to which, in step that follows a branch or a link is added. Thus generalizing the process consider m buses or nodes already contained in the partial network in which (x) and (y) are any buses (Fig. 4.2).

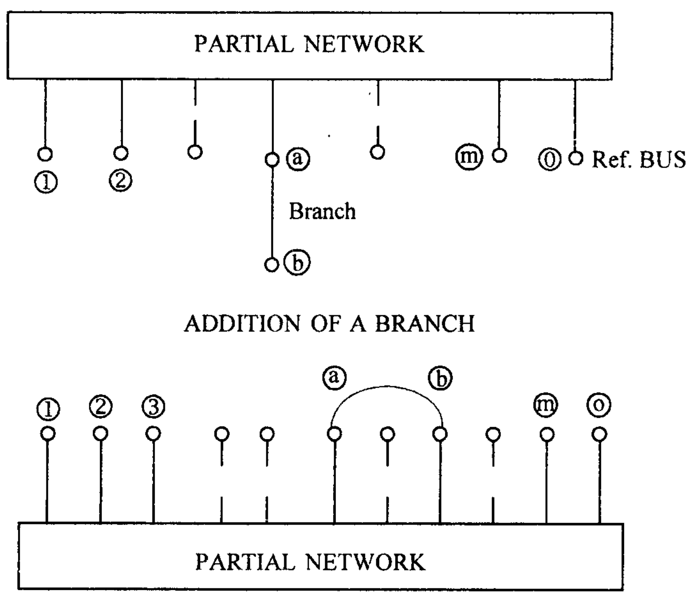


Fig. 4.2

Recalling Eqn. (4.1)

$$\bar{V}_{BUS} = [Z_{BUS}]\bar{I}_{BUS}$$

in the partial network $[Z_{BUS}]$ will be of $[m \times m]$ dimension while $\bar{V}_{BUS}$ and $\bar{I}_{BUS}$ will be of $(m \times 1)$ dimension.

The voltage and currents are indicated in (Fig. 4.3)

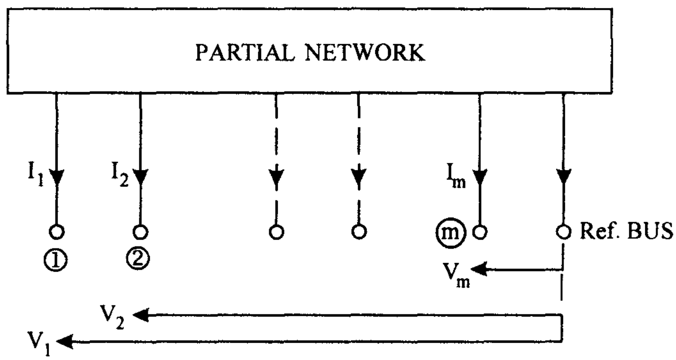


Fig. 4.3 Partial Network.

The performance equation (4.1) for the partial network is represented in the matrix form as under.

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdots \\ \cdots \\ \cdots \\ V_m \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ \cdots \\ \cdots \\ I_m \end{bmatrix} \qquad \ldots\ldots(4.1)$$

4.2 Addition of a Branch

Consider an element a-b added to the node (a) existing in the partial network. An additional node (b) is created as in (Fig. 4.4)

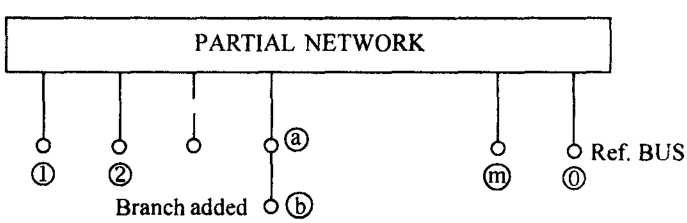


Fig. 4.4 Addition of a Branch.

The performance equation will be

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdots\cdots \\ V_a \\ \cdots\cdots \\ V_m \\ \hline V_b \end{bmatrix} = \left[\begin{array}{ccccccc|c} Z_{11} & Z_{12} & \cdots & Z_{1a} & \cdots & Z_{1m} & & Z_{1b} \\ Z_{21} & Z_{22} & \cdots & Z_{2a} & \cdots & Z_{2m} & & Z_{2b} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ Z_{a1} & Z_{a2} & \cdots & Z_{aa} & \cdots & Z_{am} & & Z_{ab} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{ma} & \cdots & Z_{mm} & & Z_{mb} \\ \hline Z_{b1} & Z_{b2} & \cdots & Z_{ba} & \cdots & Z_{bm} & & Z_{bb} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \cdots\cdots \\ I_a \\ \cdots\cdots \\ I_m \\ \hline I_b \end{bmatrix} \qquad \ldots\ldots(4.2)$$

The last row and the last column in the Z-matrix are due to the added node b

$Z_{bi} = Z_{ib}$............. where i = 1,2,...............m for all passive bilateral elements.

The added branch element a-b may have mutual coupling with any of the elements of the partial network.

Calculation of Mutual Impedances

It is required to find the self and mutual impedance elements of the last row and last column of eq. (4.2). For this purpose a known current, say I = 1 p.u. is injected into bus K and the voltage is measured as shown in (Fig. 4.5) at all other buses. We obtain the relations.

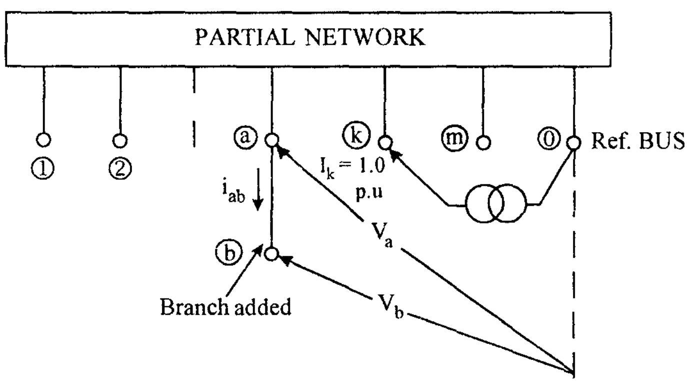


Fig. 4.5 Partial Network with Branch Added (Calculations of Mutual Impedances).

$$\left.\begin{aligned} V_1 &= Z_{1k} I_k \\ V_2 &= Z_{2k} I_k \\ &\cdots\cdots\cdots\cdots \\ V_a &= Z_{ak} I_k \\ &\cdots\cdots\cdots\cdots \\ V_m &= Z_{mk} I_k \\ &\cdots\cdots\cdots\cdots \\ V_b &= Z_{bk} I_k \end{aligned}\right\} \qquad \ldots\ldots(4.3)$$

Since I_k is selected as 1 p.u. and all other bus currents are zero.

Z_{bk} can be known setting I_k=1.0, from the measured value of V_b.

$$\left.\begin{array}{l} V_1 = Z_{1k} \\ V_2 = Z_{2k} \\ \dots\dots\dots\dots \\ V_a = Z_{ak} \\ \dots\dots\dots\dots \\ V_m = Z_{mk} \\ \dots\dots\dots\dots \\ V_b = Z_{bk} \end{array}\right\} \qquad \text{.....(4.3a)}$$

We have that

$$V_b = V_a - e_{ab} \qquad \text{.....(4.4)}$$

Also,

$$\begin{bmatrix} i_{ab} \\ i_{xy} \end{bmatrix} = \begin{bmatrix} y_{ab-ab} & y_{ab-xy} \\ y_{xy-ab} & y_{xy-xy} \end{bmatrix} \begin{bmatrix} e_{ab} \\ e_{xy} \end{bmatrix} \qquad \text{.....(4.5)}$$

y_{ab-ab} = self admittance of added branch a-b

$\bar{y}_{ab-xy}$ = mutual admittance between added branch ab and the elements x-y of partial network

$\bar{y}_{xy-ab}$ = transpose of y_{ab-xy}

y_{xy-xy} = primitive admittance of the partial network

i_{ab} = current in element a-b

e_{ab} = voltage across the element a-b

It is clear from the (Fig. 4.5) that

$$i_{ab} = 0 \qquad \text{.....(4.6)}$$

But, e_{ab} is not zero, since it may be mutually coupled to some elements in the partial network.

Also,

$$e_{xy} = V_x - V_y \qquad \text{.....(4.7)}$$

where V_x and V_y are the voltages at the buses x and y in the partial network.

The current in a-b

$$i_{ab} = y_{ab-ab}\, e_{ab} + \bar{y}_{ab-xy}\, \bar{e}_{xy} = 0 \qquad \text{.....(4.8)}$$

From equation (4.6)

$$y_{ab\text{-}ab}\, e_{ab} + \bar{y}_{ab\text{-}xy}\, \bar{e}_{xy} = 0$$

$$e_{ab} = \frac{-\bar{y}_{ab-xy}\, \bar{e}_{xy}}{y_{ab-ab}} \qquad \text{.....(4.9)}$$

substituting equation (4.7)

$$e_{ab} = \frac{-\bar{y}_{ab-xy}(\bar{V}_x - \bar{V}_y)}{y_{ab-ab}} \qquad \text{.....(4.10)}$$

From equation (4.4)

$$V_b = V_a + \frac{\bar{y}_{ab-xy}(\bar{V}_x - \bar{V}_y)}{y_{ab-ab}} \qquad \text{.....(4.11)}$$

Using equation (4.3) a general expression for the mutual impedance Z_{bi} between the added branch and other elements becomes

$$\boxed{\begin{array}{l} Z_{bi} = Z_{ai} + \dfrac{\bar{y}_{ab-xy}(\bar{Z}_{xi} - \bar{Z}_{yi})}{y_{ab-ab}} \\ i = 1,2,\ldots\ldots.,m; \quad i \neq b \end{array}} \qquad \text{.....(4.12)}$$

Calculation of self impedance of added branch Z_{ab}.

In order to calculate the self impedance Z_{bb} once again unit current $I_b = 1$ p.u. will be injected into bus b and the voltages at all the buses will be measured. Since all other currents are zero.

$$\left.\begin{array}{l} V_1 = Z_{1b} I_b \\ V_2 = Z_{2b} I_b \\ \ldots\ldots\ldots\ldots\ldots \\ V_a = Z_{ab} I_b \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ V_m = Z_{mb} I_b \\ V_q = Z_{bb} I_b \end{array}\right\} \qquad \text{.....(4.13)}$$

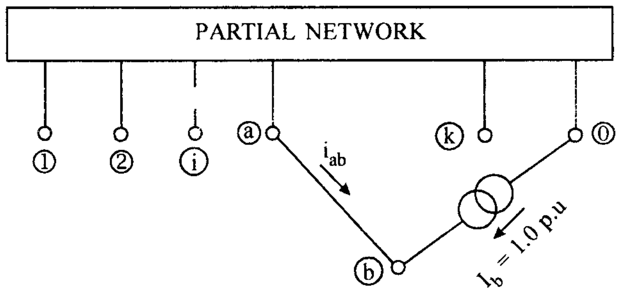


Fig. 4.6 Partial Network with Branch Added (Calculations of Self Impedance).

The voltage across the elements of the partial network are given by equation (4.7). The currents are given by equation (4.5)

Also, $\quad i_{ab} = -I_b \quad = -1$ .....(4.14)

From equation (4.8)

$$i_{ab} = y_{ab-ab}e_{ab} + \bar{y}_{ab-xy}\bar{e}_{xy} = -1$$

But $\quad \bar{e}_{xy} = V_x - V_y$

Therefore $\quad -1 = y_{ab-ab}\, e_{ab} + y_{eb-xy}\,(V_x - V_y)$

Hence $$e_{ab} = \left[\frac{-1 + \bar{y}_{ab-xy}(\bar{V}_x - \bar{V}_y)}{y_{ab-ab}}\right] \quad .....(4.15)$$

Note : $\quad \bar{V}_x - \bar{V}_y = Z_{xb}I_b - Z_{yb}\,I_b$

$= (Z_{xb} - Z_{yb})\,I_b$

$= (Z_{xb} - Z_{yb})$ .....(4.16)

substituting from equation (4.16) into (4.15)

$$e_{ab} = \frac{-\left[1 + \bar{y}_{ab-xy}(Z_{xb} - Z_{yb})\right]}{y_{ab-ab}} \quad .....(4.16a)$$

From equation (4.4) $V_b = V_a - e_{ab}$

Therefore $$V_b = V_a + \frac{\left[1 + \bar{y}_{ab-xy}(Z_{xb} - Z_{yb})\right]}{y_{ab-ab}}$$

y_{ab-ab} is the self admittance of branch added a-b also

$\bar{y}_{ab-xy}$ is the mutual admittance vector between a-b and x-y.

$$V_b = Z_{bb}\, I_b,$$
$$Va = Z_{ab}\, I_b,$$
and $$I_b = 1 \text{ p.u.}$$

Hence, $$Z_{bb} = Z_{ab} + \frac{1 + \bar{y}_{ab-xy}(\bar{Z}_{xb} - \bar{Z}_{yb})}{y_{ab-ab}} \quad .....(4.17)$$

Special Cases :

If there is no mutual coupling from equation (4.12)

with $$Z_{ab-ab} = \frac{1}{y_{ab-ab}}$$

And since $$\bar{y}_{ab-xy} = \bar{0}$$

$$\boxed{Z_{bi} = Z_{ai}}; \quad i = 1,2,\ldots\ldots\ldots\ldots\ldots, m; \; i \neq b$$

From equation (4.17)

$$\boxed{Z_{bb} = Z_{ab} + z_{ab-ab}}$$

If there is no mutual coupling and a is the reference bus equation (4.12) further reduces to

with $$Z_{ai} = 0 \qquad i = 1,2,\ldots\ldots\ldots\ldots\ldots,m \; ; i \neq b$$
$$Z_{bi} = 0$$
From eqn. (4.17)
$$Z_{ab} = 0$$
and $$Z_{bb} = Z_{ab-ab}.$$

4.3 Addition of a Link

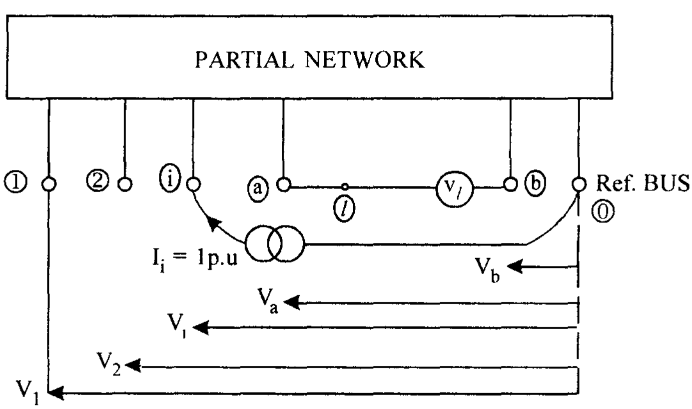


Fig. 4.7 Addition of a Link (Calculation of Mutual Impedance).

Consider the partial network shown in (Fig. 4.7)

Consider a link connected between a and b as shown.

The procedure for building up Z_{BUS} for the addition of a branch is already developed. Now, the same method will be used to develop an algorithm for the addition of a link. Consider a fictitious node l between a and b. Imagine an voltage source V_l in series with it between l and b as shown in figure (4.6).

Voltage V_l is such that the current through the link ab (ie) $i_{ab} = 0$

e_{ab} = voltage across the link a-b

V_l = source voltage across l-b = e_{lb}

Thus we may consider that a branch a-l is added at the node (a) since the current through the link is made zero by introducing a source voltage V_l.

Now consider the performance equation

$$\bar{E}_{BUS} = [Z_{BUS}].\bar{I}_{BUS} \qquad .....(4.18)$$

once again the partial network with the link added

$$\left[\begin{array}{c} V_1 \\ V_2 \\ \cdots \\ V_i \\ \cdots \\ V_a \\ \cdots \\ V_m \\ \hline V_l \end{array}\right] = \begin{array}{c} \\ 1 \\ 2 \\ \cdots \\ i \\ \cdots \\ a \\ \cdots \\ m \\ l \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & \cdots & i & \cdots & a & \cdots & m & l \end{array} \\ \left[\begin{array}{cccccccc|c} Z_{11} & Z_{12} & \cdots & Z_{1i} & \cdots & Z_{1a} & \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2a} & \cdots & Z_{2m} & Z_{2l} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{ia} & \cdots & Z_{im} & Z_{il} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{a1} & Z_{a2} & \cdots & Z_{i2} & \cdots & Z_{aa} & \cdots & Z_{am} & Z_{al} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mi} & \cdots & Z_{ma} & \cdots & Z_{mm} & Z_{ml} \\ \hline Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{la} & \cdots & Z_{lm} & Z_{ll} \end{array}\right] \end{array} \bullet \left[\begin{array}{c} I_1 \\ I_2 \\ \cdots \\ I_i \\ \cdots \\ I_a \\ \cdots \\ I_m \\ \hline I_l \end{array}\right] \qquad .....(4.19)$$

Also, the last row and the last column in Z-matrix are due to the added fictitious node l.

$$v_l = V_l - V_b \qquad .....(4.20)$$

Calculation of Mutual Impedances

The element Z_{li}, in general, can be determined by injecting a current at the i_{th} bus and measuring the voltage at the node i with respect to bus b. Since all other bus currents are zero we obtain from the above consideration.

$$V_l = Z_{li}\, I_i;\ k = 1, 2, \ldots\ldots\ldots\ldots m \qquad .....(4.21)$$

and $$V_l = Z_{li}\, I_i \qquad .....(4.22)$$

Letting $I_l = 1.0$ p.u. Z_{ll} can be seen as v_l which is the same as v_l

But $\quad v_l = V_a - V_b - e_{al}$ .....(4.23)

It is already stated that the current through the added link $i_{ab} = 0$

Treating a-l as a branch, current in this element in terms of primitive admittances and the voltages across the elements

$$i_{al} = y_{al-al} \cdot e_{al} + \bar{y}_{al-xy} \cdot e_{xy} \qquad .....(4.24)$$

Where

$\bar{y}_{al-xy}$ are the mutual admittances of any element x-y in the partial network with respect to al and e_{xy} is the voltage across the element x-y in the partial network.

But $\quad i_{al} = i_{ab} = 0$

Hence, equation (4.23) gives

$$e_{al} = \left(\frac{-\bar{y}_{al-xy} \cdot \bar{e}_{xy}}{y_{al-al}} \right) \qquad .....(4.24)$$

Note that $\quad y_{al-al} = y_{ab-ab}$ .....(4.26)

and $\quad \bar{y}_{al-xy} = \bar{y}_{ab-xy}$ .....(4.27)

Therefore $$e_{al} = \left(\frac{-\bar{y}_{ab-xy} \cdot \bar{e}_{xy}}{y_{ab-ab}} \right) \qquad .....(4.28)$$

$$v_l = v_a - v_b + \frac{\bar{y}_{ab-xy} \, \bar{e}_{xy}}{y_{ab-ab}}$$

(i.e.) $$Z_{li} I_l = Z_{ai} I_l - Z_{bi} I_l + \frac{\bar{y}_{ab-xy} \, \bar{v}_x - \bar{v}_y}{y_{ab-ab}}$$

since $\quad I_l = 1.0$ p.u.

$i = 1, \ldots\ldots\ldots\ldots\ldots, m$

$i \neq l$

Also, $\quad \bar{e}_{xy} = \bar{v}_x - \bar{v}_y$ .....(4.29)

$$Z_{li} = Z_{ai} - Z_{bi} + \frac{\bar{y}_{ab-xy} \left(Z_{xi} - Z_{yi} \right) I_l}{y_{ab-ab}}$$

Thus, using equation 4.20 and putting $I_l = 1.0$ p.u.

$$Z_{li} = Z_{ai} - Z_{bi} + \left(\frac{\bar{y}_{ab-xy}(\bar{Z}_{xi} - \bar{Z}_{yi})}{y_{ab-ab}} \right) \quad \text{.....(4.30)}$$

$i = 1,2,\ldots\ldots\ldots\ldots, m$

$i \neq l$

In this way, all the mutual impedance in the last row and last column of equation (4.19) can be calculated.

Computation of Self impedance

Now, the value of Z_{ll}, the self impedance in equation (4.30) remains to be computed. For this purpose, as in the case of a branch, a unit current is injected at bus l and the voltage with respect to bus b is measured at bus l. Since all other bus currents are zero.

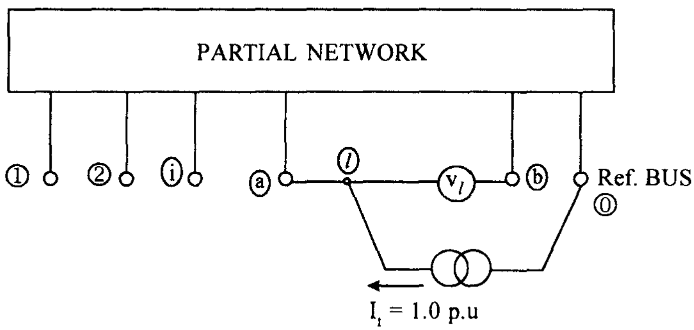


Fig. 4.8 Addition of a Link (Calculation of Self Impedance).

$$V_k = Z_{kl} I_l; \quad k = 1, 2\ldots\ldots\ldots\ldots, m \quad \text{.....(4.31)}$$

and $$v_l = Z_{ll}\, I_l \quad \text{.....(4.32)}$$

But $$I_l = 1 \text{ p.u.} = -i_{al} \quad \text{.....(4.33)}$$

The current i_{al} in terms of the primitive admittances and voltages across the elements

$$i_{al} = y_{al-al}\, e_{al} + \bar{y}_{al-xy}\, \bar{e}_{xy} \quad \text{.....(4.34)}$$

$$= -1$$

Again, as

$$\bar{y}_{al-xy} = \bar{y}_{ab-xy} \text{ and} \quad \text{.....(4.35)}$$

$$y_{al-al} = y_{ab-ab} \quad \text{.....(4.36)}$$

Then, from eqn. (4.34) $-1 = y_{al-al}\, e_{al} + \bar{y}_{ab-xy}\, \bar{e}_{xy}$

$$e_{al} = \frac{-(1 + \bar{y}_{ab-xy}\,\bar{e}_{xy})}{y_{ab-ab}} \qquad \text{......(4.37)}$$

Substituting

$$e_{xy} = V_x - V_y$$
$$= \bar{Z}_{xl} I_l - \bar{Z}_{yl} \cdot I_l$$
$$= Z_{xl} - Z_{yl} \text{ (since } I_l = 1.0 \text{ p.u)}$$

$$Z_{ll} = Z_{al} - Z_{bl} + \frac{1 + \bar{y}_{ab-xy}(\bar{Z}_{xl} - \bar{Z}_{yl})}{y_{ab-ab}} \qquad \text{.....(4.38)}$$

Case (i) : no mutual impedance

If there is no mutual coupling between the added link and the other elements in the partial network

$$\bar{y}_{ab-xy} \text{ are all zero}$$

$$\frac{1}{y_{ab-ab}} = z_{ab-ab} \qquad \text{.....(4.39)}$$

Hence we obtain

$$Z_{li} = Z_{ai} - Z_{bi};\ i = 1, 2, \ldots\ldots\ldots\ldots m \qquad \text{.....(4.40)}$$
$$i \neq l$$

$$Z_{ll} = Z_{al} - Z_{bl} + z_{ab\text{-}ab} \qquad \text{.....(4.41)}$$

Case (ii) : no mutual impedance and a is reference node

If there is no mutual coupling and a is the reference node

$$\left.\begin{aligned} Z_{al} &= 0; \\ Z_{li} &= -Z_{bi} \end{aligned}\right\} \qquad \text{.....(4.42)}$$

Also $$Z_{ll} = -Z_{bl} + z_{ab\text{-}ab} \qquad \text{.....(4.43)}$$

Thus all the elements introduced in the performance equation of the network with the link added and node l created are determined.

It is required now to eliminate the node l.

For this, we short circuit the series voltage source v_l, which does not exist in reality From eqn. (4.19)

$$\bar{V}_{BUS} = [Z_{BUS}]\bar{I}_{BUS} + \bar{Z}_{il} I_l \qquad .....(4.44)$$

$$V_l = \bar{Z}_{lj}.\bar{I}_{BUS} + Z_{ll} I_l; \; j = 1,2,................, m$$

$$= 0 \text{ (since the source is short circuited)} \qquad .....(4.45)$$

Solving for I_l from equation (4.45)

$$I_l = \frac{-\bar{Z}_{lj}.\bar{I}_{BUS}}{Z_{ll}} \qquad .....(4.46)$$

Substituting in equation (4.44)

$$V_{BUS} = [Z_{BUS}]\; \bar{I}_{BUS} - \frac{Z_{il}.Z_{lj}}{Z_{ll}} \bar{I}_{BUS}$$

$$= \left[Z_{BUS} - \frac{Z_{il}.Z_{lj}}{Z_{ll}} \right] I_{BUS} \qquad .....(4.47)$$

This is the performance equation for the partial network including the link a-b incorporated.

From equation (4.47) we obtain

$$Z_{BUS}\text{ (modified)} = \left[Z_{BUS}\text{(before addition of link)} - \frac{\bar{Z}_{il}\,\bar{Z}_{lj}}{Z_{ll}} \right]$$

and for any element

$$Z_{ij}\text{(modified)} = \left[Z_{ij}\text{(before addition of link} - \frac{Z_{il} Z_{lj}}{Z_{ll}} \right] \qquad .....(4.48)$$

Removal of Elements or Changes in Element

Consider the removal of an element from a network. The modified impedance can be obtained by adding in parallel with the element a link, whose impedance is equal to the negative of the impedance to be removed.

In a similar manner, if the impedance of the element is changed, then the modified impedance matrix obtained by adding a link in parallel with the element such that the equivalent impedance of the two elements is the desired value.

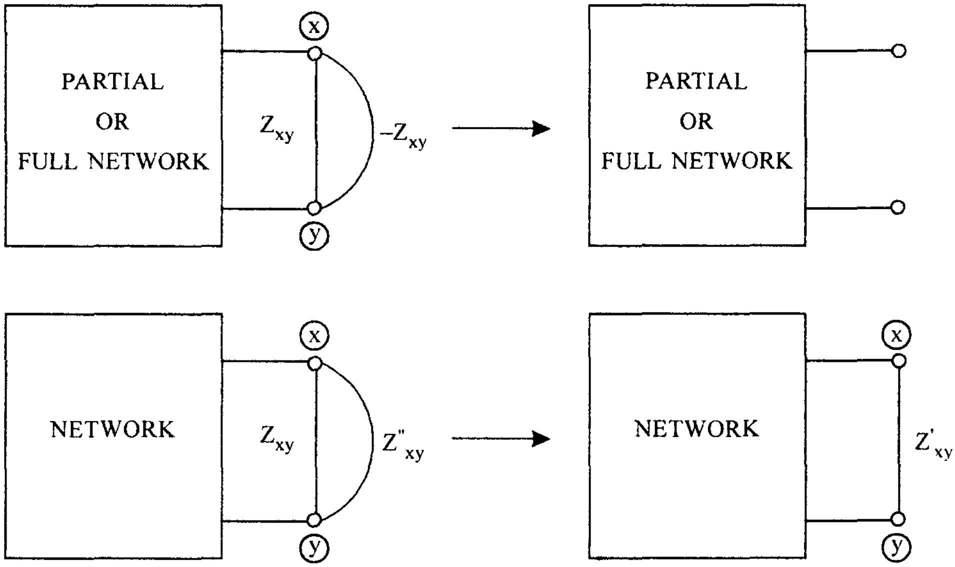


Fig. 4.9 Removal or Change in Impedance of an Element.

$$Z_{xy} \text{ changed to } Z'_{xy}$$

$$\frac{1}{Z'_{xy}} = \frac{1}{Z_{xy}} + \frac{1}{Z''_{xy}}$$

However, the above are applicable only when there is no mutual coupling between the element to be moved or changed with any element or elements of partial network.

4.4 Removal or Change in Impedance of Elements with Mutual Impedance

Changes in the configuration of the network elements introduce changes in the bus currents of the original network. In order to study the effect of removal of an element or changes in the impedance of the element can be studied by considering these changes in the bus currents.

The basic bus voltage relation is

$$\bar{V}_{BUS} = [Z_{BUS}]\bar{I}_{BUS}$$

with changes in the bus currents denoted by the vector $\Delta\bar{I}_{BUS}$ the modified voltage performance relation will become

$$\bar{V}'_{BUS} = [Z'_{BUS}]\left(\bar{I}_{BUS} + \Delta\bar{I}_{BUS}\right) \qquad .....(4.49)$$

$\overline{V}_{BUS}$ is the new bus voltage vector. It is desired now to calculate the impedances Z_{ij}^{1} of the modified impedance matrix $\left[\overline{Z}_{BUS}^{1}\right]$.

The usual method is to inject a known current (say 1 p.u) at the bus j and measure the voltage at bus i.

Consider an element p-q in the network. Let the element be coupled to another element in the network r-s. If now the element p-q is removed from the network or its impedance is changed then the changes in the bus currents can be represented by

$$\left.\begin{aligned} \Delta I_p &= \Delta i_{pq} \\ \Delta I_q &= -\Delta i_{pq} \\ \Delta I_r &= \Delta i_{rs} \\ \Delta I_s &= -\Delta i_{rs} \end{aligned}\right\} \qquad .....(4.50)$$

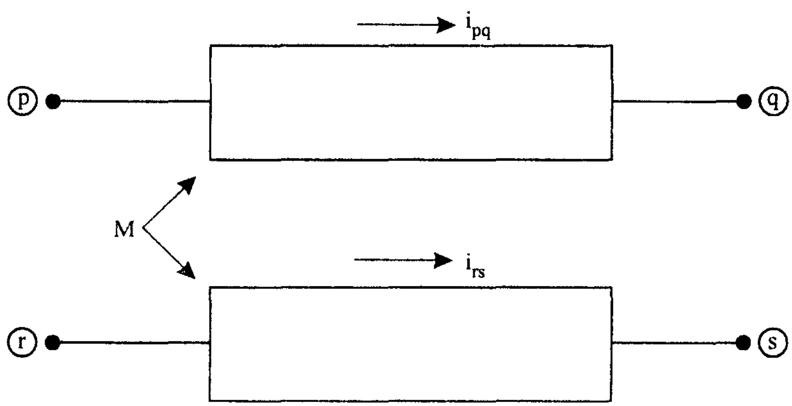


Fig. 4.10

Inject a current of 1 p.u. at any j^{th} bus

$$I_j = 1.0$$
$$I_k = 0 \; [k = 1, 2, \ldots\ldots\ldots\ldots, n; \; k \neq j]$$

Then

$$V_i^1 = \sum_{k=1}^{n} Z_{ik}\left(I_k + \Delta I_k\right)$$

i = 1, 2..................n. With the index k introduced, equation may be understood from (Fig. 4.10).

$$\left.\begin{aligned}\Delta I_k &= \Delta i_{pq}\,; k = p\\ \Delta I_k &= -\Delta i_{pq}\,; k = q\\ \Delta I_k &= \Delta i_{rs}\,; k = r\\ \Delta I_k &= -\Delta i_{rs}\,; k = s\end{aligned}\right\} \qquad .....(4.50(a))$$

From equation

$$V_i^1 = Z_{ij}^1 \bullet 1 + Z_{ij}\,\Delta i_{pq} - Z_{iq}\Delta i_{pq} + Z_{ir}\,\Delta i_{rs} - Z_{is}\Delta i_{rs}$$

$$= Z_{ij} + \left(Z_{ip} - Z_{iq}\right)\Delta i_{pq} + \left(Z_{ir} - Z_{is}\right)\Delta\, i_{rs} \qquad .....(4.51)$$

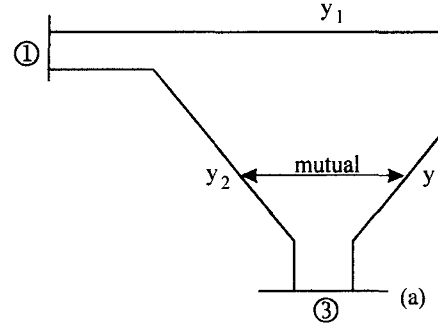
If α, β are used as the subscripts for the elements of both p-q and r–s then

$$V_i^1 = Z_{ij} + \left(Z_{i\alpha} - Z_{i\beta}\right)\Delta\bar{i}_{\alpha\beta} \quad i = 1, 2, 3 \ldots\ldots\ldots\ldots, n \qquad .....(4.52)$$

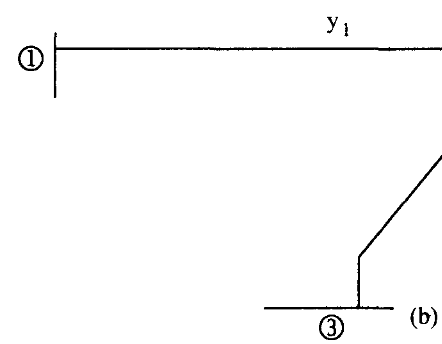
From the performance equation of the primitive network.

$$\Delta i_{\alpha\beta} = \left(\left[y_{sm}\right] - \left[y_{sm}\right]^{-1}\right) v_{\gamma\delta} \qquad .....(4.53)$$

Where $[y_{sm}]$ and $[y_{sm}]^{-1}$ are the square sub matrices of the original and modified primitive admittance matrices. Consider as an example, the sytem in Fig 4.11(a), y matrix is shown. If y_2 element is removed, the y^1 matrix is shown in Fig. 4.11(b).



$$y = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array}\begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & y_{23} \\ 0 & y_{32} & y_{33} \end{bmatrix} \end{array}$$



$$y^1 = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array}\begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Fig. 4.11

Then, Y_{sm} and Y_{sm}^{I} are given : $Y_{sm} = \begin{bmatrix} y_{22} & y_{23} \\ y_{32} & y_{33} \end{bmatrix}$ and $Y'_{sm} = \begin{bmatrix} y_{22} & 0 \\ 0 & 0 \end{bmatrix}$

Thus, the rows and column of the sub matrices Y_{sm} and Y_{sm}^{I} correspond to the network elements p-q and r-s (Fig. 4.10)

The subscripts of the elements of $\left(|y_{sm}| - |y^{I}_{sm}|\right)$ are $\alpha\beta$ and $\gamma\delta$. We know that

$$v^{I}_{\gamma\delta} = v^{I}_{\gamma} - v^{I}_{\delta} \qquad .....(4.54)$$

substituting from eqn. (4.52)

for $\bar{v}^{I}_{\gamma}$ and $\bar{v}^{I}_{\delta}$

$$v_{\gamma\delta}{}^{1} = \bar{Z}_{\gamma j} - \bar{Z}_{\delta j} + ([Z_{\gamma\alpha}] - [Z_{\gamma\alpha}] - [Z_{\delta\alpha}] + [Z_{\gamma\beta}])\Delta\bar{i}_{\alpha\beta} \qquad ......(4.55)$$

Substituting from eqn.(4.55) for $v^{I}_{\gamma\delta}$ into eqn.(4.53)

$$\Delta\bar{i}_{\alpha\beta} = \left(|y_{sm}| - |y'_{sm}|\right)\left(Z_{\gamma j} - Z_{\delta j}\right) + \left[\left(\left[Z_{\gamma\alpha}\right] - \left[Z_{\delta\alpha}\right] - \left[Z_{\gamma\beta}\right] + \left[Z_{\delta\beta}\right]\right)\right] \Delta\bar{i}_{\alpha\beta} \qquad .....(4.56)$$

Solving eqn. (5.56) for $\Delta\bar{i}_{\alpha\beta}$

$$\Delta\bar{i}_{\alpha\beta} = \left\{U - [y_{sm}] - \left[y^{I}_{sm}\right]\left[\bar{Z}_{\gamma\alpha}\right] - \left[Z_{\delta\alpha}\right] - \left[Z_{\delta\alpha}\right] + \left[Z_{\delta\beta}\right]\right\}^{-1}\left\{\left(ysm - y^{I}_{sm}\right)\left(z_{\gamma j} - z_{\delta j}\right)\right\} .....(4.57)$$

Where U is unit matrix

Designating $(\Delta y_{sm}) = [y_{sm}] - \left[y^{I}_{sm}\right]$ giving the changes in the admittance matrix .....(4.58)

and the term

$$\left\{U - [\Delta y_{sm}] \left(\left[Z_{\gamma\alpha}\right] - \left[Z_{\delta\alpha}\right] - \left[Z_{\gamma\beta}\right] - \left[Z_{\delta\beta}\right]\right)\right\}$$

by F, the multiplying factor .....(4.59)

$$\Delta\bar{i}_{\alpha\beta} = [F]^{-1}[\Delta y_{sm}]\left[\bar{Z}_{\gamma j} - \bar{Z}_{\delta j}\right] \qquad .....(4.60)$$

substituting equation (4.60) in equation (4.52)

$$V^{I}_{i} = Z_{ij} + \left(\bar{Z}_{i\alpha} - \bar{Z}_{i\beta}\right)[F]^{-1}\,[\Delta y_{sm}]^{I}\left[Z_{\gamma j} - Z_{\delta j}\right] \qquad .....(4.61)$$

The above equation gives the bus voltage V^{I}_{i} at the bus i as a result of injecting 1 p.u. current at bus j and the approximate current changes.

The ij the element of modified bus impedance matrix is then,

$$Z^{I}_{ij} = Z_{ij} + \left(\bar{Z}_{i\alpha} - \bar{Z}_{i\beta}\right)[F]^{-1}[\Delta y_{sm}]\left[\bar{Z}_{\gamma i} - \bar{Z}_{\delta j}\right]$$

The process is to be repeated for each j = 1, 2, ...n to obtain all element of Z^{I}_{BUS}.

Worked Examples

E. 4.1 A transmission line exists between buses 1 and 2 with per unit impedance 0.4. Another line of impedance 0.2 p.u. is connected in parallel with it making it a doubl-circuit line with mutual impedance of 0.1 p.u. Obtain by building algorithm method the impedance of the two-circuit system.

Solution :

Consider the system with one line

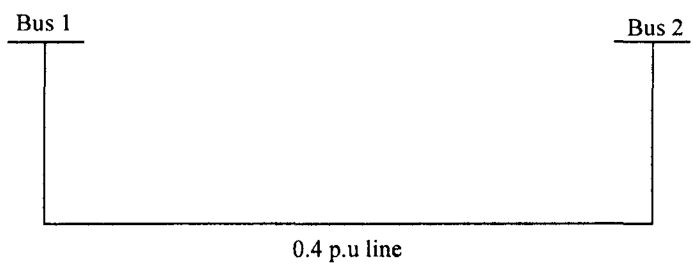


Fig. E.4.1

Taking bus (1) as reference the Z_{BUS} is obtained as

$$Z_{BUS} = \begin{array}{c|c|c} & (1) & (2) \\ \hline (1) & 0 & 0 \\ \hline (2) & 0 & 0.4 \end{array} = \begin{array}{c|c} & (2) \\ \hline (2) & 0.4 \end{array}$$

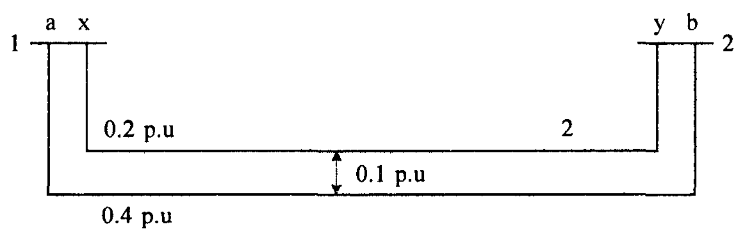


Fig. E.4.2

Now consider the addition of the second line in parallel with it

The addition of the second line is equivalent to addition of a link. The augmented impedance matrix with the fictitious node *l* introduced.

$$Z_{zl} = \begin{array}{c} \\ (2) \\ (l) \end{array} \begin{array}{c} \begin{array}{cc} (2) & (l) \end{array} \\ \begin{bmatrix} 0.4 & z_{2l} \\ z_{l2} & z_{ll} \end{bmatrix} \end{array}$$

$$Z_{ab\text{-}xy} = \begin{array}{c c} & \begin{array}{cc} 1-2(1) & 1-2(2) \end{array} \\ \begin{array}{c} 1-2(2) \\ 1-2(1) \end{array} & \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \end{array}$$

a = 1; x =1

b = 2; y = 2

$$z_{li} = z_{al} - z_{bl} + \frac{y_{ab-xy}(z_{xl} - z_{yl})}{y_{ab-ab}}$$

$$y_{ab-xy} = [z_{ab-xy}]^{-1}\left[\begin{pmatrix} 0.2 & -0.1 \\ -0.1 & 0.4 \end{pmatrix}\right] \bullet \frac{1}{(0.08-0.01)} = \begin{array}{c c} & \begin{array}{cc} 1-2(1) & 1-2(2) \end{array} \\ \begin{array}{c} 1-2(1) \\ 1-2(2) \end{array} & \begin{bmatrix} 2.857 & -1.4286 \\ -1.4286 & 5.7143 \end{bmatrix} \end{array}$$

$$z_{l_l} = z_{ai} - z_{bl} + \frac{\bar{y}_{ab-xy}\left(\bar{z}_{xl} - \bar{z}_{yl}\right)}{y_{ab-ab}}$$

$$z_{2l} = z_{l2} = 0 - 0.4 + \frac{(-1.4286)(0-0.2)}{5.7143} = -0.35$$

$$z_{ll} = z_{al} - a_{bl} + \frac{\left[1 + \bar{y}_{ab-xy}\left(\bar{z}_{xl} - \bar{z}_{yl}\right)\right]}{y_{ab-ab}} = 0-(-0.35) + \frac{[1+(-1.4286)(0-(-0.35)]}{5.7143}$$

$$= 0.35 + 0.2625 = 0.6125$$

$$z_{augmented} = \begin{array}{c c} & \begin{array}{cc} (2) & (l) \end{array} \\ \begin{array}{c} (2) \\ (l) \end{array} & \begin{bmatrix} 0.4 & -0.35 \\ -0.35 & 0.6125 \end{bmatrix} \end{array}$$

Now, eliminating the fictions bus l z_{22} (modified) = $z^{1}_{22} = z_{22} - \frac{z_{2l} \bullet z_{l2}}{z_{ll}}$

$$= 0.4 - \frac{(-0.35)(-0.35)}{0.6125} = 0.4 - 0.2 = 0.2$$

$$Z_{BUS} = \begin{array}{c c} & (2) \\ (2) & \boxed{2} \end{array}$$

E. 4.2 The double circuit line in the problem E 4.1 is further extended by the addition of a transmission line from bus (1). The new line by virtue of its proximity to the existing lines has a mutual impedance of 0.05 p.u. and a self – impedance of 0.3 p.u. obtain the bus impedance matrix by using the building algorithm.

Solution : Consider the extended system

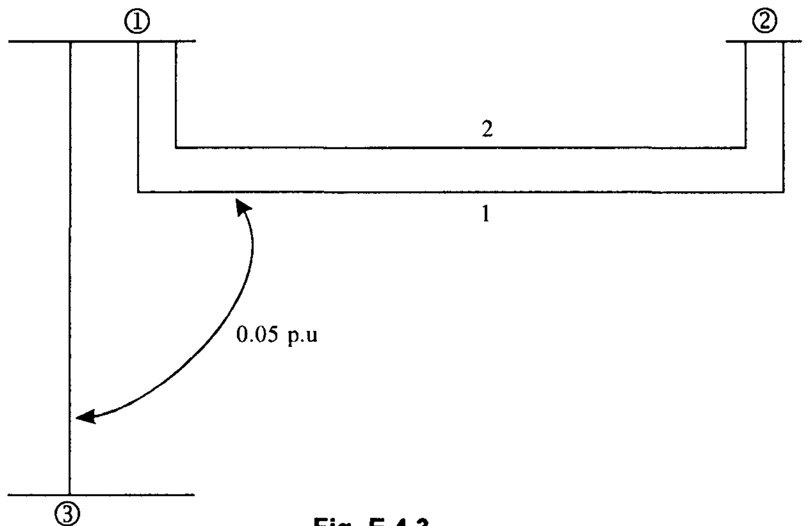


Fig. E.4.3

Now a =1 and b = 3

Also a is the reference bus

$$z_{bi} = z_{ai} + \frac{\bar{y}_{ab-xy}(\bar{z}_{xi} - \bar{z}_{yi})}{y_{ab-ab}}$$

The primitive impedance matrix $z_{ab\text{-}xy}$ is given by

$$z_{ab-xy} = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \\ 1-3 \end{array} \begin{array}{c} \begin{array}{ccc} 1-2(1) & 1-2(2) & 1-3 \end{array} \\ \begin{bmatrix} 0.4 & 0.1 & 0.05 \\ 0.1 & 0.2 & 0 \\ 0.05 & 0 & 0.3 \end{bmatrix} \end{array}$$

$$\left[y_{ab-xy}\right] = \left[z_{ab-xy}\right]^{-1}$$

$$= \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \\ 1-3 \end{array} \begin{array}{c} \begin{array}{ccc} 1.2(1) & 1.2(2) & 1.3 \end{array} \\ \begin{bmatrix} 2.9208 & -1.4634 & -04878 \\ -1.4634 & 5.7317 & 0.2439 \\ -0.4878 & 0.2439 & 3.4146 \end{bmatrix} \end{array}$$

Setting b = 3; i = 2; a = 1

$$z_{32} = z_{12} + \frac{\left[y_{1-31-2(1)}\, y_{1-31-2(2)}\right]\begin{bmatrix} z_{12} - z_{22} \\ z_{12} - z_{22} \end{bmatrix}}{y_{13-13}} = \frac{0 + \left[(-0.4878)(0.2439)\right]\begin{bmatrix} 0-0.2 \\ 0-0.2 \end{bmatrix}}{3.4146}$$

$$= \frac{0.04828}{3.4146} = 0.014286$$

$$Z_{33} = Z_{13} + \frac{1 + \left[y_{1-31-2(1)}\, y_{1-31-2(2)}\right]\begin{bmatrix} z_{13} - z_{23} \\ z_{13} - z_{23} \end{bmatrix}}{y_{1-3-13}} = 0 + \frac{1 + \left[-0.4878\ 0.2439\right]\begin{bmatrix} -0.014286 \\ -0.014286 \end{bmatrix}}{3.4146}$$

$$= \frac{1.0034844}{3.4146} = 0.29388$$

$$Z_{BUS} = \begin{array}{c|cc} & (2) & (3) \\ \hline (2) & 0.2 & 0.01428 \\ (3) & 0.01428 & 0.29388 \end{array}$$

E 4.3 The system E4.2 is further extended by adding another transmission line to bus 3 with self impedance of 0.3 p.u. Obtain the Z_{BUS}

Solution :

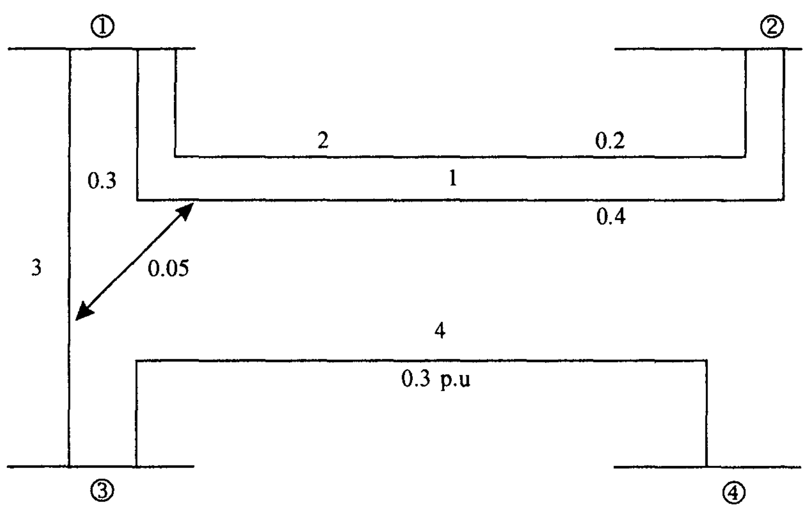


Fig. E.4.4

Consider the system shown above with the line 4 added to the previous system.

This is the case of the addition of a branch. Bus (3) is not the reference bus.

$$a = 3$$
$$b = 4$$

There is no mutual coupling.

$$z_{bi} = z_{ai} \qquad i = 1, 2, \ldots\ldots\ldots m \; i \neq b$$

$$z_{bb} = z_{ab} + z_{ab\text{-}ab}$$

setting i = 1, 2 and 3 respectively we can compute.

$$z_{41} = z_{41} = 0 \text{ (ref. Node)} = z_{31}$$
$$z_{42} = z_{32} = 0 = 0.01428$$
$$z_{41} = z_{33} = 0.29288$$
$$z_{4i} = z_{31} + z_{44\text{-}44} = 0.29288 + 0.3 = 0.59288$$

$$Z_{BUS} = \begin{array}{c} \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{ccc} (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 0.2 & 0.1428 & 0.1428 \\ 0.01428 & 0.29288 & 0.29288 \\ 0.01428 & 0.29288 & 0.59288 \end{bmatrix} \end{array}$$

E. 4.4 The system in E 4.3 is further extended and the radial system is converted into a ring system joining bus (2) to bus (4) for reliability of supply. Obtain the Z_{BUS}.

The self impedance of element 5 is 0.1 p.u

Solution :

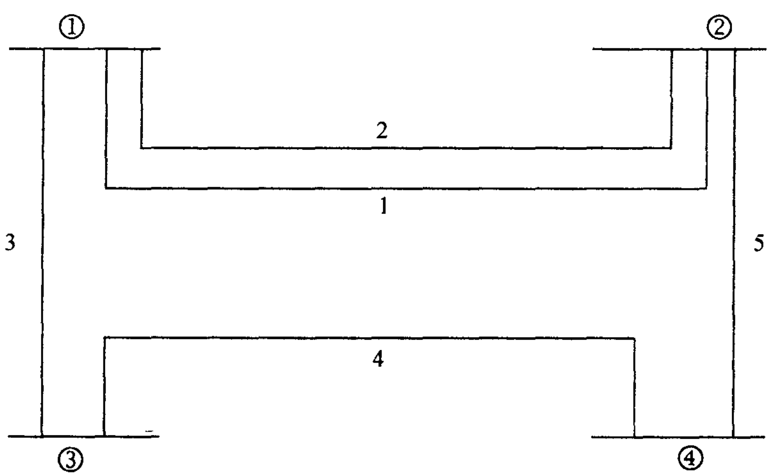


Fig. E.4.5

The ring system is shown in figure now let a = 2 and b = 4 addition of the line 5 is addition of a link to the existing system. Hence initially a fictitious node l is created. However, there is no mutual impedance bus (2) is not a reference node.

$$z_{li} = z_{ai} - z_{bi}$$

$$z_{ll} = z_{al} - z_{bl} + z_{ab\text{-}ab}$$

with i = 2 $\quad z_{21} = z_{22} - z_{42} = z_{12} = 0.2 - 0.01428 = 0.18572$

with i = 3 $\quad z_{31} = z_{23} - z_{43} = z_{13} = 0.01428 - 0.29288 = -0.27852$

setting i = 4 $\quad z_{41} = z_{14} = z_{24} - z_{44} = 0.1428 - 0.59288 = -0.5786$

$$z \text{ (augmented)} = z_{ll} + z_{al} - a_{bl} + z_{ab\text{-}ab}$$

$$= z_{21} - z_{41} + 0.1 \quad = 0.18572 + 0.5786 + 0.1 \quad = 0.86432$$

$$\begin{array}{c} \\ (2) \\ (3) \\ (4) \\ (l) \end{array} \begin{array}{c} \begin{array}{cccc} (2) & (3) & (4) & (l) \end{array} \\ \begin{bmatrix} 0.2 & 0.01428 & 0.01428 & 0.18572 \\ 0.01428 & 0.29288 & 0.29288 & -0.27852 \\ 0.01428 & 0.29288 & 0.59288 & -0.5786 \\ 0.18572 & -0.27582 & -0.5786 & 0.86432 \end{bmatrix} \end{array}$$

Now it remains to eliminate the fictitious node l.

$$z_{22} \text{ (modified)} = z_{22} - \frac{z_{21}\, z_{12}}{z_{ll}} = 0.2 - \frac{(0.18572)(0.18572)}{0.86432} = 0.16$$

$$z_{23} \text{ (modified)} = z_{23} - \frac{z_{21}\, z_{13}}{z_{ll}}$$

$$= 0.01428 - \frac{(0.18572)(-0.27852)}{0.86432} = 0.01428 + 0.059467 = 0.0741267$$

$$z_{24} \text{ (modified)} = z_{33} - \frac{z_{21}\, z_{14}}{z_{ll}}$$

$$= 0.01428 - \frac{(0.18572)(-0.5786)}{0.86432} = 0.01428 + 0.1243261 = 0.1386$$

$$z_{33} \text{ (modified)} = z_{33} = \frac{z_{31}\, z_{13}}{z_{ll}}$$

$$= 0.59288 - \frac{(-0.27582)(-0.27852)}{0.86432} = 0.59288 - 0.0897507 = 0.50313$$

z_{34} (modified) = z_{43} (modified)

$$= 0.2928 - \frac{(-0.27852)(-0.5786)}{0.86432} = 0.29288 - 0.186449 = 0.106431$$

$$z_{44}\text{ (modified)} = z_{44} - \frac{z_{41}\, z_{14}}{z_{11}}$$

$$= 0.59288 - \frac{(-0.5786)(-0.57861)}{0.86432} = 0.59288 - 0.38733 = 0.2055$$

The Z_{BUS} for the entire ring system is obtained as

$$Z_{BUS} = \begin{array}{c} \\ (2) \\ (3) \\ (4) \end{array}\begin{array}{c} \begin{array}{ccc} (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 0.16 & 0.0741267 & 0.1386 \\ 0.0741267 & 0.50313 & 0.106431 \\ 0.1386 & 0.106431 & 0.2055 \end{bmatrix} \end{array}$$

E 4.5 Compute the bus impedance matrix for the system shown in figure by adding element by element. Take bus (2) as reference bus

Solution :

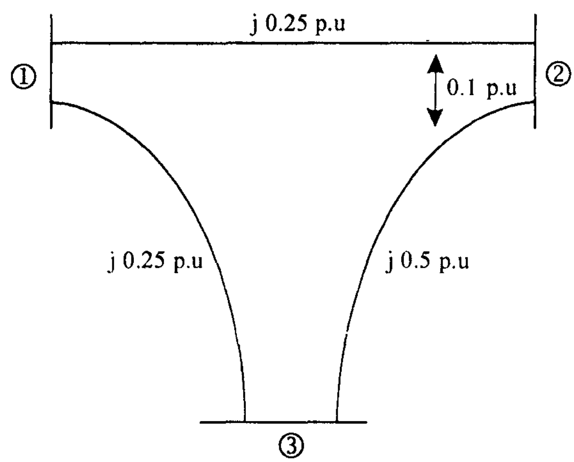


Fig. E.4.6

Step-1 Taking bus (1) as reference bus

$$Z_{BUS} = (2) \begin{array}{c} (2) \\ \boxed{j0.25} \end{array}$$

Step-2 Ass line joining buses (2) and (3). This is addition of a branch with mutuals.

a = (2); b = (3)

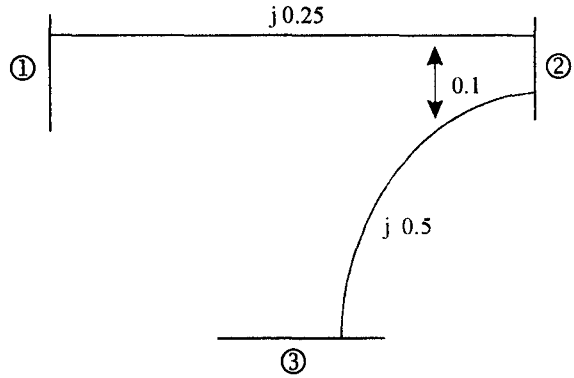


Fig. E.4.7

$$Z_{BUS} = \begin{array}{c|cc} & (2) & (3) \\ \hline (2) & j0.25 & z_{23} \\ (3) & z_{32} & z_{33} \end{array}$$

$$z_{bi} = z_{ai} + \frac{\bar{y}_{ab-xy}(\bar{z}_{xi} - \bar{z}_{yi})}{y_{ab-ab}}$$

$$z_{32} = z_{22} + \frac{y_{23-12}(z_{12} - z_{22})}{y_{23-23}}$$

The primitive impedance matrix.

$$z_{(primitive)} = \begin{bmatrix} j0.5 & -j0.1 \\ -j0.1 & j0.25 \end{bmatrix}$$

$$y_{ab-xy} = [z]_{primitive}^{-1} = \begin{bmatrix} -j4.347 & j0.869 \\ j0.869 & -j2.1739 \end{bmatrix}$$

Hence $$z_{32} = j0.25 + \frac{j0.869(0 - j0.25)}{-j2.1739} = j0.25 + j0.099 = j0.349$$

$$z_{33} = j0.349 + \frac{1 + 0.869(0 - j0.349)}{-j2.1739} = j0.349 + j0.5 = j0.946$$

$$Z_{BUS} = \begin{array}{c|cc} & (2) & (3) \\ \hline (2) & j0.25 & j0.349 \\ (3) & j0.349 & j0.946 \end{array}$$

Step-3 : Add the live joining (1) and (3) buses. This is addition of a link to the existing system with out mutual impedance.

A fictitious bus l is created.

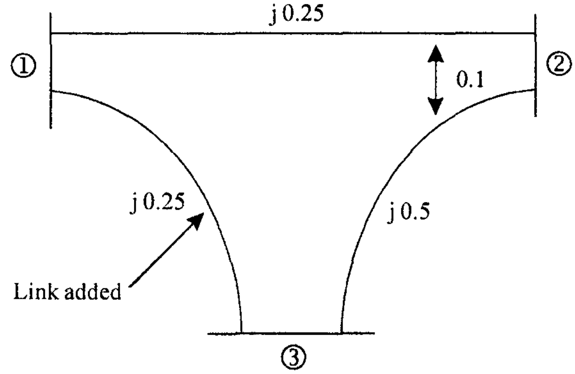


Fig. E.4.8

$z_{li} = -z_{bl}$

$z_{ll} = -z_{\,bi} + z_{ab-ab}$

$z_{l2} = -z_{32} = -j\ 0.349$

$z_{l3} = -z_{33} = -j\ 0.9464$

$z_{ll} = -z_{3l} + z_{13-13}$

$= +j\ 0.9464 + j\ 0.25$

$= j\ 1.196$

The augmented impedance matrix

$$Z_{BUS} = \begin{array}{c|ccc} & (2) & (3) & (4) \\ \hline (2) & j0.25 & j0.349 & -j0.349 \\ (3) & j0.349 & j0.946 & -j0.9464 \\ (l) & -j0.349 & -j0.9464 & j1.196 \end{array}$$

The factious node (l) is now eliminated.

$$z_{22}\ (\text{modified}) = z_{22} - \frac{z_{2l}\, z_{l2}}{z_{\,ll}}$$

$$= j0.25 - \frac{(-j0.349)(-j0.349)}{j1.196} = 0.1481$$

$$z_{23}\text{ (modified)} = z_{32}\text{ (modified)} = z_{23} - \frac{z_{21}\, z_{13}}{z_{11}}$$

$$= j0.349 - \frac{(-j0.349)(-j0.9464)}{j1.196} = 0.072834$$

$$z_{33}\text{ (modified)} = z_{33} - \frac{z_{31}\, z_{13}}{z_{11}}$$

$$= j\,0.94645 - \frac{(-j0.9464)^2}{j1.196} = 0.1976$$

Hence
$$Z_{BUS} = \begin{bmatrix} j0.1481 & j0.0728 \\ j0.0728 & j0.1976 \end{bmatrix}$$

E. 4.6 Using the building algorithm construct z_{BUS} for the system shown below. Choose 4 as reference BUS.

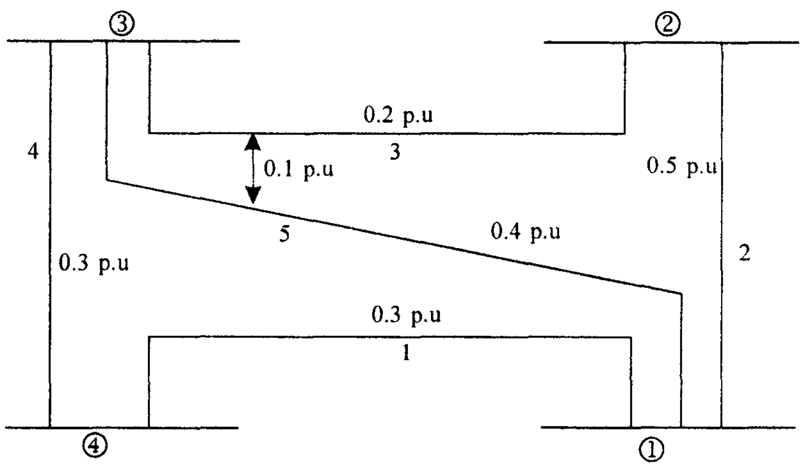


Fig. E.4.9

Solution :

Step-1 Start with element (1) which is a branch a = 4 to b = 1. The elements of the bus impedance matrix for the partial network containing the single branch are

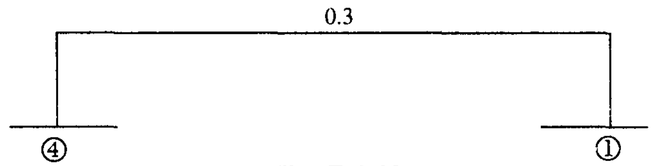


Fig. E.4.10

Taking bus (4) as reference bus

$$Z_{BUS} = \begin{array}{c|cc} & (4) & (1) \\ \hline (4) & 0 & 0 \\ (1) & 0 & 0.3 \end{array}$$

Since node 4 chosen as reference. The elements of the first row and column are zero and need not be written thus

$$Z_{BUS} = \begin{array}{c|c} & (1) \\ \hline (1) & 0.3 \end{array}$$

Step-2 Add element (2) which is a branch a = 1 to b = 2. This adds a new bus.

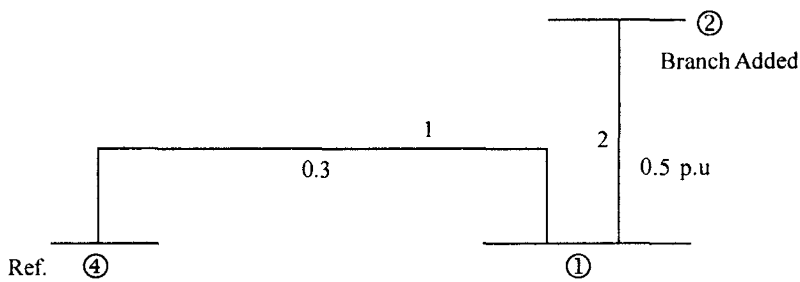


Fig. E.4.11

$$Z_{BUS} = \begin{array}{c|cc} & (1) & (2) \\ \hline (1) & 0.3 & z_{12} \\ (2) & z_{21} & z_{22} \end{array}$$

$z_{12} = z_{21} = z_{11} = 0.3$

$z_{22} = z_{12} + z_{1212} = 0.3 + 0.5 = 0.8$

$$Z_{BUS} = \begin{array}{c|cc} & (1) & (2) \\ \hline (1) & 0.3 & 0.3 \\ (2) & 0.3 & 0.8 \end{array}$$

Step-3 Add element (3) which is a branch a = 2 to b = 3. This adds a new BUS. The BUS impedance matrix.

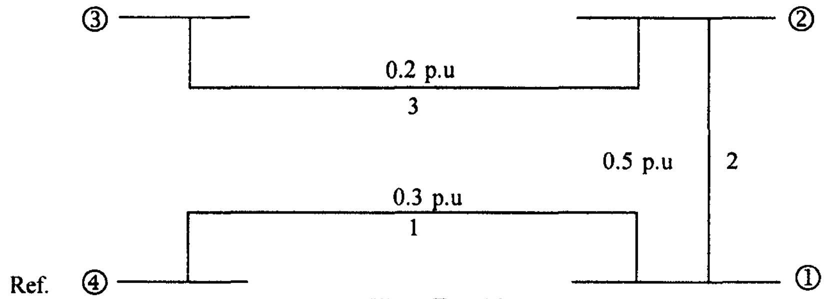


Fig. E.4.12

$$Z_{BUS} = \begin{array}{c|ccc} & (1) & (2) & (3) \\ \hline (1) & 0.3 & 0.3 & z_{13} \\ (2) & 0.3 & 0.8 & z_{23} \\ (3) & z_{31} & z_{23} & z_{33} \end{array}$$

$$z_{13} = z_{31} = z_{21} = 0.3$$

$$z_{32} = z_{23} = z_{22} = 0.8$$

$$z_{33} = z_{23} + z_{2323} = 0.8 + 0.2 = 1.0$$

Hence,

$$Z_{BUS} = \begin{array}{c|ccc} & (1) & (2) & (3) \\ \hline (1) & 0.3 & 0.3 & 0.3 \\ (2) & 0.3 & 0.8 & 0.8 \\ (3) & 0.3 & 0.8 & 1.0 \end{array}$$

Step-4 Add element (4) which is a link a = 4; b=3. The augmented impedance matrix with the fictitious node l will be.

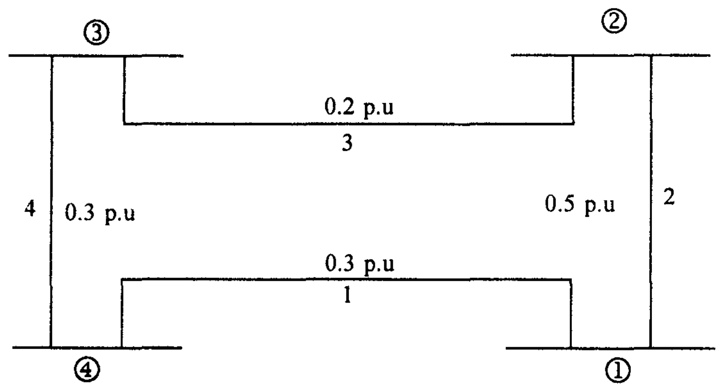


Fig. E.4.13

$$
\begin{array}{c|c}
 & \begin{array}{cccc} (1) & (2) & (3) & (l) \end{array} \\
\begin{array}{c} (1) \\ (2) \\ (3) \\ (l) \end{array} & \begin{bmatrix} 0.3 & 0.3 & 0.3 & z_{1l} \\ 0.3 & 0.8 & 0.8 & z_{2l} \\ 0.3 & 0.8 & 1.0 & z_{3l} \\ z_{l1} & z_{l2} & z_{l3} & z_{ll} \end{bmatrix}
\end{array}
$$

$$z_{1l} = z_{11} - z_{31} = -0.3$$

$$z_{2l} = z_{12} - z_{32} = -0.8$$

$$z_{3l} = z_{13} - z_{33} = -1.0$$

$$z_{ll} = -z_{31} + z_{4343} = -(-1) + 0.3 = 1.3$$

The augmented matrix is

$$
\begin{array}{c|c}
 & \begin{array}{cccc} (1) & (2) & (3) & (l) \end{array} \\
\begin{array}{c} (1) \\ (2) \\ (3) \\ (l) \end{array} & \begin{bmatrix} 0.3 & 0.3 & 0.3 & -0.3 \\ 0.3 & 0.8 & 0.8 & -0.8 \\ 0.3 & 0.8 & 1.0 & -1.0 \\ -0.3 & -0.8 & -1.0 & 1.3 \end{bmatrix}
\end{array}
$$

To eliminate the l^{th} row and column

$$z_{11}^1 = z_{11} - \frac{(z_{1l})(z_{l1})}{z_{ll}} = 0.3 - \frac{(-0.3)(-0.3)}{1.3} = 0.230769$$

$$z_{21}^1 = z_{12}^1 = z_{12} - \frac{(z_{1l})(z_{2l})}{z_{ll}} = 0.3 - \frac{(-0.3)(-0.8)}{1.3} = 0.1153$$

$$z_{31}^1 = z_{13}^1 = z_{13} - \frac{(z_{1l})(z_{l3})}{z_{ll}} = 0.3 - \frac{(-0.3)(-1.0)}{1.3} = 0.06923$$

$$z_{22}^1 = z_{22} - \frac{(z_{2l})(z_{l2})}{z_{ll}} = 0.8 - \frac{(-0.8)(-0.8)}{1.3} = 0.30769$$

$$z_{23}^1 = z_{32}^1 = z_{32} - \frac{(z_{3l})(z_{l2})}{z_{ll}} = 0.8 - \frac{(-1.0)(-0.8)}{1.3} = 0.18461$$

$$z_{33}^1 = z_{33} - \frac{(z_{3l})(z_{l3})}{z_{ll}} = 1.0 - \frac{(-1.0)(-1.0)}{1.3} = 0.230769$$

and, thus, $Z_{BUS} =$

	(1)	(2)	(3)
(1)	0.230769	0.1153	0.06923
(2)	0.1153	0.30769	0.18461
(3)	0.06923	0.18461	0.230769

Step-5 Add element (5) which is a link a = 3 to b = 1, mutually coupled with element (4). The augmented impedance matrix with the fictitious node l will be

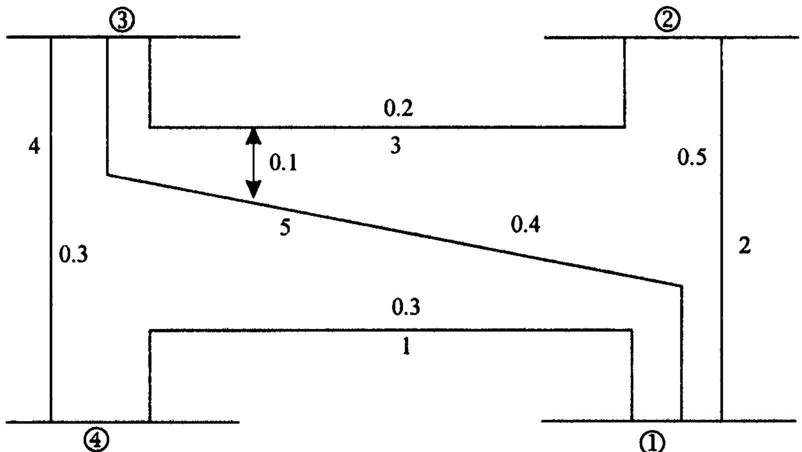


Fig. E.4.14

	(1)	(2)	(3)	(l)
(1)	0.230769	0.1153	0.06923	z_{1l}
(2)	0.1153	0.30769	0.18461	z_{2l}
(3)	0.06923	0.18461	0.230769	z_{3l}
(l)	z_{l1}	z_{l2}	z_{l3}	z_{ll}

$$z_{1l} = z_{l1} = z_{31} - z_{11} + \frac{y_{31\,23}(z_{21} - z_{31})}{y_{3131}}$$

$$z_{l2} = z_{2l} = z_{32} - z_{12} + \frac{y_{31\,23}(z_{22} - z_{32})}{y_{3131}}$$

$$z_{3l} = z_{l3} = z_{33} - z_{13} + \frac{y_{31\,23}(z_{23} - z_{33})}{y_{3131}}$$

$$z_{ll} = z_{31} - z_{11} + \frac{1 + y_{31\,23}(z_{21} - z_{31})}{y_{3131}}$$

Invert the primitive impedance matrix of the partial network to obtain the primitive admittance matrix.

$$[z_{xy\,xy}] = \begin{array}{c} \\ 4-1(1) \\ 1-2(2) \\ 2-3(3) \\ 4-3(4) \\ 3-1(5) \end{array} \begin{array}{c} \begin{array}{ccccc} 4-1(1) & 1-2(2) & 2-3(3) & 4-3(4) & 3-1(5) \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0.1 & 0 & 0.4 \end{bmatrix} \end{array}$$

$$[z_{xy\,xy}]^{-1} = [y_{xy\,xy}] = \begin{array}{c} \\ 4-1(1) \\ 1-2(2) \\ 2-3(3) \\ 4-3(4) \\ 3-1(5) \end{array} \begin{array}{c} \begin{array}{ccccc} 4-1(1) & 1-2(2) & 2-3(3) & 4-3(4) & 3-1(5) \end{array} \\ \begin{bmatrix} 3.33 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 5.714 & 0 & -1.428 \\ 0 & 0 & 0 & 3.33 & 0 \\ 0 & 0 & -1.428 & 0 & 2.8571 \end{bmatrix} \end{array}$$

$$z_{11} = z_{12} = 0.06923 - 0.230769 + \frac{(-1.428)(0.1153 - 0.06923)}{2.8571} = -0.18456$$

$$z_{12} = z_{21} = 0.18461 - 0.1153 + \frac{(-1.428)(0.30769 - 0.18461)}{2.8571} = 0.00779$$

$$z_{31} = z_{13} = 0.230769 - 0.06923 + \frac{(-1.428)(0.18461 - 0.230769)}{2.8571} = 0.1846$$

$$z_{ll} = 0.1846 - (-0.1845) + \frac{1 + (-1.428)(0.00779 - 0.1846)}{2.8571} = 0.8075$$

	(1)	(2)	(3)	(l)
(1)	0.230769	0.1153	0.06923	−0.18456
(2)	0.1153	0.30769	0.18461	0.00779
(3)	0.06923	0.18461	0.230769	0.186
(l)	−0.18456	0.00779	0.1846	0.8075

To Eliminate l^{th} row and column :

$$z_{21}^{1} = z_{11} - \frac{(z_{11})(z_{11})}{z_{11}} = 0.230769 - \frac{(-0.18456)(-0.18456)}{0.8075} = 0.18858$$

$$z_{12}^{1} = z_{21}^{1} = z_{12} = -\frac{(z_{11})(z_{12})}{z_{11}} = 0.1153 - \frac{(-0.18456)(0.00779)}{0.8075} = 0.11708$$

$$z_{22}^{1} = z_{22} = -\frac{(z_{21})(z_{12})}{z_{11}} = 0.30769 - \frac{(-0.00779)(0.00779)}{0.8075} = 0.30752$$

$$z_{13}^{1} = z_{31}^{1} = z_{13} = -\frac{(z_{11})(z_{13})}{z_{11}} = 0.06923 - \frac{(-0.18456)(0.1846)}{0.8075} = 0.11142$$

$$z_{33}^{1} = z_{33} - \frac{(z_{31})(z_{13})}{z_{11}} = 0.230769 - \frac{(-0.1846)(0.1846)}{0.8075} = 0.18857$$

$$z_{32}^{1} = z_{23}^{1} = z_{23} - \frac{(z_{21})(z_{31})}{z_{11}} = 0.18461 - \frac{(0.00779)(0.1846)}{0.8075} = 0.18283$$

and $Z_{BUS} =$

	(1)	(2)	(3)
(1)	0.18858	0.11708	0.11142
(2)	0.11708	0.30752	0.18283
(3)	0.11142	0.18283	0.18858

E 4.7 Given the network shown in Fig. E.4.15.

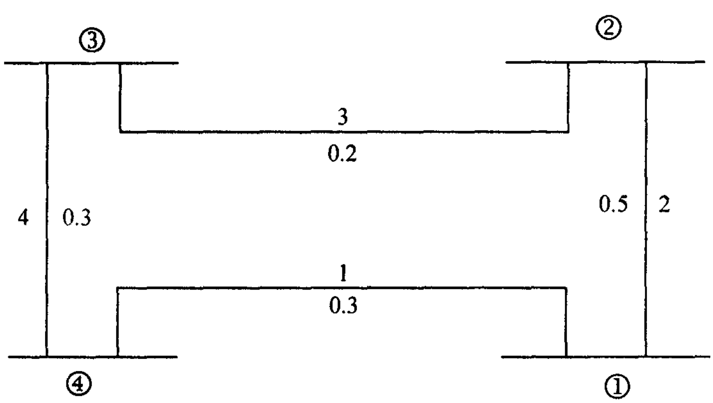


Fig. E.4.15

Its z_{BUS} is as follows.

Z_{BUS} = (1)	0.230769	0.1153	0.0623
(2)	0.1153	0.30769	0.18461
(3)	0.06923	0.18461	0.230769

If the line 4 is removed determine the z $_{BUS}$ for the changed network.

Solution : Add an element parallel to the element 4 having an impedance equal to impedance of element 4 with negative sign.

$$\frac{1}{Z_{new}} = \frac{1}{Z_{added}} + \frac{1}{Z_{existing}} = -\frac{1}{0.3} + \frac{1}{0.3} = 0$$

This amount to addition of a link.

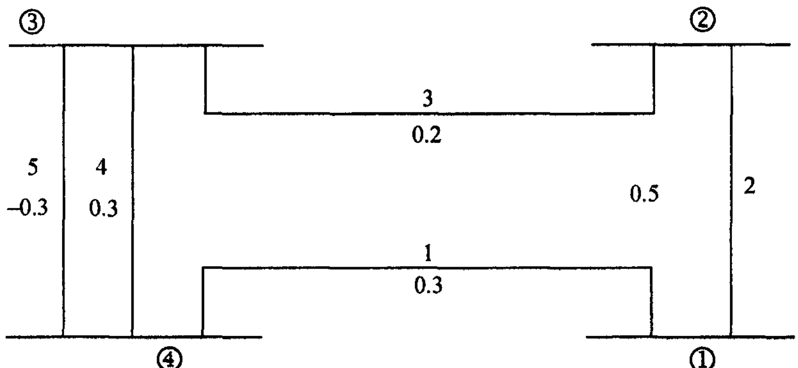


Fig. E.4.16

	(1)	(2)	(3)	(l)
Z_{BUS} = (1)	0.230769	0.1153	0.0623	z_{1l}
(2)	0.1153	0.30769	0.18461	z_{2l}
(3)	0.06923	0.18461	0.230769	z_{3l}
(l)	z_{l1}	z_{l2}	z_{l3}	z_{ll}

where

$$z_{l1} = z_{l1} = -z_{31} = -0.06923$$

$$z_{2l} = z_{l2} = -z_{32} = -0.18461$$

$$z_{3l} = z_{l3} = -z_{33} = -0.230769$$

$$z_{ll} = -z_{31} + z_{4343}$$

$$= (-0.230769) + (-0.3) = -0.06923$$

The augmented z_{BUS} is then

$$Z_{BUS} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (l) \end{array} \begin{array}{cccc} (1) & (2) & (3) & (l) \\ \hline 0.230769 & 0.1153 & 0.0623 & -0.06923 \\ 0.1153 & 0.30769 & 0.18461 & -0.18461 \\ 0.06923 & 0.18461 & 0.230769 & -0.230769 \\ -0.06923 & -0.18461 & -0.230769 & -0.06923 \end{array}$$

Eliminating the fictitious node l

$$z_{32}^1 = z_{23}^1 = z_{23} - \frac{(z_{21})(z_{13})}{z_{11}} = 0.18461 - \frac{(0.18461)(-0.2307669)}{-0.06923} = 0.8$$

$$z_{11}^1 = z_{11} - \frac{(z_{21})(z_{11})}{z_{11}} = (0.230769) - (-0.06923) = 0.3$$

$$z_{21}^1 = z_{12}^1 = z_{12} - \frac{(z_{21})(z_{21})}{z_{11}} = (0.1153) - (-0.18461) = 0.3$$

$$z_{31}^1 = z_{13}^1 = z_{13} - \frac{(z_{21})(z_{31})}{z_{11}} = (0.06923) - (-0.230769) = 0.3$$

$$z_{22}^1 = z_{22} - \frac{(z_{21})(z_{12})}{z_{11}} = (0.30769) - \frac{(-0.18461)(-0.18461)}{-0.06923} = 0.8$$

$$z_{33}^1 = z_{33} - \frac{(z_{31})(z_{13})}{z_{11}} = (0.230769) - \frac{(-0.230769)(-0.230769)}{-0.06923} = 1.0$$

The modified z_{BUS} is

$$Z_{BUS} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{ccc} (1) & (2) & (3) \\ \hline 0.3 & 0.3 & 0.3 \\ 0.3 & 0.8 & 0.8 \\ 0.3 & 0.8 & 1.0 \end{array}$$

E 4.8 Consider the system in Fig. E.4.17.

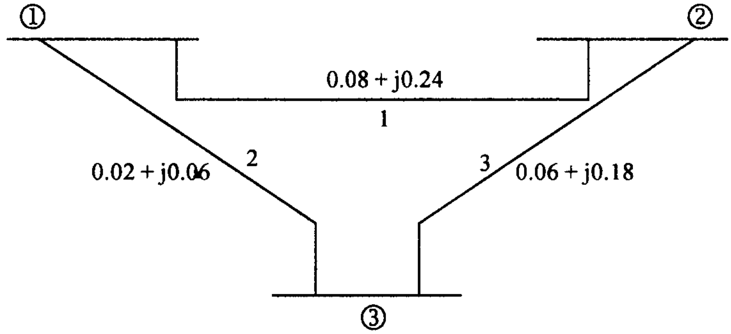


Fig. E.4.17

Obtain z_{BUS} by using building algorithm.

Solution :

Bus (1) is chosen as reference. Consider element 1 (between bus (1) and (2))

$$Z_{BUS} = \begin{array}{c|c|} & (2) \\ (2) & 0.08 + j0.24 \end{array}$$

Step-1 Add element 2 (which is between bus (1) and (3))

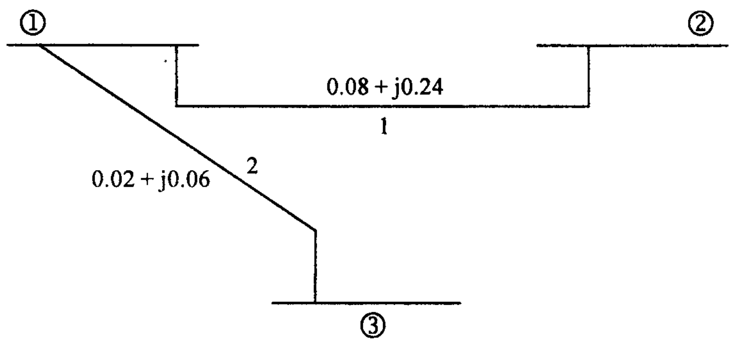


Fig. E.4.18

This is addition of a branch. A new bus (3) is created. There is no mutual impedance.

$$Z_{BUS} = \begin{array}{c|c|c|} & (2) & (3) \\ (2) & 0.08 + j0.24 & 0.0 + j0.0 \\ (3) & 0.0 + j0.0 & 0.02 + j0.06 \end{array}$$

Step-2 add element 3 which is between buses (2) and (3)

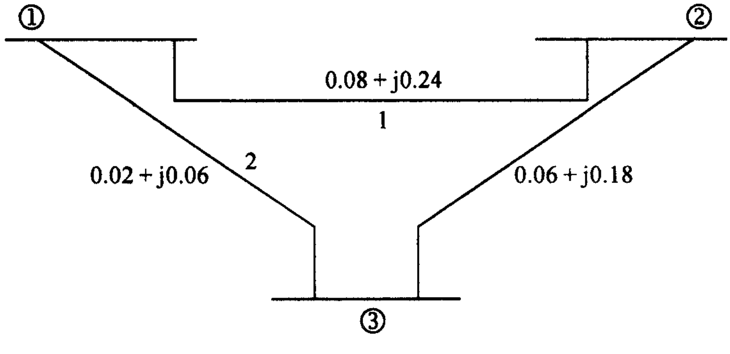


Fig. E.4.19

A link is added. Fictitious node l is introduced.

$$Z_{BUS} = \begin{array}{c|c|c|c} & (2) & (3) & (l) \\ \hline (2) & 0.08 + j0.24 & 0.0 + j0.0 & 0.08 + j0.24 \\ (3) & 0.0 + j0.0 & 0.02 + j0.06 & -(0.02 + j0.06) \\ (l) & 0.08 + j0.24 & -(0.02 + j0.006) & 0.16 + j0.48 \end{array}$$

eliminating the fictitious node l

$$z_{22}\,(\text{modified}) = z_{22} - \frac{z_{21}\,z_{12}}{z_{ll}}$$

$$= (0.08 + j0.24) - \frac{(0.08 + j0.24)^2}{0.49 + j0.48} = 0.04 + j0.12$$

$$z_{23}\,(\text{modified}) = z_{32}\,(\text{modified})$$

$$= \left[z_{23} - \frac{z_{2l}\,z_{l3}}{z_{ll}} \right] = 0.0 + j0.0 + \frac{(0.8 + j0.24)(0.02 + j0.06)}{0.16 + j0.48}$$

$$= 0.01 + j0.03$$

$$z_{33}\,(\text{modified}) = 0.0175 + j\,0.0526$$

The z_{BUS} matrix is thus

$$Z_{BUS} = \begin{array}{c} \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{cc} \quad(2)\quad & \quad\quad(3) \end{array} \\ \left[\begin{array}{c|c} 0.04 + j0.12 & 0.01 + j0.03 \\ \hline 0.01 + j0.03 & 0.0175 + j0.0526 \end{array} \right] \end{array}$$

E 4.9 Given the system of E4.4. An element with an impedance of 0.2 p.u. and mutual impedance of 0.05 p.u. with element 5. Obtained the modified bus impedance method using the method for computations of Z_{BUS} for changes in the network.

Solution :

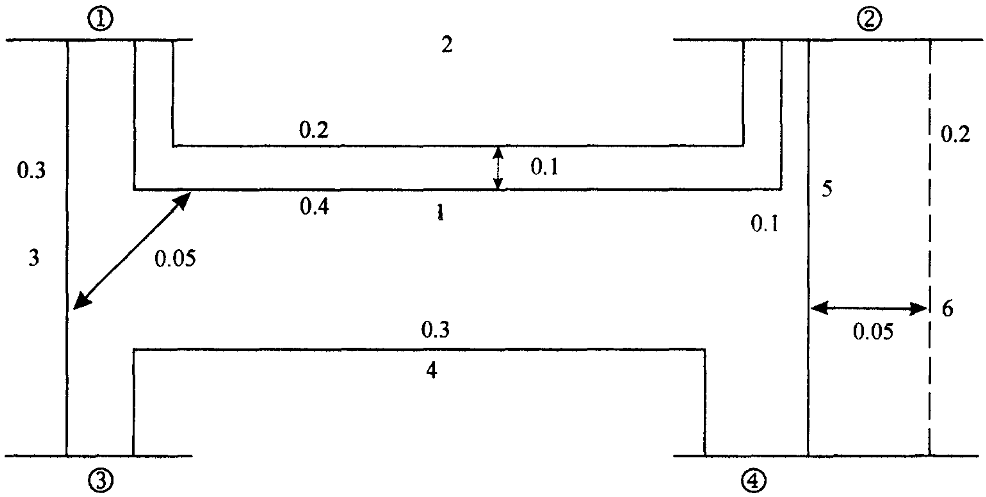


Fig. E.4.20

Element added is a link

a = 2 ; b = 4. A fictitious node is l is created.

	(2)	(3)	(4)	l
(2)	0.16	0.0741267	0.1386	Z_{2l}
(3)	0.0741267	0.50313	0.106431	Z_{3l}
(4)	0.1386	0.106437	0.2055	Z_{4l}
(l)	Z_{l2}	Z_{l3}	Z_{l4}	Z_{ll}

$$Z_{li} = Z_{ai} - Z_{bi} + \frac{\bar{y}_{ab-xy}(Z_{xi} - Z_{yi})}{y_{ab-ab}}$$

and
$$Z_{ll} = Z_{al} - Z_{bl} + \frac{1 + \bar{y}_{ab-xy}(Z_{xl} - Z_{yl})}{y_{ab-ab}}$$

The primitive impedance matrix is

$$[z] = \begin{array}{c} \\ 1-2(2) \\ 1-3 \\ 3-4 \\ 2-4(1) \\ 2-4(2) \end{array}$$

	1 – 2(1)	1.2(2)	1 – 3	3 – 4	2.4(1)	2.4(2)
	0.4	0.1	0.05	0	0	0
1 – 2(2)	0.1	0.2	0	0	0	0
1 – 3	0.05	0	0.3	0	0	0
3 – 4	0	0	0	0	0	0
2 – 4(1)	0	0	0	0.3	0.1	0.05
2 – 4(2)	0	0	0	0	0.05	0.2

The added element 6 is coupled to only one element (i.e.) element 5. It is sufficient to invert the sub matrix for the coupled element.

$Z_{ab-xy} =$

	2 – 4(1)	2 – 4(2)
2 – 4(1)	0.1	0.05
2 – 4(2)	0.05	0.2

$$y_{ab-xy} = \begin{bmatrix} 11.4285 & -2.857 \\ -2.857 & 5.7143 \end{bmatrix}$$

$$Z_{2l} = Z_{l2} = Z_{22} - Z_{42} + \frac{(-2.857)(Z_{xi} - Z_{yi})}{11.43}$$

$$= 0.16 - 0.1386 + \frac{(-2.857)(0 - 16 - 0.1386)}{5.7143} = 0.01070$$

$$Z_{3l} = Z_{l3} = 0.741267 - 0.10643 + \frac{(-2.857)(0.741 - 0.10643)}{5.7143} = -0.1614$$

$$Z_{l4} = Z_{41} = Z_{24} - Z_{44} + \frac{\bar{y}_{ab-xy}(Z_{xi} - Z_{yi})}{y_{ab-ab}}$$

$$= 0.1386 - 0.2055 + \frac{(-2.857)(0.1386 - 0.2055)}{5.7143} = -0.03345$$

$$Z_{ll} = 0.0107 - (-0.03345) + \frac{1 + (-2.857)(0.0107 - (-0.03345)}{5.7143} = 0.19707$$

The augmented matrix is then

$$\begin{array}{c} \\ (2) \\ (3) \\ (4) \\ (l) \end{array} \begin{array}{c} \begin{array}{cccc} (2) & (3) & (4) & (l) \end{array} \\ \begin{bmatrix} 0.1600 & 0.0741 & 0.1386 & 0.0107 \\ 0.0741 & 0.5031 & 0.1064 & -0.01614 \\ 0.1386 & 0.1064 & 0.2055 & -0.0.3345 \\ 0.0107 & -0.01614 & -0.03345 & 0.19707 \end{bmatrix} \end{array}$$

$$Z_{22}\,(\text{modified}) = 0.16 - \frac{(0.0107)^2}{0.19707} = 0.1594$$

$$Z_{23}\,(\text{modified}) = Z_{32}\,(\text{modified})$$

$$= 0.0741 \quad \frac{0.01070 \times (-0.01614)}{0.19707} = 0.7497$$

$$Z_{24}\,(\text{modified}) = Z_{42}\,(\text{modified})$$

$$= 0.1386 - \frac{0.01070 \times (-0.03345)}{0.19707} = 0.1404$$

$$Z_{33}\,(\text{modified}) = 0.50313 - \frac{(-0.01614)^2}{0.19707} = 0.5018$$

$$Z_{43}\,(\text{modified}) = Z_{34}\,(\text{modified})$$

$$= 0.106431 - \frac{(-0.03345)(-0.01614)}{0.19707} = 0.10369$$

$$Z_{44}\,(\text{modified}) = 0.2055 - \frac{(-0.0334)^2}{0.19707} = 0.1998$$

Hence the Z_{BUS} is obtained by

$$Z_{BUS} = \begin{array}{c} \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{ccc} (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 0.1594 & 0.07497 & 0.1404 \\ 0.07497 & 0.5018 & 0.10369 \\ 0.1404 & 0.10369 & 0.1998 \end{bmatrix} \end{array}$$

E 4.10 Consider the problem E.4.9. If the element 6 is now removed obtain the Z_{BUS}.

Solution :

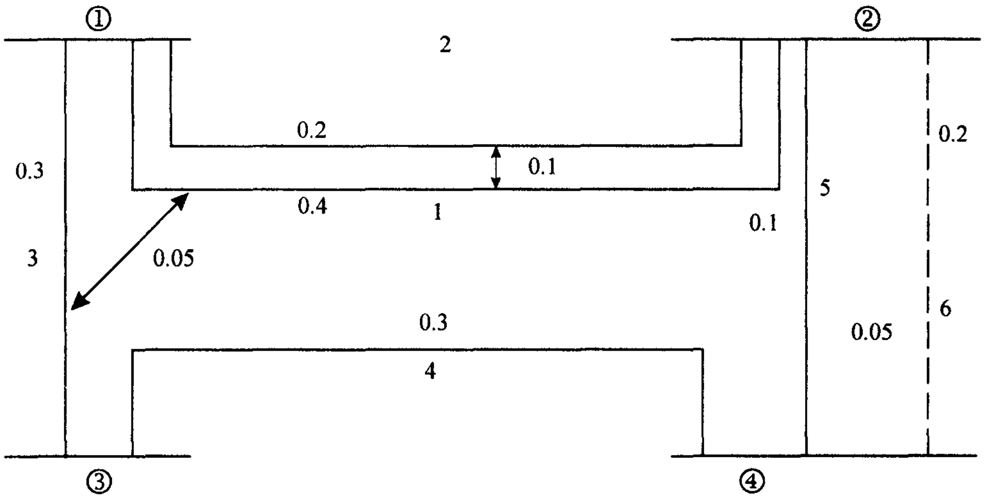


Fig. E.4.21

$$Z^{|}_{ij} = Z_{ij} + \left(\bar{Z}_{i\alpha} - \bar{Z}_{i\beta}\right)[F]^{-1}[\Delta y_i]\begin{bmatrix} \bar{Z} & -Z \\ \gamma & \delta i \end{bmatrix}$$

where $\alpha = 2$; $\beta = 4$ $i = 1,2,\ldots\ldots\ldots\ldots,n$
and $\gamma = 2$; $\delta = 4$

the original primitive admittance matrix

$$[y_{sm}] = \begin{array}{c} \\ 2-4(1) \\ 2-4(2) \end{array} \begin{array}{c} \begin{array}{cc} 2-4(1) & 2-4(2) \end{array} \\ \left[\begin{array}{c|c} 11.4285 & -2.857 \\ \hline -2.857 & 5.7143 \end{array}\right] \end{array}$$

The modified primitive admittance matrix

$$[y_{sm}]^{|} = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \end{array} \begin{array}{c} \begin{array}{cc} 1-2(1) & 1-2(2) \end{array} \\ \left[\begin{array}{c|c} +\frac{1}{0.1} & 0 \\ \hline 0 & 0 \end{array}\right] \end{array} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\left\{ [y_{sm}][y_{sm}]^{|} \right\} = \begin{bmatrix} 11.4285 & -2.857 \\ -2.857 & 5.7143 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1.4285 & -2.857 \\ -2.857 & 5.7143 \end{bmatrix} = \Delta y_{sm}$$

Computing term by term

$$Z_{\gamma\alpha} - Z_{\delta\alpha} - Z_{\gamma\beta} - Z_{\delta\beta} = \left[\begin{array}{cc} & \begin{array}{cc}(2) & (2)\end{array} \\ \begin{array}{c}(2)\\(2)\end{array} & \begin{bmatrix} 0.1594 & 0.1594 \\ 0.1594 & 0.1594 \end{bmatrix}\end{array}\right]$$

$$Z_{\delta\alpha} = Z_{24} = Z_{42} = \begin{array}{cc} & \begin{array}{cc}(2) & (2)\end{array} \\ \begin{array}{c}(4)\\(4)\end{array} & \begin{bmatrix} 0.1404 & 0.1404 \\ 0.1404 & 0.1404 \end{bmatrix}\end{array}$$

$$Z_{\gamma\alpha} = \begin{array}{cc} & \begin{array}{cc}(4) & (4)\end{array} \\ \begin{array}{c}(2)\\(2)\end{array} & \begin{bmatrix} 0.1404 & 0.1404 \\ 0.1404 & 0.1404 \end{bmatrix}\end{array}$$

$$Z_{\delta\beta} = \begin{array}{cc} & \begin{array}{cc}(4) & (4)\end{array} \\ \begin{array}{c}(4)\\(4)\end{array} & \begin{bmatrix} 0.1998 & 0.1998 \\ 0.1998 & 0.1998 \end{bmatrix}\end{array}$$

$$Z_{\gamma\alpha} - Z_{\delta\alpha} - Z_{\gamma\beta} + Z_{\delta\beta} = \begin{bmatrix} 0.1594 & 0.1594 \\ 0.1594 & 0.1594 \end{bmatrix} - \begin{bmatrix} 0.1404 & 0.1404 \\ 0.1404 & 0.1404 \end{bmatrix}$$

$$- \begin{bmatrix} 0.1404 & 0.1404 \\ 0.1404 & 0.1404 \end{bmatrix} + \begin{bmatrix} 0.1998 & 0.1998 \\ 0.1998 & 0.1998 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0784 & 0.0784 \\ 0.0784 & 0.0784 \end{bmatrix}$$

$$[\Delta y_{sm}][Z_{\gamma\alpha} - Z_{\delta\alpha} - Z_{\gamma\beta} - Z_{\delta\beta}]$$

$$= \begin{bmatrix} 1.4285 & -2.857 \\ -2.857 & 5.7143 \end{bmatrix} \begin{bmatrix} 0.0784 & 0.784 \\ 0.0784 & 0.0784 \end{bmatrix} = \begin{bmatrix} -0.1120 & -0.1120 \\ 0.2240 & 0.2240 \end{bmatrix}$$

$$F = U - \Delta y_{sm}(Z_{\gamma\alpha} - Z_{\delta\alpha} - Z_{\gamma\beta} + Z_{\delta\beta}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.1120 & -0.1120 \\ 0.2240 & 0.2240 \end{bmatrix} = \begin{bmatrix} 1.112 & 0.112 \\ -0.224 & 0.7760 \end{bmatrix}$$

$$[F]^{-1} = \begin{bmatrix} 0.87387 & -0.12612 \\ 0.25225 & 1.25225 \end{bmatrix}$$

$$[F]^{-1}\Delta y_{sm} = \begin{bmatrix} 0.87387 & -0.12612 \\ 0.25225 & 1.25225 \end{bmatrix} \begin{bmatrix} 1.4285 & -2.8571 \\ -2.8571 & 5.7143 \end{bmatrix} = \begin{bmatrix} 1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix}$$

The elements of modified Z_{BUS} are then given by

$$Z^{|}_{ij} = Z_{ij} + \left(\bar{Z}_{i\alpha} - \bar{Z}_{i\beta}\right)[F]^{-1}\,[\Delta y_{sm}]\left[\bar{Z}_{\gamma i} - \bar{Z}_{\delta i}\right]$$

For i = 2; j = 2

$$Z^{|}_{22} = Z_{22} + \left([Z_{22}\ Z_{22}] - [Z_{24}\ Z_{24}]\ [M]^{-1}\{\Delta y_{sm}\} \right)\left(\begin{bmatrix} Z_{22} \\ Z_{22} \end{bmatrix} - \begin{bmatrix} Z_{42} \\ Z_{42} \end{bmatrix} \right)$$

$$= 0.1594 + ([0.1594\ 0.1594] - [0.1404\ 0.1404])$$

$$\begin{bmatrix} 1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix} \left(\boxed{\begin{matrix} 0.1594 \\ 0.1594 \end{matrix}} - \boxed{\begin{matrix} 0.1404 \\ 0.1404 \end{matrix}} \right)$$

$$= 0.1594 + [0.019\ 0.019] \begin{bmatrix} 1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix} \begin{bmatrix} 0.019 \\ 0.019 \end{bmatrix}$$

$$= 0.1594 + [-0.03056\ 0.0611] \begin{bmatrix} 0.019 \\ 0.019 \end{bmatrix} = 0.1594 + 0.00058026 = 0.15998 = 0.16$$

Let i = 2; j = 3

$$Z^{|}_{23} = 0.07497 + [0.019\ 0.019] \begin{bmatrix} 1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix} \begin{bmatrix} -0.03146 \\ -0.03146 \end{bmatrix}$$

$$= 0.07497 + [-0.03056\ 0.0611] \begin{bmatrix} -0.03146 \\ -0.03146 \end{bmatrix} = 0.07497 - 0.00096075 = 0.074040$$

Let i = 2; j = 4

$$Z^{|}_{24} = 0.1404 + [-0.03056\ 0.0611] \begin{bmatrix} -0.0594 \\ -0.0594 \end{bmatrix} = 0.1404 - 0.001814 = 0.13858$$

Let i = 3; j = 3

$$Z^{|}_{33} = 0.5018 + [-0.03146\ \ -0.03146] \begin{bmatrix} -1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix} \begin{bmatrix} -0.03146 \\ -0.03146 \end{bmatrix}$$

$$= 0.5018 + [0.0506\ \ -0.101228] \begin{bmatrix} -0.03146 \\ -0.03146 \end{bmatrix} = 0.5018 + 0.001592 = 0.503392$$

For i = 3; j = 4

$$Z^{1}_{34} = 0.103691 + [0.05060 \quad -0.101228]\begin{bmatrix} -0.0594 \\ -0.0594 \end{bmatrix}$$

$$= 0.103691 + 0.003006 = 0.106696$$

Similarly for I = j =

$$Z^{1}_{44} = 0.1998 + [-0.0594 \quad -0.0594]\begin{bmatrix} 1.6086 & -3.2173 \\ -3.2173 & 6.435 \end{bmatrix}\begin{bmatrix} -0.0594 \\ -0.0594 \end{bmatrix}$$

$$= 0.1998 + [0.09556 \quad -0.19113]\begin{bmatrix} -0.0594 \\ -0.0594 \end{bmatrix} = 0.205476$$

Hence the Z_{BUS} with the element 6 removed will be

Z_{BUS} =

	(2)	(3)	(4)
(2)	0.15998	0.074040	0.13858
(3)	0.07404	0.503392	0.106696
(4)	0.13858	0.106696	0.20576

Problems

P 4.1

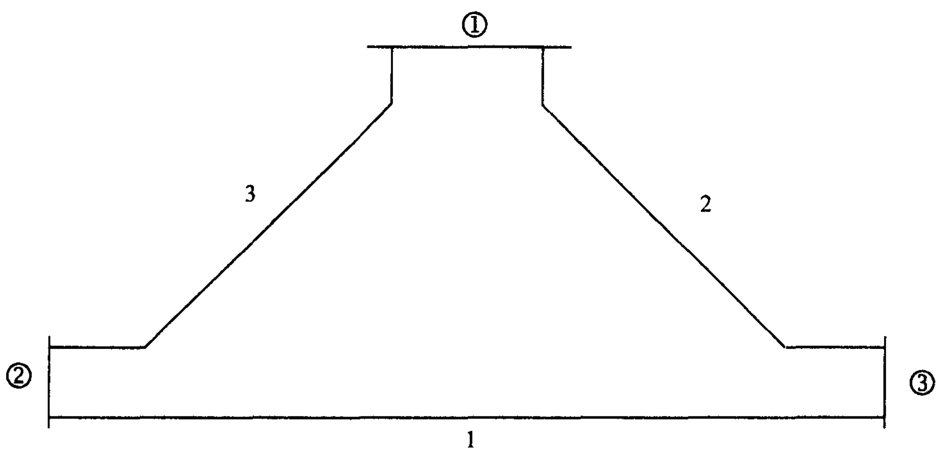


Fig. P 4.14

Form the bus impedance matrix for the system shown in Fig. P 4.1 the line data is given below.

Element Numbers	Bus Code	Self Impedance
1	(2) - (3)	0.6 p.u.
2	(1) - (3)	0.5 p.u.
3	(1) - (2)	0.4 p.u.

P 4.2 Obtain the modified Z_{BUS} if a line 4 is added parallel to line 1 across the busses (2) and (3) with a self impedance of 0.5 p.u.

P 4.3 In the problem P 4.2 if the added line element 4 has a mutual impedance with respect to line element 1 of 0.1 p.u. how will the Z_{BUS} matrix change ?

Questions

4.1 Starting from Z_{BUS} for a partial network describe step – by – step how you will obtain the Z_{BUS} for a modified network when a new line is to be added to a bus in the existing network.

4.2 Starting from Z_{BUS} for a partial network describe step by step how you will obtain the Z_{BUS} for a modified network when a new line is to be added between two buses of the existing network.

4.3 What are the advantages of Z_{BUS} building algorithm?

4.4 Describe the Procedure for modification of Z_{BUS} when a line is added or removed which has no mutual impedance.

4.5 Describe the procedure for modifications of Z_{BUS} when a line with mutual impedance is added or removed.

4.6 Derive the necessary expressions for the building up of Z_{BUS} when (i) new element is added. (ii) new element is added between two existing buses. Assume mutual coupling between the added element and the elements in the partial network.

4.7 Write short notes on

Removal of a link in Z_{BUS} with no mutual coupling between the element deleted and the other elements in the network.

4.8 Derive an expression for adding a link to a network with mutual inductance.

4.9 Derive an expression for adding a branch element between two buses in the Z_{BUS} building algorithm.

4.10 Explain the modifications necessary in the Z_{BUS} when a mutually coupled element is removed or its impedance is changed.

4.11 Develop the equation for modifying the elements of a bus impedance matrix when it is coupled to other elements in the network, adding the element not creating a new bus.

5 POWER FLOW STUDIES

Power flow studies are performed to determine voltages, active and reactive power etc. at various points in the network for different operating conditions subject to the constraints on generator capacities and specified net interchange between operating systems and several other restraints. Power flow or load flow solution is essential for continuous evaluation of the performance of the power systems so that suitable control measures can be taken in case of necessity. In practice it will be required to carry out numerous power flow solutions under a variety of conditions.

5.1 Necessity for Power Flow Studies

Power flow studies are undertaken for various reasons, some of which are the following :

1. The line flows
2. The bus voltages and system voltage profile
3. The effect of change in configuration and incorporating new circuits on system loading
4. The effect of temporary loss of transmission capacity and (or) generation on system loading and accompanied effects.
5. The effect of in-phase and quadrative boost voltages on system loading
6. Economic system operation
7. System loss minimization
8. Transformer tap setting for economic operation
9. Possible improvements to an existing system by change of conductor sizes and system voltages.

For the purpose of power flow studies a single phase representation of the power network is used, since the system is generally balanced. When systems had not grown to the present size, networks were simulated on network analyzers for load flow solutions. These analyzers are of analogue type, scaled down miniature models of power systems with resistances, reactances, capacitances, autotransformers, transformers, loads and generators. The generators are just supply sources operating at a much higher frequency than 50 Hz to limit the size of the components. The loads are represented by constant impedances. Meters are provided on the panel board for measuring voltages, currents and powers. The power flow solution in obtained directly from measurements for any system simulated on the analyzer.

With the advent of the modern digital computers possessing large storage and high speed the mode of power flow studies have changed from analog to digital simulation. A large number of algorithms are developed for digital power flow solutions. The methods basically distinguish between themselves in the rate of convergence, storage requirement and time of computation. The loads are gerally represented by constant power.

Network equations can be solved in a variety of ways in a systematic manner. The most popular method is node voltage method. When nodal or bus admittances are used complex linear algebraic simultaneous equations will be obtained in terms of nodal or bus currents. However, as in a power system since the nodal currents are not known, but powers are known at almost all the buses, the resulting mathematical equations become non-linear and are required to be solved by interactive methods. Load flow studies are required as has been already explained for power system planning, operation and control as well as for contingency analysis. The bus admittance matrix is invariably utilized in power flow solutions

5.2 Conditions for Successful Operation of a Power System

There are the following :

1. There should the adequate real power generation to supply the power demand at various load buses and also the losses
2. The bus voltage magnitudes are maintained at values very close to the rated values.
3. Generators, transformers and transmission lines are not over loaded at any point of time or the load curve.

5.3 The Power Flow Equations

Consider an n-bus system the bus voltages are given by

$$\underline{V} = \begin{bmatrix} V_1 \quad \angle\delta_1 \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ V_n \quad \angle\delta_n \end{bmatrix} \qquad .....(5.1)$$

The bus admittance matrix

$$[Y] = [G] + j\,[B] \qquad .....(5.2)$$

where $y_{ik} = |y_{ik}| \angle \theta_{ik}$

$= g_{ik} + j\, b_{ik}$

$$\underline{V}_i = |V_i| \angle \delta_i = |V_2| (\text{Cos } \delta_i + j \sin \delta_i) \quad .....(5.3)$$

$$V_k^* = |V_k| \angle{-\delta_k} = |V_k| (\text{Cos } \delta_k + j \sin \delta_k) \quad .....(5.4)$$

The current injected into the network at bus 'i'

$I_i = Y_{i1} V_1 + Y_{i2} V_2 + ... + Y_{in} V_n \rightarrow$ where n is the number of buses

$$\therefore \quad I_i = \sum_{k=1}^{n} Y_{ik} V_k \quad .....(5.5)$$

The complex power into the system at bus i

$$S_i = P_i + j\, Q_i = V_i I_i^*$$

$$= V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$= \sum_{k=1}^{n} |V_i V_k Y_{ik}| \exp (\delta_i - \delta_k - \theta_{ik}) \quad .....(5.6)$$

Equating the real and imaginary parts

$$P_i = \sum_{k=1}^{n} |V_i V_k Y_{ik}| \text{Cos } (\delta_i - \delta_k - \theta_{ik}) \quad .....(5.7)$$

and

$$Q_i = \sum_{k=1}^{n} |V_i V_k Y_{ik}| \text{Sin } (\delta_i - \delta_k - \theta_{ik}) \quad .....(5.8)$$

where $i = 1, 2, ..., n$

Excluding the slack bus, the above power flow equations are 2 (n – 1) and the variables are P_i, Q_i, $|V_i|$ and $\angle d_i$

Simultaneous solution to the 2 (n – 1) equations

$$P_{Gi} - P_{Di} - \sum_{\substack{k=1 \\ k \neq \text{slack bus}}}^{n} |V_i V_k Y_{ik}| \text{Cos } (\delta_i - \delta_k - \theta_{ik}) = 0 \quad .....(5.9)$$

$$Q_{Gi} - Q_{Di} - \sum_{\substack{k=1 \\ k \neq \text{slack bus}}}^{n} |V_i V_k Y_{ik}| \text{Sin } (\delta_i - \delta_k - \theta_{ik}) = 0 \quad .....(5.10)$$

Constitutes the power flow or load flow solution.

The voltage magnitudes and the phase angles at all load buses are the quantities to be determined. They are called state variables or dependent variables. The specified or scheduled values at all buses are the independent variables.

Y matrix interactive methods are based on solution to power flow equations using their current mismatch at a bus given by

$$\Delta I_i = I_i - \sum_{k=1}^{n} Y_{ik} V_k \quad .....(5.11)$$

or using the voltage form

$$\Delta V_i = \frac{\Delta I_i}{Y_{ii}} \qquad .....(5.12)$$

At the end of the interactive solution to power flow equation, ΔI_i or more usually ΔV_i should become negligibly small so that they can be neglected.

5.4 Classification of Buses

(a) ***Load bus :*** A bus where there is only load connected and no generation exists is called a load bus. At this bus real and reactive load demand P_d and Q_d are drawn from the supply. The demand is generally estimated or predicted as in load forecast or metered and measured from instruments. Quite often, the reactive power is calculated from real power demand with an assumed power factor. A load bus is also called a P, Q bus. Since the load demands P_d and Q_d are known values at this bus. The other two unknown quantities at a load bus are voltage magnitude and its phase angle at the bus. In a power balance equation P_d and Q_d are treated as negative quantities since generated powers P_g and Q_g are assumed positive.

(b) ***Voltage controlled bus or generator bus :***

A voltage controlled bus is any bus in the system where the voltage magnitude can be controlled. The real power developed by a synchronous generator can be varied by changing the prime mover input. This in turn changes the machine rotor axis position with respect to a synchronously rotating or reference axis or a reference bus. In other words, the phase angle of the rotor δ is directly related to the real power generated by the machine. The voltage magnitude on the other hand, is mainly, influenced by the excitation current in the field winding. Thus at a generator bus the real power generation P_g and the voltage magnitude $|V_g|$ can be specified. It is also possible to produce vars by using capacitor or reactor banks too. They compensate the lagging or leading vars consumed and then contribute to voltage control. At a generator bus or voltage controlled bus, also called a PV-bus the reactive power Q_g and δ_g are the values that are not known and are to be computed.

(c) ***Slack bus***

In a network as power flows from the generators to loads through transmission lines power loss occurs due to the losses in the line conductors. These losses when included, we get the power balance relations

$$P_g - P_d - P_L = 0 \qquad .....(5.14)$$

$$Q_g - Q_d - Q_L = 0 \qquad .....(5.15)$$

where P_g and Q_g are the total real and reactive generations, P_d and Q_d are the total real and reactive power demands and P_L and Q_L are the power losses in the transmission network. The values of P_g, Q_g, P_d and Q_d are either known or estimated. Since the flow of cements in the various lines in the transmission lines are not known in advance, P_L and Q_L remains unknown before the analysis of the network. But, these losses have to be supplied by the generators in the system. For this purpose, one of the generators or generating bus is specified as 'slack bus' or 'swing bus'. At this bus the generation P_g and Q_g are not specified. The voltage magnitude is specified at this bus. Further, the voltage phase angle δ is also fixed at this bus. Generally it is specified as 0° so that all voltage phase angles are measured with respect to voltage at this bus. For this reason slack bus is also known as reference bus. All the system losses are supplied by the generation at this bus. Further the system voltage profile is also influenced by the voltage specified at this bus. The three types of buses are illustrated in Fig. 5.1.

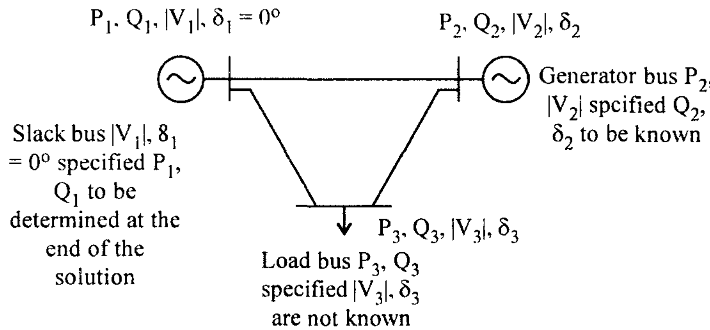


Fig. 5.1

Bus classification is summarized in Table 5.1.

Table 5.1

Bus	**Specified variables**	**Computed variables**
Slack – bus	Voltage magnitude and its phase angle	Real and reactive powers
Generator bus (PV – bus or voltage controlled bus)	Magnitudes of bus voltages and real powers (limit on reactive powers)	Voltage phase angle and reactive power.
Load bus	Real and reactive powers	Magnitude and phase angle of bus voltages

5.5 Bus Admittance Formation

Consider the transmission system shown in Fig. 5.1.

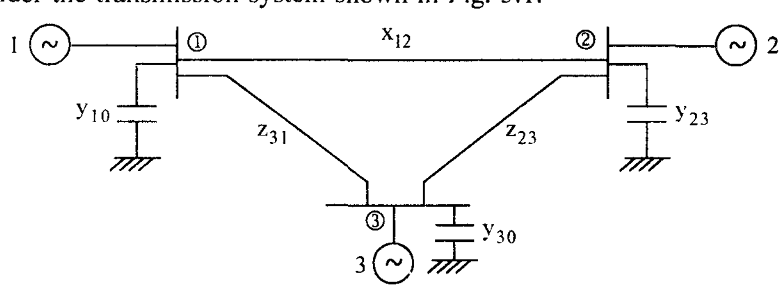


Fig. 5.2 Three bus transmission system.

The line impedances joining buses 1, 2 and 3 are denoted by z_{12}, z_{22} and z_{31} respectively. The corresponding line admittances are y_{12}, y_{22} and y_{31}

The total capacitive susceptances at the buses are represented by y_{10}, y_{20} and y_{30}.

Applying Kirchoff's current law at each bus

$$\left.\begin{aligned} I_1 &= V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13} \\ I_2 &= V_2 y_{20} + (V_2 - V_1) y_{21} + (V_2 - V_3) y_{23} \\ I_3 &= V_3 y_{30} + (V_3 - V_1) y_{31} + (V_3 - V_2) y_{32} \end{aligned}\right\} \quad .....(5.15)$$

In matrix from

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{10} + y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{20} + y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{30} + y_{13} + y_{23} \end{bmatrix} \times$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}, \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad .....(5.16)$$

where

$$\left.\begin{aligned} Y_{11} &= y_{10} + y_{12} + y_{13} \\ Y_{22} &= y_{20} + y_{12} + y_{23} \\ Y_{33} &= y_{30} + y_{13} + y_{23} \end{aligned}\right\} \quad .....(5.17)$$

are the self admittances forming the diagonal terms and

$$\left.\begin{aligned} Y_{12} &= Y_{21} = -y_{12} \\ Y_{13} &= Y_{31} = -y_{13} \\ Y_{23} &= Y_{32} = -y_{23} \end{aligned}\right\} \quad .....(5.18)$$

are the mutual admittances forming the off-diagonal elements of the bus admittance matrix. For an n-bus system, the elements of the bus admittance matrix can be written down merely by inspection of the network as

diagonal terms

$$Y_{ii} = y_{i0} + \sum_{\substack{k=1 \\ k \neq i}}^{n} y_{ik} \quad .....(5.19)$$

off and diagonal terms

$$Y_{ik} = -y_{ik}$$

If the network elements have mutual admittance (impedance), the above formulae will not apply. For a systematic formation of the y-bus, linear graph theory with singular transformations may be used.

5.6 System Model for Load Flow Studies

The variable and parameters associated with bus i and a neighboring bus k are represented in the usual notation as follows :

$$V_i = |V_i| \exp j\,\delta_i = V_i\,(\text{Cos}\,\delta_i + j\,\sin\delta_i) \qquad .....(5.20)$$

Bus admittance,

$$Y_{ik} = |Y_{ik}| \exp j\,\theta_{ik} = |Y_{ik}|\,(\text{Cos}\,q_{ik} + j\,\sin\theta_{ik}) \qquad .....(5.21)$$

Complex power,

$$S_i = P_i + j\,Q_i = V_i\;I^*_i \qquad .....(5.22)$$

Using the indices G and L for generation and load,

$$P_i = P_{Gi} - P_{Li} = \text{Re}\,[Vi\;\;I^*_i] \qquad .....(5.23)$$

$$Q_i = Q_{Gi} - Q_{Li} = \text{Im}\,[Vi\;\;I^*_i] \qquad .....(5.24)$$

The bus current is given by

$$I_{BUS} = Y_{BUS} \cdot V_{BUS} \qquad .....(5.25)$$

Hence, from eqn. (5.22) and (5.23) from an n-bus system

$$I_i^* = \frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{k=1\\k\neq 1}}^{n} Y_{ik} V_k \qquad .....(5.26)$$

and from eqn. (5.26)

$$V_i = -\frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1\\k\neq 1}}^{n} Y_{ik}\,V_k\right] \qquad .....(5.27)$$

Further,

$$P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^*\,V_k^* \qquad .....(5.28)$$

In the polar form

$$P_i + jQ_i = \sum_{k=1}^{n} |V_i\quad V_k\quad Y_{ik}| \exp j(\delta_i \quad - \delta_k \quad - \theta_{ik}) \qquad .....(5.29)$$

so that

$$P_i = \sum_{k=1}^{n} |V_i\quad V_k\quad Y_{ik}| \cos(\delta_i \quad - \delta_k \quad - \theta_{ik}) \qquad .....(5.30)$$

and

$$Q_i = \sum_{k=1}^{n} |V_i\quad V_k\quad Y_{ik}| \sin(\delta_i \quad - \delta_k \quad - \theta_{ik}) \qquad .....(5.31)$$

$i = 1, 2, \ldots n;\; i \neq$ slack bus

The power flow eqns. (5.30) and (5.31) are nonlinear and it is required to solve 2(n-1) such equations involving $|V_i|$, δ_i, P_i and Q_i at each bus i for the load flow solution. Finally, the powers at the slack bus may be computed from which the losses and all other line flows can be ascertained. Y-matrix interactive methods are based on solution to power flow relations using their current mismatch at a bus given by

$$\Delta I_i = I_i - \sum_{k=1}^{n} Y_{ik} V_k \quad .....(5.32)$$

or using the voltage from

$$\Delta V_i = \frac{\Delta I_i}{Y_{ii}} \quad .....(5.33)$$

The convergence of the iterative methods depends on the diagonal dominance of the bus admittance matrix. The self-admittances of the buses, are usually large, relative to the mutual admittances and thus, usually convergence is obtained. Junctions of very high and low series impedances and large capacitances obtained in cable circuits long, EHV lines, series and shunt compensation are detrimental to convergence as these tend to weaken the diagonal dominance in the Y-matrix. The choice of slack bus can affect convergence considerably. In difficult cases, it is possible to obtain convergence by removing the least diagonally dominant row and column of Y. The salient features of the Y-matrix iterative methods are that the elements in the summation terms in eqn. (5.26) or (5.27) are on the average only three, even for well-developed power systems. The sparsity of the Y-matrix and its symmetry reduces both the storage requirement and the computation time for iteration. For a large, well conditioned system of n-buses, the number of iterations required are of the order of n and total computing time varies approximately as n^2.

Instead of using eqn (5.25), one can select the impedance matrix and rewrite the equation as

$$V = Y^{-1} I = Z \,.\, I \quad .....(5.34)$$

The Z-matrix method is not usually very sensitive to the choice of the slack bus. It can easily be verified that the Z-matrix is not sparse. For problems that can be solved by both Z-matrix and Y-matrix methods, the former are rarely competitive with the Y-matrix methods.

5.7 Gauss-Seidel Iterative Method

Gauss-Seidel iterative method is very simple in concept but may not yield convagence to the required solution. However, when the initial solution or starting point is very close to the actual solution convergence is generally ontained. The following example illustrator the method.

Consider the equations :

$$2x + 3y = 22$$

$$3x + 4y = 31$$

The free solution to the above equations is $x = 5$ and $y = 4$

If an interactive solution using Gauss-Seidel method is required then let us assume a starting value for x = 4.8 which is nearer to the true value of 5 we obtain from the given equations

$$y = \frac{22-2x}{3} \text{ and } x = \frac{34-4y}{3}$$

Iteration 1 : Let x = 4.8 ; y = 4.13

with y = 4.13; x = 6.2

Iteration 2	x = 6.2;	;	y = 3.2
	y = 3.2;	;	x = 6.06
Iteration 3 :	x = 6.06	;	y = 3.29
	y = 3.29	;	x = 5.94
Iteration 4 :	x = 5.96	;	y = 3.37
	y = 3.37	;	x = 5.84
Iteration 5 :	x = 5.84	;	y = 3.44
	y = 3.44	;	x = 5.74
Iteration 6 :	x = 5.74	;	y = 3.5
	y = 3.5	;	x = 5.66

The iteractive solution slowly converges to the true solution. The convergence of the method depending upon the starting values for the iterative solution.

In many cases the conveyence may not be obtained at all. However, in case of power flow studies, as the bus voltages are not very far from the rated values and as all load flow studies are performed with per unit values assuming a flat voltage profile at all load buses of (1 + j0) p.u. yields convergence in most of the cases with appriate accleration factors chosen.

5.8 Gauss – Seidel Iterative Method of Load Flow Solution

In this method, voltages at all buses except at the slack bus are assumed. The voltage at the slack bus is specified and remains fixed at that value. The (n-1) bus voltage relations.

$$\mathbf{V}_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{\mathbf{V}_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik}\, \mathbf{V}_k\right] \qquad .....(5.35)$$

i = 1, 2, ... n; i ≠ slack bus

are solved simultaneously for an improved solution. In order to accelerate the convergence, all newly-computed values of bus voltages are substituted in eqn. (5.35). Successively the bus voltage equation of the (m + 1)th iteration may then be written as

$$\mathbf{V}_i^{(m+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{\mathbf{V}_i^{(m)*}} - \sum_{k=1}^{i-1} Y_{ik}\, \mathbf{V}_k^{(m+1)} - \sum_{k=i-1}^{n} Y_{ik}\, \mathbf{V}_k^{(m)}\right] \qquad .....(5.36)$$

The method converges slowly because of the loose mathematical coupling between the buses. The rate of convergence of the process can be increased by using acceleration factors to the solution obtained after each iteration. A fixed acceleration factor α ($1 \le \alpha \le 2$) is normally used for each voltage change.

$$\Delta \mathbf{V}_i = \alpha \frac{\Delta \mathbf{S}_i^*}{\mathbf{V}_i^* \mathbf{Y}_{ii}} \qquad \text{.....(5.37)}$$

The use of the acceleration factor amounts to a linear extrapolation of V_i. For a given system, it is quite often found that a near-optimal choice of α exists as suggested in literature over a range of operating conditions. Even though a complex value of α is suggested in literature, it is more convenient to operate with real values given by

$$\left|V_i^{(m)}\right| \angle \delta_i = |\alpha| \left|V_i^{(m)}\right| \angle \delta_i \qquad \text{.....(5.38)}$$

Alternatively, different acceleration factors may be used for real and imaginary parts of the voltage.

Treatment of a PV - bus

The method of handling a PV-bus requires rectangular coordinate representation for the voltages. Letting

$$\mathbf{V}_i = v_i' + jv_i'' \qquad \text{.....(5.39)}$$

Where v_i' and v_i'' are the real and imaginary components of $\mathbf{V}_i$ the relationship.

$$v_i'^2 + v_i''^2 = |V_i|^2_{\text{scheduled}} \qquad \text{.....(5.40)}$$

must be satisfied. so that the reactive bus power required to establish the scheduled bus voltage can be computed. The estimates of voltage components, $v_i'^{(m)}$ and $v_i''^{(m)}$ after m iterations must be adjusted to satisfy eqn. (5.40). The Phase angle of the estimated bus voltage is

$$\delta_i^{(m)} = \tan^{-1}\left[\frac{v_i''^{(m)}}{v_i'^{(m)}}\right] \qquad \text{.....(5.41)}$$

Assuming that the phase angles of the estimated and scheduled voltages are equal, then the adjusted estimates of $V'^{(m)}$ and $V_i''^{(m)}$ are

$$v'^{(m)}_{i(\text{new})} = |V_i|_{\text{scheduled}} \cos \delta_i^{(m)} \qquad \text{.....(5.42)}$$

and

$$v''^{(m)}_{i(\text{new})} = |V_i|_{\text{scheduled}} \sin \delta_i^{(m)} \qquad \text{.....(5.43)}$$

These values are used to calculate the reactive power $Q_i^{(m)}$. Using these reactive powers $Q_i^{(m)}$ and voltages $V_{i(\text{new})}^{(m)}$ a new estimate $V_i^{(m+1)}$ is calculated. The flowchart for computing the solution of load flow using gauss-seidel method is given in Fig. 5.3.

While computing the reactive powers, the limits on the reactive source must be taken into consideration. If the calculated value of the reactive power is beyond limits. Then its value is fixed at the limit that is violated and it is no longer possible to hold the desired magnitude of the bus voltage, the bus is treated as a PQ bus or load bus.

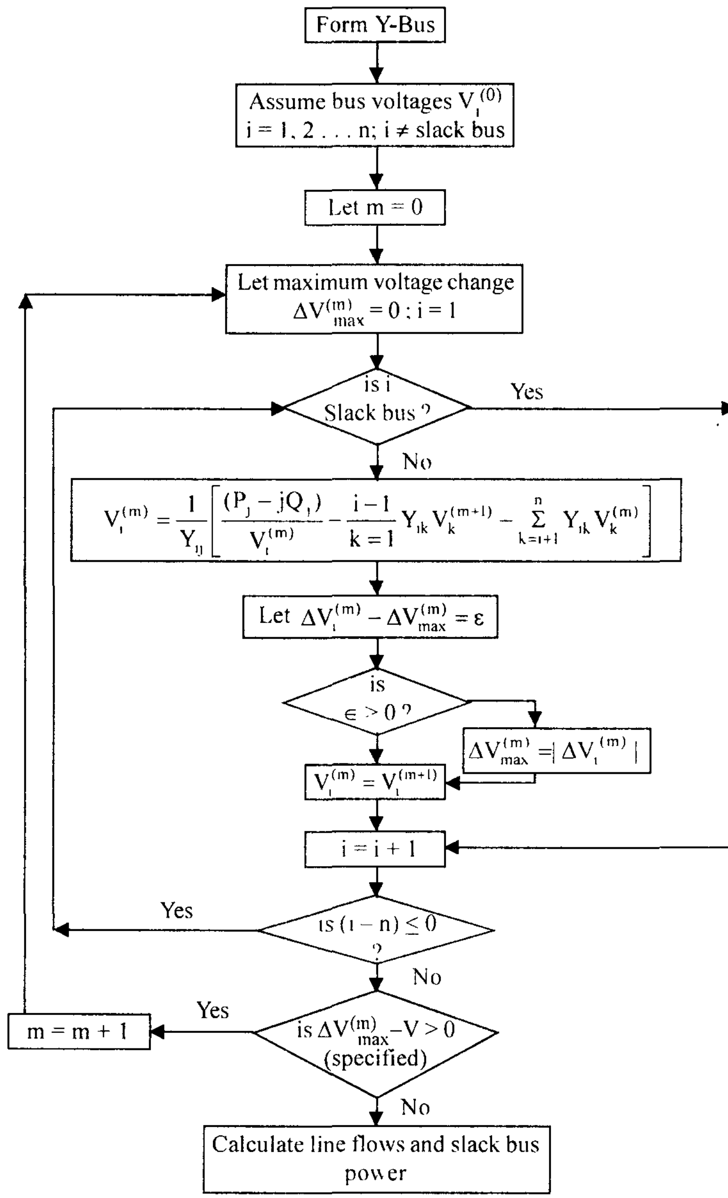


Fig. 5.3 Flowchart for Gauss – Seidel iterative method for load flow solution using Y-Bus.

5.9 Newton-Raphson Method

The generated Newton-Raphson method is an interactive algorithm for solving a set of simultaneous nonlinear equations in an equal number of unknowns. Consider the set of nonlinear equations.

$$f_i(x_1, x_2, \ldots\ldots, x_n) = y_i, \; i = 1, 2, \ldots., n \qquad \ldots..(5.44)$$

with initial estimates for x_i

$$x_1^{(0)}, x_2^{(0)}, \ldots\ldots, x_n^{(0)}$$

which are not far from the actual solution. Then using Taylor's series and neglecting the higher order terms, the corrected set of equations are

$$\left(x_1^{(0)} + \Delta x_1, x_2^{(0)} + \Delta x_2, \ldots\ldots, x_n^{(0)} + \Delta x_n\right) = y_i \qquad \ldots..(5.45)$$

where Δx_i are the corrections to $x_i = (i = 1, 2, \ldots\ldots, n)$

A set of linear equations, which define a tangent hyperplane to the function $f_i(x)$ at the given iteration point $\left(x_i^{(0)}\right)$ are obtained as

$$\Delta Y = J \Delta X \qquad \ldots..(5.46)$$

where ΔY is a column vector determined by

$$y_i - f_i\left(x_1^{(0)}, \ldots\ldots, x_n^{(0)}\right)$$

ΔX is the column vector of correction terms Δx_i, and J is the Jacobian matrix for the function f given by the first order partial derivatives evaluated at $x_i^{(0)}$ The corrected solution is obtained as

$$x_i^{(1)} = x_i^{(0)} + \Delta x_i \qquad \ldots..(5.47)$$

The square Jacobian matrix J is defined by

$$J_{ik} = \frac{\partial f_i}{\partial x_k} \qquad \ldots..(5.48)$$

The above method of obtaining a converging solution for a set of nonlinear equations can be used for solving the load flow problem. It may be mentioned that since the final voltage solutions are not much different from the nominal values, Newton – Raphson method is particularly suited to the load flow problem. The matrix J is highly sparse and is particularly suited to the load flow application and sparsity – programmed ordered triangulation and back substitution methods result in quick and efficient convergence to the load flow solution. This method possesses quadratic convergence and thus converges very rapidly when the solution point is close.

There are two methods of solution for the load flow using Newton – Raphson method. The first method uses rectangular coordinates for the variables, while the second method uses the polar coordinate formulation.

5.9.1 The Rectangular Coordinates Method

The power entering the bus i is given by

$$S_i = P_i + j\,Q_i$$

$$= V_i\, I_i^* = V_i \sum_{k=1}^{n} Y_{ik}^*\, V_{ik}^*, \; i = 1,2,....,n \qquad .....(5.49)$$

Where $\quad V_i = v_i' + jv_i''$

and $\quad Y_{ik} = G_{ik} + j\,B_{ik}$

$$(P_i + jQ_i) = \left((v_i' + jv_i'') \sum_{k=1}^{n} (G_{ik} - jB_{ik})(v_k' - v_k'') \right) V_K' - j\,V_K'' \qquad .....(5.50)$$

Expanding the right side of the above equation and separating out the real and imaginary parts.

$$P_i = \sum_{k=1}^{n} \left[v_i' \left(G_{ik}\, v_k' - B_{ik}\, v_k'' \right) + v_i'' \left(G_{ik} v_k'' - B_{ik}\, v_k' \right) \right] \qquad .....(5.51)$$

$$Q_i = \sum_{k=1}^{n} \left[v_i' \left(G_{ik}\, v_k' - G_{ik}\, v_k'' \right) - v_i'' \left(G_{ik}\, v_k'' + G_{ik}\, v_k' \right) \right] \qquad .....(5.52)$$

These are the two power relations at each bus and the linearized equations of the form (5.46) are written as

$$\begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \\ \Delta Q_1 \\ \vdots \\ \Delta Q_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial v_1''} & \cdots & \frac{\partial P_1}{\partial v_{n-1}''} & \frac{\partial P_1}{\partial v_n''} & \cdots & \frac{\partial P_1}{\partial v_{n-1}''} \\ \vdots & \cdots & & & \cdots & \vdots \\ \frac{\partial P_{n-1}}{\partial v_1'} & \cdots & \frac{\partial P_{n-1}}{\partial v_{n-1}'} & \frac{\partial P_{n-1}}{\partial v_n''} & \cdots & \frac{\partial P_{n-1}}{\partial v_{n-1}''} \\ \frac{\partial Q_1}{\partial v_1''} & \cdots & \frac{\partial Q_{n-1}}{\partial v_{n-1}''} & \frac{\partial Q_1}{\partial v_1'} & \cdots & \frac{\partial Q_{n-1}}{\partial v_{n-1}''} \\ \vdots & \cdots & & & \cdots & \vdots \\ \frac{\partial Q_{n-1}}{\partial v_1'} & \cdots & \frac{\partial Q_{n-1}}{\partial v_{n-1}'} & \frac{\partial Q_{n-1}}{\partial v_1''} & \cdots & \frac{\partial Q_{n-1}}{\partial v_{n-1}''} \end{bmatrix} \begin{bmatrix} \Delta v_1' \\ \vdots \\ \Delta v_{n-1}' \\ \Delta v_1'' \\ \vdots \\ \Delta v_{n-1}'' \end{bmatrix} \qquad .....(5.53)$$

Matrix equation (5.53) can be solved for the unknowns $\Delta v_i'$ and $\Delta v_i''$ $(i = 1, 2,\ldots, n-1)$, leaving the slack bus at the n^{th} bus where the voltage is specified. Equation (2.33) may be written compactly as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta v' \\ \Delta v'' \end{bmatrix} \qquad .....(5.54)$$

where H, N, M and L are the sub-matrices of the Jacobian. The elements of the Jacobian are obtained by differentiating Eqns. (5.51) and (5.52). The off-diagonal and diagonal elements of

H matrix are given by

$$\frac{\partial P_i}{\partial v'_k} = G_{ik} v'_k + B_{ik} v''_k, i \neq k \qquad .....(5.55)$$

$$\frac{\partial P_i}{\partial v'_k} = 2G_{ik} v'_k - B_{ik} v''_k + B_{ii} v''_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(G_{ik} v'_k - B_{ik} v''_k \right) \qquad .....(5.56)$$

The off-diagonal and diagonal elements of N are :

$$\frac{\partial P_i}{\partial v'_k} = G_{ik} v''_k - B_{ik} v'_k, k \neq i \qquad .....(5.57)$$

$$\frac{\partial P_i}{\partial v''_k} = -B_{ii} v'_i + 2G_{ii} v''_k + B_{ii} v'_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(G_{ik} v''_k + B_{ii} v'_k \right) \qquad .....(5.58)$$

The off-diagonal and diagonal elements of sub-matrix M are obtained as,

$$\frac{\partial Q_i}{\partial v'_k} = G_{ik} v''_i - B_{ik} v'_i, k \neq i \qquad .....(5.59)$$

$$\frac{\partial Q_i}{\partial v'_i} = G_{ii} v''_i - G_{ii} v''_i - 2B_{ii} v'_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(G_{ik} v''_k + B_{ik} v'_k \right) \qquad .....(5.60)$$

Finally, the off-diagonal and diagonal elements of L are given by

$$\frac{\partial Q_i}{\partial v'_k} = -G_{ik} v'_i - B_{ik} v''_i, k \neq i \qquad .....(5.61)$$

$$\frac{\partial Q_i}{\partial v'_i} = G_{ii} v''_i - 2B_{ii} v'_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(G_{ik} v''_k - B_{ik} v'_k \right) \qquad .....(5.62)$$

It can be noticed that

$$L_{ik} = -H_{ik}$$

and $$N_{ik} = M_{ik}$$

This property of symmetry of the elements reduces computer time and storage.

Treatment of Generator Buses

At all generator buses other than the swing bus, the voltage magnitudes are specified in addition to the real powers. At the ith generator bus

$$|V_i|^2 = v_i'^2 + v_i''^2 \qquad .....(5.63)$$

Then, at all the generator nodes, the variable ΔQ_i will have to be replaced by

$$\Delta|V_i|^2$$

But,
$$|\Delta V_i|^2 = \frac{\partial(|\Delta_i|^2)}{\partial v_i'}\Delta V_i' + \frac{\partial(|\Delta_i|^2)}{\partial v_i''}\Delta V_i''$$

$$= 2\ v_i'' + \Delta v_i' + 2v_i''\Delta v_i' \qquad .....(5.64)$$

This is the only modification required to be introduced in eqn. (5.60)

5.9.2 The Polar Coordinates Method

The equation for the complex power at node i in the polar form is given in eqn. (5.60) and the real and reactive powers at bus i are indicated in eqn. (5.30) and (5.31). Reproducing them here once again for convenience.

$$P_i = \sum_{k=1}^{n} |V_i\ V_k\ Y_{ik}|\cos(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.65)$$

and
$$Q_i = \sum_{k=1}^{n} |V_i\ V_k\ V_{ik}|\sin(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.66)$$

The Jocobian is then formulated in terms of $|V|$ and δ instead of V_i' and V_i'' in this case. Eqn. (5.46) then takes the form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \qquad .....(5.67)$$

The off-diagonal and diagonal elements of the sub-matrices H, N, M and L are determined by differentiating eqns. (5.30) and (5.31) with respect to δ and $|V|$ as before. The off-diagonal and diagonal elements of H matrix are

$$\frac{\partial P_i}{\partial \delta_k} = |V_i\ V_k\ V_{ik}|\sin(\delta_i - \delta_k - \theta_{ik}), i \neq k \qquad .....(5.68)$$

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{k=1}^{n} |V_i\ V_k\ V_{ik}|\cos(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.69)$$

The Jocobian is then formulated in terms of $|V|$ and δ instead of V_i' and V_i'' in this case. Eqn. (5.46) then takes the form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \qquad .....(5.70)$$

The off-diagonal and diagonal elements of the sub-matrices H, N, M and L are determined by differentiating eqns. (5.30) and (5.31) with respect to δ and |V| as before. The off-diagonal and diagonal elements of H matrix are

$$\frac{\partial P_i}{\partial \delta_k} = |V_i \quad V_k \quad Y_{ik}| \sin(\delta_i \quad - \delta_k \quad - \theta_{ik}), i \neq k \qquad .....(5.71)$$

$$\frac{\partial P_i}{\partial \delta_k} = \sum_{K=1}^{n} |V_i \quad V_k \quad Y_{ik}| \cos(\delta_i \quad - \delta_k \quad - \theta_{ik}), i \neq k \qquad .....(5.72)$$

The off-diagonal and diagonal elements of N matrix are

$$\frac{\partial P_i}{\partial |V_k|} = |V_i \ Y_{ik}| \sin(\delta_i \quad - \delta_k \quad - \theta_{ik}) \qquad .....(5.73)$$

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i Y_{ii}| \text{Cos}\theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k Y_{ik}| \text{Cos}(\delta_i - \delta_k - \delta_{ik}) \qquad .....(5.74)$$

The off-diagonal and diagonal elements of M matrix are

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i \quad V_k \quad Y_{ik}| \text{Cos}(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.75)$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i \quad V_k \quad Y_{ik}| \text{Cos}(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.76)$$

Finally, the off-diagonal and diagonal elements of L matrix are

$$\frac{\partial Q_i}{\partial |V_k|} = |V_i \ Y_{ik}| \sin(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.77)$$

$$\frac{\partial Q_i}{\partial |V_i|} = |2V_i Y_{ii}| \text{Cos}\theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k Y_{ik}| \text{Sin}(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.78)$$

It is seen from the elements of the Jacobian in this case that the symmetry that existed in the rectangular coordinates case is no longer present now. By selecting the variable as $\Delta\delta$ and $\Delta|V| / |V|$ instead equation (5.70) will be in the form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix} \qquad .....(5.79)$$

In this case it will be seen that

$$H_{ik} = L_{ik}$$

and $$N_{ik} = -M_{ik}$$

or, in other words, the symmetry is restored. The number of elements to be calculated for an n – dimensional Jacobian matrix are only $n + n^2/2$ instead of n^2, thus again saving computer time and storage. The flow chart for computer solution is given in Fig. 5.4.

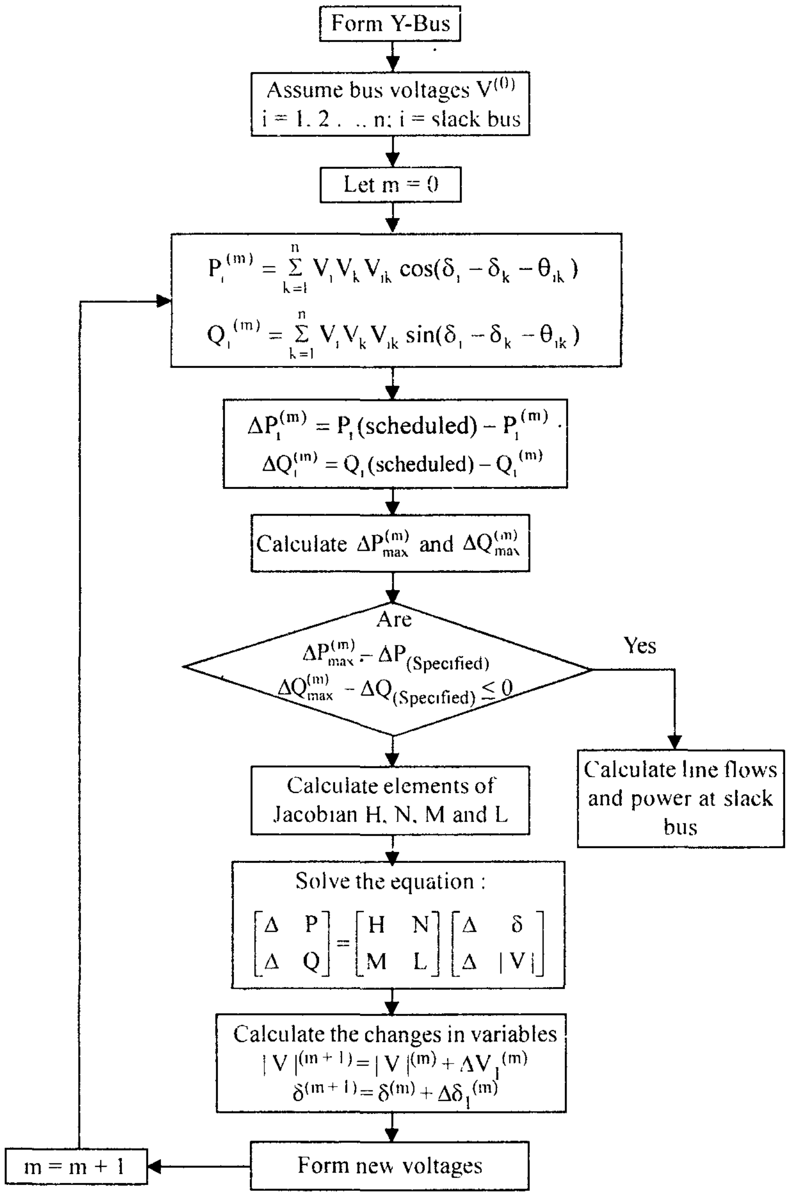


Fig. 5.4 Flow chart for Newton – Raphson method (Polar coordinates) for load flow solution.

Treatment of Generator Nodes

For a PV-bus, the reactive power equations are replaced at the i^{th} generator bus by

$$|V_i|^2 = v_i'^2 + v_i''^2 \qquad .....(5.80)$$

The elements of M are given by

$$M_{ik} = \frac{\partial(|V|)^2}{\partial\delta_k} = 0; i \neq k \qquad .....(5.81)$$

and
$$M_{ii} = \frac{\partial|V_i|^2}{\partial\delta_i} = 0 \qquad .....(5.82)$$

Then elements of L are given by

$$L_{ik} = \frac{\partial\left(|V_i|^2\right)}{\partial|V_k|}|V_k| = 0; i \neq k \qquad .....(5.83)$$

and
$$L_{ii} = \frac{\partial(|V_i|)^2}{\partial|V_i|}|V_i| = 2|V_i|^2 \qquad .....(5.84)$$

Newtons method converges in 2 to 5 iterations from a flat start ($\{V\} = 1.0$ p.u and $\delta = 0$) independent of system size. Previously stored solution can be used as starting values for rapid convergence. Iteration time can be saved by using the same triangulated Jacobian matrix for two or more iterations. For typical, large size systems, the computing time for one Newton – Raphson iteration is roughly equivalent to seven Gauss – Seidel iterations.

The rectangular formulation is marginally faster than the polar version because there are no time consuming trigonometric functions. However, it is observed that the rectangular coordinates method is less reliable than the polar version.

5.10 Sparsity of Network Admittance Matrices

For many power networks, the admittance matrix is relatively sparse, where as the impedance matrix is full. In general, both matrices are nonsingular and symmetric. In the admittance matrix. each non-zero off diagonal element corresponds to a network branch connecting the pair of buses indicated by the row and column of the element. Most transmission networks exhibit irregularity in their connection arrangements, and their admittance matrices are relatively sparse. Such sparse systems possess the following advantages:

1. Their storage requirements are small, so that larger systems can be solved.
2. Direct solutions using triangularization techniques can be obtained much faster unless the independent vector is extremely sparse.
3. Round off errors are very much reduced.

The exploitation of network sparsity requires sophisticated programming techniques.

5.11 Triangular Decompostion

Matrix inversion is a very inefficient process for computing direct solutions, especially for large, sparse systems. Triangular decomposition of the matrix for solution by Gussian elimination is more suited for load flow solutions. Generally, the decomposition is accomplished by elements below the main diagonal in successive columns. Elimination by successive rows is more advantageous from computer programming point of view.

Consider the system of equations

$$\mathbf{A}\ \mathbf{x} = \mathbf{b} \qquad \text{.....(5.84)}$$

where **A** is a nonsingular matrix, **b** is a known vector containing at least one non-zero element and **x** is a column vector of unknowns.

To solve eqn. (5.84) by the triangular decomposition method, matrix **A** is augmented by **b** as shown

$$\begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} & b_2 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} & b_n \end{bmatrix}$$

The elements of the first row in the augmented matrix are divided by a_{11} as indicated by the following step with superscripts denoting the stage of the computation.

$$a_{1j}^{(1)} = \left(\frac{1}{a_{11}}\right) a_{1j}, j = 2, \ldots\ldots\ldots n \qquad \text{.....(5.85)}$$

$$b_1^{(1)} = \left(\frac{1}{a_{11}}\right) b_1 \qquad \text{.....(5.86)}$$

In the next stage a_{21} is eliminated from the second row using the relations

$$a_{2j}^{(1)} = a_{2j} - a_{21}\, a_{1j}^{(1)},\ j = 2, \ldots\ldots\ldots, n \qquad \text{.....(5.87)}$$

$$b_2^{(1)} = b_2 - a_{21}\ b_1^{(1)} \qquad \text{.....(5.88)}$$

$$a_{2j}^{(2)} = \left(\frac{1}{a_{22}^{(2)}}\right) a_{2j}^{(1)},\ j = 3, \ldots\ldots\ldots, n$$

$$b_2^{(2)} = \left(\frac{1}{a_{22}^{(1)}}\right) b_2^{(1)} \qquad \text{.....(5.89)}$$

The resulting matrix then becomes

$$\begin{bmatrix} 1 & 1_{12}^{(1)} & a_{13}^{(1)} & \ldots\ldots & a_{1n}^{(1)} & b_1^{(1)} \\ 0 & 1 & a_{23}^{(2)} & \ldots\ldots & a_{2n}^{(2)} & b_2^{(2)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & \ldots\ldots & a_{nn} & b_n \end{bmatrix}$$

using the relations

$$b_{3j}^{(1)} = a_{3j} - a_{31}a_{1j}^{(1)} \quad j = 2,\ldots\ldots\ldots\ldots, n \qquad \ldots..(5.90)$$

$$b_{(3)}^{1} = b_3 - a_{31}b_1^{(1)} \qquad \ldots..(5.91)$$

$$a_{3j}^{(2)} = a_{3j}^{(1)} - a_{32}^{(1)}\, a_{2j}^{(2)}, j = 3,\ldots\ldots\ldots\ldots, n \qquad \ldots..(5.92)$$

$$b_3^{(2)} = b_3^{(1)} - a_{32}^{(1)}b_3^{(2)}, j = 4,\ldots\ldots, n \qquad \ldots..(5.93)$$

$$a_3^{(3)} = \left(\frac{1}{a_{33}^{(2)}}\right)a_{3j}^{(1)}, j = 4,\ldots\ldots, n \qquad \ldots..(5.94)$$

$$b_3^{(3)} = \left(\frac{1}{a_{33}^{(2)}}\right)b_3^{(2)} \qquad \ldots..(5.95)$$

The elements to the left of the diagonal in the third row are eliminated and further the diagonal element in the third row is made unity.

After n steps of computation for the nth order system of eqn. (5.84), the augmented matrix will be obtained as

$$\begin{bmatrix} 1 & a_{12}^{(1)} & \ldots\ldots & a_{1n}^{(1)} & b_1^{(1)} \\ 0 & 1 & \ldots\ldots & a_{2n}^{(2)} & b_2^{(2)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 0 & 0 & \ldots\ldots & 1 & b_n^{(n)} \end{bmatrix}$$

By back substitution, the solution is obtained as

$$x_n = b_n^{(n)} \qquad \ldots..(5.96)$$

$$x_{n-1} = b_{n-1}^{(n-1)} - a_{n-1}^{(n-1)}, n\ldots\ldots x_n \qquad \ldots..(5.97)$$

$$x_i = b_i^{(1)} - \sum_{j=i+1}^{n} a_{ij}^{(1)} x_i \qquad \ldots..(5.98)$$

For matrix inversion of an n^{th} order matrix, the number of arithmetical operations required is n^3 while for the triangular decomposition it is approximately $\left(\frac{n^3}{3}\right)$.

5.12 Optimal Ordering

When the **A** matrix in eqn. (5.84) is sparse, it is necessary to see that the accumulation of non – zero elements in the upper triangle is minimized. This can be achieved by suitably ordering the equations, which is referred to as optimal ordering.

Consider the network system having five nodes as shown in Fig. 5.5.

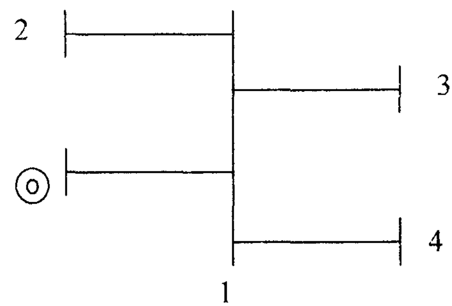


Fig. 5.5 A Five Bus system.

The y-bus matrix of the network will have entries as follows

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \times & \times & \times & \times \\ 2 & \times & \times & 0 & 0 \\ 3 & \times & 0 & \times & 0 \\ 4 & \times & 0 & 0 & \times \end{array} = Y \qquad .....(5.99)$$

After triangular decomposition the matrix will be reduced to the form

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & \times & \times & \times \\ 2 & 0 & 1 & \times & \times \\ 3 & 0 & 0 & 1 & \times \\ 4 & 0 & 0 & 0 & 1 \end{array} = Y \qquad .....(5.100)$$

By ordering the nodes as in Fig. 5.6 the bus admittance matrix will be of the form

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \times & 0 & 0 & \times \\ 2 & 0 & \times & 0 & \times \\ 3 & 0 & 0 & \times & \times \\ 4 & \times & \times & \times & \times \end{array} = Y \qquad .....(5.101)$$

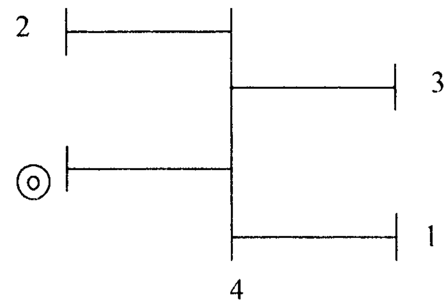


Fig. 5.6 Renumbered five bus system.

As a result of triangular decomposition, the **Y**-matrix will be reduced to

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & \times \\ 2 & 0 & 1 & 0 & \times \\ 3 & 0 & 0 & 1 & \times \\ 4 & 0 & 0 & 0 & 1 \end{array} = Y \qquad \text{.....(5.102)}$$

Thus, comparing the matrices in eqn. (5.100) and (5.102) the non-zero off diagonal entries are reduced from 6 to 3 by suitably numbering the nodes.

Tinney and Walker have suggested three methods for optimal ordering.

1. Number the rows according to the number of non-zero, off-diagonal elements before elimination. Thus, rows with less number of off diagonal elements are numbered first and the rows with large number last.
2. Number the rows so that at each step of elimination the next row to be eliminated is the one having fewest non-zero terms. This method required simulation of the elimination process to take into account the changes in the non-zero connections affected at each step.
3. Number the rows so that at each step of elimination, the next row to be eliminated is the one that will introduce fewest new non-zero elements. This requires simulation of every feasible alternative at each step.

Scheme 1. is simple and fast. However, for power flow solutions, scheme 2. has proved to be advantageous even with its additional computing time. If the number of iterations is large, scheme 3. may prove to be advantageous.

5.13 Decoupled Methods

All power systems exhibit in the steady state a strong interdependence between active powers and bus voltage angles and between reactive power and voltage magnitude. The coupling between real power and bus voltage magnitude and between reactive power and bus voltage phase angle are both relatively weak. This weak coupling is utilized in the development of the so called decoupled methods. Recalling equitation (5.79)

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ |V|/\Delta|V| \end{bmatrix}$$

by neglecting N and M sub matrices as a first step, decoupling can be obtained so that

$$|\Delta P| = |H| \, . \, |\Delta\delta| \qquad .....(5.103)$$

and

$$|\Delta Q| = |L| \, . \, |\Delta|V| / |V| \qquad .....(5.104)$$

The decoupled method converges as reliability as the original Newton method from which it is derived. However, for very high accuracy the method requires more iterations because overall quadratic convergence is lost. The decoupled Newton method saves by a factor of four on the storage for the J – matrix and its triangulation. But, the overall saving is 35 to 50% of storage when compared to the original Newton method. The computation per iteration is 10 to 20% less than for the original Newton method.

5.14 Fast Decoupled Methods

For security monitoring and outage-contingency evaluation studies, fast load flow solutions are required. A method developed for such an application is described in this section.

The elements of the sub-matrices H and L (eqn. (5.79)) are given by

$$\begin{aligned} H_{ik} &= |(V_i V_k Y_{ik})| \sin(\delta_i - \delta_k - \theta_{ik}) \\ &= |(V_i V_k Y_{ik})| \sin\delta_{ik} \, \mathrm{Cos}\,\theta_{ik} - \cos\delta_{ik} \, \mathrm{Cos}\,\theta_{ik}) \\ &= |V_i V_k| \, [G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}] \qquad .....(5.105) \end{aligned}$$

where

$$\delta_i - \delta_k = \delta_{ik}$$

$$\begin{aligned} H_{kk} &= -\sum |V_i \quad V_k \quad Y_{ik}| \sin(\delta_i - \delta_k - \theta_{ik}) \\ &= +|V_i|^2 |Y_{ii}| \sin\theta_{ik} - |V_i|^2 |V_{ik}| \sin\theta_{ik} \\ &\quad - \sum |V_i \quad V_k \quad Y_{ik}| \sin(\delta_i - \delta_k - \theta_{ik}) \\ &= V_i^2 B_{ii} - Q_i \qquad .....(5.106) \end{aligned}$$

$$L_{kk} = 2\,V_i Y_{ii} \sin\theta_{ii} + \sum V_k Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik}) \qquad .....(5.107)$$

With $\Delta|V| / |V|$ formulation on the right hand side,

$$\begin{aligned} L_{KK} &= 2\,|V_i^2 Y_{ii}| \sin\theta_{ii} + \sum |V_i \quad V_k \quad Y_{ik}| \sin(\delta_i - \delta_k - \theta_{ik}) \\ &= |V_i^2| B_{ii} + Q_i \qquad .....(5.108) \end{aligned}$$

Assuming that

$$\mathrm{Cos}\,\delta_{ik} \cong 1$$

$$\text{Sin } \delta_{ik} \cong 0$$

$$\text{Gik} \sin \delta_{ik} \leq B_{ik}$$

and

$$Q_i \leq B_{ii} \, |V_i^2|$$

$$\left.\begin{aligned} H_{ik} &= -|V_i V_k| B_{ik} \\ H_{kk} &= |V_i^2| B_{ii} \\ L_{kk} &= |V_i^2| B_{ii} \\ L_{ik} &= |V_i^2| B_{ik} \end{aligned}\right\} \qquad \text{.....(5.109)}$$

Rewriting eqns. (5.103) and (5.104)

$$|\Delta P| = \left[|V| B' \, | V \right] \Delta\delta \qquad \text{.....(5.110)}$$

$$|\Delta Q| = |V| B'' \, |V| \frac{\Delta |V|}{|V|} \qquad \text{.....(5.111)}$$

or

$$\frac{|\Delta P|}{|V|} = B'[\Delta\delta] \qquad \text{.....(5.112)}$$

$$\frac{|\Delta Q|}{|V|} = B''[\Delta |V|] \qquad \text{.....(5.113)}$$

Matrices B' and B'' represent constant approximations to the slopes of the tangent hyper planes of the functions $\Delta P / |V|$ and $\Delta Q / |V|$ respectively. They are very close to the Jacobian sub matrices H and L evaluated at system no-load.

Shunt reactances and off-nominal in-phase transformer taps which affect the Mvar flows are to be omitted from [B'] and for the same reason phase shifting elements are to be omitted from [B''].

Both [B'] and [B''] are real and spars e and need be triangularised only once, at the beginning of the study since they contain network admittances only.

The method converges very reliably in two to five iterations with fairly good accuracy even for large systems. A good, approximate solution is obtained after the 1st or 2nd iteration. The speed per iteration is roughly five times that of the original Newton method.

5.15 Load Flow Solution Using Z Bus

5.15.1 Bus Impedance Formation

Any power network can be formed using the following possible methods of construction.

1. A line may be added to a reference point or bus.

2. A bus may be added to any existing bus in the system other than the reference bus through a new line, and
3. A line may be added joining two existing buses in the system forming a loop.

The above three modes are illustrated in Fig. 5.7.

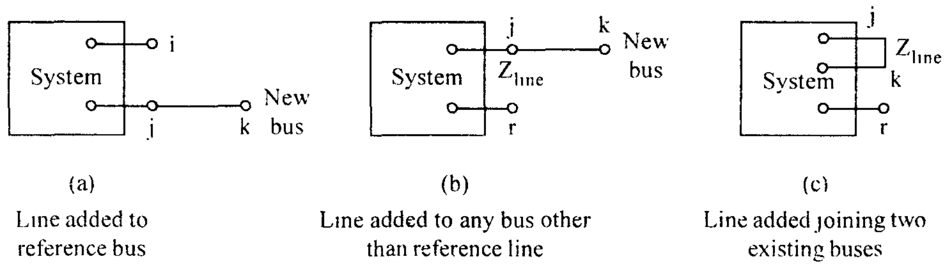


Fig. 5.7 Building of Z - Bus.

5.15.2 Addition of a Line to the Reference Bus

If unit current is injected into bus k no voltage will be produced at other buses of the systems.

$$Z_{ik} = Z_{ki} = 0, \ i \neq k \qquad .....(5.114)$$

The driving point impedance of the new bus is given by

$$Z_{kk} = Z_{line} \qquad .....(5.115)$$

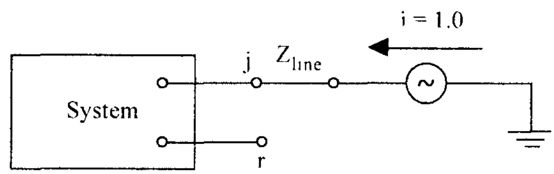


Fig. 5.8 Addition of the line to reference line

5.15.3 Addition of a Radial Line and New Bus

Injection of unit current into the system through the new bus k produces voltages at all other buses of the system as shown in Fig. 5.9.

These voltages would of course, be same as that would be produced if the current were injected instead at bus i as shown.

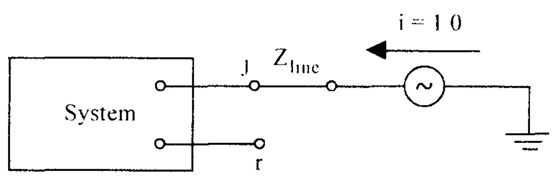


Fig. 5.9 Addition of a radial line and new bus.

Therefore,

$$Z_{km} = Z_{im}$$

therefore,

$$Z_{mk} = Z_{mi},\ m \neq k \qquad .....(5.116)$$

The dimension of the existing Z – Bus matrix is increased by one. The off diagonal elements of the new row and column are the same as the elements of the row and column of bus i of the existing system.

5.15.4 Addition of a Loop Closing Two Existing Buses in the System

Since both the buses are existing buses in the system the dimension of the bus impedance matrix will not increase in this case. However, the addition of the loop introduces a new axis which can be subsequently eliminated by Kron's reduction method.

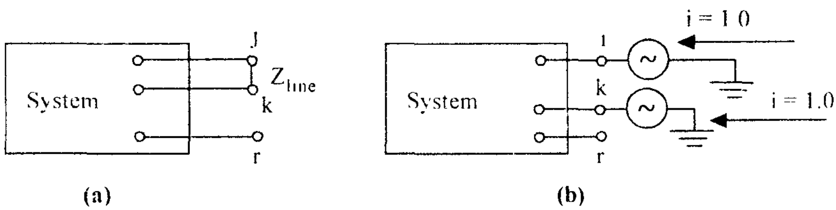


Fig. 5.10 (a) Addition of a loop (b) Equivalent representation

The systems in Fig. 5.10(a) can be represented alternatively as in Fig. 5.10(b).

The link between i and k requires a loop voltage

$$V_{loop} = 1.0\,(Z_{ii} - Z_{2k} + Z_{kk} - Z_{ik} + Z_{line}) \qquad .....(5.117)$$

for the circulation of unit current

The loop impedance is

$$Z_{loop} = Z_{ii} + Z_{kk} - 2Z_{ik} + Z_{line} \qquad .....(5.118)$$

The dimension of **Z** matrix is increased due to the introduction of a new axis due to the loop l

$$Z_{ll} = Z_{loop}$$

and

$$Z_{m-l} = Z_{mi} - Z_{mk}$$

$$Z_{l-m} = Z_{im} - Z_{km};\ m \neq l$$

The new loop axis can be eliminated now. Consider the matrix

$$\begin{bmatrix} Z_p & Z_q \\ Z_r & Z_s \end{bmatrix}$$

It can be proved easily that

$$Z'_p = Z_p - Z_q Z_s^{-1} Z_r \qquad .....(5.118)$$

using eqn. (5.118) all the additional elements introduced by the loop can be eliminated. The method is illustrated in example E.5.4.

5.15.5 Gauss – Seidel Method Using Z-bus for Load Flow Solution

An initial bus voltage vector is assumed as in the case of Y – bus method. Using these voltages, the bus currents are calculated using eqn. (5.25) or (5.26).

$$I = \frac{P_i - jQ_i}{V^*} - y_i\, v_i \qquad .....(5.119)$$

where y_i is the total shunt admittance at the bus i and $y_{ii}\, v_i$ is the shunt current flowing from bus i to ground.

A new bus voltage estimate is obtained for an n-bus system from the relation.

$$V_{bus} = Z_{bus}\, I_{bus} + V_R \qquad .....(5.120)$$

Where V_R is the $(n - 1) \times 1$ dimensional reference voltage vector containing in each element the slack bus voltage. It may be noted that since the slack bus is the reference bus, the dimension of the Z_{bus} is $(n - 1 \times (n - 1)$.

The voltages are updated from iteration to iteration using the relation

$$V_i^{m+1} = V_S + \sum_{\substack{k=1 \\ k \neq S}}^{i-1} Z_{ik}\, I_k^{m+1} + \sum_{\substack{k=i \\ k \neq S}}^{n} Z_{ik}\, I_k^{(m)} \qquad .....(5.121)$$

Then

$$V_k^{(m)} = \frac{P_k - jQ_k}{\left(V_k^{(m)}\right)^*} - y_k\, V_k^{(m+1)}$$

i = 1, 2,........, n
S = slack bus

5.16 Convergence Characteristics

The number of iterations required for convergence to solution depends considerably on the correction to voltage at each bus. If the correction DV_i at bus i is multiplied by a factor a, it is found that acceleration can be obtained to convergence rate. Then multiplier a is called acceleration factor. The difference between the newly computed voltage and the previous voltage at the bus is multiplied by an appropriate acceleration factor. The value of a that generally improves the convergence is greater than are. In general $1 < a < 2$ and a typical value for a = 1.5 or 1.6 the use of acceleration factor amounts to a linear extrapolation of bus voltage V_i. For a given system, it is quite often found that a near optimal choice of a exists as suggested in literature over a range of operating condition complex value is also suggested for a. Same suggested different a values for real and imaginary parts of the bus voltages.

The convergence of iterative methods depends upon the diagonal dominance of the bus admittance matrix. The self admittances of the buses (diagonal terms) are usually large relative to the mutual admittances (off-diagonal terms). For this reason convergence is obtained for power flow solution methods.

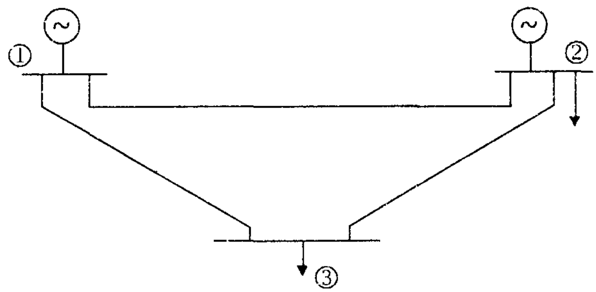
Junctions of high and low series impedances and large capacitances obtained in cable circuits, long EHV lines, series and shunt. Compensation are detrimental to convergence as these tend to weaken the diagonal dominance in Y bus matrix. The choice of swing bus may also affect convergence considerably. In difficult cases it is possible to obtain convergence by removing the least diagonally dominant row and column of Y-bus. The salient features of Y-bus matrix iterative methods are that the element in the summation term in equation ( ) or ( ) are on the average 2 or 3 only even for well developed power systems. The sparsity of the Y-matrix and its symmetry reduces both the storage requirement and the computation time for iteration.

For large well conditioned system of n buses the number of iterations required are of the order n and the total computing time varies approximately as n^2.

In contrast, the Newton-Raphson method gives convergence in 3 to 4 iterations. No acceleration factors are needed to the used. Being a gradient method solution is obtained must faster than any iterative method.

Worked Examples

E 5.1 A three bus power system is shown in Fig. E5.1. The system parameters are given in Table E5.1 and the load and generation data in Table E5.2. The voltage at bus 2 is maintained at 1.03p.u. The maximum and minimum reactive power limits of the generation at bus 2 are 35 and 0 Mvar respectively. Taking bus 1 as slack bus obtain the load flow solution using Gauss – Seidel iterative method using Y_{Bus}



E 5.1 A three bus power system

Table E 5.1 Impedance and Line charging Admittances

Bus Code i-k	Impedance (p.u.) Z_{ik}	Line charging Admittance (p.u) y_i
1-2	0.08 + j0 24	0
1-3	0.02 + j0.06	0
2-3	0.06 + j0.018	0

Table E 5.2 Scheduled Generation, Loads and Voltages

Bus No i	Bus voltage V_i	Generation		Load	
		MW	Mvar	MW	Mvar
1	1.05 + j0.0	—	—	0	0
2	1.03 + j0.0	20	—	50	20
3	——	0	0	60	25

Solution :

The line admittance are obtained as

$$y_{12} = 1.25 - j3.75$$

$$y_{23} = 1.667 - j5.00$$

$$y_{13} = 5.00 - j15.00$$

The bus admittance matrix is formed using the procedure indicated in section 2.1 as

$$Y_{Bus} = \begin{bmatrix} 6.25 & -j18.75 & -1.25 & +j3.75 & -5.0 & +j15.0 \\ -1.25 & +j3.73 & 2.9167 & -j8.75 & -j1.6667 & +j5.0 \\ -5.0 & +j15.0 & -1.6667 & +j5.0 & 6.6667 & -j20.0 \end{bmatrix}$$

Gauss – Seidel Iterative Method using Y_{BUS}

The voltage at bus 3 is assumed as 1 + j0. The initial voltages are therefore

$$V_1^{(0)} = 1.05 + j0.0$$

$$V_2^{(0)} = 1.03 + j0.0$$

$$V_3^{(0)} = 1.00 + j0.0$$

Base MVA = 100

Iteration 1 : It is required to calculate the reactive power Q_2 at bus 2, which is a P-V or voltage controlled bus

$$\delta_2^{(0)} = \tan^{-1}\left(\frac{e_2^{''}}{e_2^{'}}\right) = 0\,.$$

$$e_{2(new)}^{'} = |V_2|_{sch} \cos\delta_2 = (1.03)(1.0) = 1.03$$

$$e_{2(new)}^{''} = V_{2\,sch} \sin\delta_2^{(0)} = (1.03)(0.0) = 0.00$$

$$Q_2^{(0)} = \left[\left(e_{2(new)}^{'}\right)^2 B_{22} + \left(e_{2(new)}^{''}\right)^2 B_{22}\right] +$$

$$\sum_{\substack{k=1 \\ k\neq 2}}^{3} \left[\left(e_{2(new)}^{''} e_k^{'} G_{2k} + e_k^{''} B_{2k}\right) - \left(e_{2(new)}^{'} e_k^{''} G_{2k} - e_k^{''} B_{2k}\right)\right]$$

Substituting the values

$$\begin{aligned} Q_2^{(0)} &= \left[(1.03.)^2 8.75 + (0)^2 8.75\right] + 0(1.05)(-1.25) + 0.(-3.75) \\ &\quad - 1.03[(0)(-1.25) - (1.05)(-3.75)] \\ &\quad + (0)[(1)(-1.6667) + (0)(-5.0)] \\ &\quad - 1.03[(0)(-1.6667) - (1)(-5)] \\ &= 0.07725 \end{aligned}$$

Mvar generated at bus 2

= Mvar injection into bus 2 + load Mvar

= 0.07725 + 0.2 = 0.27725 p.u.

= 27.725 Mvar

This is within the limits specified.

The voltage at bus i is

$$V_i^{(m+1)} = \frac{+1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{(m)*}} - \sum_{k=1}^{i-1} y_{ik} V_k^{(m+1)} - \sum_{k=i+1}^{n} y_{ik} V_k^{(m)}\right]$$

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_1 - jQ_2}{V_i^{(0)*}} - Y_{21}V_1 - Y_{23}V_3^{(0)}\right]$$

$$= \frac{1}{(2.9167 - j8.75)}$$

$$\left[\frac{-0.3 - 0.07725}{1.03 - j0.0} - (-1.25 + j3.75)(1.05 + j0.0) + (-1.6667 + j5.0)(1 + j0.0)\right]$$

$$V_2^{(1)} = 1.01915 - j0.032491$$

$$= 1.0196673\angle -1.826^0$$

An acceleration factor of 1.4 is used for both real and imaginary parts.

The accelerated voltages is obtained using

$$v_2' = 1.03 + 1.4(1.01915 - 1.03) = 1.01481$$
$$v_2'' = 0.0 + 1.4(-0.032491 - 0.0) = -0.0454874$$

$$V_2^{(1)}(\text{accelerated}) = 1.01481 - j0.0454874$$

$$= 1.01583\angle -2.56648^0$$

The voltage at bus 3 is given by

$$V_2^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31}V_1 - Y_{32}V_2^{(1)}\right]$$

$$= \frac{1}{6.6667 - j20}$$

$$\left[\left(\frac{-0.6 + j0.25}{1 - j0}\right) - (-5 + j15)(1.05 + j0) - (-1.6667 + j5)(1.01481 - j0.0454874)\right]$$

$$= 1.02093 - j0.0351381$$

The accelerated value of $V_3^{(1)}$ obtained using

$$v_3' = 1.0 + 1.4(1.02093 - 1.0) = 1.029302$$

$$v_3'' = 0 + 1.4(-0.0351384 - 0) = -0.0491933$$

$$V_3^{(1)} = 1.029302 - j0.049933$$

$$= 1.03048\angle -2.73624^0$$

The voltages at the end of the first iteration are

$$V_1 = 1.05 + j0.0$$

$$V_2^{(1)} = 1.01481 - j0.0454874$$

$$V_3^{(1)} = 1.029302 - j0.0491933$$

Check for convergence : An accuracy of 0.001 is taken for convergence

$$\left[\Delta v_2'\right]^{(0)} = \left[v_2'\right]^{(1)} - \left[v_2'\right]^{(0)} = 1.01481 - 1.03 = -0.0152$$

$$\left[\Delta v_2''\right]^{(0)} = \left[v_2''\right]^{(1)} - \left[v_2''\right]^{(0)} = -0.0454874 - 0.0 = -0.0454874$$

$$\left[\Delta v_3''\right]^{(0)} = \left[v_3'\right]^{(1)} - \left[v_3'\right]^{(0)} = 1.029302 - 1.0 = 0.029302$$

$$\left[\Delta v_2''\right]^{(0)} = \left[\Delta v_2''\right]^{(1)} - \left[\Delta v_2''\right]^{(0)} = -0.0491933 - 0.0 = -0.0491933$$

The magnitudes of all the voltage changes are greater than 0.001.

Iteration 2 : The reactive power Q_2 at bus 2 is calculated as before to give

$$\delta_2^{(1)} = \tan^{-1}\frac{\left[v_2''\right]^{(1)}}{\left[v_2'\right]^{(1)}} = \tan^{-1}\left[\frac{-0.0454874}{1.01481}\right] = -2.56648^0$$

$$\left[v_2'\right]^{(1)} = \left|V_{2sch}\right| \cdot \cos\delta_2^{(1)} = 1.03\cos(-2.56648^0) = 1.02837$$

$$\left[v_2''\right]^{(1)} = \left|V_{2sch}\right| \cdot \sin\delta_2^{(1)} = 1.03\sin(-2.56648^0) = -0.046122$$

$$\left[V_{2new}\right]^{(1)} = 1.02897 - j0.046122$$

$$\begin{aligned} Q_2^{(1)} &= (1.02897)^2(8.75) + (-0.046122)^2(8.75) \\ &\quad +(-0.046122)[1.05(-1.25) + (0)(-3.75)] \\ &\quad -(1.02897)[(0)(-1.25) - (1.05)(-3.75)] + \\ &\quad -(1.02897)[(-0.0491933)(-1.6667) - (1.029302)(-5)] \\ &= -0.0202933 \end{aligned}$$

Mvar to be generated at bus 2

= Net Mvar injection into bus 2 + load Mvar

= – 0.0202933 + 0.2 = 0.1797067 p.u. = 17.97067Mvar

This is within the specified limits. The voltages are, therefore, the same as before

$$V_1 = 1.05 + j0.0$$

$$V_2^{(1)} = 1.02897 - j0.0.46122$$

$$V_3^{(1)} = 1.029302 - j0.0491933$$

The New voltage at bus 2 is obtained as

$$V_2^{(2)} = \frac{1}{2.9167 - j8.75}\left[\frac{-0.3 + j0.0202933}{1.02827 + j0.046122}\right.$$

$$-(-1.25 + j3.75)(1 - 05 + j0)$$

$$\left. -(-1.6667 + j5)\cdot(1.029302 - j0.0491933)\right]$$

$$= 1.02486 - j0.0568268$$

The accelerated value of $V_2^{(2)}$ is obtained from

$$v_2' = 1.02897 + 1.4(1.02486 - 1.02897) = 1.023216$$

$$v_2'' = -0.046122 + 1.4(-0.0568268) - (-0.046122 = -0.0611087)$$

$$v_2^{(2)'} = 1.023216 - j0.0611087$$

The new voltage at bus 3 is calculated as

$$V_3^{(2)} = \frac{1}{6.6667 - j20}\left[\frac{-6.6 + j0.25}{1.029302 + j0.0491933}\right.$$

$$-(-5 + j15)(1.05 + j0.0)$$

$$\left. -(-1.6667 + j5.0)\cdot(1.023216 - j0.0611)\right]$$

$$= 1.0226 - j0.0368715$$

The accelerated value of $V_2^{(2)}$ obtained from

$$v_3' = 1.029302 + 1.4(1.0226 - 1.029302) = 1.02$$

$$v_3'' = (-0.0491933) + 1.4(-0.0368715) + (0.0491933) = -0.03194278$$

$$V_3^{(2)} = 1.02 - j0.03194278$$

The voltages at the end of the second iteration are

$$V_1 = 1.05 + j0.0$$

$$V_2^{(2)} = 1.023216 - j0.0611087$$

$$V_3^{(2)} = 1.02 - j0.03194278$$

The procedure is repeated till convergence is obtained at the end of the sixth iteration. The results are tabulated in Table E5.1(a)

Table E5.1 (a) Bus Voltage

Iteration	Bus 1	Bus 2	Bus 3
0	1.05 + j0	1.03 + j0	1.0 + j0
1	1.05 + j0	1.01481 – j0.04548	1.029302 – j0.049193
2	1.05 + j0	1.023216 – j0.0611087	1.02 – j0.0319428
3	1.05 + j0	1.033476 – j0.0481383	1.027448 – j0.03508
4	1.05 + j0	1.0227564 – j0.051329	1.0124428 – j0.0341309
5	1.05 + j0	1.027726 – j0.0539141	1.0281748 – j0.0363943
6	1.05 + j0	1.029892 – j0.05062	1.020301 – j0.0338074
7	1.05 + j0	1.028478 – j0.0510117	1.02412 – j0.034802

Line flow from bus 1 to bus 2

$$S_{12} = V_1\left(V_1^* - V_2^*\right)Y_{12}^* = 0.228975 + j0.017396$$

Line flow from bus 2 to bus 1

$$S_{21} = V_2\left(V_2^* - V_1^*\right)Y_{21}^* = -0.22518 - j0.0059178$$

Similarly, the other line flows can be computed and are tabulated in Table E5.1(b). the slack bus power obtained by adding the flows in the lines terminating at the slack bus, is

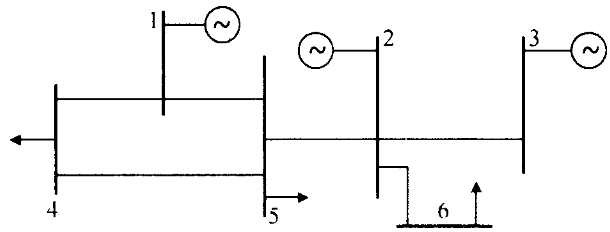
$$P_1 + jQ_1 = 0.228975 + j0.017396 + 0.684006 + j0.225$$

$$= (0.912981 + j0.242396)$$

Table E5.1(b) Line Flows

Line	P	Power Flow	Q
1-2	+ 0.228975		0.017396
2-1	– 0.225183		0.0059178
1-3	0.68396		0.224
3-1	– 0.674565		–0.195845
2-3	– 0.074129		0.0554
3-2	0.07461		–0.054

E 5.2 Consider the bus system shown in Fig. E 5.2.



E 5.2 A six bus power system.

The following is the data :

Line impedance (p.u.)	Real		Imaginary	
1 4	0.57000	E-1	0.845	E-1
1-5	1.33000	E-2	3.600	E-2
2-3	3.19999	E-2	1 750	E-1
2-5	1.73000	E-2	0.560	E-1
2-6	3 00000	E-2	1 500	E-1
4-5	1 94000	E-2	0 625	E-1

Scheduled generation and bus voltages :

Bus Code P	Assumed bus voltage	Generation		Load	
		MW p.u.	Mvar p.u	MW p.u.	Mvar p.u
1	1 05 +j0 0 (specified)	---	---	---	---
2	---	1.2	0.05	---	---
3	---	1.2	0.05	---	---
4	---	---	---	1.4	0.05
5	---	---	---	0.8	0.03
6	---	---	---	0.7	0.02

(a) Taking bus – 1 as slack bus and using an accelerating factor of 1.4, perform load flow by Gauss – Seidel method. Take precision index as 0.0001.

(b) Solve the problem also using Newton-Raphson polar coordinate method.

Solution :

The bus admittance matrix is obtained as :

Bus Code	Admittance (p.u.)	
P – Q	**Real**	**Imaginary**
1 – 1	14.516310	–32 57515
1 – 4	–5 486446	8 13342
1 – 5	–9.029870	24.44174
2 – 2	7.329113	–28.24106
2 – 3	–1.011091	5.529494
2 – 5	–5 035970	16.301400
2 – 6	–1.282051	6 410257
3 – 2	–1.011091	5.529404
3 – 3	1.011091	–5.529404
4 – 1	–5.486446	8 133420
4 – 4	10.016390	–22 727320
4 – 5	–4.529948	14.593900
5 – 1	–9 029870	24.441740
5 – 2	–5.035970	16 301400
5 – 4	–4.529948	14 593900
5 – 5	18.595790	–55.337050
6 – 2	–1.282051	6 410257
6 – 6	1.282051	–6.410254

All the bus voltages, $V^{(0)}$, are assumed to be 1 + j0 except the specified voltage at bus 1 which is kept fixed at 1.05 + j0. The voltage equations for the fist Gause-Seidel iteration are:

$$V_2^{(1)} = \frac{1}{Y_2}\left[\frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{23}V_3^{(0)} - Y_{25}V_5^{(0)} - Y_{26}V_6^{(0)}\right]$$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{32}V_2^{(1)}\right]$$

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{(0)*}} - Y_{41}V_1 - Y_{45}V_5^{(0)}\right]$$

$$V_5^{(1)} = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{(0)*}} - Y_{51}V_1 - Y_{51}V_2^{(1)} - Y_{54}V_4^{(1)}\right]$$

$$V_6^{(1)} = \frac{1}{Y_{66}}\left[\frac{P_6 - jQ_6}{V_6^{(0)*}} - Y_{62}V_2^{(1)}\right]$$

Substituting the values. the equation for solution are

$$V_2^{(1)} = \left(\frac{1}{7.329113} - j28.24100\right) \times \left[\frac{1.2 - j0.05}{1 - j0}\right.$$
$$- (-1.011091 + j5.529404) \times (1 + j0) - (-5.03597 + j16.3014)(1 + j0)$$
$$- (1 - 282051 + j16.3014)(1 + j0)$$
$$= 1.016786 + j0.0557924$$

$$V_3^{(1)} = \left(\frac{1}{1.011091} - j5.52424\right) \times \left[\frac{1.2 - j0.05}{1 - j0}\right]$$
$$- (-1.011091 + j5.529404) \times (1.016786 + j0.0557924)$$
$$= 1.089511 + j0.3885233$$

$$V_4^{(1)} = \left(\frac{1}{10.01639} - j22.72732\right) \times \left[\frac{-1.4 + j0.005}{1 - j0}\right]$$
$$- (-5.486446 + j8.133342) \times (1.05 + j0.)$$
$$- (-4.529948 + j14.5939)(1 + j0)$$
$$= 0.992808 - j0.0658069$$

$$V_5^{(1)} = \left(\frac{1}{18.59579} - j55.33705\right) \times \left[\frac{-0.8 + j0.03}{1 - j0}\right]$$
$$- (-9.02987 + j24.44174) \times (1.05 + j0)$$
$$- (-5.03597 + j16.3014)(1.016786 + j0.0557929)$$
$$- (-4.529948 + j14.5939)(0.992808 - j0.0658069)$$
$$= 1.028669 - j0.01879179$$

$$V_6^{(1)} = \left(\frac{1}{1.282051} - j6.410257\right) \times \left[\frac{-0.7 + j0.02}{1 - j0}\right]$$
$$- (-1.282051 - j6.410257) \times (1.016786 + j0.0557924)$$
$$= 0.989904 - j0.0669962$$

The results of these iterations is given in Table 5.3 (a)

Table E5.3(a)

It.No	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
0	1 + j0.0	1 + j0.0	1 + j0 0	1 + j0.0	1 + j0 0
1	1.016789 + j0.0557924	1.089511 + j0 3885233	0.992808 – j0 0658069	1.02669 – j0.01879179	0 989901 – j0.0669962
2	1.05306 + j0.1018735	1.014855 + j0.2323309	1.013552 – j0.0577213	1.042189 + j0.0177322	1.041933 + j0.0192121
3	1 043568 + j0 089733	1 054321 + j0 3276035	1.021136 – j0 0352727	1 034181 + j0.00258192	1 014571 – j0 02625271
4	1.047155 + j0.101896	1 02297 + j0.02763564	1.012207 – j0.0500558	1.035391 + j0 00526437	1 02209 + j0 00643566
5	1 040005 + j0.093791	1 03515 + j0.3050814	1.61576 – j0 04258692	0.033319 + j0.003697056	1 014416 – j0.01319787
6	1 04212 + j0.0978431	1.027151 + j0 2901358	1.013044 – j0 04646546	10.33985 + j0.004504417	1.01821 – j0 001752973
7	1.040509 + j0.0963405	1.031063 + j0.2994083	1.014418 – j0.0453101	1 033845 + j0 00430454	1 016182 – j0 00770669
8	1.041414 + j0 097518	1.028816 + j0 294465	1.013687 – j0 0456101	1 033845 + j0.004558826	1 017353 – j0 0048398
9	1.040914 + j0.097002	1.030042 + j0.2973287	1-014148 – j0 04487629	1.033711 + j0.004413647	1 016743 – j0.0060342
10	1.041203 + j0.0972818	1.02935 + j0 2973287	1.013881 – j0.04511174	1.03381 + j0.004495542	1 017089 – j0.00498989
11	1.041036 + j0.097164	1.029739 + j0.296598	1.01403 – j0 04498312	1 03374 + j0.004439559	1.016877 – j0 00558081
12	1.041127 + j0.0971998	1.029518 + j0 2960784	1 013943 – j0 04506212	1.033761 + j0 00447096	1.016997 – j0.00524855
13	1.041075 + j0.0971451	1.029642 + j0.2963715	1.019331 – j0 04501488	1.033749 + j0.004454002	1.016927 – j0 00543323
14	1.041104 + j0.0971777	1.02571 + j0.2962084	1.0013965 – j0.04504223	1.033756 + j0.004463713	1 016967 – j0.00053283

In the polar form, all the voltages at the end of the 14th iteration are given in Table E5.3(b).

Table E5.3(b)

Bus	Voltage magnitude (p.u.)	Phase angle (^{0})
1	1 05	0
2	0 045629	5.3326
3	1 071334	16.05058
4	1.014964	–2.543515
5	1 033765	2.473992
6	1.016981	–3.001928

(b) Newton – Raphson polar coordinates method

The bus admittance matrix is written in polar form as

$$Y_{BUS} = \begin{bmatrix} 19.7642\angle-71.6^0 & 3.95285\angle-108.4^0 & 15.8114\angle-108.4^0 \\ 3.95285\angle-108.4^0 & 9.22331\angle-71.6^0 & 5.27046\angle-108.4^0 \\ 15.8114\angle-108.4^0 & 5.27046\angle-108.4^0 & 21.0819\angle-71.6^0 \end{bmatrix}$$

Note that

$$\angle Y_{ii} = -71.6^0$$

and $\quad \angle Y_{ik} = -180^0 - 71.6^0 = 108.4^0$

The initial bus voltages are

$$V_1 = 1.05\ \angle 0^0$$

$$V_2^{(0)} = 1.03\angle 0^0$$

$$V_2^{(0)} = 1.0\angle 0^0$$

The real and reactive powers at bus 2 are calculated as follows :

$$P_2 = |V_2V_1Y_{21}|\cos\left(\delta_2^{(0)} - \delta_1 - \theta_{21}\right) + |V_2^2\,Y_{22}|\cos(-\theta_{22}) +$$

$$|V_2V_3Y_{23}|\cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$= (1.03)\,(1.05)\,(3.95285)\cos(108^0.4) + (1.03)^2\,(9.22331)\cos(-108^0.4)$

$+ (1.03)^2\,(9.22331)\cos(71^0.6) + (1.03)(1.0)(5.27046)\cos(-108^0.4)$

$= 0.02575$

$$Q_2 = |V_2 V_1 Y_{21}| \sin\left(\delta_2^{(0)} - \delta_1 - \theta_{21}\right) + |V_2^2 Y_{22}| \sin(-\theta_{22}) +$$

$$|V_2 V_3 Y_{23}| \sin\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$= (1.03)(1.05)(3.95285) \sin(-108^0.4) + (1.03)^2 (9.22331) \sin(71.6^0)$

$+ (1.03)(1.0)(5.27046) \sin(108.4^0)$

$= 0.07725$

Generation of p.u Mvar at bus 2

$= 0.2 + 0.07725$

$= 0.27725 = 27.725$ Mvar

This is within the limits specified. The real and reactive powers at bus 3 are calculated in a similar way.

$$P_3 = \left|V_3^{(0)} V_1 Y_{31}\right| \cos\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) + \left|V_3^{(0)} V_2 Y_{32}\right| \cos\left(\delta_3^{(0)} - \delta_2 - \theta_{32}\right) +$$

$$\left|V_3^{(0)2} Y_{33}\right| \cos(-\theta_{33})$$

$= (1.0)(1.05)(15.8114) \cos(-108.4^0) +$

$(1.0)(1.03)(5.27046) \cos(-108.4^0)$

$+ (1.0)^2 (21.0819) \cos(71.6^0)$

$= -0.3$

$$Q_3 = \left|V_3^{(0)} V_1 Y_{31}\right| \sin\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) + \left|V_3^{(0)} V_2 Y_{32}\right| \sin\left(\delta_3^{(0)} - \delta_2 - \theta_{32}\right) +$$

$$\left|V_3^{(0)2} Y_{33}\right| \sin(-\theta_{33})$$

$= (1.0)\ 1.05\ (15.8114) \sin(-108.4^0) + (1.0)(1.03)(5.27046)$

$\sin(-108.4^0)$

$+ (1.0)^2 (21.0891) \sin(71.6^0)$

$= -0.9$

The difference between scheduled and calculated powers are

$$\Delta P_2^{(0)} = -0.3 - 0.02575 = -0.32575$$

$$\Delta P_3^{(0)} = -0.6 - (-0.3) = -0.3$$

$$\Delta Q_2^{(0)} = -0.25 - (-0.9) = -0.65$$

It may be noted that ΔQ_2 has not been computed since bus 2 is voltage controlled bus.

since $\left|\Delta P_2^{(0)}\right|, \left|\Delta P_3^{(0)}\right|$ and $\left|\Delta Q_3^{(0)}\right|$

are greater than the specified limit of 0.01, the next iteration is computed.

Iteration 1 : Elements of the Jacobian are calculated as follows.

$$\frac{\partial P_2}{\partial \delta_3} = \left|V_2\, V_3^{(0)}\, Y_{23}\right| \sin\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$= (1.03)\,(1.0)\,(5.27046)\,\sin\,(-108.4^0) = -5.15$$

$$\frac{\partial P_2}{\partial \delta_2} = -\left|V_2\, V_1\, Y_{21}\right| \sin\left(\delta_2^{(0)} - \delta_1^{(0)} - \theta_{21}\right) +$$

$$\left|V_2\, V_3^{(0)}\, Y_{23}\right| \sin\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$= -\,(1.03)\,(1.05)\,(3.95285)\,\sin\,(108.4^0) +$$

$$(1.03)\,(1.0)\,(5.27046)\,\sin\,(-108.4^0)$$

$$= 9.2056266$$

$$\frac{\partial P_2}{\partial \delta_3} = \left|V_2\, Y_{23}\right| \cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$= (1.03)\,(5.27046)\,\cos\,(108.4^0)$$

$$= -\,1.7166724$$

$$\frac{\partial P_3}{\partial \delta_3} = \left|V_3^{(0)} V_1\, Y_{31}\right| \sin\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$$= (0.0)\,(1.03)\,(5.27046)\,\sin\,(-108.4^0)$$

$$= -\,5.15$$

$$\frac{\partial P_3}{\partial \delta_3} = \left|V_3^{(0)} V_1\, Y_{31}\right| \sin\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) + \left|V_3^{(0)} V_2\, Y_{32}\right| \sin\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$$= -\,(1.0)\,(1.05)\,(15.8114)\,\sin\,(-108.4^0) - 5.15$$

$$= 20.9$$

$$\frac{\partial P_3}{\partial V_3} = 2\left|V_3\, Y_{33}\right| \cos\theta_{33} + \left|V_1\, Y_{31}\right| \cos\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right) +$$

$$\left|V_2\, Y_{32}\right| \cos\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$$= 2(1.0)\,(21.0819)\,\cos\,(71.6^0) + (1.05)\,(15.8114)\,\cos\,(-108.4^0) +$$

$$(1.03)\,(5.27046)\,\cos\,(-108.4^0)$$

$$= 6.366604$$

$$\frac{\partial Q_3}{\partial \delta_2} = -\left|V_3^{(0)} V_1 \; Y_{32}\right| \cos\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$= (1.0)\,(1.03)\,(5.27046)\cos(-108.4^0)$

$= 1.7166724$

$$\frac{\partial Q_3}{\partial \delta_3} = \left|V_3^{(0)} V_1 \; Y_{32}\right| \cos\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) +$$

$$\left|V_3^{(0)} V_2 \; Y_{32}\right| \cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{32}\right)$$

$= (1.0)\,(1.05)\,(15.8114)\cos(-108.4^0) - 1.7166724$

$= -6.9667$

$$\frac{\partial Q_3}{\partial \delta_3} = 2\left|V_3^{(0)} \; Y_{33}\right| + \left|V_1 \; Y_{31}\right| \sin\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) +$$

$$\left|V_2 \; Y_{32}\right| \sin\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$= 2(1.0)\,(21.0819)\sin(71.6^0) + (1.05)\,(15.8114)\sin(-108.4^0) +$

$(1.03)\,(5.27046)\sin(-108.4^0)$

$= 19.1$

From eqn. (5.70)

$$\begin{bmatrix} -0.32575 \\ -0.3 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 9.20563 & -5.15 & -1.71667 \\ -5.15 & 20.9 & 6.36660 \\ 1.71667 & -6.9967 & 19.1 \end{bmatrix} \begin{bmatrix} |\Delta\delta_2| \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

Following the method of triangulation and back substations

$$\begin{bmatrix} -0.35386 \\ -0.3 \\ -0.035386 \end{bmatrix} = \begin{bmatrix} 1 & -0.55944 & -0.18648 \\ -5.15 & 20.9 & 6.36660 \\ +1.71667 & -6.9667 & 19.1 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

$$\begin{bmatrix} -0.35386 \\ -0.482237 \\ +0.710746 \end{bmatrix} = \begin{bmatrix} 1 & -0.55944 & -0.18648 \\ 0 & 18.02 & 5.40623 \\ 0 & -6.006326 & 19.42012 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

Finally,
$$\begin{bmatrix} -0.35386 \\ -0.0267613 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 1 & -0.55944 & -0.18648 \\ 0 & 1 & 0.3 \\ 0 & 0 & 21.22202 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

Thus,
$$\Delta|V_3| = (0.55)/(21.22202) = 0.025917$$
$$\Delta\delta_3 = -0.0267613 - (0.3)(0.025917)$$
$$= -0.0345364 \text{ rad}$$
$$= -1.98^0$$
$$\Delta\delta_2 = -0.035286 - (-0.55944)(-0.034536) - (-0.18648)(0.025917)$$
$$= -0.049874 \text{ rad}$$
$$= -2.8575^0$$

At the end of the firs iteration the bus voltages are

$$V_1 = 1.05\angle 0^0$$
$$V_2 = 1.03\angle 2.85757^0$$
$$V_3 = 1.025917\angle -1.9788^0$$

The real and reactive powers at bus 2 are computed :

$$P_2^{(1)} = (1.03)(1.05)(3.95285)[\cos(-2.8575) - 0(-108.4^0)$$
$$+ (1.03)^2(1.025917)(5.27046)\cos[(-2.8575) - (-1.9788) - 108.4^0$$
$$-0.30009$$
$$Q_2^{(1)} = (1.03)(1.05)(3.95285)[\sin(-2.8575) - 0(-108.4^0)$$
$$+ (1.03)^2(9.22331)\sin[(-2.85757) - (-1.9788) - 108.4^0)]$$
$$= 0.043853$$

Generation of reactive power at bus 2

$$= 0.2 + 0.043853 = 0.243856 \text{ p.u. Mvar}$$
$$= 24.3856 \text{ Mvar}$$

This is within the specified limits.

The real and reactive powers at bus 3 are computed as

$$P_3^{(1)} = (1.025917)(1.05)(15.8117)\cos[(-1.09788) - 0 - 108.4^0)]$$
$$+ (1.025917)(1.03)(5.27046)\cos[(-1.0988) - (-2.8575) - 108.4]$$
$$+ (1.025917)^2(21.0819)\cos(71.6^0)$$
$$= -0.60407$$

$$Q_3^{(1)} = (1.025917)(1.05)(15.8114)\sin[(-1.977) - 108.4^0)]$$
$$+ (1.025917)(1.03)(5.27046)\sin[(-1.9788) - (-2.8575) - 108.4^0)]$$
$$+ (1.025917)^2(21.0819)\sin(71.6^0)$$
$$= -0.224$$

The differences between scheduled powers and calculated powers are

$$\Delta P_2^{(1)} = -0.3 - (-0.30009) = 0.00009$$
$$\Delta P_3^{(1)} = -0.6 - (-0.60407) = 0.00407$$
$$\Delta Q_3^{(1)} = -0.25 - (-0.2224) = -0.0276$$

Even though the first two differences are within the limits the last one, $Q_2^{(1)}$ is greater than the specified limit 0.01. The next iteration is carried out in a similar manner. At the end of the second iteration even ΔQ_3 also is found to be within the specified tolerance. The results are tabulated in table E5.4(a) and E5.4(b)

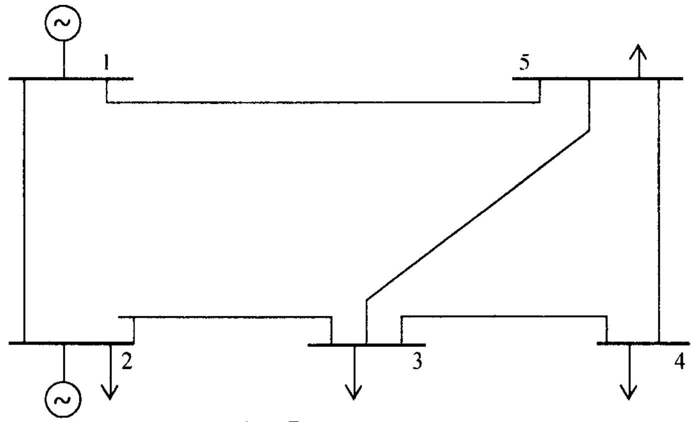
Table E5.4 (a) Bus voltages

Iteration	Bus 1	Bus 2	Bus 3
0	$1.05\angle 0^0$	$1.03\angle 0^0$	$1.\angle 0^0$
1	$1.05\angle 0^0$	$1.03\angle -2.85757$	$1.025917\angle -1.9788$
2	$1.05\angle 0^0$	$1.03\angle -2.8517$	$1.02476\angle -1.947$

Table E5.4 (b) Line Flows

Line	P	Power Flow	Q
1-2	0.2297		0.016533
2-1	−0.22332		0.0049313
1-3	0.68396		0.224
3-1	−0.674565		−0.0195845
2-3	−0.074126		0.0554
3-2	0.07461		−0.054

E5.3 For the given sample power system find load flow solution using N-R polar coordinates method, decoupled method and fast decoupled method.



(a) Power system

Bus Code	Line impedance Zpq	Line charging
1-2	0.02 + j0.24	j 0.02
2-3	0.04 + j0.02	j 0.02
3-5	0.15 + j0.04	j 0.025
3-4	0.02 + j0.06	j 0.01
4-5	0.02 + j0.04	j 0.01
5-1	0.08 + j0.02	j 0.2

(b) Line-data

Bus Code (Slack)	Generation		Load	
	Mw	Mvar	MW	Mvar
1	0	0	0	0
2	50	25	15	10
3	0	0	45	20
4	0	0	40	15
5	0	0	50	25

(c) Generation and load data

	1	2	3	4	5
1	11.724 – j24.27	–10 + j20	0 + j0	0 + j0	–1.724 + j4.31
2	–10 + j20	10.962 – j24.768	–0.962 + j4.808	0 + j0	0 + j0
3	0 + j0	–0.962 + j4.808	6.783 – j21.944	–5 + j15	–0.822 + j2.192
4	0 + j0	0 + j0	–5 + j15	15 – j34.98	–10 + j20
5	–1.724 + j4.31	0 + j0	–0.82 + j2.192	–10 + j20	12.546 – j26.447

(d) Bus admittance matrix

Fig. E 5.3

Solution :

The Residual or Mismatch vector for iteration no:1 is

dp[2] = 0.04944

dp[3] = –0.041583

dp[4] = –0.067349

dp[5] = –0.047486

dQ[2] = –0.038605

dQ[3] = –0.046259

dQ[4] = –0.003703

dQ[5] = –0.058334

The New voltage vector after iteration 1 is :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 1.984591 F :- 0.008285

Bus no 3 E : 0.882096 F :- 0.142226

Bus no 4 E : 0.86991 F :- 0.153423

Bus no 5 E : 0.875810 F :- 0.142707

The residual or mismatch vector for iteration no : 2 is

dp[2] = 0.002406

dp[3] = –0.001177

dp[4] = –0.004219

dp[5] = –0.000953

dQ[2] = –0.001087

dQ[3] = –0.002261

dQ[4] = –0.000502

dQ[5] = –0.002888

The New voltage vector after iteration 2 is :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984357 F :- 0.008219

Bus no 3 E : 0.880951 F :- 0.142953

Bus no 4 E : 0.868709 F :- 0.154322

Bus no 5 E : 0.874651 F :- 0.143439

The residual or mismatch vector for iteration no : 3 is dp[2] = 0.000005

dp[3] = –0.000001

dp[4] = –0.000013

dp[5] = –0.000001

dQ[2] = –0.000002

dQ[3] = –0.000005

dQ[4] = –0.000003

dQ[5] = –0.000007

The final load flow solution (for allowable error.0001) :

bus no 1 Slack P = 1.089093	Q = 0.556063	E = 1.000000	F = 0.000000
bus no 2 pq P = 0.349995	Q = 0.150002	E = 0.984357	F = –0.008219
bus no 3 pq P = –0.449999	Q = –0.199995	E = 0.880951	F = –0.1429531
bus no 4 pq P = –0.399987	Q = –0.150003	E = 0.868709	F = –0.154322
bus no 5 pq P = –0.500001	Q = –0.249993	E = 0.874651	F = –0.143439

Decoupled load flow solution (polar coordinate method)

The residual or mismatch vector for iteration no : 0 is

dp[2] = 0.350000

dp[3] = –0.450000

dp[4] = –0.400000

dp[5] = –0.500000

dQ[2] = –0.190000

dQ[3] = –0.145000

dQ[4] = –0.130000

dQ[5] = –0.195000

The new voltage vector after iteration 0 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.997385 F :- 0.014700

Bus no 3 E : 0.947017 F :- 0.148655

Bus no 4 E : 0.941403 F :- 0.161282

Bus no 5 E : 0.943803 F :- 0.150753

The residual or mismatch vector for iteration no:1 is

dp [2] = 0.005323

dp[3] = –0.008207

dp[4] = –0.004139

dp[5] = –0.019702

dQ[2] = –0.067713

dQ[3] = –0.112987

dQ[4] = –0.159696

dQ[5] = –0.210557

The new voltage vector after iteration 1 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.982082 F :- 0.013556

Bus no 3 E : 0.882750 F :- 0.143760

Bus no 4 E : 0.870666 F :- 0.154900

Bus no 5 E : 0.876161 F :- 0.143484

The residual or mismatch vector for iteration no:2 is

dp[2] = 0.149314

dp[3] = –0.017905

dp[4] = –0.002305

dp[5] = –0.006964

dQ[2] = –0.009525

dQ[3] = –0.009927

dQ[4] = –0.012938

dQ[5] = 0.007721

The new voltage vector after iteration 2 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.981985 F :- 0.007091

Bus no 3 E : 0.880269 F :- 0.142767

Bus no 4 E : 0.868132 F :- 0.154172

Bus no 5 E : 0.874339 F :- 0.143109

The residual or mismatch vector for iteration no:3 is

dp[2] = 0.000138

dp[3] = 0.001304

dp[4] = 0.004522

dp[5] = –0.006315

dQ[2] = 0.066286

dQ[3] = 0.006182

dQ[4] = –0.001652

dQ[5] = –0.002233

The new voltage vector after iteration 3 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984866 F :- 0.007075

Bus no 3 E : 0.881111 F :- 0.142710

Bus no 4 E : 0.868848 F :- 0.154159

Bus no 5 E : 0.874862 F :- 0.143429

The residual or mismatch vector for iteration no:4 is

dp[2] = –0.031844

dp[3] = 0.002894

dp[4] = –0.000570

dp[5] = 0.001807

dQ[2] = –0.000046

dQ[3] = 0.000463

dQ[4] = 0.002409

dQ[5] = –0.003361

The new voltage vector after iteration 4 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984866 F :- 0.008460

Bus no 3 E : 0.881121 F :- 0.142985

Bus no 4 E : 0.868849 F :- 0.1546330

Bus no 5 E : 0.874717 F :- 0.143484

The residual or mismatch vector for iteration no:5 is

dp[2] = 0.006789

dp[3] = –0.000528

dp[4] = –0.000217

dp[5] = –0.0000561

dQ[2] = –0.000059

dQ[3] = –0.000059

dQ[4] = –0.000635

dQ[5] = –0.000721

The new voltage vector after iteration 5 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984246 F :- 0.008169

Bus no 3 E : 0.880907 F :- 0.142947

Bus no 4 E : 0.868671 F :- 0.154323

Bus no 5 E : 0.874633 F :- 0.143431

The residual or mismatch vector for iteration no : 6 is

dp[2] = 0.000056

dp[3] = 0.000010

dp[4] = 0.000305

dp[5] = –0.000320

dQ[2] = 0.003032

dQ[3] = –0.000186

dQ[4] = –0.000160

dQ[5] = –0.000267

The new voltage vector after iteration 6 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984379 F :- 0.008165

Bus no 3 E : 0.880954 F :- 0.142941

Bus no 4 E : 0.868710 F :- 0.154314

Bus no 5 E : 0.874655 F :- 0.143441

The residual or mismatch vector for iteration no:7 is

dp[2] = - 0.001466

dp[3] = 0.000106

dp[4] = -0.000073

dp[5] = 0.000156

dQ[2] = 0.000033

dQ[3] = 0.000005

dQ[4] = 0.000152

dQ[5] = -0.000166

The new voltage vector after iteration 7 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.954381 F :- 0.008230

Bus no 3 E : 0.880958 F :- 0.142957

Bus no 4 E : 0.868714 F :- 0.154325

Bus no 5 E : 0.874651 F :- 0.143442

The residual or mismatch vector for iteration no : 8 is

dp[2] = –0.000022

dp[3] = 0.000001

dp[4] = –0.000072

dp[5] = –0.000074

dQ[2] = –0.000656

dQ[3] = 0.000037

dQ[4] = –0.000048

dQ[5] = –0.000074

The new voltage vector after iteration 8 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984352 F :- 0.008231

Bus no 3 E : 0.880947 F :- 0.142958

Bus no 4 E : 0.868706 F :- 0.154327

Bus no 5 E : 0.874647 F :- 0.143440

The residual or mismatch vector for iteration no:9 is

dp[2] = 0.000318

dp[3] = –0.000022

dp[4] = 0.000023

dp[5] = –0.000041

dQ[2] = –0.000012

dQ[3] = –0.000000

dQ[4] = 0.000036

dQ[5] = –0.000038

The new voltage vector after iteration 9 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984352 F :- 0.008217

Bus no 3 E : 0.880946 F :- 0.142954

Bus no 4 E : 0.868705 F :- 0.154324

Bus no 5 E : 0.874648 F :- 0.143440

The residual or mismatch vector for iteration no:10 is

dp[2] = 0.000001

dp[3] = –0.000001

dp[4] = 0.000017

dp[5] = –0.000017

dQ[2] = 0.000143

dQ[3] = –0.000008

dQ[4] = 0.000014

dQ[5] = –0.000020

The new voltage vector after iteration 10 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984658 F :- 0.008216

Bus no 3 E : 0.880949 F :- 0.142954

Bus no 4 E : 0.868707 F :- 0.154324

Bus no 5 E : 0.874648 F :- 0.143440

The residual or mismatch vector for iteration no:11is

dp[2] = –0.000069

dp[3] = 0.000005

dp[4] = –0.000006

dp[5] = 0.000011

dQ[2] = 0.000004

dQ[3] = –0.000000

dQ[4] = 0.000008

dQ[5] = –0.000009

The final load flow solution after 11 iterations

(for allowable arror.0001)

The final load flow solution (for allowable error.0001) :

Bus no 1	Slack	P = 1.089043	Q = 0.556088	E = 1.000000	F = 0.000000
Bus no 2	pq	P = 0.350069	Q = 0.150002	E = 0.984658	F = -0.008216
Bus no 3	pq	P = –0.450005	Q = –0.199995	E = 0.880949	F = -0.142954
Bus no 4	pq	P = –0.399994	Q = –0.150003	E = 0.868707	F = -0.154324
Bus no 5	pq	P = –0.500011	Q = –0.249991	E = 0.874648	F = –0.143440

Fast decoupled load flow solution (polar coordinate method)

The residual or mismatch vector for iteration no:0 is

dp[2] = 0.350000

dp[3] = –0.450000

dp[4] = 0.400000

dp[5] = –0.500000

dQ[2] = 0.190000

dQ[3] = –0.145000

dQ[4] = 0.130000

dQ[5] = –0.195000

The new voltage vector after iteration 0 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.997563 F :- 0.015222

Bus no 3 E : 0.947912 F :- 0.151220

Bus no 4 E : 0.942331 F :- 0.163946

Bus no 5 E : 0.944696F :- 0.153327

The residual or mismatch vector for iteration no:1 is

dp[2] = 0.004466

dp[3] = –0.000751

dp[4] = 0.007299

dp[5] = –0.012407

dQ[2] = 0.072548

dQ[3] = –0.118299

dQ[4] = 0.162227

dQ[5] = –0.218309

The new voltage vector after iteration 1 :

Bus no 1 E : 1.000000 F . 0.000000

Bus no 2 E : 0.981909 F :- 0.013636

Bus no 3 E : 0.882397 F :- 0.143602

Bus no 4 E : 0.869896 F :- 0.154684

Bus no 5 E : 0.875752 F :- 0.143312

The residual or mismatch vector for iteration no: 2 is

dp[2] = 0.153661

dp[3] = -0.020063

dp[4] = 0.005460

dp[5] = –0.009505

dQ[2] = 0.011198

dQ[3] = –0.014792

dQ[4] = -0.000732

dQ[5] = –0.002874

The new voltage vector after iteration 2 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.982004 F :- 0.007026

Bus no 3 E : 0.880515 F :- 0.142597

Bus no 4 E : 0.868400 F :- 0.153884

Bus no 5 E : 0.874588 F :- 0.143038

The residual or mismatch vector for iteration no: 3 is

dp[2] = –0.000850

dp[3] = –0.002093

dp[4] = 0.000155

dp[5] = –0.003219

dQ[2] = 0.067612

dQ[3] = –0.007004

dQ[4] = –0.003236

dQ[5] = –0.004296

The new voltage vector after iteration 3 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984926 F :- 0.007086

Bus no 3 E : 0.881246 F :- 0.142740

Bus no 4 E : 0.869014 F :- 0.154193

Bus no 5 E : 0.874928 F :- 0.143458

The residual or mismatch vector for iteration no: 4 is

dp[2] = –0.032384

dp[3] = 0.003011

dp[4] = –0.001336

dp[5] = –0.002671

dQ[2] = –0.000966

dQ[3] = –0.000430

dQ[4] = –0.000232

dQ[5] = –0.001698

The new voltage vector after iteration 4 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984862 F :- 0.008488

Bus no 3 E : 0.881119 F :- 0.143053

Bus no 4 E : 0.868847 F :- 0.154405

Bus no 5 E : 0.874717 F :- 0.143501

The residual or mismatch vector for iteration no: 5 is

dp[2] = 0.000433

dp[3] = 0.000006

dp[4] = –0.000288

dp[5] = 0.000450

dQ[2] = –0.014315

dQ[3] = –0.000936

dQ[4] = –0.000909

dQ[5] = –0.001265

The new voltage vector after iteration 6 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984230 F :- 0.008463

Bus no 3 E : 0.881246 F :- 0.143008

Bus no 4 E : 0.869014 F :- 0.154357

Bus no 5 E : 0.874607 F :- 0.143433

The residual or mismatch vector for iteration no: 6 is

dp[2] = 0.006981

dp[3] = –0.000528

dp[4] = 0.000384

dp[5] = –0.000792

dQ[2] = 0.000331

dQ[3] = 0.000039

dQ[4] = –0.000155

dQ[5] = 0.000247

The residual or mismatch vector for iteration no: 7 is

dp[2] = –0.000144

dp[3] = –0.000050

dp[4] = 0.000080

dp[5] = –0.000068

dQ[2] = 0.003107

dQ[3] = –0.000162

dQ[4] = –0.000255

dQ[5] = –0.000375

The new voltage vector after iteration 7 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984386 F :- 0.008166

Bus no 3 E : 0.880963 F :- 0.142943

Bus no 4 E : 0.868718 F :- 0.154316

Bus no 5 E : 0.874656F :- 0.143442

The residual or mismatch vector for iteration no: 8 is

dp[2] = –0.001523

dp[3] = –0.000105

dp[4] = –0.000115

dp[5] = –0.000215

dQ[2] = 0.000098

dQ[3] = –0.000024

dQ[4] = –0.000037

dQ[5] = –0.000038

The new voltage vector after iteration 8 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984380 F :- 0.008233

Bus no 3 E : 0.880957 F :- 0.142961

Bus no 4 E : 0.868714 F :- 0.154329

Bus no 5 E : 0.874651 F :- 0.143442

The residual or mismatch vector for iteration no: 9 is

dp[2] = –0.000045

dp[3] = 0.000015

dp[4] = –0.000017

dp[5] = 0.000008

dQ[2] = 0.000679

dQ[3] = 0.000031

dQ[4] = –0.000072

dQ[5] = –0.000105

The new voltage vector after iteration 9 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984350 F :- 0.008230

Bus no 3 E : 0.880945 F :- 0.142958

Bus no 4 E : 0.868704 F :- 0.154326

Bus no 5 E : 0.874646 F :- 0.143440

The residual or mismatch vector for iteration no: 10 is

dp[2] = 0.000334

dp[3] = –0.000022

dp[4] = 0.000033

dp[5] = 0.000056

dQ[2] = 0.000028

dQ[3] = 0.000007

dQ[4] = –0.000007

dQ[5] = 0.000005

The new voltage vector after iteration 10 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984352 F :- 0..008216

Bus no 3 E : 0.880946 F :- 0.142953

Bus no 4 E : 0.898705 F :- 0.154323

Bus no 5 E : 0.874648 F :- 0.143440

The residual or mismatch vector for iteration no: 11 is

dp[2] = –0.000013

dp[3] = –0.000004

dp[4] = 0.000003

dp[5] = –0.000000

dQ[2] = 0.000149

dQ[3] = –0.000007

dQ[4] = 0.000020

dQ[5] = –0.000027

The new voltage vector after iteration 11 :

Bus no 1 E : 1.000000 F : 0.000000

Bus no 2 E : 0.984358 F :- 0.008216

Bus no 3 E : 0.880949 F :- 0.142954

Bus no 4 E : 0.868707 F :- 0.154324

Bus no 5 E : 0.874648 F :- 0.143440

The residual or mismatch vector for iteration no: 12 is

dp[2] = –0.000074

dp[3] = 0.000005

dp[4] = –0.000009

dp[5] = –0.000014

dQ[2] = 0.000008

dQ[3] = –0.000002

dQ[4] = –0.000001

dQ[5] = –0.000000

The load flow solution

Bus no 1	Slack	P = 1.089040	Q = 0.556076	E = 1.000000	F = 0.000000
Bus no 2	pq	P = 0.350074	Q = 0.150008	E = 0.984358	F = –0.008216
Bus no 3	pq	P = –0.450005	Q = –0.199995	E = 0.880949	F = –0.142954
Bus no 4	pq	P = -0.399991	Q = –0.150001	E = 0.868707	F = –0.154324
Bus no 5	pq	P = –0.500014	Q = –0.250000	E = 0.874648	F = –0.143440

E5.4 Obtain the load flow solution to the system given in example E5.1 using Z-Bus. Use Gauss – Seidel method. Take accuracy for convergence as 0.0001.

Solution :

The bus impedance matrix is formed as indicated in section 5.15. The slack bus is taken as the reference bus. In this example, as in example 5.1 bus 1 is chosen as the slack bus.

(i) Add element 1-2. This is addition of a new bus to the reference bus

$$Z_{BUS} = \begin{array}{c|c|} & (2) \\ (2) & 0.05 + j0.24 \end{array}$$

(ii) Add element 1-3. This is also addition of a new bus to the reference bus

$$Z_{BUS} = \begin{array}{c|c|c|} & (2) & (3) \\ (2) & 0.08 + j0.24 & 0.0 + j0.0 \\ (3) & 0.0 + j0.0 & 0.02 + j0.06 \end{array}$$

(iii) Add element 2-3. This is the addition of a link between two existing buses 2 and 3.

$$Z_{2\text{-loop}} = Z_{\text{loop-2}} = Z_{22} - Z_{23} = 0.08+j0.24$$

$$Z_{3\text{-loop}} = Z_{\text{loop-3}} = Z_{32} - Z_{33} = -(0.02+j0.06)$$

$$Z_{\text{loop-loop}} = Z_{22} + Z_{33} - 2\,Z_{23} + Z_{23\ 23}$$

$$= (0.08+j0.24)+(0.02+j0.06)(0.06 + j0.18)$$

$$= 0.16 +j0.48$$

$$Z_{BUS} = \begin{array}{c|c|c|c|} & (2) & (3) & (\ell) \\ (2) & 0.08 + j0.024 & 0 + j0 & 0.08 + j0.24 \\ (3) & 0.0 + j0.0 & 0.02 + j0.06 & -(0.02 + j0.06) \\ \ell & 0.08 + j0.24 & -(0.02 + j0.006) & 0.16 + j0.48 \end{array}$$

The loop is now eliminated

$$Z_{22}^{'} = Z_{22} - \frac{Z_{2-\text{loop}}\,Z_{\text{loop}-2}}{Z_{\text{loop-loop}}}$$

$$= (0.08 + j0.24) - \frac{(0.8 + j0.24)^2}{0.16 + j0.48}$$

$$= 0.04 + j0.12$$

$$Z'_{23} = Z'_{32} = \left[Z_{23} - \frac{Z_{2\text{-loop}}\, Z_{\text{loop-3}}}{Z_{\text{loop-loop}}} \right]$$

$$= (0.0 + j0.0) - \frac{(0.8 + j0.24)(-0.02 - j0.06)}{0.16 + j0.48}$$

$$= 0.01 + j0.03$$

Similarly $\qquad Z'_{33} = 0.0175 + j0.0526$

The Z - Bus matrix is thus

$$Z_{Bus} = \left[\begin{array}{c|c} 0.04 + j0.12 & 0.01 + j0.03 \\ \hline 0.01 + j0.03 & 0.017 + j0.0525 \end{array} \right] =$$

$$\left[\begin{array}{c|c} 0.1265\angle 71.565^0 & 0.031623\angle 71.565^0 \\ \hline 0.031623\angle 71.565^0 & 0.05534\angle 71.565^0 \end{array} \right]$$

The voltages at bus 2 and 3 are assumed to be

$$V_2^{(0)} = 1.03 + j0.0$$

$$V_3^{(0)} = 1.0 + j0.0$$

Assuming that the reactive power injected into bys 2 is zero,

$$Q_2 = 0.0$$

The bus currents $I_2^{(0)}$ and $I_3^{(0)}$ are computed as

$$I_2^{(0)} = \frac{-0.3 + j0.0}{1.03 - j0.0} = -0.29126 - j0.0 = 0.29126\angle 180^0$$

$$I_3^{(0)} = \frac{-0.6 + j0.25}{1.0 + j0.0} = -0.6 - j0.25 = 0.65\angle 157.38^0$$

Iteration 1 : The voltage at bus 2 is computed as

$$V_2^{(1)} = V_1 + Z_{22}\, I_2^{(0)} + Z_{23}\, I^{(0)}$$

$$= 1.05\angle 0^0 + (0.1265\angle 71.565^0(0.29126\angle 180^0 +$$

$$(0.031623\angle 71.565^0)(0.65\angle 157.38^0$$

$$= 1.02485 - j0.05045$$

$$= 1.02609\angle -2.8182$$

The new bus current $I_2^{(0)}$ is now calculated.

$$\Delta I_2^{(0)} = \frac{V_2^{(1)}}{Z_{22}}\left[\frac{|V_{sch}|}{|V_2^{(1)}|} - 1\right]$$

$$= \frac{1.02609\angle -2.8182}{0.1265\angle 71.565^0} \times \left(\frac{1.03}{1.02609} - 1\right) = 0.0309084\angle -74.3832^0$$

$$\begin{aligned} Q^{(0)} &= \text{Im}\left[V_2^{(1)}\,\Delta I_2^{(0)*}\right] \\ &= \text{Im}\left[1.02609\angle -2.8182^0\right](0.0309084\angle 74.383^0 \\ &= 0.03 \end{aligned}$$

$$Q_2^{(1)} = Q_2^{(0)} + \Delta Q_2^{(0)} = 0.0 + 0.03 = 0.03$$

$$I_2^{(1)} = \frac{-0.3 - j0.3}{1.02609\angle -2.8182^0} = 0.29383\angle 182.8918^0$$

Voltage at bus 3 is now calculated

$$\begin{aligned} V_3^{(1)} &= V_1 + Z_{32}\, I_2^{(1)} + Z_{33}\, I_3^{(0)} \\ &= 1.05\angle 0.0^0 + (0.031623\angle 71.565^0)(0.29383\angle 182.832^0) + \\ &\quad (0.05534\angle 71.565^0)(0.65\angle 157.38^0) \\ &= (1.02389 - j0.036077) = 1.0245\angle -2.018^0 \end{aligned}$$

$$I_3^{(1)} = \frac{0.65\angle 157.38^0}{1.0245\angle 2.018^0} = 0.634437\angle 155.36^0$$

The voltages at the end of the first iteration are :

$$V_1 = 1.05\angle 0^0$$

$$V_2^{(1)} = 1.02609\angle -2.8182^0$$

$$V_3^{(1)} = 1.0245\angle -2.018^0$$

The differences in voltages are

$$\begin{aligned} \Delta V_2^{(1)} &= (1.02485 - j0.05045) - (1.03 + j0.0) \\ &= -0.00515 - j0.05045 \end{aligned}$$

$$\begin{aligned} \Delta V_3^{(1)} &= (1.02389 - j0.036077) - (1.0 + j0.0) \\ &= (0.02389 - j0.036077) \end{aligned}$$

Both the real and imaginary parts are greater than the specified limit 0.001.

Iteration 2 :

$$V_2^{(2)} = V_1 + Z_{22} I_2^{(1)} + Z_{23} I_3^{(1)}$$

$$= 1.02 \angle 0^0 + (0.1265\angle 71.565^0)(0.29383 \angle 182.892^0) +$$

$$(0.031623\angle 71.565^0)(0.63447\angle 155.36^0)$$

$$= 1.02634 - j0.050465$$

$$= 1.02758 \angle -2.81495^0$$

$$\Delta I_2^{(1)} = \frac{1.02758\angle -2.81495^0}{1.1265\angle -71.565^0}\left[\frac{1.03}{1.02758} - 1\right]$$

$$= 0.01923\angle -74.38^0$$

$$\Delta Q_2^{(1)} = \text{Im}\left[V_2^{(2)}\left(\Delta I_2^{(1)}\right)^*\right]$$

$$= \text{Im}(1.02758\angle -2.81495)(0.01913\angle 74.38^0)$$

$$= 0.0186487$$

$$Q_2^{(2)} = Q_2^{(1)} + \Delta Q_2^{(1)}$$

$$= 0.03 + 0.0186487 = 0.0486487$$

$$I_2^{(2)} = \frac{-0.3 - j0.0486487}{1.02758\angle 2.81495^0}$$

$$= 0.295763\angle 186.393^0$$

$$V_3^{(2)} = 1.05\angle 0^0 + (0.31623\angle 71.565^0(0.295763\angle 186.4^0 +$$

$$0.05534\angle 71.565^0)(0.634437\angle 155.36^0)$$

$$I_3^{(2)} = \frac{0.65\angle 157.38^0}{1.02466\angle 1.9459^0} = 0.6343567\angle 155.434^0$$

$$\Delta V_2^{(1)} = (1.02634 - j0.050465) - (1.02485 - j0.05041) = 0.00149 - j0.000015$$

$$\Delta V_3^{(1)} = (1.024 - j0.034793) - (1.02389 - j0.036077) = 0.00011 + j0.00128$$

As the accuracy is still not enough, another iteration is required.

Iteration 3 :

$$V_2^{(3)} = 1.05\angle 0^0 + (0.1265\angle 71.565^0)(0.295763\angle 186.4^0) +$$

$$(0.031623\angle 71.565^0)(0.63487\angle 155.434^0)$$

$$= 1.0285187 - j0.051262$$

$$= 1.0298\angle -2.853^0$$

$$I_2^{(2)} = \frac{1.0298\angle-2.853^0}{0.1265\angle71.565^0}\left[\frac{1.03}{1.0298}-1\right] = 0.001581\angle74.418^0$$

$$\Delta Q_2^{(2)} = 0.00154456$$

$$Q_2^{(3)} = 0.0486487 + 0.001544 = 0.0502$$

$$I_2^{(3)} = \frac{-0.3 - j0.0502}{0.0298\angle2.853^0} = 0.29537\angle186.647^0$$

$$V_3^{(3)} = 1.05\angle0^0 + (0.031623\angle71.565^0) + (0.29537\angle186.647^0) +$$

$$(0.05534\angle71.565^0)(0.634357\angle155.434^0)$$

$$= 1.024152 - j0.034817 = 1.02474\angle-1.9471^0$$

$$I_3^{(3)} = \frac{-0.65 - \angle157.38^0}{1.02474\angle1.9471^0} = 0.6343\angle155.433^0$$

$$\Delta V_2^{(2)} = (1.0285187 - j0.051262) - (1.02634 - j0.050465)$$

$$= 0.0021787 - 0.000787$$

$$\Delta V_3^{(2)} = (1.024152 - j0.034817) - (1.024 - j0.034793)$$

$$= 0.000152 - j0.00002$$

Iteration 4 :

$$V_2^{(4)} = 1.02996\angle-2.852^0$$

$$\Delta I_2^{(3)} = 0.0003159\angle-74.417^0$$

$$\Delta Q_2^{(3)} = 0.0000867$$

$$Q_2^{(4)} = 0.0505$$

$$I_2^{(4)} = 0.29537\angle186.7^0$$

$$V_2^{(4)} = 1.02416 - J0.034816 = 1.02475\angle-1.947^0$$

$$\Delta V_2^{(3)} = 0.000108 + j0.000016$$

$$\Delta V_3^{(3)} = 0.00058 + j0.000001$$

The final voltages are

$$V_1 = 1.05 + j0.0$$

$$V_2 = 1.02996\angle-2.852^0$$

$$V_3 = 1.02475\angle-1.947^0$$

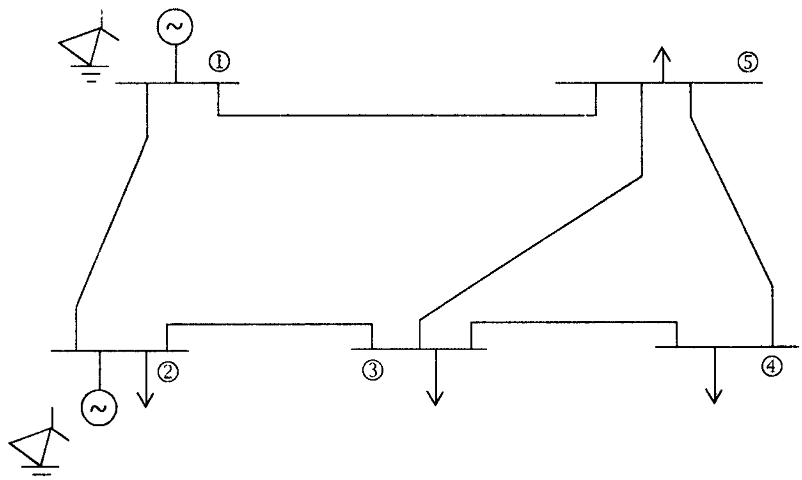
The line flows may be calculated further if required.

Problems

P5.1 Obtain a load flow solution for the system shown in Fig. P5.1 use

(i) Gauss – Seidel method

(ii) N-R polar coordinates method



Bus code p-q	Impedance Z_{pq}	Line charges Y pq/s
1 – 2	0.02 + j0 2	0.0
2 – 3	0.01 + j0.025	0.0
3 – 4	0.02 + j0.4	0.0
3 – 5	0.02 + 0.05	0.0
4 – 5	0.015 + j0 04	0.0
1-5	0 015 + j0 04	0.0

Values are given in p.u. on a base of 100Mva.

The scheduled powers are as follows

Bus Code (P)	Generation		Load	
	Mw	Mvar	MW	Mvar
1 (slack bus)	0	0	0	0
2	80	35	25	15
3	0	0	0	0
4	0	0	45	15
5	0	0	55	20

Take voltage at bus 1 as $1\angle 0^0$ p.u.

P5.2 Repeat problem P5.1 with line charging capacitance $Y_{pq}/2 = j0.025$ for each line

P5.3 Obtain the decoupled and fast decouple load flow solution for the system in P5.1 and compare the results with the exact solution.

P5.4 For the 51 bus system shown in Fig. P5.1, the system data is given as follows in p.u. Perform load flow analysis for the system

Line data	Resistance	Reactance	Capacitance
2 – 3	0.0287	0.0747	0.0322
3 – 4	0.0028	0.0036	0.0015
3 – 6	0.0614	0.1400	0.0558
3 – 7	0 0247	0.0560	0.0397
7 – 8	0.0098	0.0224	0.0091
8 – 9	0 0190	0.0431	0.0174
9 – 10	0.0182	0.0413	0.0167
10 – 11	0.0205	0.0468	0.0190
11 – 12	0.0660	0.0150	0.0060
12 – 13	0.0455	0.0642	0.0058
13 – 14	0 1182	0.2360	0.0213
14 – 15	0.0214	0.2743	0.0267
15 – 16	0.1336	0.0525	0.0059
16 – 17	0.0580	0.3532	0.0367

Contd.....

Line data	Resistance	Reactance	Capacitance
17 – 18	0.1550	0.1532	0.0168
18 – 19	0 1550	0.3639	0.0350
19 – 20	0.1640	0.3815	0.0371
20 – 21	0.1136	0.3060	0.0300
20 – 23	0.0781	0.2000	0.0210
23 – 24	0.1033	0.2606	0.0282
12 – 25	0.0866	0.2847	0.0283
25 – 26	0.0159	0.0508	0 0060
26 – 27	0 0872	0.2870	0 0296
27 – 28	0.0136	0.0436	0.0045
28 – 29	0.0136	0.0436	0.0045
29 – 30	0.0125	0 0400	0.0041
30 – 31	0.0136	0 0436	0 0045
27 – 31	0 0136	0 0436	0.0045
30 – 32	0 0533	0 1636	0.0712
32 – 33	0.0311	0.1000	0 0420
32 – 34	0.0471	0 1511	0.0650
30 – 51	0.0667	0.1765	0.0734
51 – 33	0 0230	0.0622	0.0256
35 – 50	0.0240	0 1326	0.0954
35 – 36	0 0266	0 1418	0.1146
39 – 49	0.0168	0 0899	0.0726
36 – 38	0.0252	0.1336	0.1078
38 – 1	0 0200	0.1107	0 0794
38 – 47	0.0202	0.1076	0 0869
47 – 43	0.0250	0.1336	0.1078
42 – 43	0.0298	0.1584	0.1281
40 – 41	0.0254	0.1400	0.1008

Contd.....

Line data	Resistance	Reactance	Capacitance
41 - 43	0.0326	0.1807	0.1297
43 - 45	0.0236	0.1252	0 1011
43 – 44	0.0129	0 0715	0.0513
45 – 46	0.0054	0.0292	0.0236
44 – 1	0.0330	0.1818	0.1306
46 – 1	0.0343	0.2087	0.1686
1 – 49	0.0110	0.0597	0.1752
49 – 50	0 0071	0.0400	0.0272
37 – 38	0.0014	0.0077	0.0246
47 – 39	0.0203	0.1093	0.0879
48 – 2	0.0426	0.1100	0.0460
3 – 35	0 0000	0 0500	0.0000
7 - 36	0.0000	0 0450	0.0000
11 - 37	0.0000	0.0500	0.0000
14 – 47	0.0000	0.0900	0.0000
16 - 39	0.0000	0.0900	0.0000
18 – 40	0.0000	0.0400	0.0000
20 - 42	0.0000	. 0.0800	0.0000
24 – 43	0.0000	0.0900	0.0000
27 – 45	0.0000	0.0900	0.0000
26 – 44	0.0000	0.0500	0.0000
30 – 46	0.0000	0.0450	0.0000
1 – 34	0.0000	0.0630	0.0000
21 – 2	0.0000	0.2500	0.0000
4 – 5	0.0000	0.2085	0.0000
19 – 41	0.0000	0.0800	0.0000

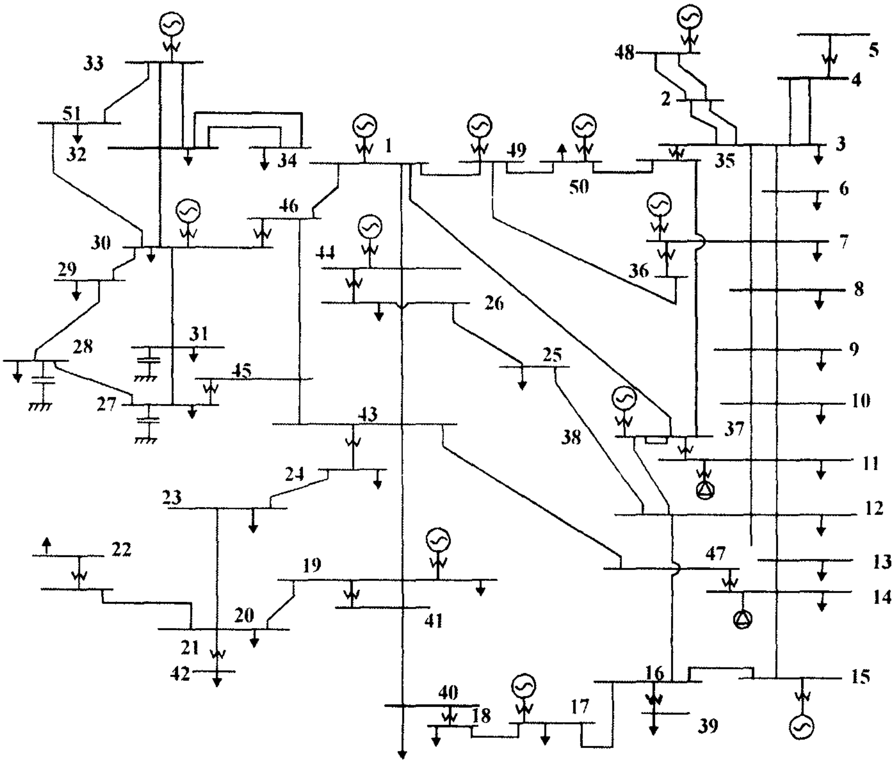


Fig. P5.4 51 Bus Power System.

Bus P – Q	TAP
3 – 35	1.0450
7 – 36	1.0450
11 – 37	1.0500
14 – 47	1.0600
16 – 39	1.0600
18 – 40	1.0900
19 – 41	1.0750
20 – 42	1.0600
24 – 43	1.0750
30 – 46	1.0750
1 – 34	1.0875
21 – 22	1.0600
5 – 4	1.0800
27 – 45	1 0600
26 – 44	1.0750

Bus Data – Voltage and Scheduled Powers

Bus no	Voltage magnitude (p.u.)	Voltage phase angle	Real power (p.u.)	Reactive power (p.u.)
1	1 0800	0 0000	0.0000	0.0000
2	1 0000	0 0000	– 0.5000	– 0.2000
3	1.0000	0 0000	– 0 9000	– 0.5000
4	1.0000	0.0000	0.0000	0.0000
5	1.0000	0.0000	– 0.1190	0.0000
6	1.0000	0.0000	– 0.1900	– 0.1000
7	1.0000	0 0000	– 0.3300	– 0.1800
8	1 0000	0 0000	– 0 4400	– 0 2400
9	1 0000	0.0000	– 0.2200	– 0.1200
10	1 0000	0.0000	– 0.2100	– 0 1200
11	1.0000	0.0000	– 0.3400	– 0.0500
12	1.0000	0 0000	– 0.2400	– 0.1360
13	1 0000	0.0000	– 0.1900	– 0.1100
14	1.0000	0 0000	– 0.1900	– 0.0400
15	1.0000	0.0000	0.2400	0.0000
16	1 0000	0.0000	– 0.5400	– 0.3000
17	1 0000	0.0000	– 0.4600	– 0.2100
18	1 0000	0.0000	– 0.3700	– 0.2200
19	1.0000	0.0000	– 0.3100	– 0.0200
20	1 0000	0.0000	– 0 3400	– 0.1600
21	1 0000	0 0000	0 0000	0.0000
22	1 0000	0.0000	– 0 1700	– 0 0800
23	1 0000	0.0000	– 0.4200	– 0.2300
24	1.0000	0 0000	– 0.0800	– 0.0200
25	1.0000	0.0000	– 0.1100	– 0.0600
26	1.0000	0.0000	– 0.2800	– 0.1400
27	1.0000	0 0000	– 0.7600	– 0.2500
28	1 0000	0 0000	– 0 8000	– 0.3600

Contd.....

Bus no	Voltage magnitude (p.u.)	Voltage phase angle	Real power (p.u.)	Reactive power (p.u.)
29	1.0000	0.0000	– 0.2500	– 0.1300
30	1.0000	0.0000	– 0.4700	0.0000
31	1.0000	0.0000	– 0.4200	– 0.1800
32	1.0000	0.0000	– 0.3000	– 0.1700
33	1.0000	0.0000	0.5000	0.0000
34	1.0000	0.0000	– 0 5800	– 0.2600
35	1.0000	0.0000	0.0000	0.0000
36	1.0000	0.0000	0.0000	0.0000
37	1.0000	0.0000	0.0000	0.0000
38	1.0000	0.0000	1 7000	0.0000
39	1.0000	0.0000	0.0000	0.0000
40	1.0000	0.0000	0.0000	0.0000
41	1.0000	0.0000	0.0000	0.0000
42	1.0000	0.0000	0.0000	0.0000
43	1.0000	0 0000	0.0000	0.0000
44	1.0000	0.0000	1 7500	0.0000
45	1.0000	0.0000	0 0000	0.0000
46	1.0000	0.0000	0.0000	0.0000
47	1.0000	0.0000	0.0000	0.0000
48	1.0000	0 0000	0.5500	0.0000
49	1.0000	0.0000	3 5000	0 0000
50	1.0000	0.0000	1.2000	0.0000
51	1.0000	0.0000	– 0.5000	– 0.3000

Bus No.	Voltage at VCB	Reactive power limit
15	1.0300	0.1800
30	1.0000	0.0400
33	1.0000	0.4800
38	1.0600	0.9000
44	1 0500	0 4500
48	1 0600	0 2000
49	1.0700	0.5600
50	1.0700	1.500

P 5.4 The data for a 13 machine, 71 bus, 94 line system is given. Obtain the load flow solution.

Data :

No. of buses	71
No. of lines	94
Base power (MVA)	200
No. of machines	13
No. of shunt loads	23

BUS NO	GENERATION		LOAD	POWER
1	-	-	0.0	0 0
2	0 0	0.0	0.0	0.0
3	506.0	150.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	100.0	32.0	0 0	0 0
7	0.0	0 0	12.8	8.3
8	300.0	125.0	0.0	0.0
9	0.0	0.0	185.0	130.0
10	0.0	0.0	80.0	50.0
11	0.0	0.0	155.0	96.0
12	0.0	0.0	0.0	0.0

Contd.....

BUS NO	GENERATION		LOAD	POWER
13	0.0	0.0	100.0	62.0
14	0.0	0.0	0.0	0.0
15	180.0	110.0	0 0	0.0
16	0.0	0.0	73.0	45.5
17	0.0	0.0	36.0	22.4
18	0.0	0.0	16.0	9.0
19	0.0	0.0	32 0	19.8
20	0 0	0.0	27 0	16.8
21	0.0	0.0	32.0	19 8
22	0.0	0.0	0.0	0 0
23	0 0	0.0	75.0	46.6
24	0.0	0.0	0 0	0.0
25	0.0	0 0	133.0	82 5
26	0 0	0.0	0 0	0 0
27	300 0	75.0	0.0	0 0
28	0.0	0.0	30.0	20.0
29	260.0	70.0	0.0	0.0
30	0.0	0.0	120.0	0.0
31	0.0	0.0	160.0	74.5
32	0.0	0.0	0.0	99 4
33	0.0	0.0	0.0	0 0
34	0.0	0.0	112.0	69.5
35	0.0	0.0	0.0	0 0
36	0.0	0.0	50.0	32.0
37	0 0	0.0	147 0	92.0
38	0 0	0.0	93 5	88 0
39	25.0	30.0	0.0	0.0
40	0.0	0.0	0.0	0.0
41	0.0	0.0	225.0	123.0
42	0.0	0.0	0.0	0.0
43	0.0	0.0	0.0	0.0

Contd.....

BUS NO	GENERATION		LOAD	POWER
44	180.0	55.0	0.0	0.0
45	0.0	0.0	0.0	0.0
46	0.0	0.0	78.0	38.6
47	0.0	0.0	234.0	145.0
48	340.0	250.0	0.0	0.0
49	0.0	0.0	295.0	183.0
50	0 0	0.0	40.0	24.6
51	0.0	0.0	227 0	142.0
52	0.0	0.0	0.0	0.0
53	0.0	0.0	0.0	0.0
54	0.0	0.0	108.0	68.0
55	0.0	0.0	25.5	48.0
56	0.0	0.0	0.0	0.0
57	0 0	0 0	55 6	35.6
58	0.0	0.0	42 0	27.0
59	0.0	0.0	57.0	27.4
60	0.0	0.0	0.0	0.0
61	0.0	0.0	0.0	0.0
62	0.0	0.0	40.0	27.0
63	0.0	0.0	33.2	20.6
64	300.0	75.0	0.0	0.0
65	0.0	0.0	0.0	0.0
66	96.0	25.0	0.0	0.0
67	0.0	0.0	14.0	6.5
68	90.0	25.0	0.0	0.0
69	0.0	0.0	0.0	0.0
70	0.0	0.0	11.4	7.0
71	0.0	0.0	0.0	0 0

LINE DATA

Line No	From Bus	To Bus	Line impedance		1/2 Y charge	Turns Ratio
1	9	8	0.0000	0.0570	0.0000	1.05
2	9	7	0.3200	0.0780	0.0090	1 00
3	9	5	0.0660	0.1600	0.0047	1.00
4	9	10	0.0520	0.1270	0.0140	1.00
5	10	11	0.0660	0.1610	0.0180	1.00
6	7	10	0.2700	0.0700	0.0070	1 00
7	12	11	0.0000	0.0530	0.0000	0.95
8	11	13	0.0600	0 1480	0.0300	1.00
9	14	13	0.0000	0.0800	0.0000	1.00
10	13	16	0.9700	0.2380	0.0270	1.00
11	17	15	0.0000	0.0920	0.0000	1.05
12	7	6	0 0000	0.2220	0.0000	1.05
13	7	4	0.0000	0.0800	0.0000	1.00
14	4	3	0.0000	0.0330	0.0000	1.05
15	4	5	0.0000	0.1600	0.0000	1.00
16	4	12	0.0160	0.0790	0.0710	1.00
17	12	14	0.0160	0.0790	0.0710	1.00
18	17	16	0.0000	0.0800	0.0000	0.95
19	2	4	0.0000	0.0620	0.0000	1.00
20	4	26	0.0190	0.0950	0.1930·	0.00
21	2	1	0.0000	0.0340	0.0000	1.05
22	31	26	0.0340	0.1670	0 1500	1.00
23	26	25	0.0000	0.0800	0.0000	0.95
24	25	23	0 2400	0.5200	0.1300	1.00
25	22	23	0 0000	0 0800	0.0000	0.95
26	24	22	0.0000	0.0840	0.0000	0.95
27	22	17	0.0480	0.2500	0.0505	1.00
28	2	24	0 0100	0.1020	0.3353	1.00
29	23	21	0.0366	0.1412	0.0140	1.00
30	21	20	0.7200	0.1860	0.0050	1.00
31	20	19	0 1460	0.3740	0.0100	1.00

Contd.

Line No	From Bus	To Bus	Line impedance		1/2 Y charge	Turns Ratio
32	19	18	0.0590	0.1500	0.0040	1.00
33	18	16	0.0300	0 0755	0.0080	1.00
34	28	27	0.0000	0.0810	0.0000	1.05
35	30	29	0.0000	0.0610	0.0000	1 05
36	32	31	0.0000	0.0930	0 0000	0.95
37	31	30	0.0000	0 0800	0.0000	0.95
38	28	32	0 0051	0.0510	0.6706	1.00
39	3	33	0.0130	0 0640	0.0580	1.00
40	31	47	0.0110	0.0790	0 1770	1.00
41	2	32	0.0158	0.1570	0.5100	1.00
42	33	34	0 0000	0.0800	0.0000	0 95
43	35	33	0.0000	0.0840	0.0000	0.95
44	35	24	0.0062	0 0612	0.2120	1.00
45	34	36	0.0790	0.2010	0.0220	1.00
46	36	37	0.1690	0 4310	0.0110	1.00
47	37	38	0.0840	0 1880	0.0210	1.00
48	40	39	0 0000	0.3800	0.0000	1.05
49	40	38	0.0890	0.2170	0.0250	1.00
50	38	41	0.1090	0.1960	0.2200	1.00
51	41	51	0.2350	0.6000	0.0160	1.00
52	42	41	0.0000	0.0530	0.0000	0.95
53	45	42	0.0000	0.0840	0.0000	0.95
54	47	49	0.2100	0.1030	0.9200	1.00
55	49	48	0.0000	0.0460	0.0000	1 05
56	49	50	0 0170	0 0840	0.0760	1 00
57	49	42	0.0370	0.1950	0 0390	1.00
58	50	51	0 0000	0.0530	0 0000	0.95
59	52	50	0.0000	0.0840	0.0000	0.95
60	50	55	0.0290	0.1520	0.0300	1.00
61	50	53	0.0100	0.0520	0.0390	1.00
62	53	54	0.0000	0.0800	0.0000	0.95
63	57	54	0 0220	0.0540	0.0060	1.00

Contd....

Line No	From Bus	To Bus	Line impedance		1/2 Y charge	Turns Ratio
64	55	56	0.0160	0.0850	0.0170	1.00
65	56	57	0 0000	0.0800	0.0000	1.00
66	57	59	0.0280	0.0720	0.0070	1.00
67	59	58	0.0480	0.1240	0.0120	1.00
68	60	59	0.0000	0.0800	0.0000	1.00
69	53	60	0.0360	0.1840	0.3700	1.00
70	45	44	0.0000	0.1200	0.0000	1.05
71	45	46	0.0370	0.0900	0.0100	1.00
72	46	41	0.0830	0.1540	0.0170	1 00
73	46	59	0.1070	0.1970	0.0210	1.00
74	60	61	0.0160	0.0830	0.0160	1.00
75	61	62	0.0000	0.0800	0.0000	0.95
76	58	62	0 0420	0 1080	0.0020	1.00
77	62	63	0.0350	0.0890	0.0090	1 00
78	69	68	0.0000	0.2220	0.0000	1.05
79	69	61	0.0230	0.1160	0.1040	1.00
80	67	66	0.0000	0.1880	0.0000	1.05
81	65	64	0 0000	0.0630	0.0000	1.05
82	65	56	0.0280	0.1440	0.0290	1.00
83	65	61	0.0230	0.1140	0.0240	1.00
84	65	67	0.0240	0.0600	0.0950	1.00
85	67	63	0.0390	0.0990	0.0100	1.00
86	61	42	0.0230	0.2293	0.0695	1.00
87	57	67	0.0550	0.2910	0 0070	1.00
88	45	70	0 1840	0.4680	0 0120	1.00
89	70	38	0.1650	0.4220	0.0110	1.00
90	33	71	0.0570	0.2960	0.0590	1.00
91	71	37	0.0000	0.0800	0.0000	0.95
92	45	41	0.1530	0.3880	0.1000	1.00
93	35	43	0.0131	0 1306	0.4293	1.00
94	52	52	0.0164	0.1632	0.5360	1.00

Shunt Load Data

S.No	Bus No	Shunt	Load
1	2	0 00	– 0.4275
2	13	0.00	0.1500
3	20	0.00	0.0800
4	24	0.00	– 0.2700
5	28	0.00	– 0.3375
6	31	0.00	0.2000
7	32	0.00	– 0.8700
8	34	0.00	0.2250
9	35	0.00	– 0.3220
10	36	0.00	0.1000
11	37	0.00	0.3500
12	38	0.00	0.2000
13	41	0.00	0.2000
14	43	0.00	– 0 2170
15	46	0.00	0.1000
16	47	0 00	0.3000
17	50	0.00	0.1000
18	51	0.00	0.1750
19	52	0.00	– 0.2700
20	54	0.00	0.1500
21	57	0.00	0.1000
22	59	0.00	0.0750
23	21	0.00	0.0500

Questions

5.1 Explain the importance of load flow studies.

5.2 Discuss breifly the bus classification.

5.3 What is the need for a slack bus or reference bus ? Explain.

5.4 Explain Gauss-Seidel method of load flow solution.

5.5 Discuss the method of Newton-Raphson method in general and explain its applicalibility for power flow solution.

5.6 Explain the Polar-Coordinates method of Newton-Raphson load flow solution.

5.7 Give the Cartesian coordinates method or rectangular coordinates method of Newton-Raphson load flow solution.

5.8 Give the flow chart for Q.No. 6.

5.9 Give the flow chart for Q.No. 7.

5.10 Explain sparsity and its application in power flow studies.

5.11 How are generator buses are P, V buses treated in load flow studies ?

5.12 Give the algorithm for decoupled load flow studies.

5.13 Explain the fast decouped load flow method.

5.14 Compare the Gauss-Seidel and Newton-Raphson method for power flow solution.

5.15 Compare the Newton-Raphson method, decoupled load flow method and fast decouped load flow method.

6 SHORT CIRCUIT ANALYSIS

Electrical networks and machines are subject to various types of faults while in operation. During the fault period, the current flowing is determined by the internal e.m.f's of the machines in the network, and by the impedances of the network and machines. However, the impedances of machines may change their values from those that exist immediately after the fault occurrence to different values during the fault till the fault is cleared. The network impedance may also change, if the fault is cleared by switching operations. It is, therefore, necessary to calculate the short-circuit current at different instants when faults occur. For such fault analysis studies and in general for power system analysis it is very convenient to use per unit system and percentage values. In the following this system is explained.

6.1 Per Unit Quantities

The per unit value of any quantity is the ratio of the actual value in any units to the chosen base quantify of the same dimensions expressed as a decimal.

$$\text{Per unit quantity} = \frac{\text{Actual value in any units}}{\text{base or reference value in the same units}}$$

In power systems the basic quantities of importance are voltage, current, impedance and power. For all per unit calculations a base KVA or MVA and a base KV are to be chosen. Once the base values or reference values are chosen, the other quantities can be obtained as follows :

Selecting the total or 3-phase KVA as base KVA, for a 3-phase system

$$\text{Base current in amperes} = \frac{\text{base KVA}}{\sqrt{3}\ [\text{base KV (line-to-line)}]}$$

$$\text{Base impedance in ohms} = \left[\frac{\text{base KV (line-to-line)}^2 \times 1000}{\sqrt{3}\,[(\text{base KVA})/3]}\right]$$

$$\text{Base impedance in ohms} = \frac{(\text{base KV (line-to-line)}^2}{\text{base MVA}}$$

Hence, $$\text{Base impedance in ohm} = \frac{(\text{base KV (line-to-line)}^2 \times 1000}{\text{base KVA}}$$

where base KVA and base MVA are the total or three phase values.

If phase values are used

$$\text{Base current in amperes} = \frac{\text{base KVA}}{\text{base KV}}$$

$$\text{Base impedance in ohm} = \frac{\text{base voltage}}{\text{base current}}$$

$$= \frac{(\text{base KV})^2 \times 1000}{\text{base KVA per phase}}$$

$$\text{Base impedance in ohm} = \frac{(\text{base KV})^2}{\text{base MVA per phase}}$$

In all the above relations the power factor is assumed unity, so that

base power KW = base KVA

Now, $$\text{Per unit impedance} = \frac{(\text{actual impedance in ohm}) \times \text{KVA}}{(\text{base KV})^2 \times 1000}$$

Some times, it may be required to use the relation

$$(\text{actual impedance in ohm}) = \frac{(\text{Per unit impedance in ohms})\ (\text{base KV})^2 \times 1000}{\text{base KVA}}$$

Very often the values are in different base values. In order to convert the per unit impedance from given base to another base, the following relation can be derived easily.

Per unit impedance on new base

$$Z_{new}\ \text{p-u} = Z_{given}\ \text{p.u}\left(\frac{\text{new KVA base}}{\text{given KVA base}}\right)\left(\frac{\text{given KV base}}{\text{new KV base}}\right)^2$$

6.2 Advantages of Per Unit System

1. While performing calculations, referring quantities from one side of the transformer to the other side serious errors may be committed. This can be avoided by using per unit system.
2. Voltages, currents and impedances expressed in per unit do not change when they are referred from one side of transformer to the other side. This is a great advantage.
3. Per unit impedances of electrical equipment of similar type usually lie within a narrow range, when the equipment ratings are used as base values.
4. Transformer connections do not affect the per unit values.
5. Manufacturers usually specify the impedances of machines and transformers in per unit or percent of name plate ratings.

6.3 Three Phase Short Circuits

In the analysis of symmetrical three-phase short circuits the following assumptions are generally made.

1. Transformers are represented by their leakage reactances. The magnetizing current, and core losses are neglected. Resistances, shunt admittances are not considered. Star-delta phase shifts are also neglected.
2. Transmission lines are represented by series reactances. Resistances and shunt admittances are neglected.
3. Synchronous machines are represented by constant voltage sources behind subtransient reactances. Armature resistances, saliency and saturation are neglected.
4. All non-rotating impedance loads are neglected.
5. Induction motors are represented just as synchronous machines with constant voltage source behind a reactance. Smaller motor loads are generally neglected.

Per unit impedances of transformers : Consider a single-phase transformer with primary and secondary voltages and currents denoted by V_1, V_2 and I_1, I_2 respectively.

we have, $$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\text{Base impedance for primary} = \frac{V_1}{I_1}$$

$$\text{Base impedance for secondary} = \frac{V_2}{I_2}$$

$$\text{Per unit impedance referred to primary} = \frac{Z_1}{(V_1 / I_1)} = \frac{I_1 Z_1}{V_1}$$

Per unit impedance referred to secondary $= \dfrac{I_2 Z_2}{V_2}$

Again, actual impedance referred to secondary $= Z_1 \left(\dfrac{V_2}{V_1}\right)^2$

Per unit impedance referred to secondary

$$= \frac{Z_1\left(\dfrac{V_2}{V_1}\right)^2}{(V_2/I_2)} = Z_1 \cdot \frac{V_2^2}{V_1^2} \cdot \frac{I_2}{V_2} = \frac{Z_1(V_2 I_2)}{V_1^2} = Z_1 \frac{(V_1 I_1)}{V_1^2} = \frac{Z_1 I_1}{V_1}$$

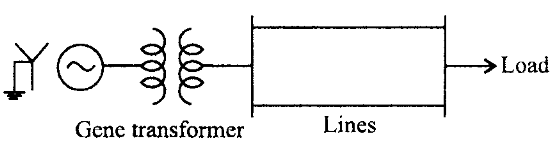
= Per unit impedance referred to primary

Thus, the per unit impedance referred remains the same for a transformer on either side.

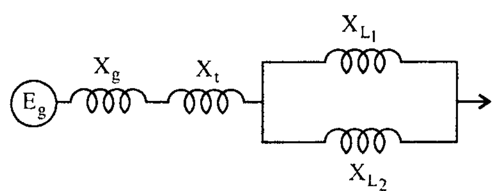
6.4 Reactance Diagrams

In power system analysis it is necessary to draw an equivalent circuit for the system. This is an impedance diagrams. However, in several studies, including short-circuit analysis it is sufficient to consider only reactances neglecting resistances. Hence, we draw reactance diagrams. For 3-phase balanced systems, it is simpler to represent the system by a single line diagram without losing the identify of the 3-phase system. Thus, single line reactance diagrams can be drawn for calculation.

This is illustrated by the system shown in Fig. 6.1 (a) & (b) and by its single line reactance diagram.



(a) A power system



(b) Equivalent single-line reactance diagram

Fig. 6.1

6.5 Percentage Values

The reactances of generators, transformers and reactors are generally expressed in percentage values to permit quick short circuit calculation.

Percentage reactance is defined as :

$$\% \, X = \frac{IX}{V} \times 100$$

where, I = full load current

V = phase voltage

X = reactance in ohms per phase

Short circuit current I_{SC} in a circuit then can be expressed as,

$$I_{SC} = \frac{V}{X} = \frac{V \,.\, I}{V \,.\, (\%X)} \times 100$$

$$= \frac{I \,.\, 100}{\%X}$$

Percentage reactance can expressed in terms of KVA and KV as following

From equation $$X = \frac{(\%X) \,.\, V}{I.100} = \frac{(\%X)V^2}{100 \,.\, V \,.\, I} = \frac{(\%X)\frac{V}{1000} \cdot \frac{V}{1000} \times 1000}{100 \,.\, \frac{V}{1000} \,.\, I}$$

$$= \frac{(\%X)\,(KV)^2 10}{KVA}$$

Alternatively $$(\%X) = X \,.\, \frac{KVA}{10\,(KV)^2}$$

As has been stated already in short circuit analysis since the reactance X is generally greater than three times the resistance, resistances are neglected.

But, in case percentage resistance and therefore, percentage impedance values are required then, in a similar manner we can define

$$\%\, R = \frac{IR}{V} \times 100$$

and $$\%\, Z = \frac{IZ}{V} \times 100$$ with usual notation.

The percentage values of R and Z also do not change with the side of the transformer or either side of the transformer they remain constant. The ohmic values of R, X and Z change from one side to the other side of the transformer.

when a fault occurs the potential falls to a value determined by the fault impedance. Short circuit current is expressed in term of short circuit KVA based on the normal system voltage at the point of fault.

6.6 Short Circuit KVA

It is defined as the product of normal system voltage and short circuit current at the point of fault expressed in KVA.

Let V = normal phase voltage in volts

I = fall load current in amperes at base KVA

% X = percentage reactance of the system expressed on base KVA.

The short circuit current,

$$I_{SC} = I.\ \frac{100}{\%X}$$

The three phase or total short circuit KVA

$$= \frac{3.V\ I_{SC}}{1000} = \frac{3.V.I\ 100}{(\%X)\ 1000} = \frac{3V\ I}{1000} \cdot \frac{100}{\%X}$$

Therefore short circuit KVA = Base KVA × $\frac{100}{(\%X)}$

In a power system or even in a single power station different equipment may have different ratings. Calculation are required to be performed where different components or units are rated differently. The percentage values specified on the name plates will be with respect to their name plate ratings. Hence, it it necessary to select a common base KVA or MVA and also a base KV. The following are some of the guide lines for selection of base values.

1. Rating of the largest plant or unit for base MVA or KVA.
2. The total capacity of a plant or system for base MVA or KVA.
3. Any arbitrary value.

$$(\%X)_{\text{on new base}} = \left(\frac{\text{Base KVA}}{\text{Unit KVA}}\right)(\%\ X \text{ at unit KVA})$$

If a transformer has 8% reactance on 50 KVA base, its value at 100 KVA base will be

$$(\%X)_{100\ KVA} = \left(\frac{100}{50}\right) \times 8 = 16\%$$

Similarly the reactance values change with voltage base as per the relation

$$X_2 = \left(\frac{V_2}{V_1}\right)^2 .\ X_1$$

where X_1 = reactance at voltage V_1

and X_2 = reactance at voltage V_2

For short circuit analysis, it is often convenient to draw the reactance diagrams indicating the values in per unit.

6.7 Importance of Short Circuit Currents

Knowledge of short circuit current values is necessary for the following reasons.

1. Fault currents which are several times larger than the normal operating currents produce large electro magnetic forces and torques which may adversely affect the stator end windings. The forces on the end windings depend on both the d.c. and a.c. components of stator currents.
2. The electro dynamic forces on the stator end windings may result in displacement of the coils against one another. This may result in loosening of the support or damage to the insulation of the windings.
3. Following a short circuit, it is always recommended that the mechanical bracing of the end windings to checked for any possible loosening.
4. The electrical and mechanical forces that develop due to a sudden three phase short circuit are generally severe when the machine is operating under loaded condition.
5. As the fault is cleared with in 3 cycles generally the heating efforts are not considerable.

Short circuits may occur in power systems due to system over voltages caused by lightning or switching surges or due to equipment insulation failure or even due to insulator contamination. Some times even mechanical causes may create short circuits. Other well known reasons include line-to-line, line-to-ground, or line-to-line faults on over head lines. The resultant short circuit has to the interrupted within few cycles by the circuit breaker.

It is absolutely necessary to select a circuit breaker that is capable of operating successfully when maximum fault current flows at the circuit voltage that prevails at that instant. An insight can be gained when we consider an R-L circuit connected to an alternating voltage source, the circuit being switched on through a switch.

6.8 Analysis of R-L Circuit

Consider the circuit in the Fig. 6.2.

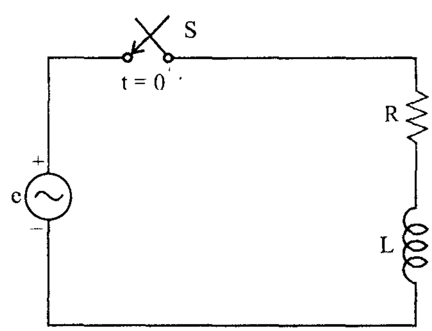


Fig. 6.2

Let $e = E_{max} \text{Sin}(\omega t + \alpha)$ when the switch S is closed at $t = 0^+$

$$e = E_{max} \text{Sin}(\omega t + \alpha) = R + L\frac{di}{dt}$$

α is determined by the magnitude of voltage when the circuit is closed.

The general solution is

$$i = \frac{E_{max}}{|Z|}\left[\text{Sin}\,(\omega t + \alpha - \theta) - e^{\frac{-Rt}{L}}\,\text{Sin}\,(\alpha - \theta)\right]$$

where $|Z| = \sqrt{R^2 + \omega^2 L^2}$

and $\theta = \text{Tan}^{-1}\,\frac{\omega L}{R}$

The current contains two components :

$$\text{a.c. component} = \frac{E_{max}}{|Z|}\,\text{Sin}\,(\omega t + \alpha - \theta)$$

and
$$\text{d.c. component} = \frac{E_{max}}{|Z|}\,e^{\frac{-Rt}{L}}\,\text{Sin}\,(\alpha - \theta)$$

If the switch is closed when $\alpha - \theta = \pi$ or when $\alpha - \theta = 0$

the d.c. component vanishes.

the d.c. component is a maximum when $\alpha - \theta = \pm\,\frac{\pi}{2}$

6.9 Three Phase Short Circuit on Unloaded Synchronous Generator

If a three phase short circuit occurs at the terminals of a salient pole synchronous we obtain typical oscillograms as shown in Fig. 6.3 for the short circuit currents the three phases. Fig. 6.4 shows the alternating component of the short circuit current when the d.c. component is eliminated. The fast changing sub-transient component and the slowly changing transient components are shown at A and C. Figure 6.5 shows the electrical torque. The changing field current is shown in Fig. 6.6.

From the oscillogram of a.c. component the quantities x''_d, x''_q, x'_d and x'_q can be determined.

If V is the line to neutral prefault voltage then the a.c. component.

$i_{ac} = \frac{V}{x''_q} = I''$, the r.m.s subtransient short circuit. Its duration is determined by T''_d, the subtransient direct axis time constant. The value of i_{ac} decreases to $\frac{V}{x'_d}$ when $t > T''_d$

with T'_d as the direct axis transient time constant when $t > T'_d$

$$i_{ac} = \frac{V}{x_d}$$

The maximum d.c. off-set component that occurs in any phase at $\alpha = 0$ is

$$i_{d.c,\,max}(t) = \sqrt{2}\frac{V}{x_d''}e^{-t/T_A}$$

where T_A is the armature time constant.

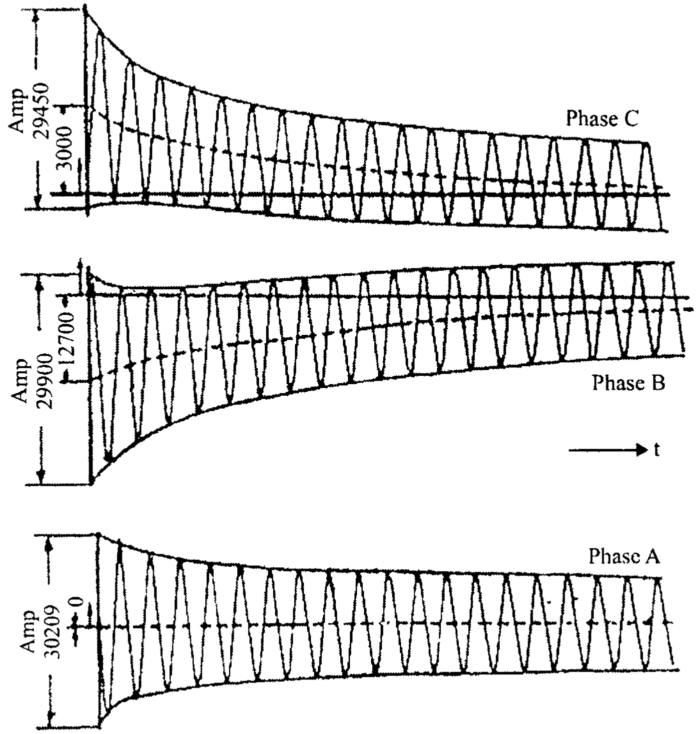


Fig. 6.3 Oscillograms of the armature currents after a short circuit.

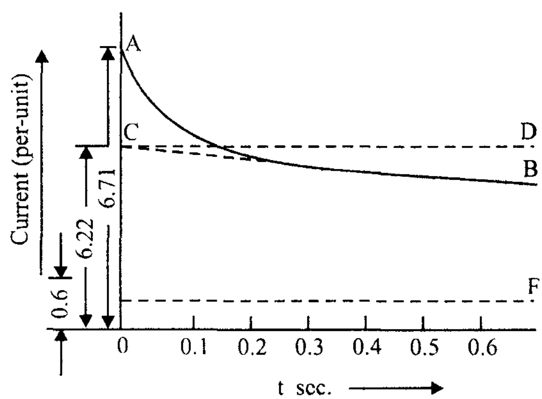


Fig. 6.4 Alternating component of the short circuit armature current

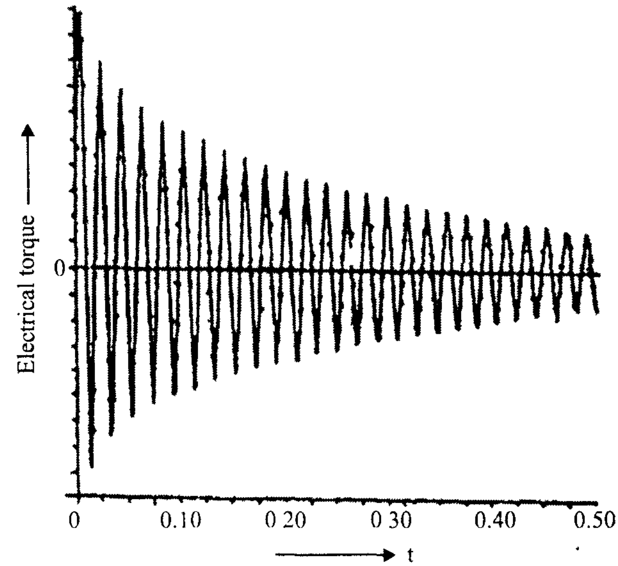


Fig. 6.5 Electrical torque on three-phase terminal short circuit.

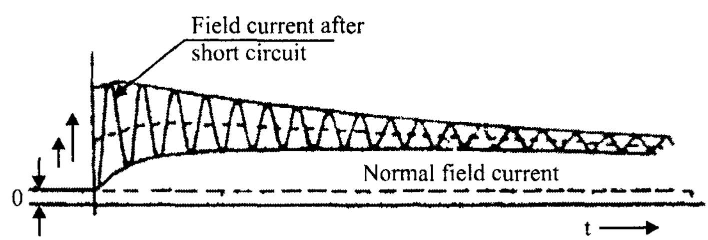


Fig. 6.6 Oscilogram of the field current after a short circuit.

6.10 Effect of Load Current or Prefault Current

Consider a 3-phase synchronous generator supplying a balanced 3-phase load. Let a three phase fault occur at the load terminals. Before the fault occurs, a load current I_L is flowing into the load from the generator. Let the voltage at the fault be v_f and the terminal voltage of the generator be v_t. Under fault conditions, the generator reactance is x''_d.

The circuit in Fig. 6.7 indicates the simulation of fault at the load terminals by a parallel switch S.

$$E''_g = V_t + j\,x''_d\,I_L = V_f + (X_{ext} + j\,x''_d)\,I_L$$

where E''_g is the subtransient internal voltage.

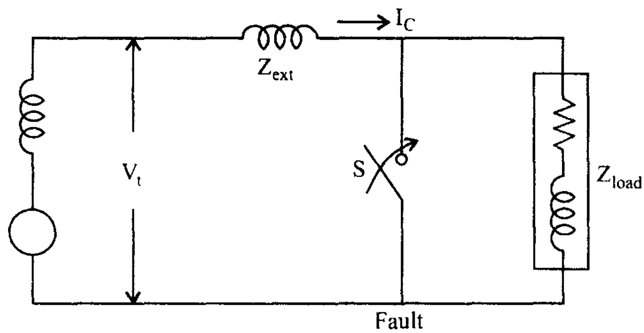


Fig. 6.7

For the transient state

$$E'_g = V_t + j\,x'_d\,I_L$$
$$= V_f + \left(Z_{ext} + j\,x'_d\right)I_L$$

E''_g or E'_g are used only when there is a prefault current I_L. Otherwise E_g, the steady state voltage in series with the direct axis synchronous reactance is to be used for all calculations. E_g remains the same for all I_L values, and depends only on the field current. Every time, of course, a new E''_g is required to be computed.

6.11 Reactors

Whenever faults occur in power system large currents flow. Especially, if the fault is a dead short circuit at the terminals or bus bars enormous currents flow damaging the equipment and its components. To limit the flow of large currents under there circumstances current limiting reactors are used. These reactors are large coils covered for high self-inductance.

They are also so located that the effect of the fault does not affect other parts of the system and is thus localized. From time to time new generating units are added to an existing system to augment the capacity. When this happens, the fault current level increases and it may become necessary to change the switch gear. With proper use of reactors addition of generating units does not necessitate changes in existing switch gear.

6.12 Construction of Reactors

These reactors are built with non magnetic core so that saturation of core with consequent reduction in inductance and increased short circuit currents is avoided. Alternatively, it is possible to use iron core with air-gaps included in the magnetic core so that saturation is avoided.

6.13 Classification of Reactors

(i) Generator reactors, (ii) Feeder reactors, (iii) Bus-bar reactors

The above classification is based on the location of the reactors. Reactors may be connected in series with the generator in series with each feeder or to the bus bars.

(i) ***Generator reactors***

The reactors are located in series with each of the generators as shown in Fig. 6.8 so that current flowing into a fault F from the generator is limited.

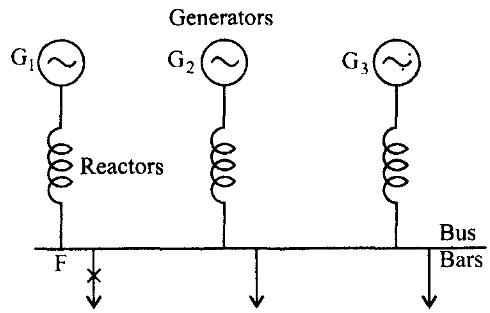


Fig. 6.8

Disadvantages

(a) In the event of a fault occuring on a feeder, the voltage at the remaining healthy feeders also may loose synchronism requiring resynchronization later.

(b) There is a constant voltage drop in the reactors and also power loss, even during normal operation. Since modern generators are designed to with stand dead short circuit at their terminals, generator reactors are now-a-days not used except for old units in operation.

(ii) ***Feeder reactors :*** In this method of protection each feeder is equipped with a series reactor as shown in Fig. 6.9.

In the event of a fault on any feeder the fault current drawn is restricted by the reactor.

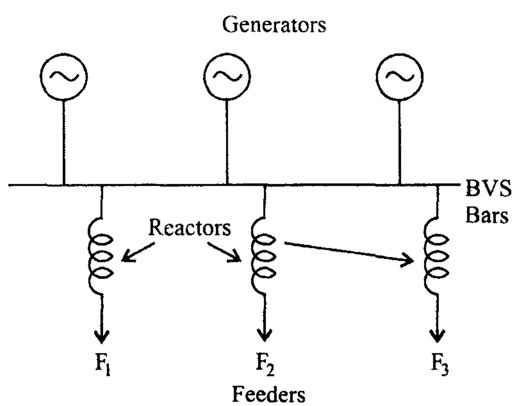


Fig. 6.9

Disadvantages : 1. Voltage drop and power loss still occurs in the reactor for a feeder fault. However, the voltage drop occurs only in that particular feeder reactor. 2. Feeder reactors do not offer any protection for bus bar faults. Neverthless, bus-bar faults occur very rarely.

As series reactors inhererbly create voltage drop, system voltage regulation will be impaired. Hence they are to be used only in special case such as for short feeders of large cross-section.

(iii) ***Bus bar reactors :*** In both the above methods, the reactors carry full load current under normal operation. The consequent disadvantage of constant voltage drops and power loss can be avoided by dividing the bus bars into sections and inter connect the sections through protective reactors. There are two ways of doing this.

(a) ***Ring system :***

In this method each feeder is fed by one generator. Very little power flows across the reactors during normal operation. Hence, the voltage drop and power loss are negligible. If a fault occurs on any feeder, only the generator to which the feeder is connected will feed the fault and other generators are required to feed the fault through the reactor.

(b) ***Tie-bar system :*** This is an improvement over the ring system. This is shown in Fig. 6.11. Current fed into a fault has to pass through two reactors in series between sections.

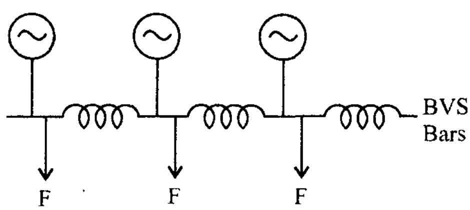


Fig. 6.10

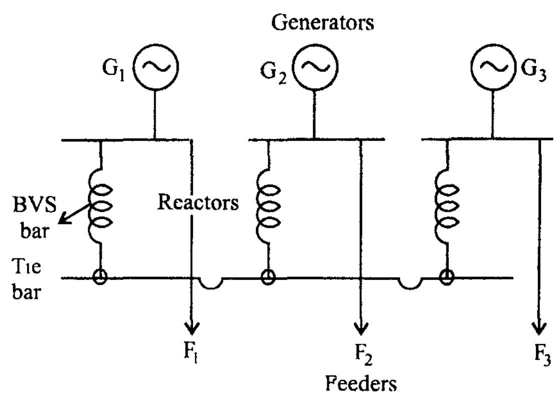


Fig. 6.11

Another advantage is that additional generation may be connected to the system without requiring changes in the existing reactors.

The only disadvantage is that this systems requires an additional bus-bar system, the tie-bar.

Worked Examples

E 6.1 Two generators rated at 10 MVA, 11 KV and 15 MVA, 11 KV respectively are connected in parallel to a bus. The bus bars feed two motors rated 7.5 MVA and 10 MVA respectively. The rated voltage of the motors is 9 KV. The reactance of each generator is 12% and that of each motor is 15% on their own ratings. Assume 50 MVA, 10 KV base and draw the reactance diagram.

Solution :

The reactances of the generators and motors are calculated on 50 MVA, 10 KV base values.

$$\text{Reactance of generator 1} = X_{G1} = 12 \cdot \left(\frac{11}{10}\right)^2 \cdot \left(\frac{50}{10}\right) = 72.6\%$$

$$\text{Reactance of generator 2} = X_{G2} = 12 \left(\frac{11}{10}\right)^2 \cdot \left(\frac{50}{10}\right) = 48.4\%$$

$$\text{Reactance of motor 1} = X_{M1} = 15 \cdot \left(\frac{9}{10}\right)^2 \left(\frac{50}{7.5}\right) = 81\%$$

$$\text{Reactance of motor 2} = X_{M2} = 15 \left(\frac{9}{10}\right)^2 \left(\frac{50}{10}\right) = 60.75\%$$

The reactance diagram is drawn and shown in Fig. E.6.1.

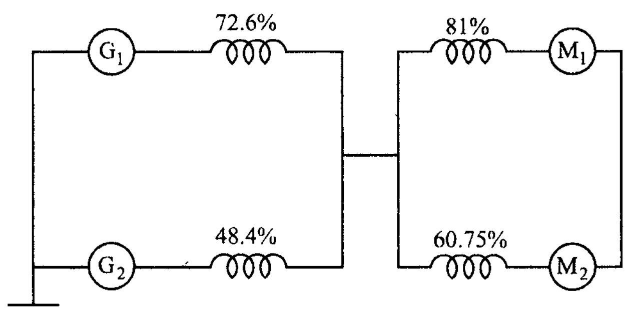


Fig. E.6.1

E.6.2 A 100 MVA, 13.8 KV, 3-phase generator has a reactance of 20%. The generator is connected to a 3-phase transformer T_1 rated 100 MVA 12.5 KV/110 KV with 10% reactance. The h.v. side of the transformer is connected to a transmission line of reactance 100 ohm. The far end of the line is connected to a step down transformer T_2, made of three single-phase transformers each rated 30 MVA, 60 KV / 10 KV with 10% reactance the generator supplies two motors connected on the l.v. side T_2 as shown in Fig. E.6.2. The motors are rated at 25 MVA and 50 MVA both at 10 KV with 15% reactance. Draw the reactance diagram showing all the values in per unit. Take generator rating as base.

Solution :

Base MVA = 100

Base KV = 13.8

$$\text{Base KV for the line} = 13.8 \times \frac{110}{12.5} = 121.44$$

$$\text{Line-to-line voltage ratio of } T_2 = \frac{\sqrt{3} \times 66\text{KV}}{10\text{KV}} = \frac{114.31}{10}$$

$$\text{Base voltage for motors} = \frac{121.44 \times 10}{114.31} = 10.62 \text{ KV}$$

% X for generators = 20 % = 0.2 p.u.

$$\% \text{ X for transformer } T_1 = 10 \times \left(\frac{12.5}{13.8}\right)^2 \times \frac{100}{100} = 8.2\%$$

% X for transformer T_2 on $\sqrt{3} \times 66$: 10 KV and 3 × 30 MVA base = 10%

% X for T_2 on 100 MVA, and 121.44 KV : 10.62 KV is

$$\% \text{ X } T_2 = 10 \times \left(\frac{10}{10.62}\right)^2 \times \left(\frac{100}{90}\right) = 9.85 \% = 0.0985 \text{ p.u.}$$

$$\text{Base reactance for line} = \left(\frac{121.44}{100}\right)^2 = 147.47 \text{ ohms}$$

$$\text{Reactance of line} = \frac{100}{147.47} = 0.678 \text{ p.u.}$$

$$\text{Reactance of motor } M_1 = 10 \times \left(\frac{10}{10.62}\right)^2 \left(\frac{90}{25}\right) = 31.92\%$$

$$= 0.3192 \text{ p.u.}$$

$$\text{Reactance of motor } M_2 = 10 \times \left(\frac{10}{10.62}\right)^2 \left(\frac{90}{50}\right) = 15.96\%$$

The reactance diagram is shown in Fig. E.6.2.

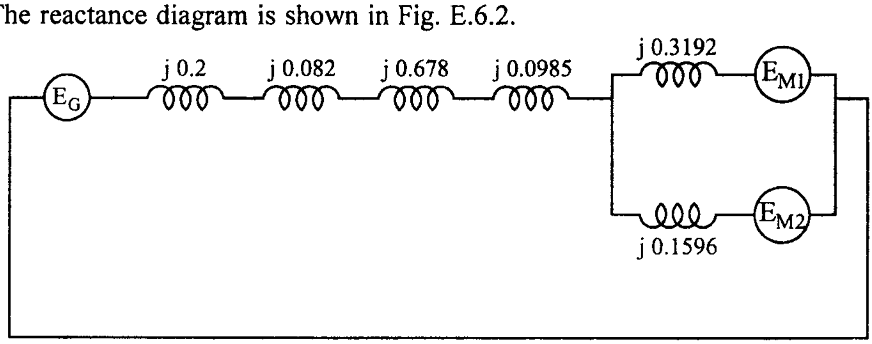


Fig. E.6.2

E.6.3 Obtain the per unit representation for the three-phase power system shown in Fig. E.6.3.

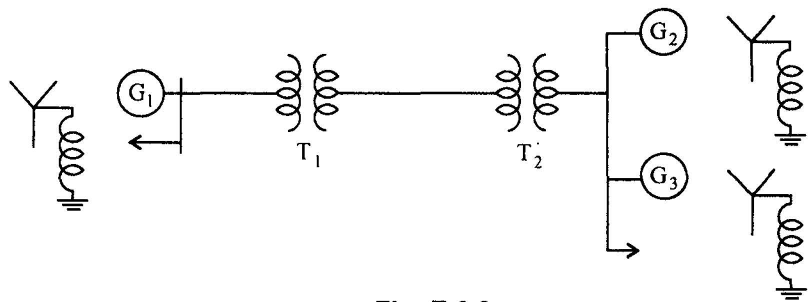


Fig. E.6.3

Generator 1 : 50 MVA, 10.5 KV; X = 1.8 ohm

Generator 2 : 25 MVA, 6.6 KV; X = 1.2 ohm

Generator 3 : 35 MVA, 6.6 KV; X = 0.6 ohm

Transformer T_1 : 30 MVA, 11/66 KV, X = 15 ohm/phase

Transformer T_2 : 25 MVA, 66/6.2 KV, as h.v. side X = 12 ohms

Transmission line : X_L = 20 ohm/phase

Solution :

Let base MVA = 50

base KV = 66 (L – L)

Base voltage on transmission as line 1 p.u. (66 KV)

Base voltage for generator 1 : 11 KV

Base voltage for generators 2 and 3 : 6.2 KV

$$\text{p.u. reactance of transmission line} = \frac{20 \times 50}{66^2} = 0.229 \text{ p.u.}$$

$$\text{p.u. reactance of transformer } T_1 = \frac{15 \times 50}{66^2} = 0.172 \text{ p.u.}$$

$$\text{p.u. reactance of transformer } T_2 = \frac{12 \times 50}{66^2} = 0.1377 \text{ p.u.}$$

$$\text{p.u. reactance of generator 1} = \frac{1.8 \times 50}{(11)^2} = 0.7438 \text{ p.u.}$$

$$\text{p.u. reactance of generator 2} = \frac{1.2 \times 50}{(6.2)^2} = 1.56 \text{ p.u.}$$

$$\text{p.u. reactance of generator 3} = \frac{0.6 \times 50}{(6.2)^2} = 0.78 \text{ p.u.}$$

E 6.4 A single phase two winding transformer is rated 20 KVA, 480/120 V at 50 HZ. The equivalent leakage impedance of the transformer referred to l.v. side is 0.0525 78.13° ohm using transformer ratings as base values, determine the per unit leakage impedance referred to the h.v. side and l.v. side.

Solution

Let base KVA = 20

Base voltage on h.v. side = 480 V

Base voltage on l.v. side = 120 V

The leakage impedance on the l.v. side of the transformer

$$= Z_{12} = \frac{V_{\text{base 2}}}{\text{VA base}} = \frac{(120)^2}{20{,}000} = 0.72 \text{ ohm}$$

p.u. leakage impedance referred to the l.v. of the transformer

$$= Z_{\text{p u 2}} = \frac{0.0525\ 78.13^{\circ}}{0.72} = 0.0729\quad 78.13^{\circ}$$

Equivalent impedance referred to h.v. side is

$$\left(\frac{400}{120}\right)^2\ [(0.0525\quad 70.13^{\circ}] = 0.84\quad 78.13^{\circ}$$

The base impedance on the h.v. side of the transformer is $\frac{(480)^2}{20{,}000} = 11.52$ ohm

p.u. leakage impedance referred to h.v. side

$$= \frac{0.84\ \ 78.13^{\circ}}{11.52} = 0.0729\quad 78.13^{\circ}\ \text{p.u.}$$

E.6.5 A single phase transformer is rated at 110/440 V, 3 KVA. Its leakage reactance measured on 110 V side is 0.05 ohm. Determine the leakage impedance referred to 440 V side.

Solution :

Base impedance on 110 V side = $\frac{(0.11)^2 \times 1000}{3}$ = 4.033 ohm

Per unit reactance on 110 V side = $\frac{0.05}{4.033}$ = 0.01239 p.u.

Leakage reactance referred to 440 V side = $(0.05)\left(\frac{440}{110}\right)^2$ = 0.8 ohm

Base impedance referred to 440 V side = $\frac{0.8}{64.53}$ = 0.01239 p.u.

E.6.6 Consider the system shown in Fig. E.6.4. Selecting 10,000 KVA and 110 KV as base values, find the p.u. impedance of the 200 ohm load referred to 110 KV side and 11 KV side.

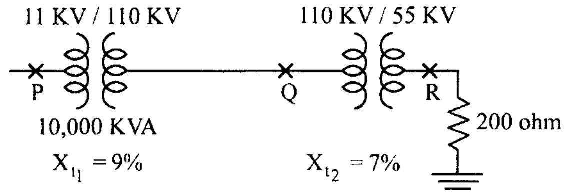


Fig. E.6.4

Solution :

Base voltage at ρ = 11 KV

$$\text{Base voltage at R} = \frac{110}{2} = 55 \text{ KV}$$

$$\text{Base impedance at R} = \frac{55^2 \times 1000}{10{,}000} = 302.5 \text{ ohm}$$

$$\text{p.u. impedance at R} = \frac{200 \text{ ohm}}{302.5 \text{ ohm}} = 0.661 \text{ ohm}$$

$$\text{Base impedance at } \phi = \frac{110^2 \times 1000}{10{,}000} = 1210 \text{ ohm}$$

Load impedance referred to $\phi = 200 \times 2^2 = 800$ ohm

$$\text{p.u. impedance of load referred to } \phi = \frac{800}{1210} = 0.661$$

$$\text{Similarly base impedance at P} = \frac{11^2 \times 1000}{10{,}000} = 121.1 \text{ ohm}$$

Impedance of load referred to P = $200 \times 2^2 \times 0.1^2 = 8$ ohm

$$\text{p.u. impedance of load at P} = \frac{8}{12.1} = 0.661 \text{ ohm}$$

E.6.7 Three transformers each rated 30 MVA at 38.1/3.81 KV are connected in star-delta with a balanced load of three 0.5 ohm, star connected resistors. Selecting a base of 900 MVA 66 KV for the h.v. side of the transformer find the base values for the l.v. side.

Solution .

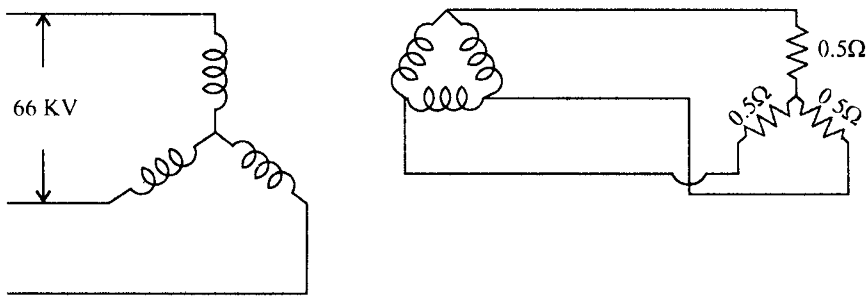


Fig. E.6.5

$$\text{Base impedance on l.v. side} = \frac{(\text{base KV}_{L-L})^2}{\text{Base MVA}} = \frac{(3.81)^2}{90} = 0.1613 \text{ ohm}$$

$$\text{p.u. load resistance on l.v. side} = \frac{0.5}{0.1613} = 3.099 \text{ p.u.}$$

$$\text{Base impedance on h.v. side} = \frac{(66)^2}{90} = 48.4 \text{ ohm}$$

$$\text{Load resistance referred to h.v. side} = 0.5 \times \left(\frac{66}{3.81}\right)^2 = 150 \text{ ohm}$$

$$\text{p.u. load resistance referred to h.v. side} = \frac{150}{48.4} = 3.099 \text{ p.u.}$$

The per unit load resistance remains the same.

E.6.8 Two generators are connected in parallel to the l.v. side of a 3-phase delta-star transformer as shown in Fig. E.6.6. Generator 1 is rated 60,000 KVA, 11 KV. Generator 2 is rated 30,000 KVA, 11 KV. Each generator has a subtransient reactance of x''_d = 25%. The transformer is rated 90,000 KVA at 11 KV Δ / 66 KV γ with a reactance of 10%. Before a fault occurred the voltage on the h.t. side of the transformer is 63 KV. The transformer in unloaded and there is no circulating current between the generators. Find the subtransient current in each generator when a 3-phase short circuit occurrs on the h.t. side of the transformer.

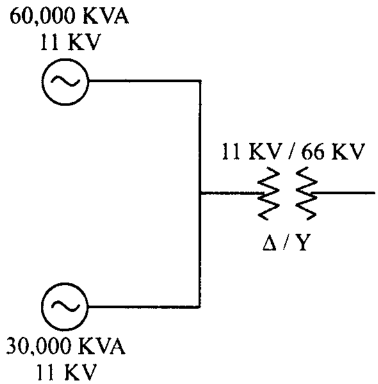


Fig. E.6.6

Solution :

Let the line voltage on the h.v. side be the base KV = 66 KV.

Let the base KVA = 90,000 KVA

Generator 1 : $x''_d = 0.25 \times \dfrac{90,000}{60,000} = 0.375$ p.u.

For generator 2 : $x''_d = \dfrac{90,000}{30,000} = 0.75$ p.u.

The internal voltage for generator 1

$$E_{g1} = \frac{0.63}{0.66} = 0.955 \text{ p.u.}$$

The internal voltage for generator 2

$$E_{g2} = \frac{0.63}{0.66} = 0.955 \text{ p.u.}$$

The reactance diagram is shown in Fig. E.6.7 when switch S is closed, the fault condition is simulated. As there is no circulating current between the generators, the equivalent reactance of the parallel circuit is $\dfrac{0.375 \times 0.75}{0.375 + 0.75} = 0.25$ p.u.

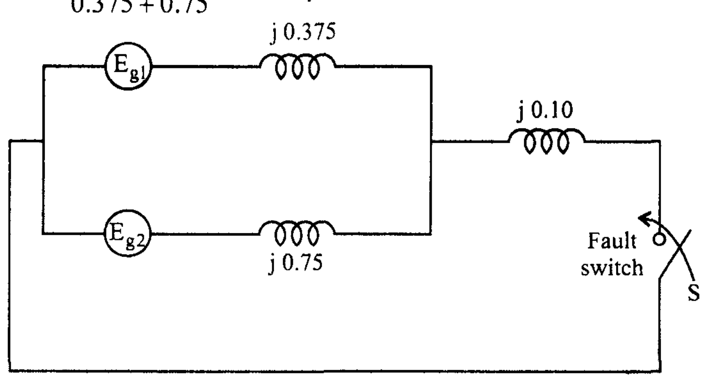


Fig. E.6.7

The subtransient current $I'' = \dfrac{0.955}{(j0.25 + j0.10)} = j\ 2.7285$ p.u.

The voltage as the delta side of the transformer is (–j 2.7285) (j 0.10) = 0.27205 p.u.

I_1'' = the subtransient current flowing into fault from generator

$$I_1'' = \frac{0.955 - 0.2785}{j\ 0.375} = 1.819 \text{ p.u.}$$

Similarly, $$I_2'' = \frac{0.955 - 0.27285}{j\ 0.75} = -j\ 1.819 \text{ p.u.}$$

The actual fault currents supplied in amperes are

$$I_1'' = \frac{1.819 \times 90{,}000}{\sqrt{3} \times 11} = 8592.78 \text{ A}$$

$$I_2'' = \frac{0.909 \times 90{,}000}{\sqrt{3} \times 11} = 4294.37 \text{ A}$$

E.6.9 R station with two generators feeds through transformers a transmission system operating at 132 KV. The far end of the transmission system consisting of 200 km long double circuit line is connected to load from bus B. If a 3-phase fault occurs at bus B, determine the total fault current and fault current supplied by each generator.

Select 75 MVA and 11 KV on LV side and 132 KV on h.v. side as base values.

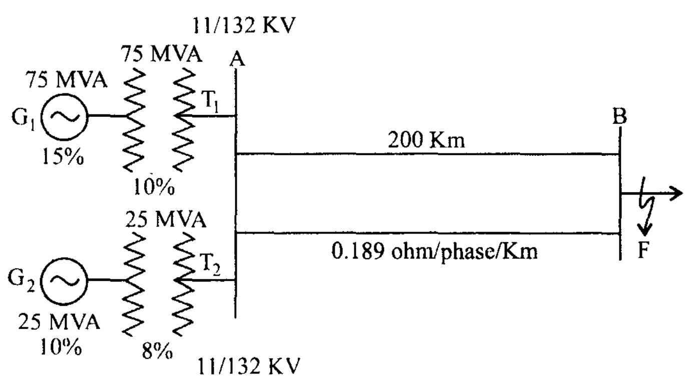


Fig. E.6.8

Solution :

p.u. x of generator 1 = j 0.15 p.u.

$$\text{p.u. x of generator 2} = j = 0.10\ \frac{75}{25}$$

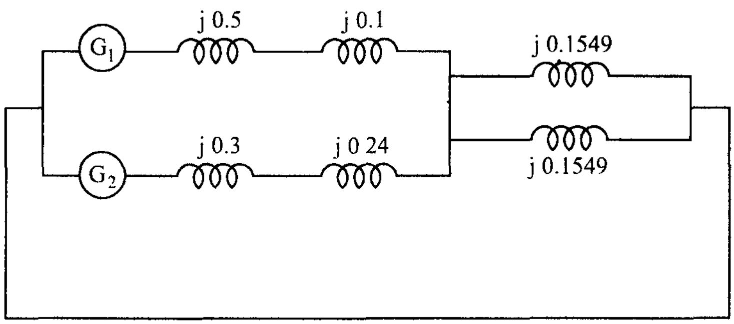
$$= j\ 0.3 \text{ p.u.}$$

p.u. x of transformer $T_1 = j\ 0.1$

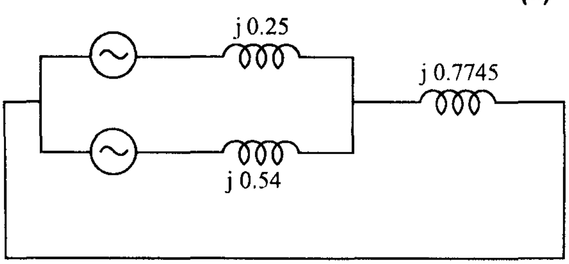
$$\text{p.u. x of transformer } T_2 = j\ 0.08 \times \frac{75}{25} = j\ 0.24$$

$$\text{p.u. x of each line} = \frac{j\ 0.180 \times 200 \times 75}{132 \times 132} = j\ 0.1549$$

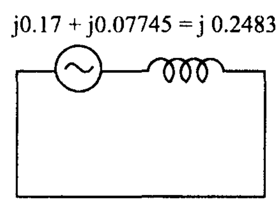
The equivalent reactance diagram is shown in Fig. E.6.9 (a), (b) & (c).



(a)



(b)



(c)

Fig. E.6.9

Fig. E.6.9 (a), (b) & (c) can be reduced further into

$$Z_{eq} = j\ 0.17 + j\ 0.07745 = j\ 0.248336$$

$$\text{Total fault current } \frac{1\ \angle 0^{\circ}}{j\ 0.248336} = -j\ 4.0268 \text{ p.u.}$$

$$\text{Base current for 132 KV circuit} = \frac{75 \times 1000}{\sqrt{3} \times 132} = 328 \text{ A}$$

Hence actual fault current = – j 4.0268 × 328 = 1321 A $\angle$–90°

Base current for 11 KV side of the transformer = $\dfrac{75 \times 1000}{\sqrt{3} \times 11}$ = 3936.6 A

Actual fault current supplied from 11 KV side = 3936.6 × 4.0248 = 15851.9 A $\angle$–90°

Fault current supplied by generator 1 = $\dfrac{1585139 \angle -90^\circ \times j\,0.54}{j\,0.54 + j\,0.25}$ = –j 10835.476 A

Fault current supplied by generator 2 = $\dfrac{15851.9 \times j\,0.25}{j\,0.79}$ = 5016.424 A $\angle$–90°

E.6.10 A 33 KV line has a resistance of 4 ohm and reactance of 16 ohm respectively. The line is connected to a generating station bus bars through a 6000 KVA step-up transformer which has a reactance of 6%. The station has two generators rated 10,000 KVA with 10% reactance and 5000 KVA with 5% reactance. Calculate the fault current and short circuit KVA when a 3-phase fault occurs at the h.v. terminals of the transformers and at the load end of the line.

Solution :

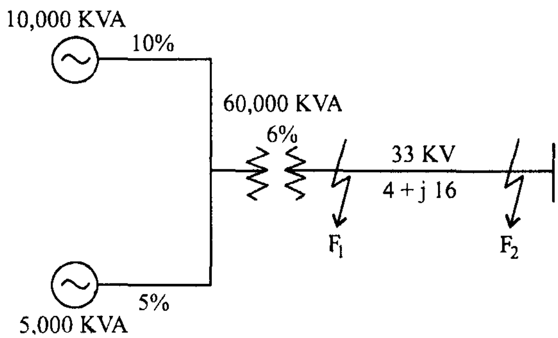


Fig. E.6.10 (a)

Let 10,000 KVA be the base KVA

Reactance of generator 1 X_{G1} = 10%

Reactance of generator 2 $X_{G2} = \dfrac{5 \times 10,000}{5000}$ = 10%

Reactance of transformer $X_T = \dfrac{6 \times 10,000}{6,000}$ = 10%

The line impedance is converted into percentage impedance

$$\% X = \frac{KVA\ .\ X}{10(KV)^2}\ ; \quad \% X_{Line} = \frac{10,000 \times 16}{10 \times (33)^2} = 14.69\%$$

$$\% R_{Line} = \frac{19000 \times 4}{10(33)^2} = 3.672\ \%$$

(i) For a 3-phase fault at the h.v. side terminals of the transformer fault impedance

$$= \left(\frac{10 \times 10}{10 + 10}\right) + 10 = 15\ \%$$

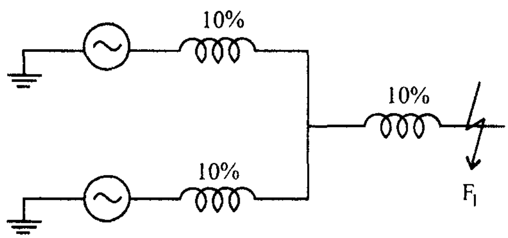


Fig. 6.10 (b)

$$\text{Short circuit KVA fed into the fault} = \frac{10,000 \times 100}{15}\ \text{KVA}$$

$$= 66666.67\ \text{KVA}$$

$$= 66.67\ \text{MVA}$$

For a fault at F_2 the load end of the line the total reactance to the fault

$$= 15 + 14.69$$

$$= 29.69\ \%$$

Total resistance to fault = 3.672 %

$$\text{Total impedance to fault} = \sqrt{3.672^2 + 29.69^2}$$

$$= 29.916\ \%$$

$$\text{Short circuit KVA into fault} = \frac{100}{29.916} \times 10,000$$

$$= 33433.63\ \text{KVA}$$

$$= 33.433\ \text{MVA}$$

E.6.11 Figure E.6.11 (a) shows a power system where load at bus 5 is fed by generators at bus 1 and bus 4. The generators are rated at 100 MVA; 11 KV with subtransient reactance of 25%. The transformers are rated each at 100 MVA, 11/112 KV and have a leakage reactance of 8%. The lines have an inductance of 1 mH / phase / km. Line L_1 is 100 km long while lines L_2 and L_3 are each of 50 km in length. Find the fault current and MVA for a 3-phase fault at bus 5.

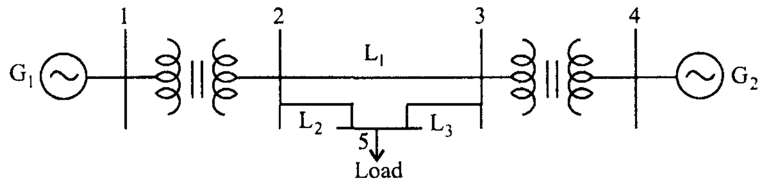


Fig. E.6.11 (a)

Solution :

Let base MVA = 100 MVA

Base voltage for l.v. side = 11 KV and

Base voltage for h.v. side = 112 KV

Base impedance for h.v. side of transformer

$$= \frac{112 \times 112}{100} = 125.44 \text{ ohm}$$

Base impedance for l.v. side of transformer

$$= \frac{11 \times 11}{100} = 1.21 \text{ ohm}$$

Reactance of line $L_1 = 2 \times p \times 50 \times 1 \times 10^{-3} \times 100 = 31.4$ ohm

Per unit reactance of line $L_1 = \dfrac{31.4}{125.44} = 0.25$ p.u.

p.u. impedance of line $L_2 = \dfrac{2\pi \times 50 \times 1 \times 10^{-3} \times 50}{125.44} = 0.125$ p.u.

p.u. impedance of line $L_3 = 0.125$ p.u.

The reactance diagram is shown in Fig. 6.11 (b).

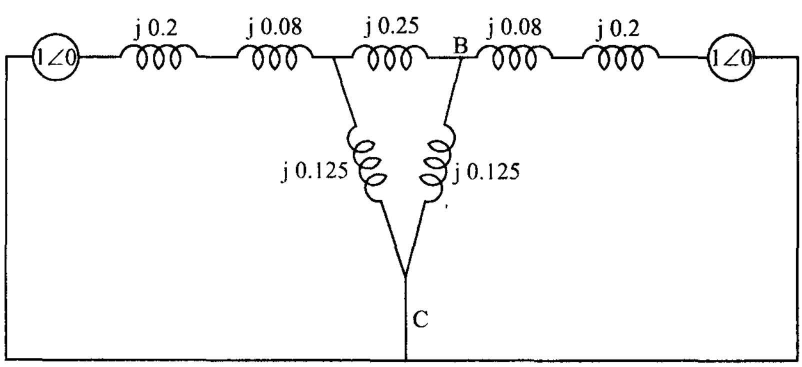


Fig. E.6.11 (b)

By performing conversion of delta into star at A, B and C, the star impedances are

$$Z_1 = \frac{j\,0.25 \times j\,0.125}{j\,0.25 + j\,0.125 + j\,0.125} = j\,0.0625$$

$$Z_2 = \frac{j\,0.25 \times j\,0.125}{j\,0.5} = j\,0.0625$$

and $$Z_3 = \frac{j\,0.125 \times j\,0.125}{j\,0.5} = j\,0.03125$$

The following reactance diagram is obtained.

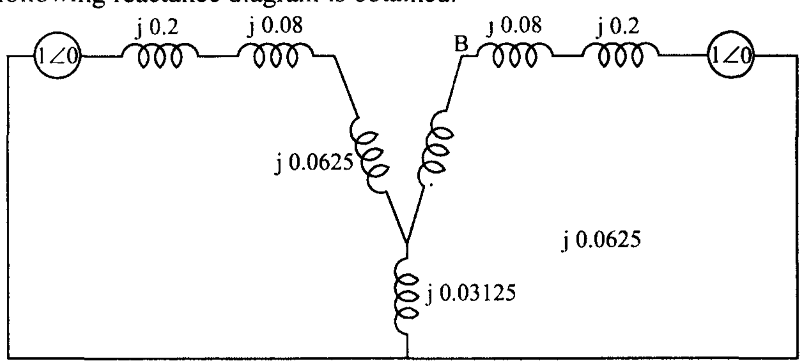


Fig. E.6.11 (c)

This can be further reduced into Fig. E.6.11 (d).

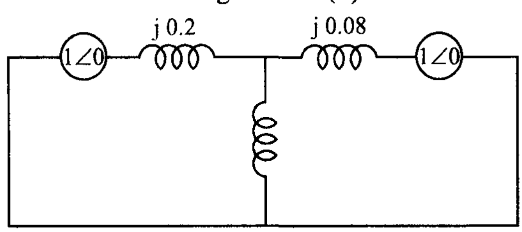


Fig. E.6.11 (d)

Finally this can be put first into Fig. E.6.11 (e) and later into Fig. E.6.11 (f).

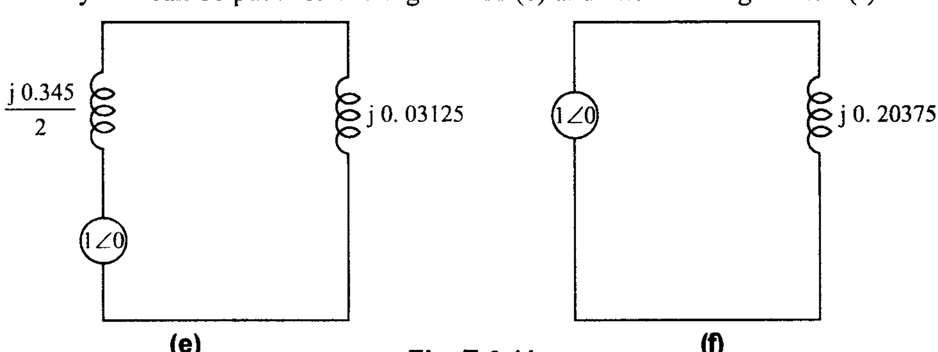


Fig. E.6.11

$$\text{Fault MVA} = \frac{1}{0.20375} = 4.90797 \text{ p.u.}$$

$$= 100 \text{ MVA} \times 4.90797 = 490.797 \text{ MVA}$$

$$\text{Fault current} = \frac{1}{j\,0.20375} = 4.90797 \text{ p.u.}$$

$$\text{Base current} = \frac{100 \times 10^6}{\sqrt{3} \times 112 \times 10^3} = 515.5 \text{ Amp}$$

$$\text{Fault current} = 4.90797 \times 515.5$$

$$= 2530 \text{ Amp}$$

E.6.12 Two motors having transient reactances 0.3 p.u. and subtransient reactances 0.2 p.u. based on their own ratings of 6 MVA, 6.8 KV are supplied by a transformer rated 15 MVA, 112 KV / 6.6 KV and its reactance is 0.18 p.u. A 3-phase short circuit occurs at the terminals of one of the motors. Calculate (a) the subtransient fault current (b) subtransient current in circuit breaker A (c) the momentary circuit rating of the breaker and (d) if the circuit breaker has a breaking time of 4 cycles calculate the current to be interrupted by the circuit breaker A.

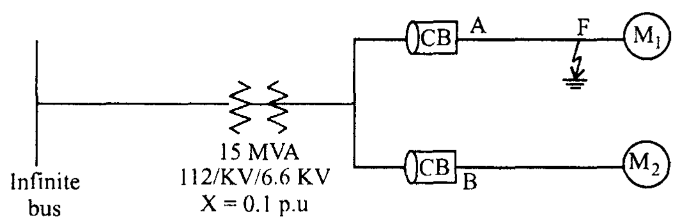


Fig. E.6.12 (a)

Solution :

Let base MVA = 15

Base KV for l.v side = 6.6 KV

Base KV for h.v side = 112 KV

For each motor $x_d'' = 0.2 \times \frac{15}{6} = 0.5$ p.u.

For each motor $x_d'' = 0.3 \times \frac{15}{6} = 0.75$ p.u.

The reactance diagram is shown in Fig. E.6.12 (b).

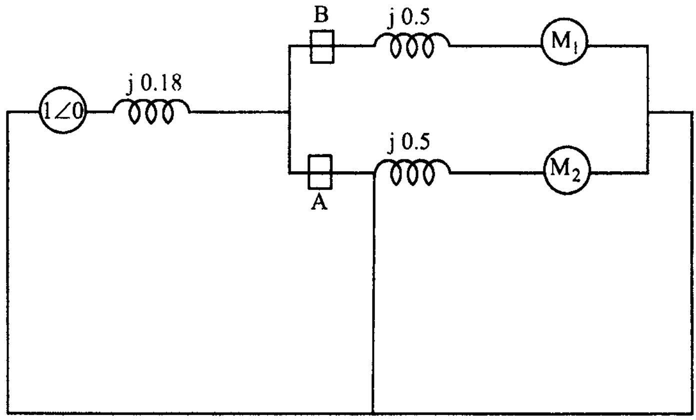


Fig. E.6.12 (b)

Under fault condition the reactance diagram can be further simplified into Fig. E.6.12 (c).

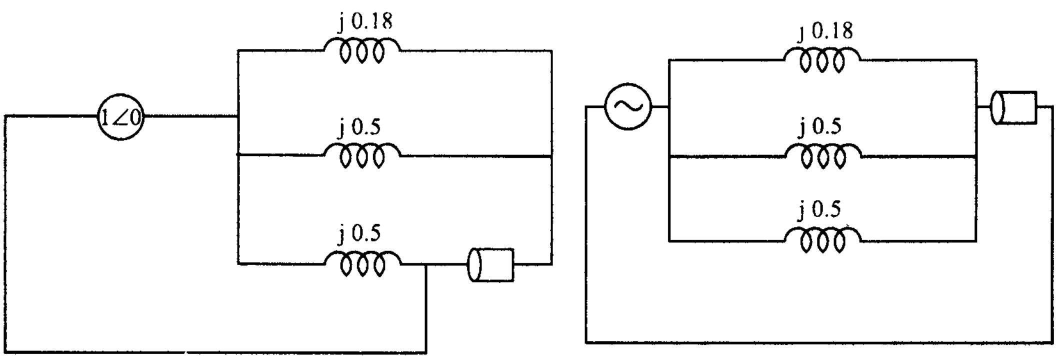


Fig. E.6.12 (c)

$$\text{Impedance to fault} = \frac{1}{\dfrac{1}{j0.18} + \dfrac{1}{j0.5} + \dfrac{1}{j0.5}}$$

$$\text{Subtransient fault current} = \frac{1\angle 0^\circ}{j\,0.1047} = -j\ 9.55 \text{ p.u.}$$

$$\text{Base current} = \frac{15 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 1312.19 \text{ A}$$

Subtransient fault current = 1312.19 × (–j 9.55)

= 12531.99 Amps (lagging)

(b) Total fault current from the infinite bus.

$$\frac{-1\angle 0^\circ}{j0.18} = -j\ 5.55 \text{ p.u.}$$

Fault current from each motor = $\frac{1 \angle 0^{\circ}}{j0.5}$ = –j 2p.u.

Fault current into breaken A is sum of the two currents from the in infinite bus and from motor 1

$$= -j\ 5.55 + (-j\ 2) = -j\ 7.55 \text{ p.u.}$$

Total fault current into breaken = –j 7.55 × 1312.19

$$= 9907 \text{ Amps}$$

(c) Manentary fault current taking into the d.c.

off-set component is approximately

$$1.6 \times 9907 = 15851.25 \text{ A}$$

(d) For the transient condition, that is, after 4 cycles the motor reactance changes to 0.3 p.u.

The reactance diagram for the transient state is shown in Fig. E.6.12 (d).

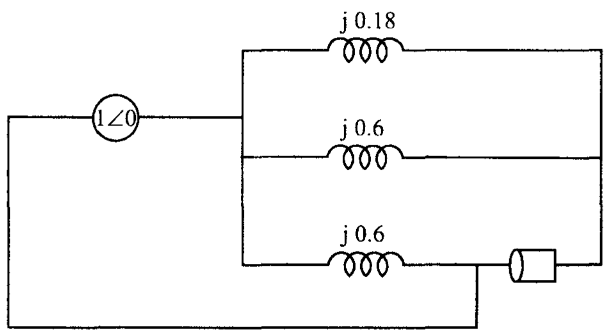


Fig. E.6.12 (d)

The fault impedance is $\dfrac{1}{\dfrac{1}{j0.15}+\dfrac{1}{j0.6}+\dfrac{1}{j0.6}}$ = j 0.1125 p.u.

The fault current = $\frac{1 \angle 0^{\circ}}{j0.1125}$ = j 8.89 p.u.

Transient fault current = –j 8.89 × 1312.19

$$= 11665.37 \text{ A}$$

If the d.c. offset current is to be considered it may be increased by a factor of say 1.1.

So that the transient fault current = 11665.37 × 1.1

$$= 12831.9 \text{ Amp.}$$

E.6.13 Consider the power system shown in Fig. E.6.13 (a).

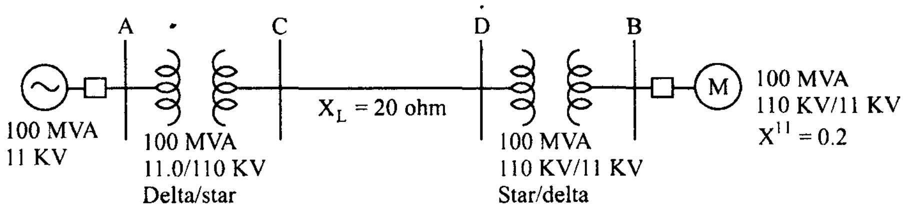


Fig. E.6.13 (a)

The synchronous generator is operating at its rated MVA at 0.95 lagging power factor and at rated voltage. A 3-phase short circuit occurs at bus A calculate the per unit value of (i) subtransient fault current (ii) subtransient generator and motor currents. Neglect prefault current. Also compute (iii) the subtransient generator and motor currents including the effect of prefault currents.

$$\text{Base line impedance} = \frac{(110)^2}{100} = 121 \text{ ohm}$$

$$\text{Line reactance in per unit} = \frac{20}{121} = 0.1652 \text{ p.u.}$$

The reactance diagram including the effect of the fault by switch S is shown in Fig. E.6.13 (b).

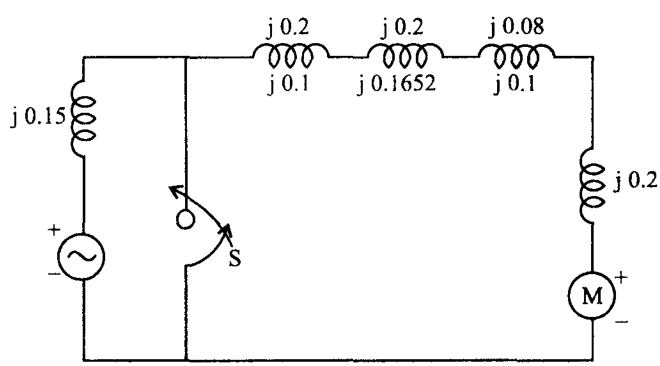


Fig. E.6.13 (b)

Looking into the network from the fault using Thevenin's theorem $Z_{th} = j\ X_{th} =$

$$j\left(\frac{0.15 \times 0.565}{0.15 + 0.565}\right) = j\ 0.1185$$

(i) The subtransient fault current

$$I''_m = \frac{0.565}{0.565+0.15} I''_f = \frac{0.565 \times j8.4388}{0.7125} = j\ 6.668$$

(ii) The motor subtransient current

$$I''_m = \frac{0.15}{0.715} I''_f = \frac{0.15}{0.715} \times 8.4388 = j\ 1.770 \text{ p.u.}$$

(iii) Generator base current $= \dfrac{100\text{MVA}}{\sqrt{3} \times 11\text{KV}} = 5.248$ KA

Generator prefault current $= \dfrac{100}{\sqrt{3} \times 11}\ [\cos^{-1} 0.95]$

$$= 5.248\ \angle -18^{o}.19 \text{ KA}$$

$$I_{load} = \frac{5.248\ \angle -18^{o}.19}{5.248} = 1\ \angle -18^{o}.19$$

$$= (0.95 - j\ 0.311) \text{ p.u.}$$

The subtransient generator and motor currents including the prefault currents are

$$I''_g = j\ 6.668 + 0.95 - j\ 0.311 = -j\ 6.981 + 0.95$$

$$= (0.95 - j\ 6.981) \text{ p.u.} = 7.045\ -82.250 \text{ p.u.}$$

$$I''_m = -j\ 1.77 - 0.95 + j0.311 = -0.95 - j\ 1.459$$

$$= 1.74\ \angle -56.93^{o}$$

E.6.14 Consider the system shown in Fig. E.6.14 (a). The percentage reactance of each alternator is expressed on its own capacity determine the short circuit current that will flow into a dead three phase short circuit at F.

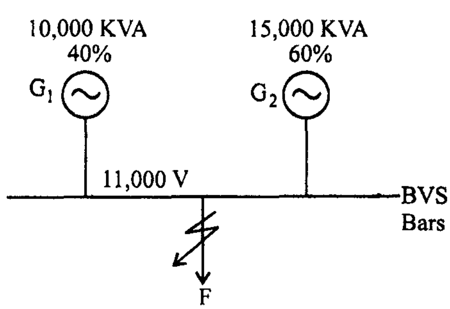


Fig. E.6.14 (a)

Solution :

Let base KVA = 25,000 and base KV = 11

$$\% \text{ X of generator 1} = \frac{25,000}{10,000} \times 40 = 100\%$$

$$\% \text{ X of generator 2} = \frac{25,000}{15,000} \times 60 = 100\%$$

$$\text{Line current at 25,000 KVA and 11 KV} = \frac{25,000}{\sqrt{3} \times 11} \times \frac{10^3}{10^3} = 1312.19 \text{ Amperes.}$$

The reactance diagram is shown in Fig. E.6.14 (b).

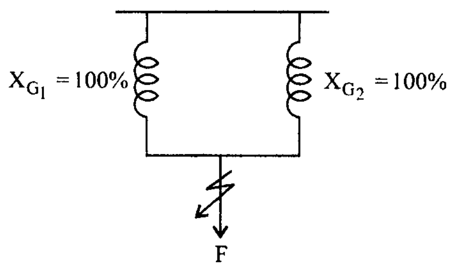


Fig. E.6.14 (b)

$$\text{The net percentage reactance upto the fault} = \frac{100 \times 100}{100 + 100} = 50\%$$

$$\text{Short circuit current} = \frac{I \times 100}{\%X} = \frac{1312.19 \times 100}{50} = 2624.30 \text{ A}$$

E.6.15 A-3-phase, 25 MVA, 11 KV alternator has internal reactance of 6%. Find the external reactance per phase to be connected in series with the alternator so that steady state short circuit current does not exceed six times the full load current.

Solution :

$$\text{Full load current} = \frac{25 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 1312.9 \text{ A}$$

$$V_{phase} = \frac{11 \times 10^3}{\sqrt{3}} = 6351.039 \text{ volts.}$$

$$\text{Total \% X} = \frac{\text{Full-load current}}{\text{Short circuit current}} \times 100 = \frac{1}{6} \times 100$$

$$= 16.67\ \%$$

External reactance needed = 16.67 – 6 = 10.67 %

Let X be the per phase external reactance required in ohms.

$$\% X = \frac{IX}{V} \times 100$$

$$10.67 = \frac{1312.19X \,.\, 100}{6351.0393}$$

$$X = \frac{6351.0393 \times 10.67}{1312.19 \times 100} = 0.516428 \text{ ohm}$$

E.6.16 A 3-phase line operating at 11 KV and having a resistance of 1.5 ohm and reactance of 6 ohm is connected to a generating station bus bars through a 5 MVA step-up transformer having reactance of 5%. The bus bars are supplied by a 12 MVA generator having 25% reactance. Calculate the short circuit KVA fed into a symmetric fault (i) at the load end of the transformer and (ii) at the h.v. terminals of the transformer.

Solution :

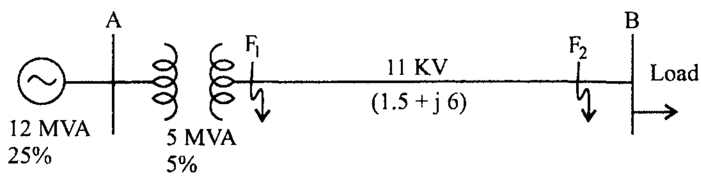


Fig. E.6.15

Let the base KVA = 12,000 KVA

%X of alternator as base KVA = 25%

$$\%X \text{ of transformer as } 12{,}000 \text{ KVA base} = \frac{12{,}000}{5{,}000} \times 5 = 12\%$$

$$\%X \text{ of line} = \frac{12{,}000}{10(11)^2} \times 6 = 59.5\%$$

$$\%R \text{ of line} = \frac{12{,}000}{10(11)^2} \times 1.5 = 14.876\ \%$$

(i) $\%X_{Total} = 25 + 12 + 59.5 = 96.5\%$

$\%R_{Total} = 14.876\%$

$$\%Z_{Total} = \sqrt{(96.5)^2 + (14.876)^2} = 97.6398\%$$

Short circuit KVA at the far end or load end $F_2 = \dfrac{12{,}000 \times 100}{97.6398} = 12290$

If the fault occurs on the h.v. side of the transformer at F_1

% X upto fault $F_1 = \%\ X_G + \%\ X_T = 25 + 12$

$= 37\ \%$

Short circuit KVA fed into the fault

$$= \frac{12{,}000 \times 100}{37} = 32432.43$$

E.6.17 A 3-phase generating station has two 15,000 KVA generators connected in parallel each with 15% reactance and a third generator of 10,000 KVA with 20% reactance is also added later in parallel with them. Load is taken as shown from the station bus-bars through 6000 KVA, 6% reactance transformers. Determine the maximum fault MVA which the circuit breakers have to interrupt on (i) l.v. side and (ii) as h.v. side of the system for a symmetrical fault.

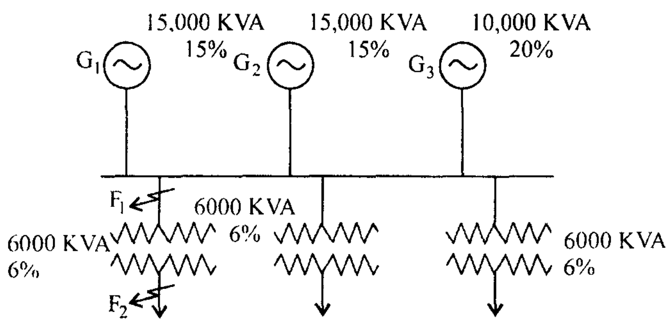


Fig. E.6.17 (a)

Solution ·

$$\%\ X \text{ of generator } G_1 = \frac{15 \times 15{,}000}{15{,}000} = 15\%$$

$$\%\ X \text{ of generator } G_2 = 15\%$$

$$\%\ X \text{ of generator } G_3 = \frac{20 \times 15000}{10{,}000} = 30\%$$

$$\%\ X \text{ of transformer } T = \frac{6 \times 15{,}000}{6000} = 15\%$$

(i) If fault occurs at F_1, the reactance is shown in Fig. E.6.17 (b).

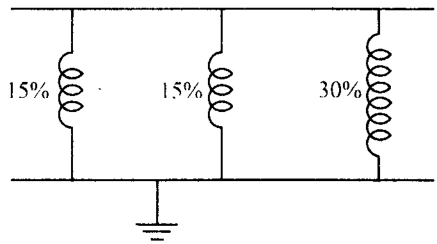


Fig. E.6.17 (b)

$$\text{The total \% C upto fault} = \frac{1}{\frac{1}{15}+\frac{1}{15}+\frac{1}{30}}$$

$$= 6\%$$

$$\text{Fault MVA} = \frac{15,000\times 100}{6} = 250,000 \text{ KVA}$$

$$= 250 \text{ MVA}$$

(ii) If the fault occurs at F_2, the reactance diagram will be as in Fig. E.6.17 (c).

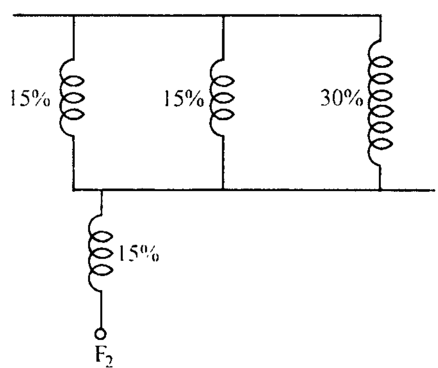


Fig. E.6.17 (c)

The total %X upto fault 6% + 15.6 = 21%

$$\text{Fault MVA} = \frac{15,000\times 100}{21\times 100} = 71.43$$

E.6.18 There are two generators at bus bar A each rated at 12,000 KVA, 12% reactance or another bus B, two more generators rated at 10,000 KVA with 10% reactance are connected. The two bus bars are connected through a reactor rated at 5000 KVA with 10% reactance. If a dead short circuit occurs between all the phases on bus bar B, what is the short circuit MVA fed into the fault ?

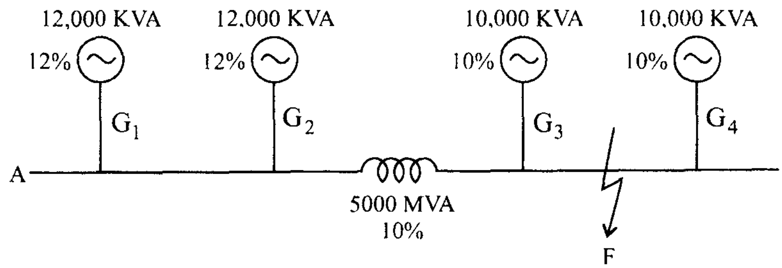


Fig. E.6.18 (a)

Solution

Let 12,000 KVA be the base KVA

% X of generator G_1 = 12 %

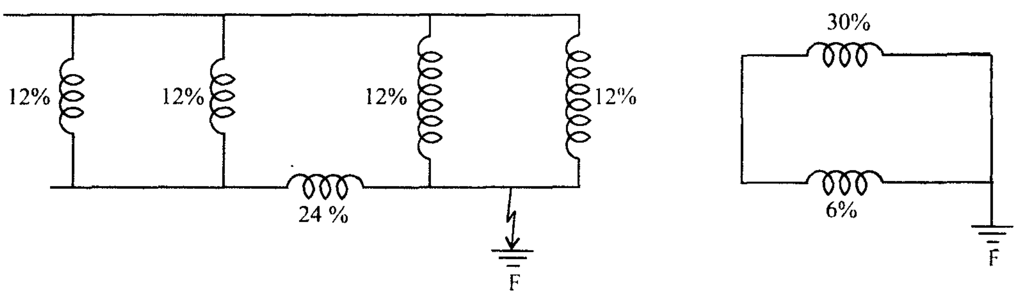
% X of generator G_2 = 12 %

$$\% \text{ X of generator } G_3 = \frac{10 \times 12000}{10,000} = 12\%$$

% X of generator G_4 = 12%

$$\% \text{ X of bus bar reactor} = \frac{10 \times 12000}{5,000} = 24\%$$

The reactance diagram is shown in Fig. E.6.18 (b).



(b) **(c)**

Fig. E.6.18

$$\% \text{ X up to fault} = \frac{30 \times 6}{30 + 6} = 50\%$$

$$\text{Fault KVA} = \frac{12,000 \times 100}{6} = 600,000 \text{ KVA}$$

$$= 600 \text{ MVA}$$

E.6.19 A power plant has two generating units rated 3500 KVA and 5000 KVA with percentage reactances 8% and 9% respectively. The circuit breakers have breaking capacity of 175 MVA. It is planned to extend the system by connecting it to the grid through a transformer rated at 7500 KVA and 7% reactance. Calculate the reactance needed for a reactor to be connected in the bus-bar section to prevent the circuit breaker from being over loaded if a short circuit occurs on any outgoing feeder connected to it. The bus bar voltage is 3.3 KV.

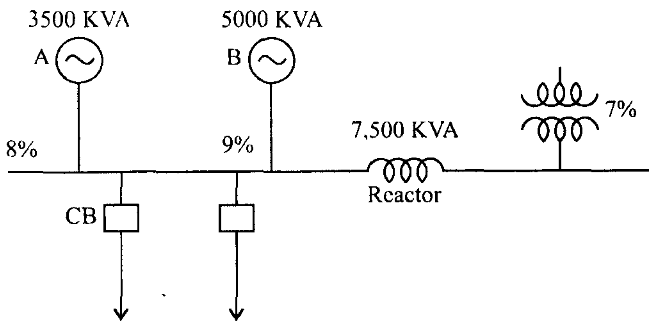


Fig. E.6.19 (a)

Solution

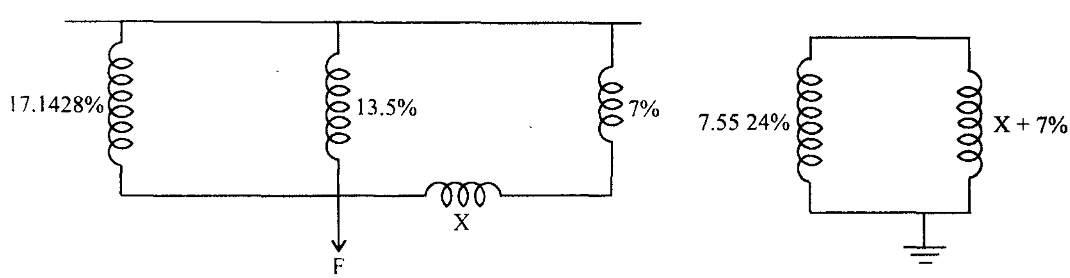
Let 7,500 KVA be the base KVA

$$\% \text{ X of generator A} = \frac{8 \times 7500}{3500} = 17.1428\%$$

$$\% \text{ X of generator B} = \frac{9 \times 7500}{5000} = 13.5\%$$

% X of transformer = 7% (as its own base)

The reactance diagram is shown in Fig. E.6.19 (b).



(b) **(c)**

Fig. E.6.19

$$\left[\text{Note}: \frac{1}{\left(\frac{1}{17.1428}+\frac{1}{13.5}\right)} = 7.5524\right]$$

The short circuit KVA should not exceed 175 MVA

$$\text{Total reactance to fault} = 1\Big/\left[\frac{1}{7.5524}+\frac{1}{X+7}\right]$$

$$= \frac{(X+7)\ (7.5524)}{X+7+7.5524}\% = \frac{(X+7)\ (7.5524)}{X+14.5524}\%$$

$$\text{Short circuit KVA} = 7500 \times 100\ \frac{X(X+14.5524)}{(X+7)(7.5524)}$$

This should not exceed 175 MVA

$$175 \times 10^3 = \frac{7500 \times 100(X+14.5524)}{(X+7)(7.5524)}$$

Solving $\quad X = 7.02\%$

Again $\quad \% X = \dfrac{KVA\ .\ (X)}{10(KV)^2} = \dfrac{7500 \times (X)}{10 \times (3.3)^2}$

$\therefore \quad X = \dfrac{7.02 \times 10 \times 3.3^2}{7500}$ = **0.102 ohm**

In each share of the bus bar a reactance of 0.102 ohm is required to be inserted.

E.6.20 The short circuit MVA at the bus bars for a power plant A is 1200 MVA and for another plant B is 1000 MVA at 33 KV. If these two are to be interconnected by a tie-line with reactance 1.2 ohm. Determine the possible short circuit MVA at both the plants.

Solution :

Let base MVA = 100

$$\% X \text{ of plant } 1 = \frac{\text{base MVA}}{\text{short circuit MVA}} \times 100$$

$$= \frac{100}{1200} \times 100 = 8.33\%$$

$$\% \text{ X of plan 2} = \frac{100}{1000} \times 100 = 10\%$$

% X of interconnecting tie line on base MVA

$$= \frac{100 \times 10^3}{10 \times (3.3)^2} \times 1.2 = 11.019 \ \%$$

For fault at bus bars for generator A

$$\% \text{ X} = 1 \Big/ \left[\frac{1}{8.33} + \frac{1}{21.019}\right]$$

$$= 5.9657 \ \%$$

$$\text{Short circuit MVA} = \frac{\text{base MVA} \times 100}{\%X}$$

$$= \frac{100 \times 100}{5.96576} = \mathbf{1676.23}$$

For a fault at the bus bars for plant B

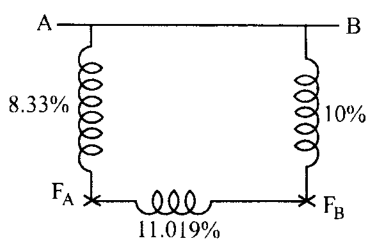


Fig. E.6.20

$$\% \text{ X} = 1 \Big/ \left[\frac{1}{19.349} + \frac{1}{10}\right] = 6.59 \ \%$$

$$\text{Short circuit MVA} = \frac{100 \times 100}{6.59} = \mathbf{1517.45}$$

E.6.21 A power plant has three generating units each rated at 7500 KVA with 15% reactance. The plant is protected by a tie-bar system. With reactances rated at 7500 MVA and 6%, determine the fault KVA when a short circuit occurs on one of the sections of bus bars. If the reactors were not present what would be the fault KVA.

Solution :

The equivalent reactance diagram is shown in Fig. E.6.21 (a) which reduces to Fig. (b) & (c).

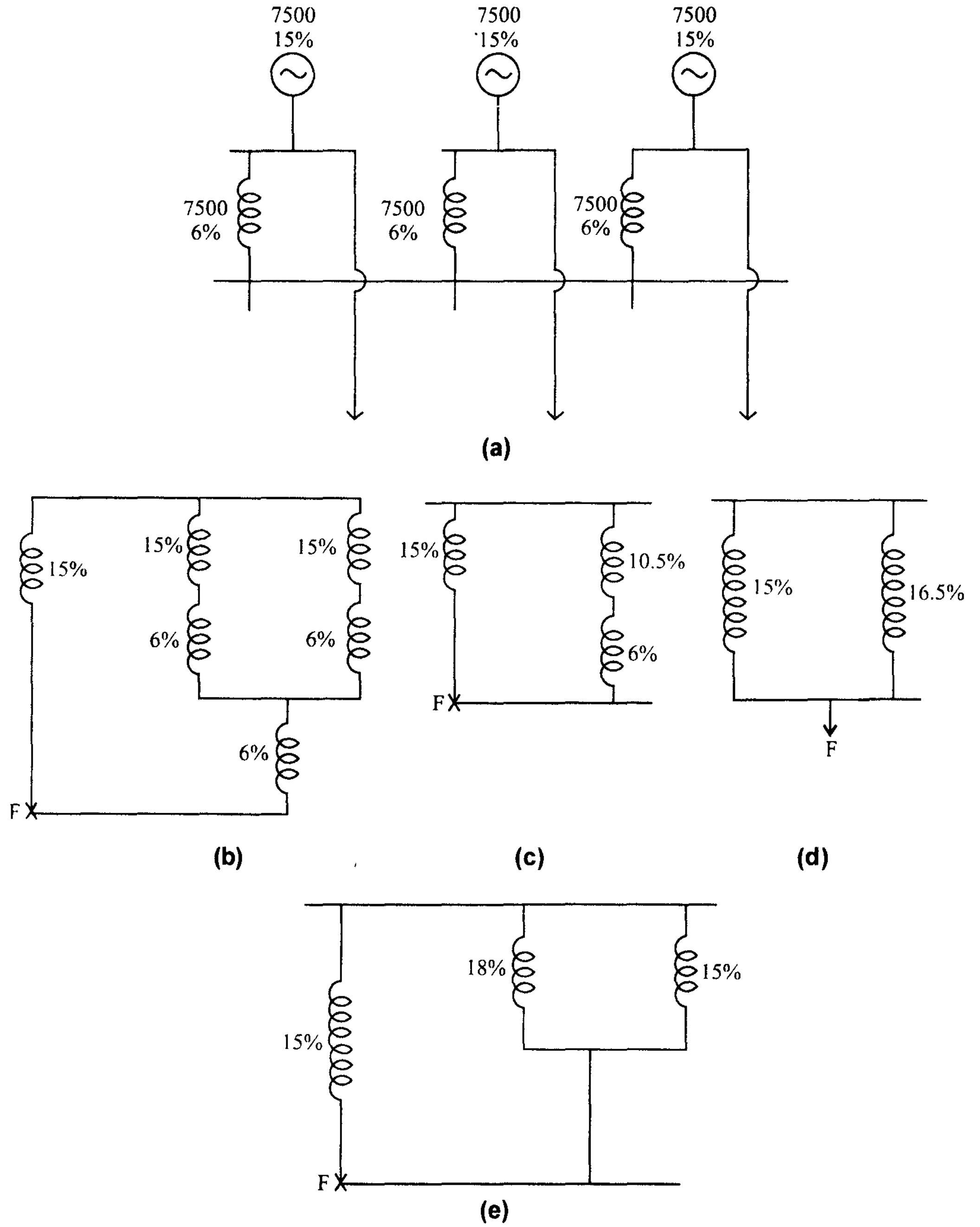


Fig. E.6.21

The total % X up to fault F = $\dfrac{15 \times 16.5}{15+16.5}$ = 7.857 %

The short circuit KVA = $\dfrac{7500 \times 100}{7.857}$ = 95456.28 KVA = 95.46 MVA

Without reactors the reactance diagram will be as shown.

The total % X up to fault F = $\dfrac{15 \times 7.5}{15+7.5}$ = 5 %

$$\text{Short circuit MVA} = \frac{7500 \times 100}{5}$$

$$= 150{,}000 \text{ KVA}$$

$$= 150 \text{ MVA}$$

Problems

P.6.1 There are two generating stations each which an estimated short circuit KVA of 500,000 KVA and 600,000 KVA. Power is generated at 11 KV. If these two stations are interconnected through a reactor with a reactance of 0.4 ohm, what will be the short circuit KVA at each station ?

P.6.2 Two generators P and Q each of 6000 KVA capacity and reactance 8.5% are connected to a bus bar at A. A third generator R of capacity 12,000 KVA with 11% reactance is connected to another bus bar B. A reactor X of capacity 5000 KVA and 5% reactance is connected between A and B. Calculate the short circuit KVA supplied by each generator when a fault occurs (a) at A and (b) at B.

P.6.3 The bus bars in a generating station are divided into three section. Each section is connected to a tie-bar by a similar reactor. Each section is supplied by a 25,000 KVA, 11 KV, 50 Hz, three phase generator. Each generator has a short circuit reactance of 18%. When a short circuit occurs between the phases of one of the section bus-bars, the voltage on the remaining section falls to 65% of the normal value. Determine the reactance of each reactor in ohms.

Questions

6.1 Explain the importance of per-unit system.

6.2 What do you understand by short-circuit KVA ? Explain.

6.3 Explain the construction and operation of protective reactors.

6.4 How are reactors classified ? Explain the merits and demerits of different types of system protection using reactors.

7 UNBALANCED FAULT ANALYSIS

Three phase systems are accepted as the standard system for generation, transmission and utilization of the bulk of electric power generated world over. The above holds good even when some of the transmission lines are replaced by d-c links. When the three phase system becomes unbalanced while in operation, analysis becomes difficult. Dr. C.L. Fortesque proposed in 1918 at a meeting of the American Institute of Electrical Engineers through a paper titled "Method of Symmetrical Coordinates applied to the solution of polyphase Networks", a very useful method for analyzing unbalanced 3-phase networks.

Faults of various types such as line-to-ground, line-to-line, three-phase short circuits with different fault impedances etc create unbalances. Breaking down of line conductors is also another source for unbalances in Power Systems Operation. The symmetrical Coordinates proposed by Fortesque are known more commonly as symmetrical components or sequence components.

An unbalanced system of n phasors can be resolved into n systems of balanced phasors. These subsystems of balanced phasors are called symmetrical components. With reference to 3-phase systems the following balanced set of three components are identified and defined.

(a) Set of three phasors equal in magnitude, displaced from each other by 120^0 in phase and having the same phase sequence as the original phasors constitute positive sequence components. They are denoted by the suffix 1.

(b) Set of three phasors equal in magnitude, displaced from each other by 120^0 in phase, and having a phase sequence opposite to that of the original phasors constitute the negative sequence components. They are denoted by the suffix 2.

(c) Set of three phasors equal in magnitude and all in phase (with no mutual phase displacement) constitute zero sequence components. They are denoted by the suffix 0. Denoting the phases as R,Y and B V_R, V_Y and V_B are the unbalanced phase voltages. These voltages are expressed in terms of the sequence componeins V_{R1}, V_{y1}, V_{B1}, V_{R2}, V_{Y2}, V_{B2} and V_{R0}, V_{Y0}, V_{B0} as follows :-

$$V_R = V_{R1} + V_{R2} + V_{RO} \quad .....(7.1)$$

$$V_y = V_{y1} + V_{y2} + V_{yO} \quad .....(7.2)$$

$$V_B = V_{B1} + V_{B2} + V_{BO} \quad .....(7.3)$$

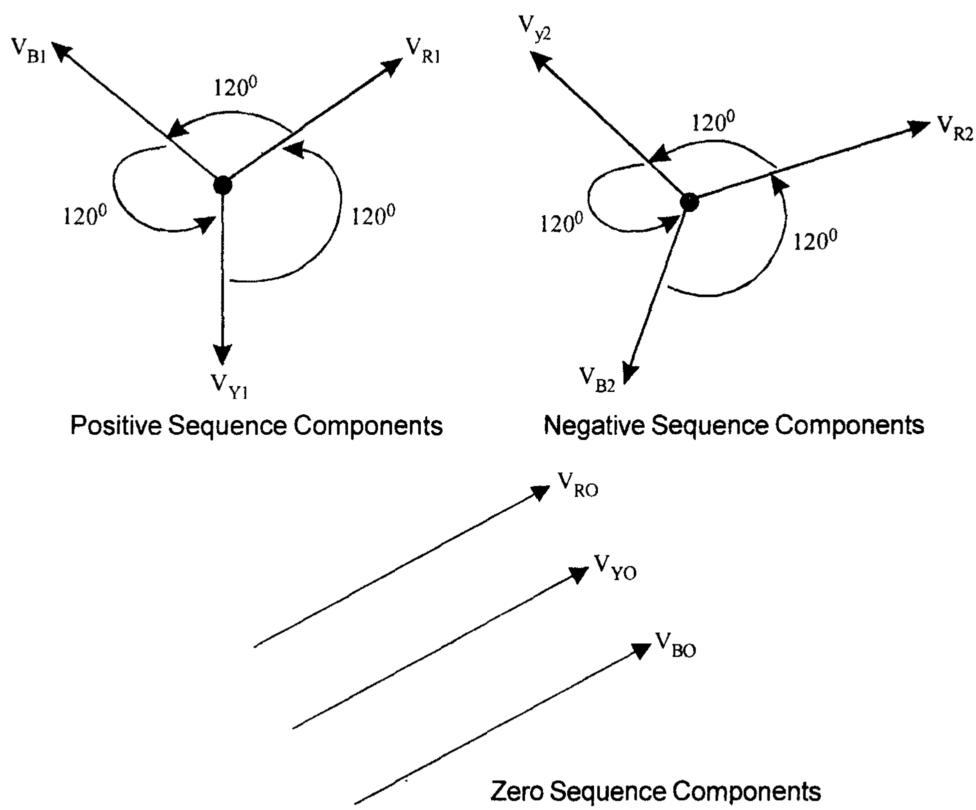


Fig. 7.1

7.1 The Operator "a"

In view of the phase displacement of 120^0, an operator "a" is used to indicate the phase displacement, just as j operator is used to denote 90^0 phase displacement.

$$a = 1\angle 120^0 = -0.5 + j0.866$$

$$a^2 = 1\angle 240^0 = -0.5 - j0.866$$

$$a^3 = 1\angle 360^0 = 1 + j0$$

so that $1 + a + a^2 = 0 + j0$

The operator is represented graphically as follows :

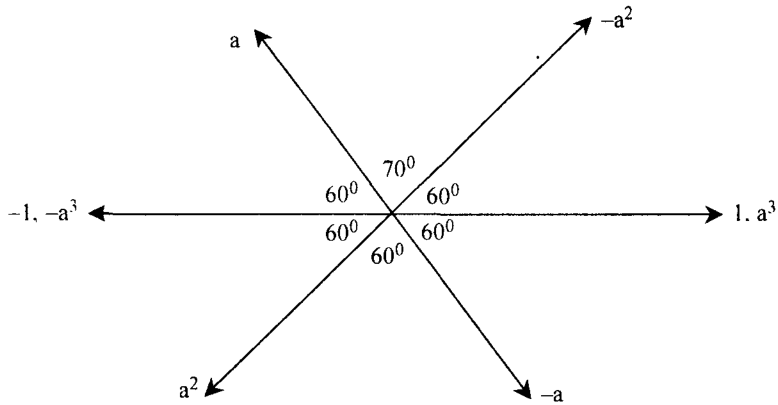


Fig. 7.2

Note that

$$a = 1\angle 120^0 = 1.e^{j\frac{2\pi}{3}}$$

$$a^2 = 1\angle 240^0 = 1.e^{j\frac{4\pi}{3}}$$

$$a^3 = 1\angle 360^0 = 1.e^{j\frac{6\pi}{3}} = 1.e^{j2\pi}$$

7.2 Symmetrical Components of Unsymmetrical Phases

With the introduction of the operator "a" it is possible to redefine the relationship between unbalanced phasors of voltages and currents in terms of the symmetrical components or sequence components as they are known otherwise. We can write the sequence phasors with the operator as follows.

$$\left.\begin{aligned} V_{R1} &= V_{R1} \\ V_{R2} &= V_{R2} \\ V_{R0} &= V_{R0} \\ V_{Y1} &= a^2 V_{R1} \\ V_{Y2} &= aV_{R2} \\ V_{Y0} &= V_{R0} \end{aligned}\right\} \quad \left.\begin{aligned} V_{B1} &= aV_{R1} \\ V_{B1} &= a^2 V_{R2} \\ V_{B0} &= V_{Ro} \end{aligned}\right\} \qquad .....(7.4)$$

The voltage and current phasors for a 3-phase unbalanced system are then represented by

$$\left.\begin{aligned} V_R &= V_{R1} + V_{R2} + V_{R0} \\ V_y &= a^2 V_{R1} + aV_{R2} + V_{R0} \\ V_B &= aV_{R1} + a^2 V_{R2} + V_{R0} \end{aligned}\right\} \quad .....(7.5)$$

$$\left.\begin{aligned} I_R &= I_{R1} + I_{R2} + I_{R0} \\ I_y &= a^2 I_{R1} + aI_{R2} + I_{R0} \\ I_B &= aI_{R1} + a^2 I_{R2} + I_{R0} \end{aligned}\right\} \quad .....(7.6)$$

The above equations can be put in matrix form considering zero sequence relation as the first for convenience.

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} \quad .....(7.7)$$

and

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} \quad .....(7.8)$$

Eqs. (7.7) and (7.8) relate the sequence components to the phase components through the transformation matrix.

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad .....(7.9)$$

consider the inverse of the transformation matrix C

$$C^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad .....(7.10)$$

Then the sequence components can be obtained from the phase values as

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_R \\ V_y \\ V_B \end{bmatrix} \quad .....(7.11)$$

and
$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ I_y \\ I_B \end{bmatrix} \qquad .....(7.12)$$

7.3 Power in Sequence Components

The total complex power flowing into a three – phase circuit through the lines R, Y, B is

$$S = P + jQ = \overline{V}\,\overline{I}^{\,*} \qquad .....(7.13)$$
$$= \overline{V}_R \overline{I}_R^{\,*} + \overline{V}_y \overline{I}_y^{\,*} + \overline{V}_z \overline{I}_z^{\,*}$$

Written in matrix notation

$$S = \begin{bmatrix} V_R & V_Y & V_B \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* \qquad .....(7.14)$$

$$= \begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^t \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* \qquad .....(7.15)$$

Also
$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = C \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} \qquad .....(7.16)$$

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* = C^* \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix}^* \qquad .....(7.17)$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^t = \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix}^t C^t \qquad .....(7.18)$$

From equation (7.14)

$$S = \begin{bmatrix} V_{RO} & V_{R1} & V_{R2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix}^* \qquad .....(7.19)$$

Note that $C^t C^* = 3\,U$

$$S = 3\,[V_{RO} \quad V_{R1} \quad V_{R2}] \begin{bmatrix} I_{RO} \\ I_{R1} \\ I_{R2} \end{bmatrix}^* \qquad .....(7.20)$$

Power in phase components is three times the power in sequence components.

The disadvantage with these symmetrical components is that the transformation matrix C is not power invariant or is not orthogonal or unitary.

7.4 Unitary Transformation for Power Invariance

It is more convenient to define "C" as a unitary matrix so that the transformation becomes power invariant.

That is power in phase components = Power in sequence components. Defining a transformation matrix T which is unitary, such that,

$$T = \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad .....(7.21)$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} \qquad .....(7.22)$$

and

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} \qquad .....(7.23)$$

so that

$$T^{-1} = [\sqrt{3}] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad .....(7.24)$$

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} = \left[\frac{\sqrt{3}}{3}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} \qquad .....(7.25)$$

and
$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \left[\frac{\sqrt{3}}{3}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} \qquad .....(7.26)$$

$$S = P + jQ = V\,I^* \qquad .....(7.27)$$

$$= \begin{bmatrix} V_R & V_Y & V_B \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* \qquad .....(7.28)$$

$$= \begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^t \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* \qquad .....(7.29)$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} \qquad .....(7.30)$$

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}^* = \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} \qquad .....(7.31)$$

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}^t = \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad .....(7.32)$$

$$S = \begin{bmatrix} V_{R0} & V_{R1} & V_{R2} \end{bmatrix} \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \left[\frac{1}{\sqrt{3}}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{RO} \\ I_{R1} \\ I_{R2} \end{bmatrix} \qquad .....(7.33)$$

$$\cdot \begin{bmatrix} V_{R0} & V_{R1} & V_{R2} \end{bmatrix} \left[\frac{1}{3}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{RO} \\ I_{R1} \\ I_{R2} \end{bmatrix} \qquad .....(7.34)$$

$$\begin{bmatrix} V_{R0} & V_{R1} & V_{R2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \end{bmatrix} [3] \begin{bmatrix} I_{RO} \\ I_{R1} \\ I_{R2} \end{bmatrix} \quad .....(7.35)$$

$$S = \begin{bmatrix} V_{R0} & V_{R1} & V_{R2} \end{bmatrix} \begin{bmatrix} I_{RO} \\ I_{R1} \\ I_{R2} \end{bmatrix} \quad .....(7.36)$$

Thus with the unitary transformation matrix

$$T = \begin{bmatrix} \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad ....(7.37)$$

we obtain power invariant transformation with sequence components.

7.5 Sequence Impedances

Electrical equipment or components offer impedance to flow of current when potential is applied. The impedance offered to the flow of positive sequence currents is called "positive sequence impedance Z_1. The impedance offered to the flow of negative sequence currents is called negative sequence impedance Z_2. When zero sequence currents flow through components of power system the impedance offered is called zero sequence impedance Z_0.

7.6 Balanced Star Connected Load

Consider the circuit in the Fig. 7.3.

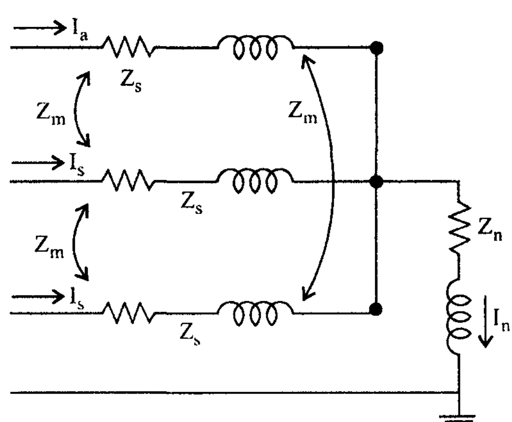


Fig. 7.3

A three phase balanced load with self and mutual impedances Z_s and Z_m drawn currents I_a, I_b and I_c as shown. Z_n is the impedance in the neutral circuit which is grounded draws and current in the circuit is I_n.

The line-to-ground voltages are given by

$$\left.\begin{aligned} V_a &= Z_s I_a + Z_m I_b + Z_m I_c + Z_n I_n \\ V_b &= Z_m I_a + Z_s I_b + Z_m I_c + Z_n I_n \\ V_c &= Z_m I_a + Z_m I_b + Z_s I_c + Z_n I_n \end{aligned}\right\} \quad .....(7.38)$$

Since, $\quad I_a + I_b + I_c = I_n$

Eliminating I_n from eqn. (7.38)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad .....(7.39)$$

Put in compact matrix notation

$$[V_{abc}] = [Z_{abc}]\,[I_{abc}] \quad .....(7.40)$$

$$V_{abc} = [A]\,V_a^{0,1,2} \quad .....(7.41)$$

and $\quad I_{abc} = [A]\,I_a^{0,1,2} \quad .....(7.42)$

Premultiplying eqn. (7.40) by $[A]^{-1}$ and using eqns. (7.41) and (7.42)

we obtain, $\quad V_a^{0,1,2} = [A]^{-1}\,[Z_{abc}]\,[A]\,I_a^{0,1,2} \quad .....(7.43)$

Defining $\quad [Z]^{0,1,2} = [A^{-1}]\,[Z_{abc}]\,[A] \quad .....(7.44)$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & 1 \end{bmatrix} \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} (Z_s + 3Z_n + 2Z_m) & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \quad .....(7.45)$$

If there is no mutual coupling

$$[Z^{0,1,2}] = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \quad .....(7.46)$$

From the above, it can be concluded that for a balanced load the three sequences are indepedent, which means that currents of one sequence flowing will produce voltage drops of the same phase sequence only.

7.7 Transmission Lines

Transmission lines are static components in a power system. Phase sequence has thus, no effect on the impedance. The geometry of the lines is fixed whatever may be the phase sequence. Hence, for transmission lines

$$Z_1 = Z_2$$

we can proceed in the same way as for the balanced 3-phase load for 3-phase transmission lines also

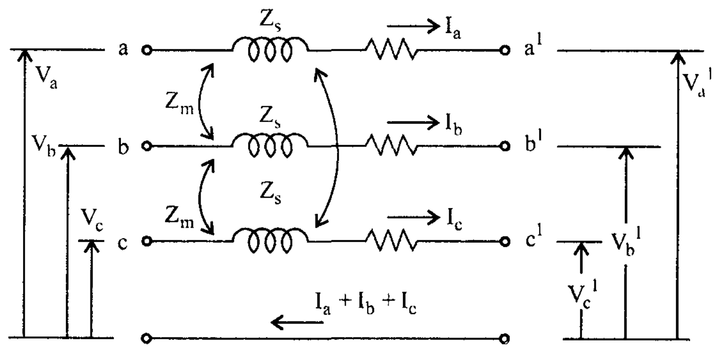


Fig. 7.4

$$\left.\begin{aligned} V_a - V_a' &= Z_s I_a + Z_m I_b + Z_m I_c \\ V_b - V_b' &= Z_m I_a + Z_s I_b + Z_m I_c \\ V_c - V_c' &= Z_m I_a + Z_m I_b + Z_s I_c \end{aligned}\right\} \qquad .....(7.47)$$

$$\begin{bmatrix} V_a - V_a' \\ V_b - V_b' \\ V_c - V_c' \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad .....(7.48)$$

$$[V_{abc}] = [V_{abc}] - [V_{abc'}] = [Z_{abc}]\,[I_{abc}] \qquad .....(7.49)$$

$$[Z^{0,1,2}] = [A^{-1}]\,[Z_{abc}]\,[A] \qquad .....(7.50)$$

$$= \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \qquad .....(7.51)$$

The zero sequence currents are in phase and flow through the line conductors only if a return conductor is provided. The zero sequence impedance is different from positive and negative sequence impedances.

7.8 Sequence Impedances of Transformer

For analysis, the magnetizing branch is neglected and the transformer is represented by an equivalent series leakage impedance.

Since, the transformer is a static device, phase sequence has no effect on the winding reactances.

Hence $$Z_1 = Z_2 = Z_l$$

where Z_l is the leakage impedance

If zero sequence currents flow then

$$Z_0 = Z_1 = Z_2 = Z_l$$

In star-delta or delta-star transformers the positive sequence line voltage on one side leads the corresponding line voltage on the other side by 30°. It can be proved that the phase shift for the line voltages to be –30° for negative sequence voltages.

The zero sequence impedance and the equivalent circuit for zero sequence currents depends upon the neutral point and its ground connection. The circuit connection for some of the common transformer connection for zero sequence currents are indicated in Fig. 7.5.

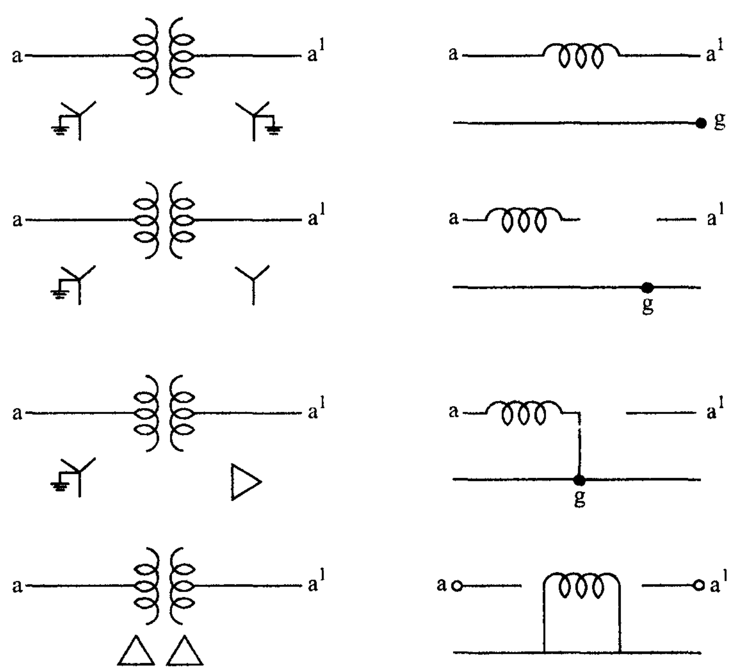


Fig. 7.5 Zero sequence equivalent circuits.

7.9 Sequence Reactances of Synchronous Machine

The positive sequence reactance of a synchronous machine may be X_d or X_d' or X_d'' depending upon the condition at which the reactance is calculated with positive sequence voltages applied.

When negative sequence cements are impressed on the stator winding, the net flux rotates at twice the synchronous speed relative to the rotor. The negative sequence reactance is approximately given by

$$X_2 = X_d'' \qquad .....(7.52)$$

The zero sequence currents, when they flow, are identical and the spatial distribution of the mmfs is sinusoidal. The resultant air gap flux due to zero sequence currents is zero. Thus, the zero sequence reactance is approximately, the same as the leakage flux

$$X_0 = X_l \qquad .....(7.53)$$

7.10 Sequence Networks of Synchronous Machines

Consider an unloaded synchronous generator shown in Fig. 7.6 with a neutral to ground connection through an impedance Z_n. Let a fault occur at its terminals which causes currents I_a, I_b and I_c to flow through its phass a, b, and c respectively. The generated phase voltages are E_a, E_b and E_c. Current I_n flows through the neutral impedance Z_n.

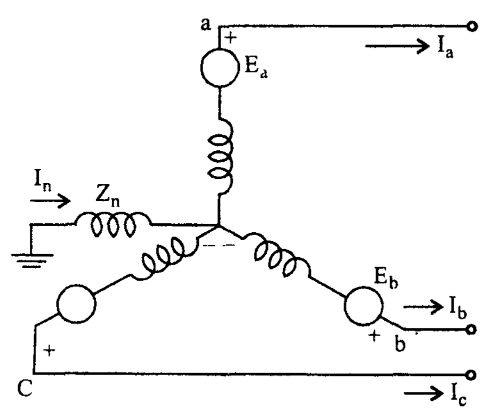


Fig. 7.6

7.10.1 Positive Sequence Network

Since the generator phase windings are identical by design and construction the generated voltages are perfectly balanced. They are equal in magnitude with a mutual phase shift of 120^0. Hence, the generated voltages are of positive sequence. Under these conditions a positive sequence current flows in the generator that can be represented as in Fig. 7.7.

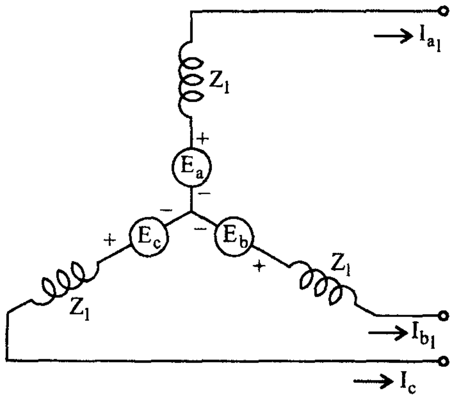


Fig. 7.7

Z_1 is the positive sequence impedance of the machine and I_a, is the positive sequence current in phase a. The positive sequence network can be represented for phase 'a' as shown in Fig. 7.8.

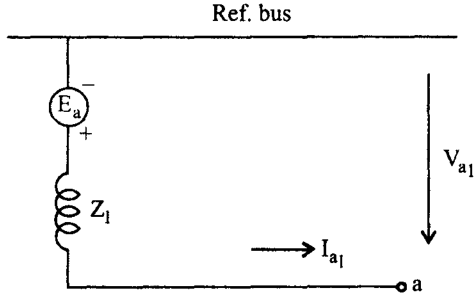


Fig. 7.8

$$V_{a_1} = E_a - I_{a_1} . Z_1 \qquad .....(7.54)$$

7.10.2 Negative Sequence Network

Synchronous generator does not produce any nagative sequence voltages. If negative sequences currents flow through the stator windings then the mmf produced will rotate at synchronous speed but in a direction opposite to the rotation of the machine rotor. This causes the negative sequence mmf to move past the direct and quadrature axes alternately. Then, the negative sequence mmf sets up a varying armature reaction effect. Hence, the negative sequence reactance is taken as the average of direct axis and quadrature axis subtransient reactances.

$$X_2 = (X_d'' + X_q'')/2 \qquad .....(7.55)$$

The negative sequence current paths and the negative sequene network are shown in Fig. 7.9.

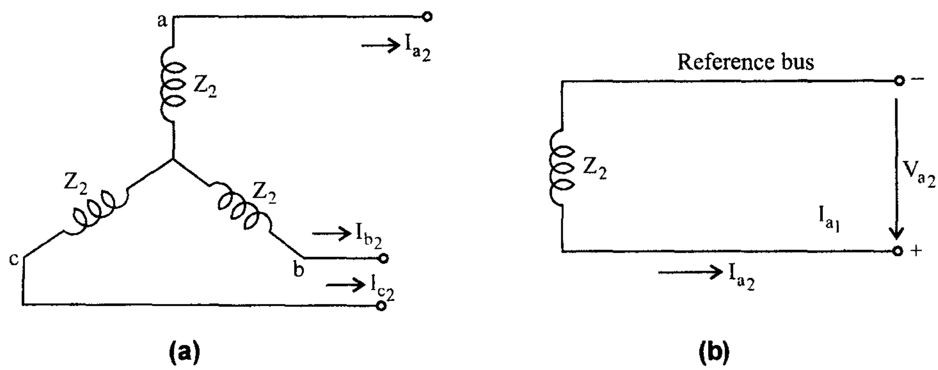


Fig. 7.9

$$V_{a_2} = -Z_2 I_{a_2} \qquad .....(7.56)$$

7.10.3 Zero Sequence Network

Zero sequence currents flowing in the stator windings produce mmfs which are in time phase. Sinusoidal space mmf produced by each of the three stator windings at any instant at a point on the axis of the stator would be zero, when the rotor is not present. However, in the actual machine leakage flux will contribute to zero sequence impedance. Consider the circuit in Fig. 7.10 (a).

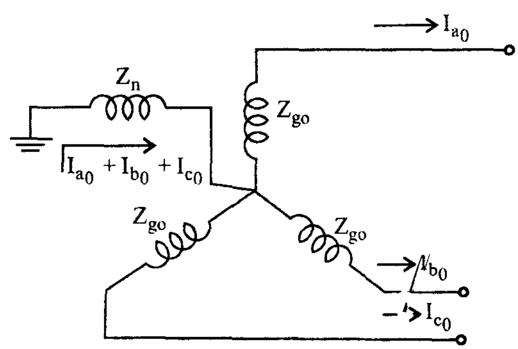


Fig. 7.10 (a)

Since $I_{a0} = I_{b0} = I_{c0}$

The current flowing through Z_n is 3 I_{a0}.

The zero sequence voltage drop

$$V_{a0} = -3\, I_{a0}\, Z_n - I_{a0}\, Z_{g0} \qquad .....(7.57)$$

Z_{g0} = zero sequence impedance per phase of the generator

Hence, $\quad Z_0^{1} = 3Z_n + Z_{g0} \qquad .....(7.58)$

so that $\quad V_{a0} = -I_{a0}\, Z_0^{1} \qquad .....(7.59)$

The zero sequence network is shown in Fig. 7.10 (b).

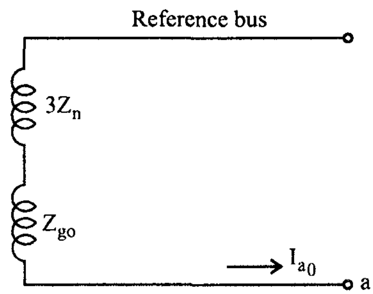


Fig. 7.10 (b)

Thus, it is possible to represent the sequence networks for a power system differently as different sequence currents flow as summarized in Fig. 7.11.

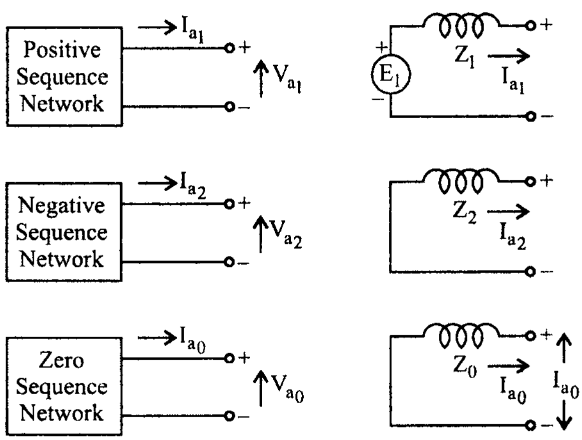


Fig. 7.11

7.11 Unsymmetrical Faults

The unsymmetrical faults generally considered are

- Line to ground fault
- Line to line fault
- Line to line to ground fault

Single line to ground fault is the most common type of fault that occurs in practice. Analysis for system voltages and calculation of fault current under the above conditions of operation is discussded now.

7.12 Assumptions for System Representation

1. Power system operates under balanced steady state conditions before the fault occurs. Therefore, the positive, negative and zero seq. networks are uncoupled before the occurrence of the fault. When an unsymmetrical fault occurs they get interconnected at the point of fault.
2. Prefault load current at the point of fault is generally neglected. Positive sequence voltages of all the three phases are equal tothe prefault voltage V_F. Prefault bus voltage in the positive sequence network is V_F.
3. Tranformer winding resistances and shunt admittances are neglected.
4. Transmission line series resistances and shunt admittances are neglected.
5. Synchronous machine armature resistance, saliency and saturation are neglected.
6. All non-rotating impedance loads are neglected.
7. Induction motors are either neglected or represented as synchronous machines.

It is conceptually easier to understand faults at the terminals of an unloaded synchronous genrator and obtain results. The same can be extended to a power system and results obtained for faults occurring at any point within the system.

7.13 Unsymmetrical Faults on an Unloaded Generator

Single Line to Ground Fault :

Consider Fig. 7.12. Let a line to ground fault occur on phase a.

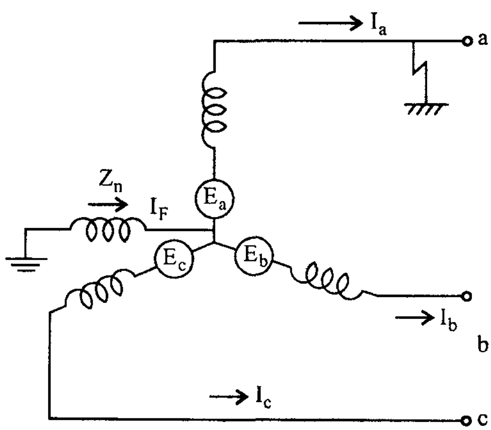


Fig. 7.12

We can write under the fault condition the following relations.

$$V_a = 0$$

$$I_b = 0$$

and $I_c = 0$

It is assumed that there is no fault impedance.

$$I_F = I_a + I_b + I_c = I_a = 3I_{a_1}$$

Now

$$\left.\begin{aligned} I_{a_1} &= \frac{1}{3}(I_a + aI_b + a^n I_c) \\ I_{a_2} &= \frac{1}{3}(I_a + a^n I_b + a\, I_c) \\ I_{a_0} &= \frac{1}{3}(I_a + I_b + I_c) \end{aligned}\right\} \quad .....(7.61)$$

Substitute eqn. (7.61) into eqns. (7.60)

$$I_b = I_c = 0 \quad .....(7.62)$$

$$I_{a_1} = I_{a_2} = I_{a_0} = \frac{1}{3} I_a \quad .....(7.63)$$

Hence the three sequence networks carry the same current and hence all can be connected in series as shown in Fig. 7.13 satisfying the relation.

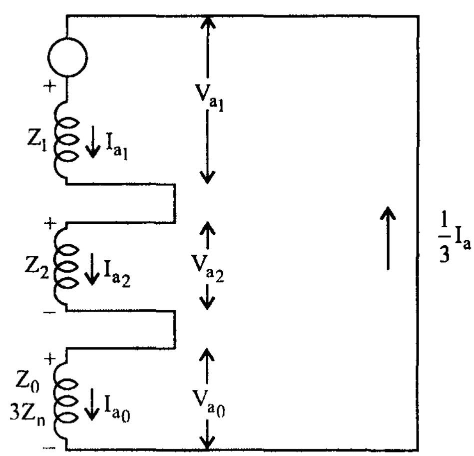


Fig. 7.13

$$V_a = E_a - I_{a_1} Z_1 - I_{a_2} Z_2 - I_{a_0} Z_0 - I_F Z_n \quad .....(7.63a)$$

Since $V_a = 0$

$$E_a = I_{a_1} Z_1 + I_{a_2} Z_2 + I_{a_0} Z_0 + I_F Z_n$$

$$E_a = I_{a1} Z_1 + I_{a_2} Z_2 + I_{a_0} Z_0 + 3I_{a_1} Z_n$$

$$E_a = I_{a1} [Z_1 + Z_2 + Z_0 + 3Z_n]$$

Hence,
$$I_a = \frac{3.E_a}{(Z_1 + Z_2 + Z_0 + 3Z_n)} \qquad .....(7.64)$$

The line voltages are now calculated.

$$V_a = 0$$

$$V_b = V_{a_0} + a^2 V_{a_1} + a V_{a_2}$$

$$= (-I_{a_0} Z_0) + a^2 (E_a - I_{a_1} Z_1) + a (-I_{a_2} Z_2)$$

$$= a^2 E_a - I_{a_1} (Z_0 + a^2 Z_1 + a Z_2)$$

Substituting the value of I_{a_1}

$$V_b = a^2 E_a - \frac{E_a}{(Z_1 + Z_2 + Z_0)} . (Z_0 + a^r Z_1 + aZ_2)$$

$$= E_a \left[a^2 - \frac{Z_0 + a^2 Z_1 + aZ_2}{Z_0 + Z_1 + Z_2} \right] = E_a \left[\frac{a^2 Z_0 + a^2 Z_1 + a^2 Z_2 - Z_0 - a^2 Z_1 - aZ_2}{Z_0 + Z_1 + Z_2} \right]$$

$\therefore$
$$V_b = E_a \left[\frac{(a^2 - a)Z_2 + (a^2 - 1)Z_0}{Z_0 + Z_1 + Z_2} \right] \qquad .....(7.65)$$

$$V_c = V_{a_0} + a V_{a_1} + a^r V_{a_2}$$

$$= (-I_{a_0} Z_0) + a (E_a - I_{a_1} Z_1) + a^r (-I_{a_2} Z_2)$$

Since $I_{a_1} = I_{a_2} = I_{a_0}$

$$V_c = a E_a - \frac{E_a}{Z_1 + Z_2 + Z_0} . (Z_0 + a Z_1 + a^2 Z_2)$$

$$= E_a \left[a - \frac{(Z_0 + aZ_1 + a^2 Z_2)}{Z_1 + Z_2 + Z_0} \right]$$

$$V_c = E_a \frac{[(a-1)Z_0 + (a - a^2)Z_2]}{Z_0 + Z_1 + Z_2} \qquad .....(7.66)$$

The phasor diagram for single line to ground fault is shown in Fig. 7.14.

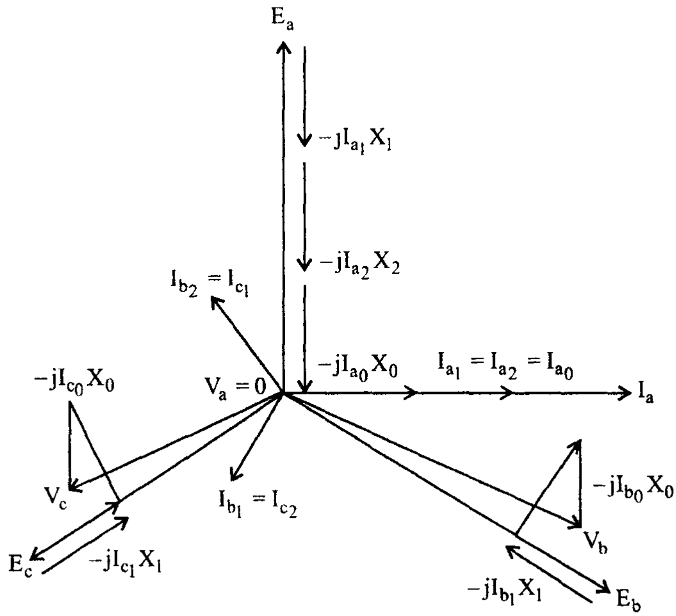


Fig. 7.14

7.14 Line-to-Line Fault

Consider a line to line fault across phases b and c as shown in Fig. 7.15.

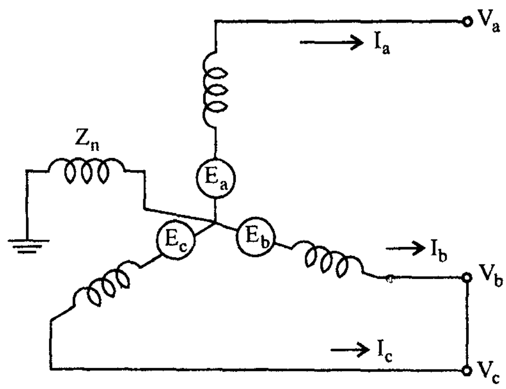


Fig. 7.15

From the Fig. 7.15 it is clear that

$$\left.\begin{aligned} I_a &= 0 \\ I_b &= -I_c \\ \text{and} \quad V_b &= V_c \end{aligned}\right\} \qquad .....(7.67)$$

Utilizing these relations

$$I_{a_1} = \frac{1}{3}(I_a + aI_b + a^2 I_c) = \frac{1}{3}(a^2 - a) I_b$$

$$= j\frac{I_b}{\sqrt{3}} \qquad .....(7.68)$$

$$I_{a_2} = \frac{1}{3}(I_a + a^2 I_b + a I_c) = \frac{1}{3}(a^2 - a) I_b$$

$$= -j\frac{I_b}{\sqrt{3}} \qquad .....(7.69)$$

$$I_{a_0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(0 + I_b - I_b) = 0 \qquad .....(7\ 70)$$

Since $V_b = V_c$

$$a^2 V_{a_1} + a V_{a_2} + V_{a_0} = a V_{a_1} + a^2 V_{a_2} + V_{a_0}$$

$$(a^2 - a) V_{a_1} = (a^2 - a) V_{a_2}$$

$\therefore$ $$V_{a_1} = V_{a_2} \qquad .....(7.71)$$

The sequence network connection is shown in Fig. 7.16.

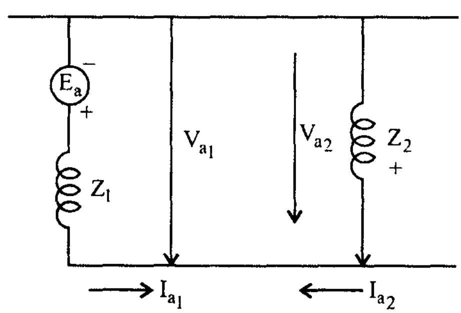


Fig. 7.16

From the diagram we obtain

$$E_a - I_{a_1} Z_1 = -I_{a_2} Z_2$$

$$= I_{a_1} Z_2$$

$$E_a = I_{a_1}(Z_1 + Z_2)$$

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2} \qquad .....(7.72)$$

$$I_b = (a^2 - a)\, I_{a_1} = -j\sqrt{3}\; I_{a_1} \qquad .....(7.73)$$

$$I_c = (a - a^2)\, I_{a_1} = j\sqrt{3}\; I_{a_1} \qquad .....(7.74)$$

Also,

$$V_{a_1} = \frac{1}{3}\,(V_a + aV_b + a^2 V_c) = \frac{1}{3}\,[V_a + (a + a^2)\, V_b]$$

$$V_{a_2} = \frac{1}{3}\,(V_a + a^2V_b + a\, V_c) = \frac{1}{3}\,[V_a + (a^2 + a)\, V_b]$$

Since $1 + a + a^2 = 0; \qquad a + a^2 = -1$

Hence

$$V_{a_1} = V_{a_2} = \frac{1}{3}\,(V_a + V_b) \qquad .....(7.75)$$

Again

$$I_a = I_{a_1} + I_{a2} = I_{a_1} - I_{a1} = 0$$

$$I_b = a^2\, I_{a_1} + aI_{a_2} = (a^2 - a)\, I_{a_1}$$

$$= \frac{(a^2 - a)E_a}{Z_1 + Z_2}$$

$$I_c = -\, I_b = \frac{(a^2 - a)E_a}{Z_1 + Z_2} \qquad .....(7.76)$$

$$V_a = V_{a_1} + V_{a_2} = E_a - I_{a_1} Z_1 + (-\, I_{a_2} Z_2)$$

$$= E_a - \frac{E_a}{Z_1 + Z_2}\,(Z_1 - Z_2) = E_a\left[1 - \frac{Z_1 - Z_2}{Z_1 + Z_2}\right]$$

$$= E_a\left[\frac{Z_1 + Z_2 - Z_1 + Z_2}{Z_1 + Z_2}\right] = E_a \cdot \frac{2Z_1}{Z_1 + Z_2} \qquad .....(7.77)$$

$$V_b = a^2\, V_{a_1} + a\, V_{a_2}$$

$$= a^2\,[E_a - I_{a_1} Z_1] + a\,(-I_{a_2} Z_2)$$

$$= a^2\, E_a - I_{a_1}\,[a^2\, Z_1 - a\, Z_2]$$

$$= E_a\left[a^2 - \frac{\left(a^2Z_1 - aZ_2\right)}{Z_1 + Z_2}\right] = E_a\left[\frac{a^nZ_1 + a^2Z_2 - a^2Z_1 + aZ_2}{Z_1 + Z_2}\right]$$

$$= \frac{E_aZ_2(a + a^2)}{Z_1 + Z_2} = \frac{E_a(-Z_2)}{Z_1 + Z_2} \qquad .....(7.78)$$

$$V_c = V_b = \frac{E_a(-Z_2)}{(Z_1 + Z_2)} \qquad .....(7.79)$$

The phasor diagram for a double line fault is shown in Fig. 7.17.

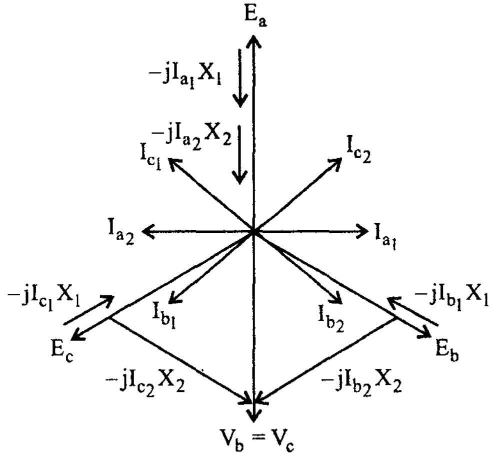


Fig. 7.17

7.15 Double Line to Ground Fault

Consider line to line fault on phases b and c also grounded as shown in Fig. 7.18.

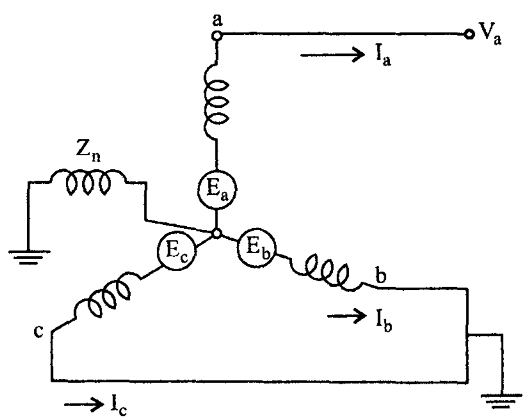


Fig. 7.18

From the Fig. 7.18.

$$\left.\begin{array}{l} I_a = 0 \\ V_b = V_c = 0 \\ I_b + I_c = I_F \end{array}\right\} \qquad .....(7.80)$$

$$V_{a_1} = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

$$= \frac{1}{3} V_a \qquad .....(7.81)$$

$$V_{a_2} = \frac{1}{3}(V_a + a^2 V_b + a V_c)$$

$$= \frac{1}{3} V_a \qquad .....(7.82)$$

Further $$V_{a_0} = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3} V_a$$

Hence $$V_{a_1} = V_{a_2} = V_{a_0} = \frac{1}{3} V_a \qquad .....(7.83)$$

But $$V_{a_1} = E_a - I_{a_1} Z_1$$

$$V_{a_2} = - I_{a_2} Z_2$$

and $$V_{a_0} = - I_{a_0} Z_0 - I_F Z_n$$

$$= - I_{a_0}(Z_0 + 3Z_n) = - I_{a_0}(Z_0{}^1) \qquad .....(7.84)$$

In may be noted that

$$I_F = I_b + I_c = a^2 I_{a_1} + a I_{a_2} + I_{a_0} + a I_{a_1} + a^2 I_{a_2} + a I_{a_0}$$

$$= (a + a^2) I_{a_1} (a + a^2) I_{a_2} + 2 I_{a_0}$$

$$= - I_{a_1} - I_{a_2} + 2I_{a_0} = - I_{a_1} - I_{a_2} - I_{a_0} + 3I_{a_0}$$

$$= -(I_{a_1} + I_{a_2} + I_{a_0}) + 3 I_{a_0} = 0 + 3 I_{a_0} = 3I_{a_0}$$

The sequence network connections are shown in Fig. 7.19.

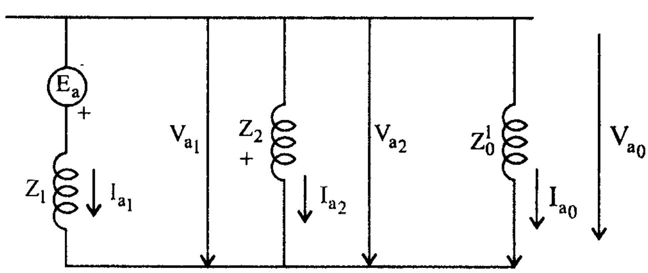


Fig. 7.19

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0^1}{Z_2 + Z_0^1}} \qquad .....(7.85)$$

$$= \frac{E_a\left(Z_2 + Z_0^1\right)}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0^1 Z_1} \qquad .....(7.86)$$

$$V_{a_2} = V_{a_1}$$

$$-I_{a_1} Z_2 = E_a - I_{a_1} Z_1$$

$$I_{a_2} = -\left(\frac{E_a - I_{a_1} Z_1}{Z_2}\right)$$

$$= -\left[E_a - \frac{E_a\left(Z_2 + Z_0^1\right).Z_1}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0 Z_1}\right].\frac{1}{Z_2}$$

$$= \frac{-E_a.Z_0^1}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0^1 Z_1} \qquad .....(7.87)$$

Similarly $-I_{a_0} Z_0^1 = -I_{a_2} Z_2$ .....(7.88)

$$I_{a_0} = -I_{a_2}\frac{Z_2}{Z_0^1} = \frac{-E_a Z_2}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1} \qquad .....(7.89)$$

$$V_a = V_{a_1} + V_{a_2} + V_{a_0}$$

$$= E_a - I_{a_1} Z_1 - I_{a_2} Z_2 - I_{a_0}(Z_0 + 3Z_n)$$

$$= E_a - \frac{E_a\left(Z_2 + Z_0\right)}{\Sigma Z_1 Z_2} Z_1 + \frac{E_a Z_0 Z_2}{\Sigma Z_1 Z_2} + \frac{E_a.Z_2\left(Z_0 + 3Z_n\right)}{\Sigma Z_1 Z_2}$$

$$= E_a \frac{3Z_2 Z_0 + 3Z_2 Z_n}{\Sigma Z_1 Z_2} = 3E_a\left(\frac{Z_2\left(Z_0 + Z_n\right)}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1}\right) \qquad .....(7.90)$$

$$V_b = V_{a_0} + a^2 V_{a_1} + a V_{a_2}$$

$$= -I_{a_0}(Z_0 + 3Z_n) + a^2[E_a - I_{a_1} Z_1] + a[-I_{a_2} Z_2]$$

$$= \frac{E_a(Z_2)\left(Z_0 + 3Z_n\right)}{\Sigma Z_1 Z_2}(Z_0 + 3Z_n) + a^2\left[E_a - \frac{\left(E_a Z_2 + Z_0\right)}{\Sigma Z_1 Z_2}\right] + a\left[\frac{E_0 Z_0 Z_2}{\Sigma Z_1 Z_2}\right]$$

$$= \frac{E_a\left[Z_2 Z_0 + 3Z_2 Z_n\right] + a^2 E_a\left[Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1 - Z_2 Z_1 - Z_0 Z_1\right]}{\Sigma Z_1 Z_2 + aE_a Z_0 Z_2}$$

$$V_b = \frac{E_a\left[Z_0 Z_2 + a^2 Z_0 Z_2 + aZ_0 Z_2 + 3Z_2 Z_n\right]}{\Sigma Z_1 Z_2}$$

$$= \frac{E_a\left[Z_0 Z_2(1 + a + a^2) + 3Z_2 Z_n\right]}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_4} = \frac{3.E_a.Z_2 Z_n}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1} \qquad .....(7.91)$$

If $\quad Z_n = 0;\ V_b = 0$

The phasor diagram for this fault is shown in Fig. 7.20.

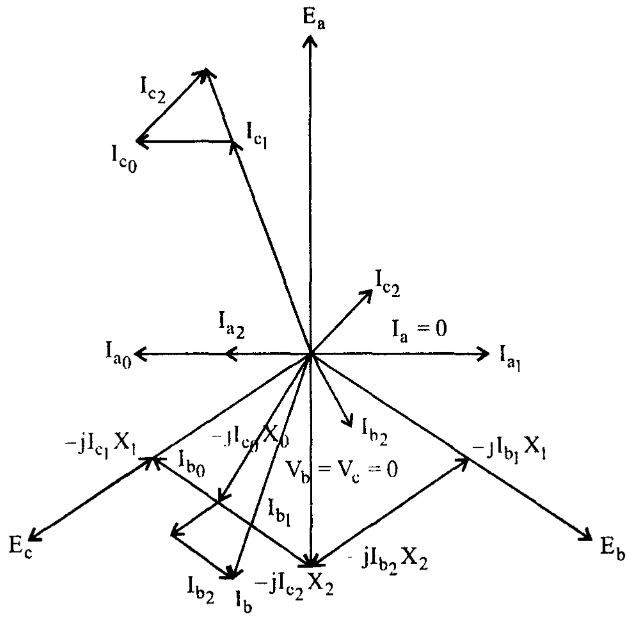


Fig. 7.20

7.16 Single-Line to Ground Fault with Fault Impedance

If in (7.13) the fault is not a dead short circuit but has an impedance Z_F then the fault in represented in Fig. 7.21. Eqn. (7.63a) wil lbe modified into

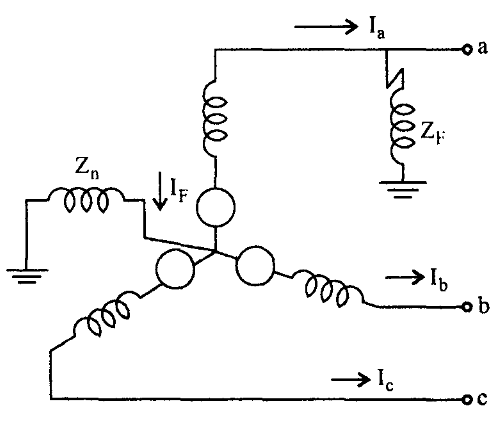


Fig. 7.21

$$V_a = E_a - J_{a_1} Z_1 - J_{a_2} Z_2 - I_{a_0} Z_0 - I_F Z_n - I_F Z_F \qquad .....(7.92)$$

Substituting $V_a = 0$ and solving for I_a

$$I_a = \frac{3E_a}{Z_1 + Z_2 + Z_0 + 3(Z_n + Z_F)} \quad .....(7.93)$$

7.17 Line-to-Line Fault with Fault Impedence

Consider the circuit in Fig. (7.22) when the fault across the phases b and c has an impedence Z_F.

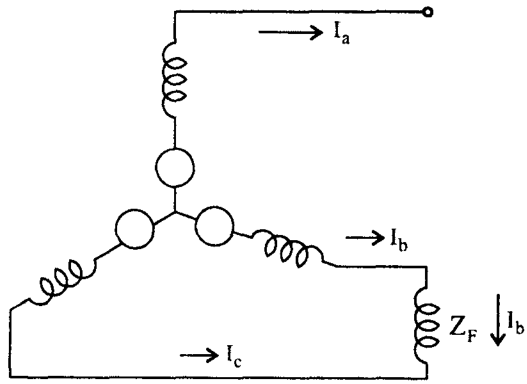


Fig. 7.22

$$I_a = 0 \quad .....(7.94)$$

and $\quad I_b = -I_c$

$$V_b - V_c = Z_F I_b \quad .....(7.95)$$

$$(V_0 + a^rV_1 + aV_2) - (V_0 + aV_1 + a^2V_2)$$
$$= Z_F (I_0 + a^rJI_1 + aJ_2) \quad .....(7.96)$$

Substituting eqn. (7.95) and (7.96) in eqn.

$$(a^2 - a)\,V_1 - (a^2 - a)\,V_2 = Z_F (a^2 - a)\,I_1$$

i.e., $\quad V_1 - V_2 = Z_F I_1 \quad .....(7.97)$

The sequence nehosh connection in this case will be as shown in Fig. (7.23).

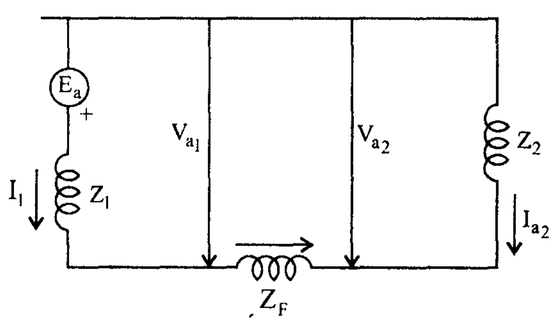


Fig. 7.23

7.18 Double Line-to-Ground Fault with Fault Impedence

This can is Illustrated in Fig. 7.24.

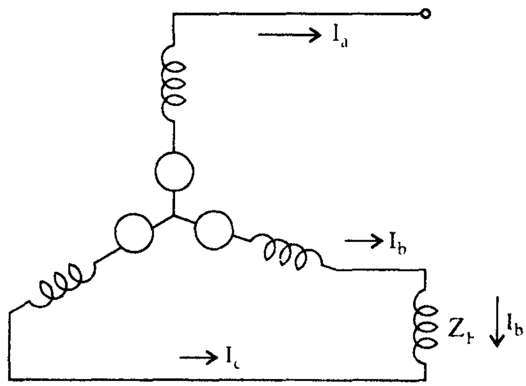


Fig. 7.24

The representative equations are

$$I_a = 0$$

$$V_b = V_c$$

$$V_b = (I_b + I_c)(Z_F + Z_n) \quad .....(7.98)$$

But, $I_0 + I_1 + I_2 = 0$

and also, $V_0 + aV_1 + a^2 V_2 = V_a + a^2 V_1 + aV_2$

So that $(a^2 - a)V_1 = (a^2 - a) V_2$

or $V_1 = V_2 \quad .....(7.99)$

Further, $(V_0 + a^2 V_1 + aV_2) = (I_0 + a^2 I_1 + aI^2 + I_0 + aJ_1 + a^2 I_2) Z_F + Z_n)$

Since $a^2 + a = -1$

$$(V_0 - V_1) = (Z_n + Z_F) [2I_0 - I_1 - I_2)\backslash$$

But since $I_0 = - I_1 - I_2$

$$V_0 - V_1 = (Z_n + Z_F)(2 I_0 + I_0) = 3(Z_F + Z_n).I_0 \quad .....(7.100)$$

Hence, the fault conditions are given by

$$I_0 + I_1 + I_2 = 0$$

$$V_1 = V_2$$

and $V_0 - V_1 = 3(Z_F + Z_n).\ I_0$

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_0^1 Z_2}{Z_0^1 + Z_2}} \text{ and so on as in case (7.15)} \quad .....(7.101)$$

where $$Z_0^1 = Z_0 + 3Z_F + 3Z_n$$

The sequence network connections are shown in Fig. 7.25.

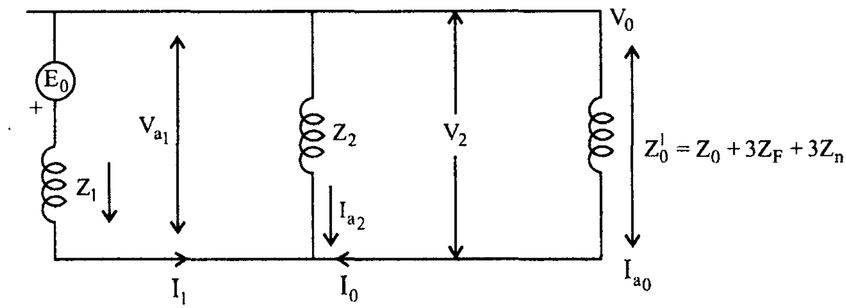


Fig. 7.25

Worked Examples

E.7.1 Calculate the sequence components of the followinmg balanced line-to-network voltages.

$$\bar{V} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 220 & \underline{/0^\circ} \\ 220 & \underline{/-120^\circ} \\ 220 & \underline{/+120^\circ} \end{bmatrix} KV$$

Solution :

$$V_0 = \frac{1}{3}(V_{an} + V_{bn} + V_{cn})$$

$$= \frac{1}{3}[200\underline{/0^\circ} + 200\underline{/-120^\circ} + 220\underline{/+120^\circ}]$$

$$= 0$$

$$V_1 = \frac{1}{3}[V_{an} + aV_{bn} + a^2V_{cn}]$$

$$= \frac{1}{3}[220\underline{/0^\circ} + 220\underline{/(-120^\circ + 120^\circ)} + 220\underline{/(120^\circ + 240^\circ)}]$$

$$= 220\underline{/0^\circ}\ KV$$

$$V_2 = \frac{1}{3}[V_{an} + a^2V_{bn} + a^2V_{cn}]$$

$$= \frac{1}{3}[220\ \underline{/0^\circ} + 220\underline{/-120^\circ + 240^\circ} + 220\ \underline{/120^\circ + 120^\circ}$$

$$= \frac{1}{3}[220 + 220\ 120^\circ + 220\ 240^\circ]$$

$$= 0$$

Note : Balanced three phase voltages do not contain negative sequence components.

E.7.2 Prone that neutral current can flow only if zero-sequence currents are present

Solution :

$$I_a = I_{a1} + I_{a2} + I_{a0}$$
$$I_b = a^2I_{a1} + aI_{a2} + I_{a0}$$
$$I_c = aI_{a1} + a^2I_{a2} + I_{a0}$$

If zero-sequence currents are not present

then $\quad I_{a0} = 0$

In that case

$$
\begin{aligned}
I_a + I_b + I_c &= I_{a_1} + I_{a_2} + a^2 I_{a_1} + a I_{a_2} + a I_{a1} + a^2 I_{a_2} \\
&= (I_{a_1} + a I_{a_1} + a^2 I_{a_1}) + (I_{a_2} + a^2 I_{a_2} + a I_{a_2}) \\
&= 0 + 0 = 0
\end{aligned}
$$

The neutral cement $I_n = I_R = I_Y + I_B = 0$. Hence, neutral cements will flow only in case of zero sequence components of currents exist in the network.

E.7.3 Given the negative sequence cements

$$
\bar{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 100 \ \underline{/0^\circ} \\ 100 \ \underline{/120^\circ} \\ 100 \ \underline{/-120^\circ} \end{bmatrix}
$$

Obtain their sequence components

Solution

$$
\begin{aligned}
I_0 &= \frac{1}{3} [I_a + I_b + I_c] \\
&= \frac{1}{3} [100 \underline{/0^\circ} + 100\underline{/120^\circ} + 100 \underline{/-120^\circ} = O \quad A
\end{aligned}
$$

$$
\begin{aligned}
I_1 &= \frac{1}{3} [I_a + aI_b + a^r I_c] \\
&= \frac{1}{3} [100\underline{/0^\circ} + 100\underline{/120^\circ + 120^\circ} + 100 \underline{/-120^\circ + 240^\circ}] \\
&= \frac{1}{3} [100\underline{/0^\circ} + 100\underline{/240^\circ} + 100\underline{/120^\circ}] \\
&= O \quad A
\end{aligned}
$$

$$
\begin{aligned}
I_2 &= \frac{1}{3} [I_a + a^2 I_b + a I_c] \\
&= \frac{1}{3} [100 \underline{/0^\circ} + 100\underline{/120^\circ + 240^\circ} + 100\underline{/-120^\circ + 240^\circ}] \\
&= \frac{1}{3} [100\underline{/0^\circ} + 100\underline{/0^\circ} + 100\underline{/0^\circ}] \\
&= 100 \text{ A}
\end{aligned}
$$

Note : Balanced currents of any sequence, positive or negative do not contain currents of the other sequences.

E.7.4 Find the symmetrical components for the given three phase currents.

$I_a = 10\angle 0^\circ$

$I_b = 10\angle -90^\circ$

$I_c = 15\angle 135^\circ$

Solution

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^1 & a \end{bmatrix}\begin{bmatrix} 10 & \angle 0^\circ \\ 10 & \angle -90^\circ \\ 10 & \angle 135^\circ \end{bmatrix}$$

$$I_0 = \frac{1}{3}\,[10\ 0^\circ + 10 - 90^\circ + 15\ 135^\circ]$$

$$= \frac{1}{3}\,[10\,(1 + j0.0) + 10\,(0 - j\,1.0) + 15\,(-0.707 + j\,0.707)$$

$$= \frac{1}{3}\,[10 - j10 - 10.605 + j\,10.605]$$

$$= \frac{1}{3}[-0.605 + j\,0.605] = \frac{1}{3}\,[0.8555]\angle 135^\circ$$

$$= 0.285\angle 135^\circ\ \text{A}$$

$$I_1 = \frac{1}{3}\,[10\angle 0^\circ + 10 - \angle 90^\circ + 120^\circ + 15\angle 135^\circ + 240^\circ\,]$$

$$= \frac{1}{3}\,[10\,(1 + j0.0) + 10\angle 30^\circ + 15\angle 15^\circ$$

$$= \frac{1}{3}\,[10 + 10\,(0.866 + j\,0.5) + 15\,(0.9659 + j0.2588]$$

$$= \frac{1}{3}[33.1485 + j\,8.849] = \frac{1}{3}\,[34.309298]\angle 15^\circ$$

$$= 11.436\angle 15^\circ\ \text{A}$$

$$I_2 = \frac{1}{3}\,[10\angle 0^\circ + 10\angle 240^\circ - 90^\circ + 15\angle 135^\circ + 120^\circ]$$

$$= \frac{1}{3}\,[10\,(1 + j0) + 10\,(-0.866 + j0.5) + 15\,(-0.2588 - j\,0.9659)$$

$$= \frac{1}{3}\,[-2.542 - j\,9.4808]$$

$$= 3.2744\angle 105^\circ\ \text{A}$$

E.7.5 In a fault study problem the following currents are measured

$$I_R = 0$$
$$I_Y = 10\,A$$
$$I_B = -\,10\,A$$

Find the symmetrical components

Solution

$$IR_1 = \frac{1}{3}\,[I_R + a\,I_y + a^2\,I_B]$$

$$= \frac{1}{3}\,[0 - a\,(10) + a^2\,(-10)] = \frac{10}{\sqrt{3}}\,A$$

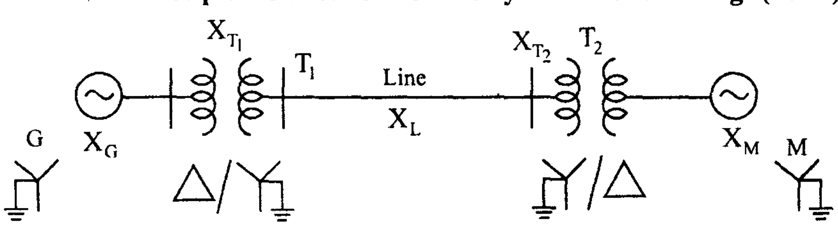
$$IR_2 = \frac{1}{3}\,[I_R + a^2\,Z_y + a\,J_B]$$

$$= \frac{1}{3}\,(a^2.10 + a\,(-10) = -\frac{10}{\sqrt{3}}$$

$$IR_0 = \frac{1}{3}(I_R + I_y + I_B)$$

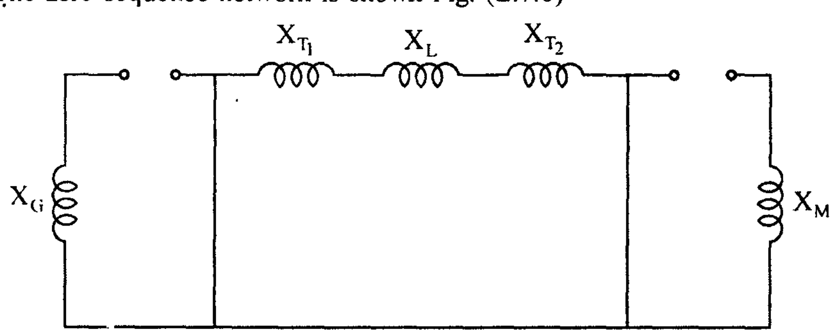
$$= \frac{1}{3}\,(10 - 10) = 0$$

E.7.6 Draw the zero sequence network for the system shown in Fig. (E.7.6).



Solution

The zero sequence network is shown Fig. (E.7.6)



Zero sequence network for the given system

E.7.7 Draw the sequence networks for the system shown in Fig. (E. 7.7).

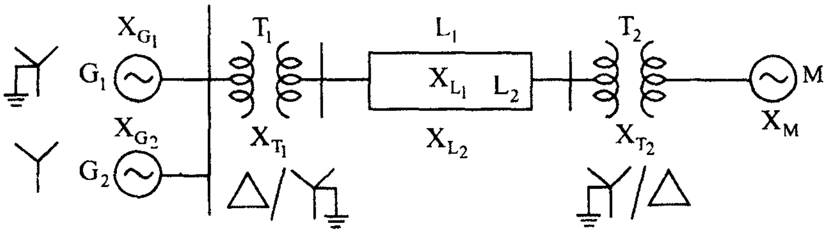
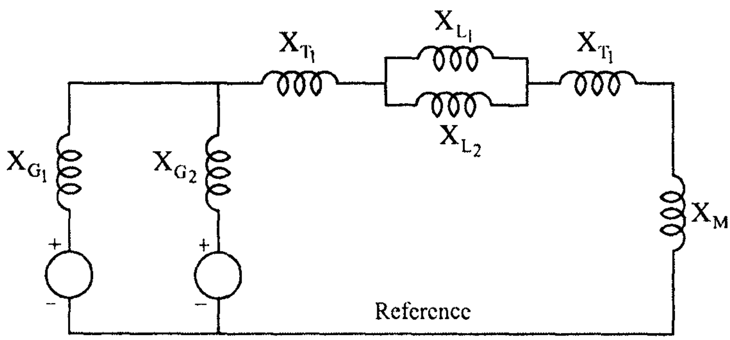
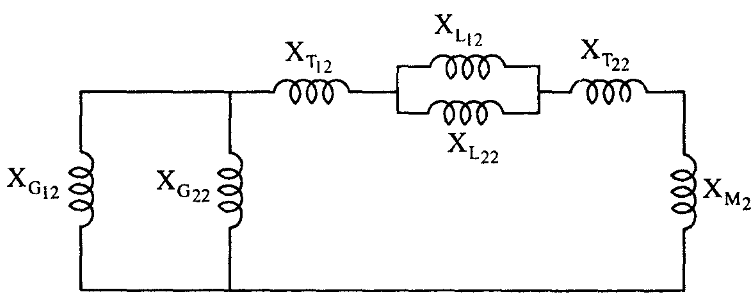


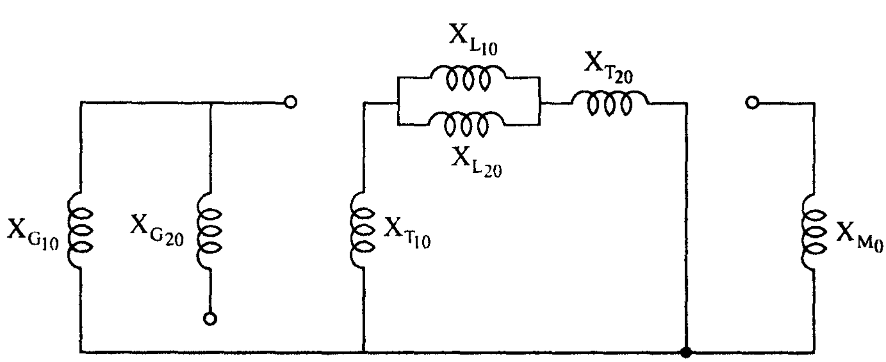
Fig. E. 7.7



Positive sequence network



Negative sequence network

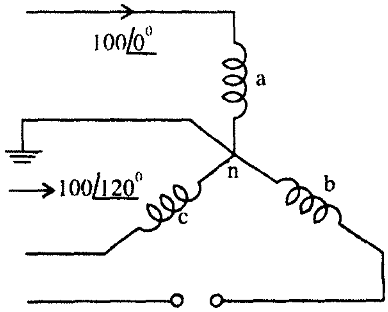


Zero sequence network

E.7.8 Consider the system shown in Fig. E. 7.8. Phase b is open due to conductor break. Calculate the sequence currents and the neutral current.

$I_a = 100\underline{/0^\circ}$ A

$I_b = 100\underline{/120^\circ}$ A



Solution

$$\bar{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 100 & \underline{/0^\circ} \\ 0 & \\ 100 & \underline{/120^\circ} \end{bmatrix} A$$

$$I_0 = \frac{1}{3}\,[100\underline{/0^\circ} + 0 + 100\underline{/120^\circ}]$$

$$= \frac{1}{3}\,[100\,(1 + j0) + 0 + 100\,(-0.5 + j\,0.866)$$

$$= \frac{100}{3}\,[0.5 + j\,0.866] - 33.3\underline{/60^\circ}\text{ A}$$

$$I_1 = \frac{1}{3}\,[100\underline{/0^\circ} + 0 + 100\underline{/120^\circ + 240^\circ}]$$

$$= \frac{1}{3}\,[100\underline{/0^\circ} + 100\underline{/0^\circ}] = \frac{200}{3} = 66.66\text{ A}$$

$$I_2 = \frac{1}{3}\,[100\underline{/0^\circ} + 0 + 100\underline{/120^\circ + 120^\circ}]$$

$$= \frac{1}{3}\,[100\,[1 + j0 - 0.5 - j\,0.866]$$

$$= \frac{100}{3}\,[-0.5 - j\,0.866] = 33.33\underline{/-60^\circ}\text{ A}$$

Nuetral current $I_n = I_0 + I_1 + I_2$

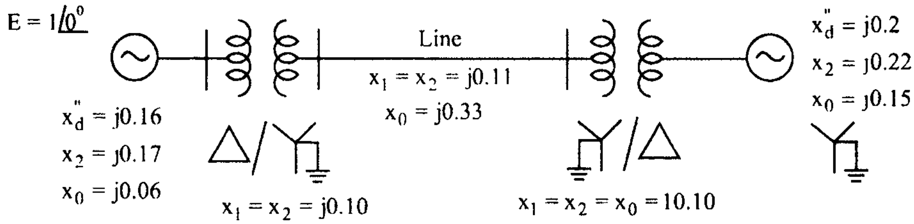
$= 100\angle 0^\circ + 0 + 100\angle 120^\circ$

$= 100\,[1 + j0 - 0.5 + j0.866]$

$= 100\angle 60^\circ$ A

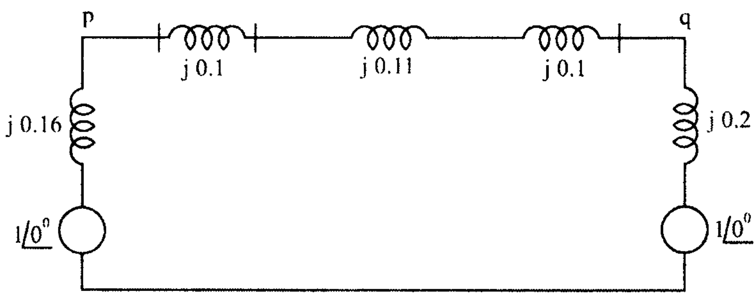
Also, $I_n = 3\,I_0 = 3\,(33.33\,(60^\circ) = 100\angle 60^\circ$ A

E.7.9 Calculate the subtransient fault current in each phase for a dead short circuit on one phase to ground at bus 'q' for the system shown in Fig. E.7.9.

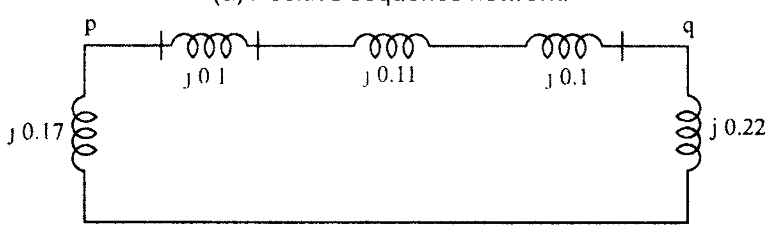


All the reactances are given in p.u on the generator base.

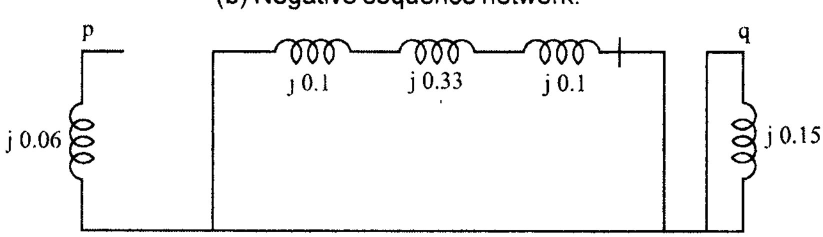
Solution :



(a) Positive sequence network.

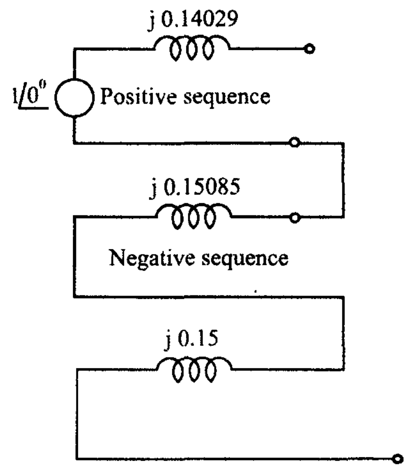


(b) Negative sequence network.



(c) zero sequence network.

The three sequence networks are shown in Fig. (a,b and c). For a line-to-ground fault an phase a, the sequence networks are connected as in Fig. E. 7.9 (d) at bus 'q'.



E.7.9 (d)

The equivalent positive sequence network reactance X_p is given form Fig. (a)

$$\frac{1}{X_p} = \frac{1}{0.47} + \frac{1}{0.2}$$

$$X_p = 0.14029$$

The equivalent negative sequence reactance X_n is given from Fig. (b)

$$\frac{1}{X_n} = \frac{1}{0.48} + \frac{1}{0.22} \quad \text{or } X_n = 0.01508$$

The zero-sequence network impedence is j 0.15 the connection of the three sequence networks is shown in Fig. E. 7.9(d).

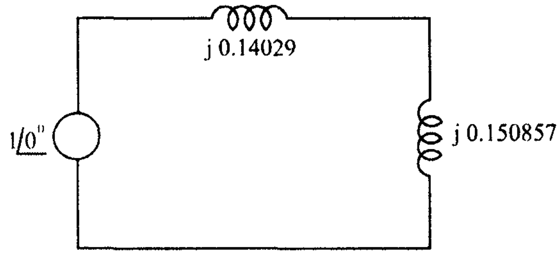
$$I_0 = I_1 = I_2 = \frac{1\angle 0^\circ}{j0.14029 + j0.150857 + j0.15}$$

$$= \frac{1\angle 0^\circ}{j0.44147} = -j2.2668 \text{ p.u}$$

E.7.10 In the system given in example (E.7.9) if a line to line fault occurs calculate the sequence components of the fault current.

Solution :

The sequence network connection for a line-to-line fault is shown in Fig. (E.7.10).



From the figure

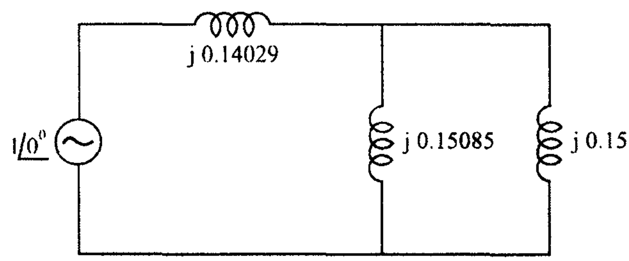
$$I_1 = I_2 = \frac{1\angle 0^{\circ}}{j0.1409 + j0.150857} + \frac{1\angle 0^{\circ}}{j0.291147}$$

$$= -j\,3.43469 \text{ p.u}$$

E.7.11 If the line-to-line fault in example E.7.9 takes place involving ground with no fault impedance determine the sequence componenets of the fault current and the neutral fault current.

Solution

The sequence network connection is shown in Fig.



$$I_1 = \frac{1\angle 0^{\circ}}{j0.14029 + \dfrac{j(0.150857)(j0.15)}{j0.150857 + j0.15}}$$

$$= \frac{1\angle 0^{\circ}}{j0.14029 + j0.0752134} = \frac{1\angle 0^{\circ}}{j0.2155034}$$

$$= -j\,4.64 \text{ p.u}$$

$$I_2 = -j(4.64)\left(\frac{j0.15}{j0.300857}\right) = -j2.31339 \text{ p.u}$$

$$I_0 = -j(4.64)\left(\frac{j0.150857}{j0.300857}\right) = -j\,2.326608 \text{ p.u}$$

The neutral fault current = $3\,j_0 = 3(-j2.326608) = -j6.9798$ p.u

E.7.12 A dead earth fault occurs on one conductor of a 3-phase cable supplied by a 5000 KVA, three-phase generator with earthed neutral. The sequence impedences of the altemator are given by

$$Z_1 = (0.4 + j4)\ \Omega;\quad Z_2 = (0.3 + j0.6)\ \Omega \text{ and}$$

$$Z_0 = (0 + j\ 0.45)\ \Omega \text{ per phase}$$

The sequence impedance of the line up to the point of fault are (0.2 + j0.3) Ω, (0.2 + j 0.3)W, (0.2 + j 0.3) Ω and (3 + j1) Ω. Find the fault current and the sequence components of the fault current. Aslo find the line-to-earth voltages on the infaulted lines. The generator line voltage is 6.6 KV.

Solution

Total positive sequence impedance is $Z_1 = (0.4 + j\ 4) + (0.2 + j\ 0.3) = (0.6 + j\ 4.3)\ \Omega$.

Total negative sequence impedence to fault is $Z_0 = (0.3 + j\ 0.6) + (0.2 + j0.3) = (0.5 + j0.9)\ \Omega$

Total zero-sequecne impedence to fault is $Z0 = (0 + j\ 0.45) + (3 + j1.0) = (3 + j\ 1.45)\ \Omega$ $Z_1 + Z_2 + Z_3 = (0.6 + j\ 4.3) + (0.5 + j\ 0.9) + (3.0 + j\ 1.45)$

$$= (4.1 + j\ 6.65)\ \Omega$$

$$I_{a1} = I_{a0} = I_{a2} = \frac{6.6 \times 1000}{\sqrt{3}} - \frac{1}{(4.1 + j6.65)} = \frac{3810.62A}{7.81233}$$

$$= 487.77\ -58^\circ.344\ A$$

$$= (255.98 - j\ 415.198)\ A$$

$$I_a = 3 \times 487.77 \angle -58^\circ.344$$

$$= 1463.31\ A \angle -58^\circ.344$$

E.7.13 A 20 MVA, 6.6 KV star connected generator has positive, negative and zero sequence reactances of 30%, 25% and 7% respectively. A reactor with 5% reactance based on the rating of the generator is placed in the neutral to groud connection. A line-to-line fault occurs at the terminals of the generator when it is operating at rated voltage. Find the initial symmetrical line-to-ground r.m.s fault current. Find also the line-to-line voltage.

Solution

$$Z_1 = j\ 0.3;\ Z_2 = j\ 0.25$$

$$Z_0 = j\ 0.07 + 3 \times j\ 0.05 = j\ 0.22$$

$$I_{a_1} = I_{a_1} = \frac{1\angle 0^\circ}{j(0.3) + j(0.25)} = \frac{1}{j0.55} = -j1.818 \text{ p.u}$$

$$= -j\ 1.818 \times \frac{20 \times 1000}{\sqrt{3} \times 6.6} = -j\ 3180 \text{ Amperes}$$

$$= -I_{a_1}$$

$I_{a_0} = 0$ as there is no ground path

$$V_a = E_a - I_{a_1} Z_1 - I_{a_2} Z_2$$

$$1 = -j\ 1.818\ (j\ 0.0.3 - j0.25)$$

$$= 0.9091 \times 3180 = 2890.9 \text{ V}$$

$$V_b = a^2 E - (a^2 I_{a_1} Z_1 + aI_{a_2} Z_2)$$

$$= (-0.5 - j\ 0.866).1 + j\ \sqrt{3}\ (-j\ 1.818)\ (j\ 0.3)$$

$$= (-j0.866 - 0.5 + j\ 0.94463)$$

$$= (-\ 0.5 + j\ 0.078\ 6328) \times 3180$$

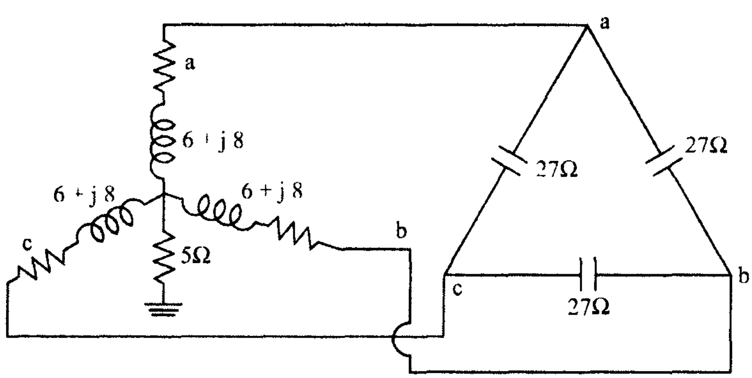
$$= (-\ 1590 + j\ 250) = 1921.63$$

$$V_c = V_b = 1921.63 \text{ V}$$

E.7.14 A balanced three phase load with an impedence of (6-j8) ohm per phase, connected in star is having in parallel a delta connected capacitor bank with each phase reactance of 27 ohm. The star point is connected to ground through an impedence of 0 + j5 ohm. Calculate the sequence impedence of the load.

Solution

The load is shown in Fig. (E.7.14).

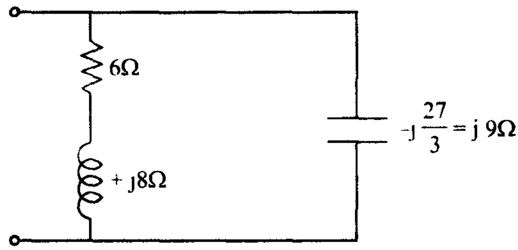


Converting the delta connected capacitor tank into star

$$C_\Delta/\text{phase} - 27 \text{ ohnm}$$

$$C_Y/\text{phase} = \frac{1}{3}\ 27 = a \text{ ohm}$$

The positive sequence network is shown in Fig. E. 7.14(a)



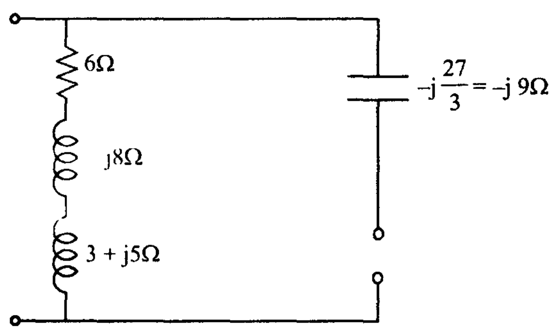
The negative sequence network is also the same as the positive sequence network

$$Z_1 = Z_2 = Z_{star} \parallel \frac{Z}{3} \text{ delta}$$

$$= \frac{(6+j8)(-j9)}{6+j8-j9} = \frac{72-j54}{6-j1} = \frac{90\underline{/36^\circ.87}}{6.082\underline{/9^\circ.46}}$$

$$= 14.7977\ \underline{/27^\circ.41} \text{ ohm}$$

The zero sequence network is shown in Fig.



$$Z_0 = Z_{star} + 3\ Z_n = 6 + j8 + 3(j5)$$

$$= (6 + j\ 23) \text{ ohm} = 23.77\ \ 80^\circ.53$$

Problems

P 7.1 Determine the symmetrical components for the three phase currents

$$I_R = 15\ \angle 0^0,\ I_Y = 15\underline{/230^0} \text{ and } I_B = 15\underline{/130^0}\text{A}$$

P 7.2 The voltages at the terminals of a balanced load consisting of three 12 ohm resistors connected in star are

$$V_{RY} = 120\ \angle 0^0\ \text{V}$$

$$V_{YB} = 96.96\ \angle -121.44^0\ \text{V}$$

$$V_{BR} = 108\ \angle 130^0\ \text{V}$$

Assuming that there is no connection to the neutral of the load determine the symmetrical components of the line currents and also from them the line currents.

P 7.3 A 50 Hz turbo generator is rated at 500 MVA 25 KV. It is star connected and solidly grounded. It is operating at rated voltaage and is on no-load. Its reactances are $x_d'' = x_1 = x_2 = 0.17$ and $x_0 = 0.06$ per unit. Find the sub-transient line current for a single line to ground fault when it is disconnected from the system.

P 7.4 Find the subtransient line current for a line-to-line fault on two phases for the generator in problem (7.3)

P 7.5 A 125 MVA, 22 KV turbo generator having $x_d'' = x_1 = x_2 = 22\%$ and $x_0 = 6\%$ has a current limiting reactor of 0.16 ohm in the neutral, while it is operating on no-load at rated voltage a double line-to ground fault occurs on two phases. Find the initial symmetrical r.m.s fault current to the ground.

Questions

7.1 What are symmetrical components ? Explain.

7.2 What is the utility of symmetrical components.

7.3 Derive an expression for power in a 3-phase circuit in terms of symmetrical components.

7.4 What are sequence impedances ? Obtain expression for sequence impedances in a balanced static 3-phase circuit.

7.5 What is the influence of transformer connections in single-phase transformers connected for 3-phase operation.

7.6 Explain the sequence networks for an synchronous generator.

7.7 Derive an expression for the fault current for a single line-to ground fault as an unloaded generator.

7.8 Derive an expression for the fault current for a double-line fault as an unloaded generator.

7.9 Derive an expression for the fault current for a double-line-to ground fault as an unloaded generator.

7.10 Draw the sequence network connections for single-line-to ground fault, double line fault and double line to ground fault conditions.

7.11 Draw the phasor diagrams for

(i) Single-line-to ground fault

(ii) Double-line fault and

(iii) Double-line to ground fault

Conditions as on unloaded generator.

7.12 Explain the effect of prefault currents.

7.13 What is the effect of fault impedance ? Explain.

8 POWER SYSTEM STABILITY

8.1 Elementary Concepts

Maintaining synchronism between the various elements of a power system has become an important task in power system operation as systems expanded with increasing inter connection of generating stations and load centres. The electromechanical dynamic behaviour of the prime mover-generator-excitation systems, various types of motors and other types of loads with widely varying dynamic characteristics can be analyzed through some what oversimplified methods for understanding the processes involved. There are three modes of behaviour generally identified for the power system under dynamic condition. They are

(a) Steady state stability
(b) Transient stability
(c) Dynamic stability

Stability is the ability of a dynamic system to remain in the same operating state even after a disturbance that occurs in the system.

Stability when used with reference to a power system is that attribute of the system or part of the system, which enables it to develop restoring forces between the elements thereof, equal to or greater than the disturbing force so as to restore a state of equilibrium between the elements.

A power system is said to be steady state stable for a specific steady state operating condition, if it returns to the same steady state operating condition following a disturbance. Such disturbances are generally small in nature.

A stability limit is the maximum power flow possible through some particular point in the system, when the entire system or part of the system to which the stability limit refers is operating with stability.

Larger disturbances may change the operating state significantly, but still into an acceptable steady state. Such a state is called a transient state.

The third aspect of stability viz. Dynamic stability is generally associated with excitation system response and supplementary control signals involving excitation system. This will be dealt with later.

Instability refers to a conditions involving loss of 'synchronism' which in also the same as 'falling out of the step' with respect to the rest of the system.

8.2 Illustration of Steady State Stability Concept

Consider the synchronous generator-motor system shown in Fig. 8.1. The generator and motor have reactances X_g and X_m respectively. They are connected through a line of reactance X_e. The various voltages are indicated.

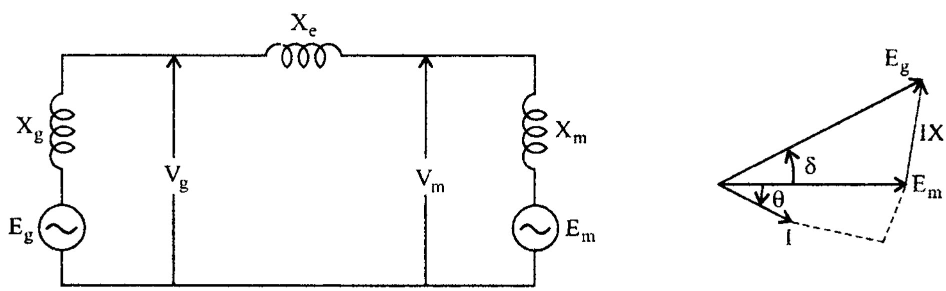


Fig. 8.1

From the Fig. 8.1

$$E_g = E_m + j \times I;$$

$$I = \frac{E_g - E_m}{jX} \quad \text{where } X = X_g + X_e + X_m$$

Power delivered to motor by the generator is

$$P = \text{Re}\,[E\, I^*]$$

$$= \text{Re}\,[E_g \angle\delta]\, \frac{\left[E_g\angle -\delta - E_m\angle 0^\circ\right]}{X\angle -90^\circ}$$

$$= \frac{E_g^2}{X} \text{ Cos } 90^\circ - \frac{E_g E_m}{X} \text{ Cos } (90 + \delta)$$

$$P = \frac{E_g E_m}{X} \text{ Sin } \delta \qquad .....(8.1)$$

P is a maximum when $\delta = 90^\circ$

$$P_{max} = \frac{E_g E_m}{X} \qquad .....(8.2)$$

The graph of P versus δ is called power angle curve and is shown in Fig. 8.2. The system will be stable so long $\frac{dP}{d\delta}$ is positive. Theoretically, if the load power is increased in very small increments from $\delta = 0$ to $\delta = \pi/2$, the system will be stable. At $\delta = \pi/2$. The steady state stability limit will be reached P_{max} is dependent on E_g, E_m and X. Thus, we obtain the following possibilities for increasing the value of p_{max} indicated in the next section.

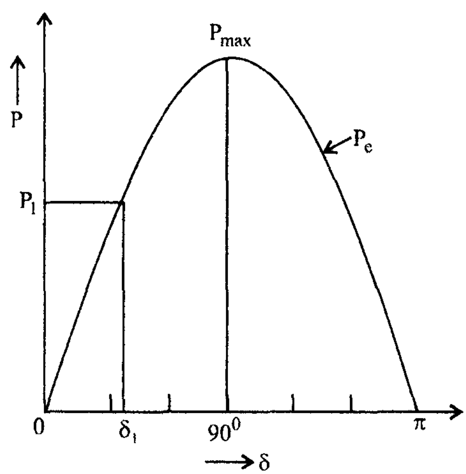


Fig. 8.2

8.3 Methods for Improcessing Steady State Stability Limit

1. Use of higher excitation voltages, thereby increasing the value of Eg.
2. Reducing the reactance between the generator and the motor. The reactance $X = X_g + X_m + X_e$ is called the transfer reactance between the two machines and this has to be brought down to the possible extent.

8.4 Synchronizing Power Coefficient

We have $$P = \frac{E_g\, E_m}{X} \operatorname{Sin} \delta$$

The quantity

$$\frac{dP}{d\delta} = \frac{E_g\, E_m}{X} \operatorname{Cos} \delta \qquad .....(8.3)$$

is called Synchronizing power coefficient or stiffness.

For stable operation $\frac{dP}{d\delta}$, the synchronizing coefficient must be positive.

8.5 Transient Stability

Steady state stability studies often involve a single machine or the equivalent to a few machines connected to an infinite bus. Undergoing small disturbances. The study includes the behaviour of the machine under small incremental changes in operating conditions about an operating point on small variation in parameters.

When the disturbances are relatively larger or faults occur on the system, the system enters transient state. Transient stability of the system involves non-linear models. Transient internal voltage $E_i^{'}$ and transient reactances $X_d^{'}$ are used in calculations.

The first swing of the machine (or machines) that occur in a shorter time generally does not include the effect off excitation system and load-frequency control system. The first swing transient stability is a simple study involving a time space not exceeding one second. If the machine remains stable in the first second, it is presumed that it is transient stable for that disturbances. However, where disturbances are larger and require study over a longer period beyond one second, multiswong studies are performed taking into effect the excitation and turbine-generator controls. The inclusion of any control system or supplementary control depends upon the nature of the disturbances and the objective of the study.

8.6 Stability of a Single Machine Connected to Infinite Bus

Consider a synchronous motor connected to an infinite bus. Initially the motor is supplying a mechanical load P_{mo} while operating at a power angle δ_0. The speed is the synchronous speed ω_s. Neglecting losses power in put is equal to the mechanical load supplied. If the load on the motor is suddenly increased to p_{m1}, this sudden load demand will be met by the motor by giving up its stored kinetic energy and the motor, therefore, slows down. The torque angle δ increases from δ_0 to δ_1 when the electrical power supplied equals the mechanical power demand at b as shown in Fig. 8.3. Since, the motor is decelerating, the speed, however, is less than N_s at b. Hence, the torque 'angle δ' increases further to δ_2 where the electrical power P_e is greater than P_{m1}, but N = Ns at point c. At this point c further increase of δ is arrested as $P_e > P_{m1}$ and N = Ns. The torque angle starts decreasing till δ_1 is reached at b but due to the fact that till point b is reached P_e is still greater than P_{m1}, speed is more than N_s. Hence, δ decreases further till point a is reacted where $N = N_s$ but $P_{m1} > P_e$. The cycle of oscillation continues. But, due to the damping in the system that includes friction and losses, the rotor is brought to the new operating point b with speed $N = N_s$.

In Fig. 8.3 area 'abd' represents deceleration and area bce acceleration. The motor will reach the stable operating point b only if the accelerating energy A_1 represented by bce equals the decelerating energy A_2 represented by area abd.

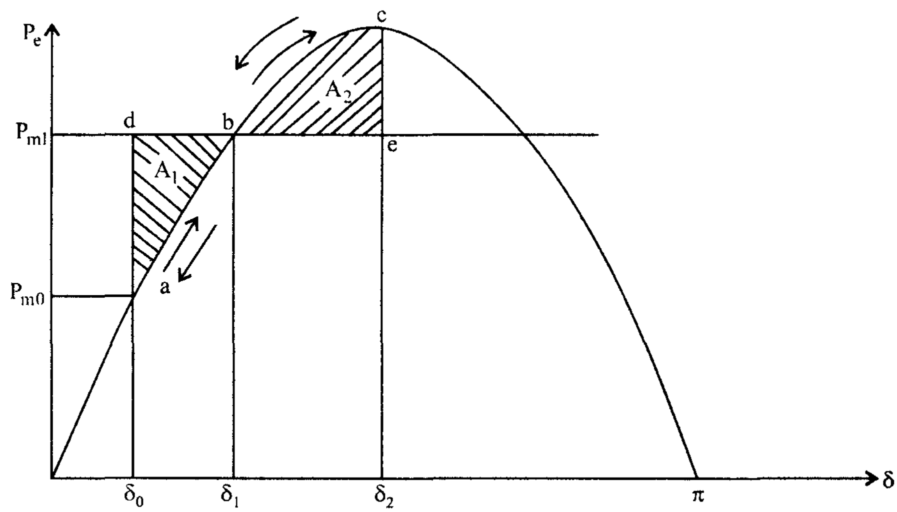


Fig. 8.3 Stability of synchronous motor connected to infinite bus.

8.7 The Swing Equation

The interconnection between electrical and mechanical side of the synchronous machine is provided by the dynamic equation for the acceleration or deceleration of the combined-prime mover (turbine) – synchronous machine roter. This is usually called swing equation.

The net torque acting on the rotor of a synchronous machine

$$T = \frac{WR^2}{g}\alpha \qquad .....(8.4)$$

where T = algebraic sum of all torques in Kg-m.

a = Mechanical angular acceleration

WR^2 = Moment of Inertia in kg-m^2

Electrical angle $\vartheta_e = \vartheta_m \cdot \frac{P}{Z}$ .....(8.5)

Where ϑ m is mechanical angle and P is the number of poles

The frequency $f = \frac{PN}{120}$ .....(8.6)

Where N is the rpm.

$$f = \frac{P}{2}\left(\frac{rpm}{60}\right)$$

$$\frac{60f}{rpm} = \frac{P}{2}$$

$$\vartheta_e = \left(\frac{60f}{rpm}\right)\vartheta_m \qquad .....(8.7)$$

The electrical angular position d in radians of the rotor with respect to a synchronously rotating reference axis is $\delta_e = \vartheta_e - \omega_0 t$ .....(8.8)

Where w_0 = rated synchronous speed in rad./sec

And t = time in seconds (Note : $\delta + \omega_0 t = \vartheta_e$)

The angular acceleration taking the second derivative of eqn. (8.8) is given by

$$\frac{d^2\delta}{dt^2} = \frac{d^2\vartheta_e}{dt^2}$$

From eqn. (8.7) differentiating twice

$$\frac{d^2\vartheta_e}{dt^2} = \left(\frac{60f}{rpm}\right)\frac{d^2\vartheta_m}{dt^2}$$

$$\therefore \qquad \frac{d^2\vartheta_m}{dt^2} = \alpha = \left(\frac{rpm}{60f}\right)\frac{d^2\vartheta_e}{dt^2}$$

From eqn. (8.4)

$$T = \frac{WR^2}{g}\left(\frac{rpm}{60f}\right)\frac{d^2\vartheta_e}{dt^2} = \frac{WR^2}{g}\left(\frac{rpm}{60f}\right)\frac{d^2\delta}{dt^2} \qquad .....(8.9)$$

Let the base torque be defined as

$$T_{Base} = \frac{\text{Base KVA}}{2\pi\left(\frac{rpm}{60}\right)} \qquad .....(8.10)$$

torque in per unit $\quad T\ \text{p.u.} = \frac{T}{T_{Base}} = \frac{WR^2}{g}\left(\frac{rpm}{60f}\right)\frac{d^2\delta}{dt^2}\cdot\frac{2\pi\left(\frac{rpm}{60}\right)}{\text{baseKVA}}$

$$= \frac{WR^2}{g}\left(\frac{rpm}{60}\right)^2\frac{2\pi}{f}\cdot\frac{1}{\text{baseKVA}}\frac{d^2\delta}{dt^2} \qquad .....(8.11)$$

Kinetic energy K.E. $\quad = \frac{1}{2}\frac{WR^2}{g}\omega_0^{\,2} \qquad .....(8.12)$

Where $\quad \omega_0 = 2\pi\frac{rpm}{60}$

Defining $\quad H = \frac{\text{kinetic energy at rated speed}}{\text{base KVA}}$

$$= \underbrace{\frac{1}{2}\frac{WR^2}{g}\left(2\pi\frac{rpm}{60}\right)^2}_{\text{K.E at rated speed}}\frac{1}{\text{base KVA}}$$

$$T = \frac{H}{\pi f}\cdot\frac{d^2\delta}{dt^2} \qquad .....(8.13)$$

The torque acting on the rotor of a generator includes the mechanical input torque from the prime mover, torque due to rotational losses [(i.e. friction, windage and core loss)], electrical output torque and damping torques due to prime mover generator and power system.

The electrical and mechanical torques acting on the rotor of a motor are of opposite sign and are a result of the electrical input and mechanical load. We may neglect the damping and rotational losses, so that the accelerating torque.

$$T_a = T_m - T_e$$

Where T_e is the air-gap electrical torque and T_m the mechanical shaft torque.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = T_m - T_e \qquad .....(8.14)$$

(i.e.,)
$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(T_m - T_e) \qquad .....(8.15)$$

Torque in per unit is equal to power in per unit if speed deviations are neglected. Then

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_m - P_e) \qquad .....(8.16)$$

The eqn. (8.15) and (8.16) are called swing equations.

It may be noted, that, since $\delta = \vartheta - \omega_0 t$

$$\frac{d\delta}{dt} = \frac{d\vartheta}{dt} - \omega_0$$

Since the rated synchronous speed in rad/sec is $2\pi f$

$$\frac{d\vartheta}{dt} = \frac{d\delta}{dt} + \omega_0$$

we may put the equation in another way.

Kinetic Energy $= \frac{1}{2}I\omega^2$ joules

The moment of inertia I may be expressed in Joule – $(Sec)^2/(rad)^2$ since ω is in rad/sec. The stored energy of an electrical machine is more usually expressed in mega joules and angles in degrees. Angular momentum M is thus described by mega joule – sec. per electrical degree

$$M = I.\omega$$

Where ω is the synchronous speed of the machine and M is called inertia constant. In practice ω is not synchronous speed while the machine swings and hence M is not strictly a constant.

The quantity H defined earlier as inertia constant has the units mega joules.

$$H = \frac{\text{stored energy in mega joules}}{\text{machine rating in mega volt ampers(G)}} \qquad .....(8.17)$$

but stored energy $= \frac{1}{2}I\omega^2 = \frac{1}{2}M\omega$

In electrical degrees $\omega = 360f\ (= 2\pi f)$ .....(8.18)

$$GH = \frac{1}{2}M(360f) = \frac{1}{2}M2\pi f = M\pi f$$

$$M = \frac{GH}{\pi f} \text{ mega joule -- sec/elec degree} \quad .....(8.19)$$

In the per unit systems $M = \dfrac{H}{\pi f}$ .....(8.20)

So that $$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_m - P_e) \quad .....(8.21)$$

which may be written also as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

This is another form of swing equation. .....(8.22)

Further $$P_e = \frac{EV}{X}\text{Sin}\delta$$

So that $$M\frac{d^2\delta}{dt^2} = P_m - \frac{EV}{X}\text{Sin}\delta \quad .....(8.23)$$

with usual notation.

8.8 Equal Area Criterion and Swing Equation

Equal area criterion is applicable to single machine connected to infinite bus. It is not directly applicable to multi machine system. However, the criterion helps in understanding the factors that influence transient stability.

The swing equation connected to infinite bus is given by

$$\frac{H}{\pi f}\frac{d^r\delta}{d t^2} = P_m - P_e = P_a \quad .....(8.24)$$

or $$\frac{2H}{w_s}\frac{d^r\delta}{d t^2} = P_m - P_e = P_a \quad .....(8.25)$$

Also $$M \frac{d^2\delta}{dt^2} = P_a \qquad .....(8.26)$$

Now as t increases to a maximum value δmax where $\frac{d\delta}{dt} = 0$, Multilying eqn (8.11) on both sides by $2\frac{d\delta}{dt}$ we obtain

$$2\frac{d^2\delta}{dt^2}\frac{d\delta}{dt} = \frac{P_a}{M}2\frac{d\delta}{dt}$$

Integrating both sides

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int P_a \, d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta}$$

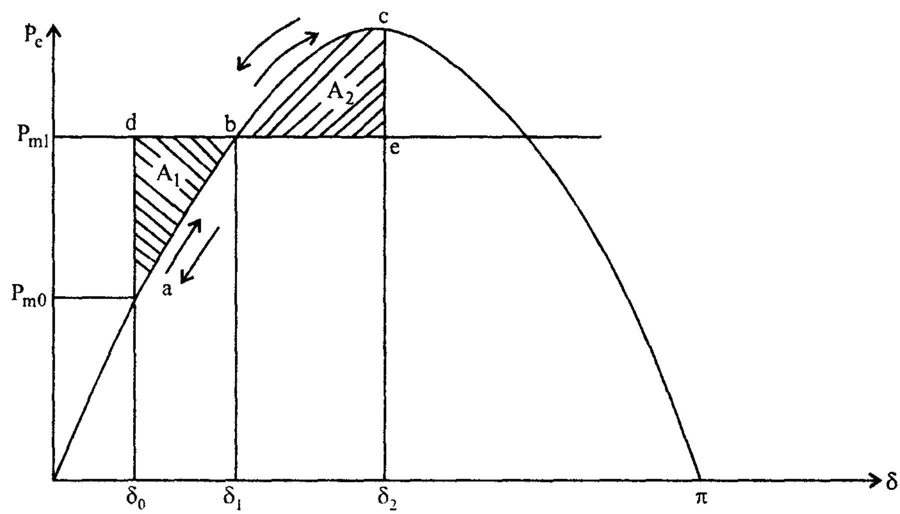


Fig. 8.4 Equal area criterion.

δ_o is the initial rotor angle from where the rotor starts swinging due to the disturbance.

For Stability $\frac{d\delta}{dt} = 0$

Hence, $$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \, d\delta} = 0$$

i.e., $$\int_{\delta_0}^{\delta} P_a \, d\delta \qquad \int_{\delta_0}^{\delta} (P_m - P_e)\, d\delta = 0$$

The system is stable, if we could locate a point c on the power angle curve such that areas A_1 and A_2 are equal. Equal area criterion states that whenever, a disturbance occurs, the acclerating and decelerating energies involved in swinging of the rotor of the synchronous machine must equal so that a stable operating point (such as b) could be located.

$A_1 - A_2 = 0$ means that,

$$\int_{\delta_0}^{\delta_1}(P_{m1} - P_e)d\delta - \int_{\delta_1}^{\delta_2}(P_e - P_m)\, d\delta = 0$$

But $$P_e = P_{max} \sin \delta$$

$$\int_{\delta_0}^{\delta_1}\left(P_{m_1} - P_{max} \sin \delta\right)d\delta - \int_{\delta_1}^{\delta_2}\left(P_{max} \sin \delta - P_{m_1}\right) d\delta = 0$$

$$P_{m_1} (\delta_1 - \delta_0) + p_{max} (\cos \delta_1 - \cos \delta_0)$$

$$P_{max} (\cos \delta_1 - \cos \delta_2) + P_{m1} (\delta_2 - \delta_1) = 0$$

i.e., $$P_{m_1} [\delta_2 - \delta_0] = P_{max} [\cos \delta_0 - \cos \delta_2)$$

$$\cos \delta_0 - \cos \delta_2 = \frac{P_{m_1}}{P_{max}} [\delta_2 - \delta_0]$$

But $$\frac{P_{m_1}}{P_{max}} = \frac{P_{max} \sin \delta_1}{P_{max}} = \sin \delta_1$$

Hence $$(\cos \delta_c - \cos \delta_2) = \sin \delta_1 [\delta_2 - \delta_0] \qquad \text{.....(8.27)}$$

The above is a transcendental equation and hence cannot be solved using normal algebraic methods.

8.9 Transient Stability Limit

Now consider that the change in P_m is larger than the change shown in Fig. 8.5. This is illustrated in Fig. 8.5.

In the case $A_1 > A_2$. That is, we fail to locate an area A_2 that is equal to area A_1. Then, as stated the machine will loose its stability since the speed cannot be restored to N_s.

Between these two cases of stable and unstable operating cases, there must be a limiting case where A_2 is just equal to A_1 as shown in Fig. 8.6. Any further increase in P_{m1} will cause

A_2 to be less than A_1. $P_{m1} - P_{m0}$ in Fig. 8.6 is the maximum load change that the machine can sustain synchronism and is thus the transient stability limit.

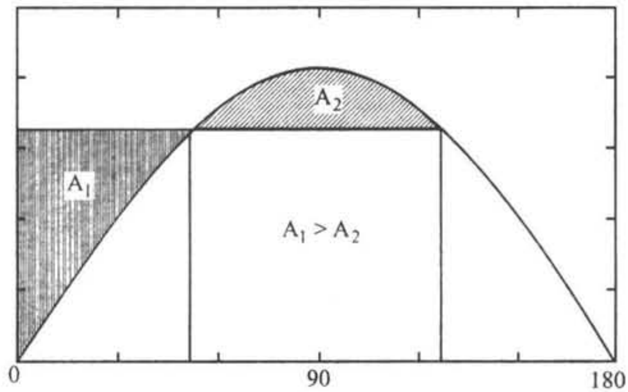


Fig. 8.5 Unstable system (A_1 > A_2).

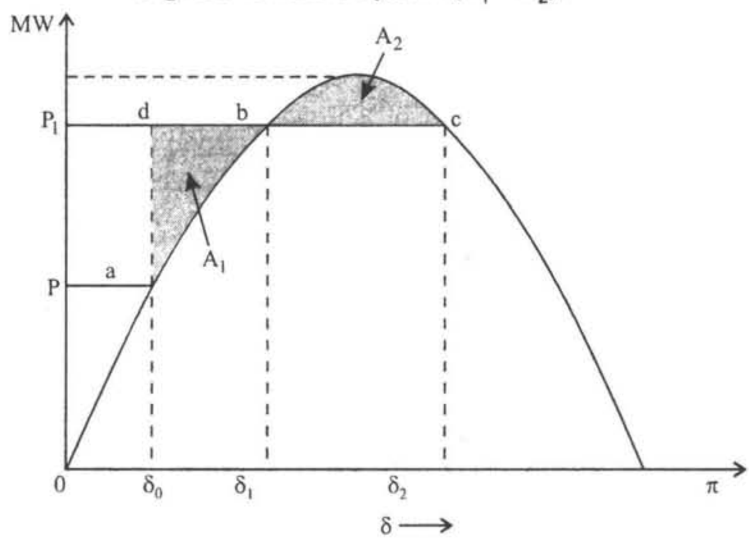


Fig. 8.6 Transient stability limit.

8.10 Frequency of Oscillations

Consider a small change in the operating angle δ_0 due to a transient disturbance by $\Delta\delta$. Corresponding to this we can write

$$\delta = \delta^o + \Delta\delta$$

and $$P_e = P^o_e + \Delta P_e$$

where ΔP_e is the change in power and P^o_e, the initial power at δ^o

$$(P_e + \Delta P_e) = P_{max} \sin \delta^o + P_{max} \cos \delta^o) \Delta\delta$$

Also, $P_m = P_e^o = P_{max} \sin \delta^o$

Hence, $(P_m - P_e^o + \Delta P_e)$

$$= P_{max} \sin \delta^o - [P_{max} \sin \delta^o - [P_{max} \cos \delta^o) \Delta\delta]$$

$$= (P_{max} \text{ as } \delta^o) \Delta\delta$$

$d\dfrac{P_e}{d\delta}$ is the synchronizing coefficient S.

The swing equation is

$$\frac{2H}{\omega}\frac{d^2\delta^o}{dt^2} = P_a = P_m - P_e^o$$

Again,
$$\frac{2H}{\omega}\frac{d^2\left(\delta^o + \Delta\delta\right)}{dt^2} = P_m - (P_e^o + \Delta\, P_e)$$

Hence,
$$\frac{2H}{\omega}\frac{d^2(\Delta\delta)}{dt^2} = -P_{max} (\cos \delta^o) \,.\, \Delta\delta = -S^o \,.\, \Delta\delta$$

where S^o is the synchronizing coefficient at P_e^o.

Therefore,
$$\frac{d^2(\Delta\delta)}{dt^2} + \left(\frac{\omega S^o}{2H}\right) \Delta\delta = 0$$

which is a linear second-order differential equation. The solution depends upon the sign of δ^o. If δ^o is positive, the equation represents simple harmonic motion.

The frequency of the undamped oscillation in

$$\omega_m = \sqrt{\frac{\omega\, \delta^o}{2\, H}} \qquad .....(8.28)$$

The frequency f is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{\omega\, \delta^o}{2\, H}} \qquad .....(8.29)$$

Transient stability and fault clearance time consider the electrical power system shown is Fig. 8.7. If a 3-phase fault occurs near the generator bus on the radial line connected to it, power transmitted over the line to the infinite bus will become zero instantaneously. The mechanical input power P_m remains constant. Let the fault be cleared at $\delta = \delta_1$. All the mechanical input energy represented by area a b c d = A_1, will be utilized in accelerating the rotor from δ_o to δ_1. Fault clearance at δ_1 angle or point c will shift the operating point from c to e instantaneously on the P - δ curve. At point f, an area A_2 = d e f g is obtained which is equal to A_1 Fig. 8.8.. The rotor comes back from f and finally settles down at 'a' where $P_m = P_e$. δ_1 is called the clearing angle and the corresponding time t_1 is called the critical clearing time t_c for the fault from the inception of it at δ_o.

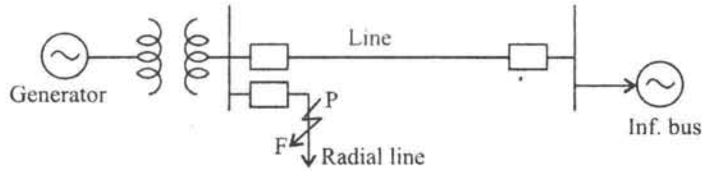


Fig. 8.7

8.11 Critical Clearing Time and Critical Clearing Angle

If, in the previous case, the clearing time is increased from t_1 to t_c such that δ_1 is δ_c as shown in Fig. 8.9. Where A_1 is just equal to A_2. Then, any furter increase in the fault cleaing time t_1

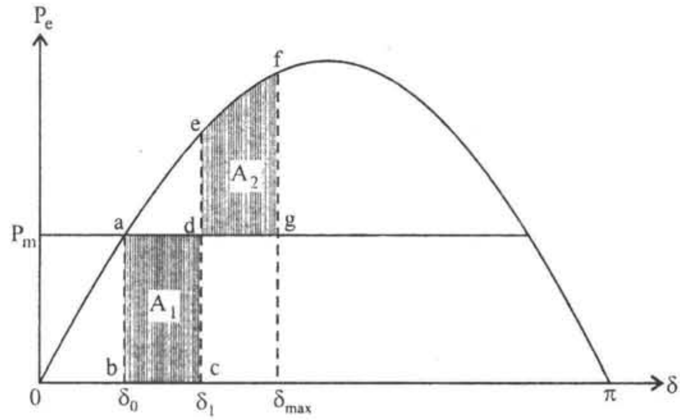


Fig. 8.8

beyond t_c, would not be able to enclosed an area A_2 equal to A_1. This is shown in Fig. 8.10. Beyond δ_c, A2 starts decreasing. Fault clearance cannot be delayed beyond t_c. This limiting fault clearance angle δ_c is called critical clearing angle and the corresponding time to clear the fault is called critical clearing time t_c.

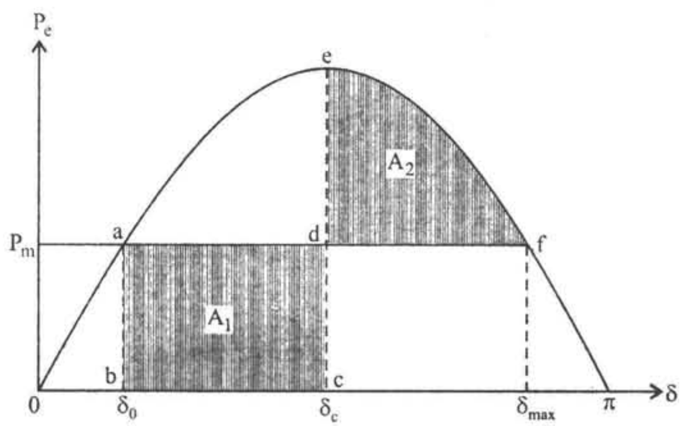


Fig. 8.9

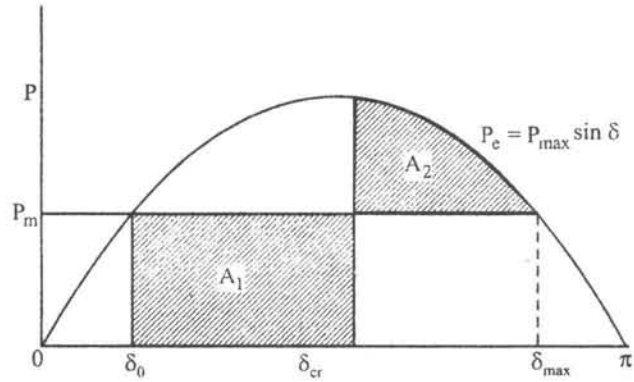


Fig. 8.10

From Fig. 8.9

$$\delta_{max} = \pi - \delta^o$$

$$P_m = P_{max} \sin \delta_o$$

$$A_1 = \int_{\delta_o}^{\delta_c} (P_m - 0)\, d\delta = P_m \left[\delta_c - \delta_o\right]$$

$$A_2 = \int_{\delta_c}^{\delta_{max}} \left(P_{max} \sin\delta - P_m\right) d\delta$$

$$= P_{max} (\cos \delta_c - \cos \delta_{max}) - P_m (\delta_{max} - \delta_c)$$

$A_1 = A_2$ gives

$$\cos \delta_c - \cos \delta_m = \frac{P_m}{P_{max}} [\delta_{max} - \delta_o]$$

$$\cos \delta_c = \frac{P_m}{P_{max}} [(\pi - \delta_o) - \delta_o] + \cos (\pi - \delta_o)$$

$$= \frac{P_m}{P_{max}} [(\pi - 2\delta_o)] - [\cos \delta_o]$$

$$\delta_c = \cos^{-1} \left[\frac{P_m}{P_{max}} (\pi - 2\delta_o) - (\cos \delta_o)\right] \qquad .....(8.30)$$

During the period of fault the swing equation is given by

$$\frac{d^r\delta}{d\, t^r} = \frac{\pi\, f}{H} (P_m - P_e). \text{ But since } P_e = 0$$

During the fault period

$$\frac{d^r\delta}{d\,t^r} = \frac{\pi\,f}{H}\;P_m$$

Integrating both sides $\int_0^t \frac{d^r\delta\;dt}{dt^2} = \int_0^t \frac{\pi f}{H}\;P_m\;dt$

$$\frac{d\delta}{dt} = \frac{\pi f}{H}\;P_m t$$

and integrating once again

$$\delta_c = \frac{\pi f}{2H}\;P_m t^2 + K$$

At $t = 0$; $\delta = \delta_o$, Hence $K = \delta_o$

Hence $$\delta_c = \frac{\pi f}{2H}\;P_m t^2 + \delta_o \qquad \text{.....(8.31)}$$

Hence the critical cleaning time $t_c = \sqrt{\frac{2\,H\,(\delta_o - \delta_c)}{P_m\;\pi.f}}$ sec. .....(8.32)

8.12 Fault on a Double-Circit Line

Consider a single generator or generating station suplying power to a load or an infinite bus through a double circuit line as shown in Fig. 8.11.

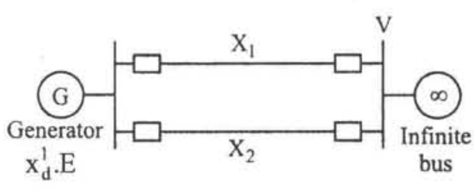


Fig. 8.11 Double-circuit line and fault.

The eletrical power transmitted is given by $P_{e_{12}} = \frac{EV}{x_d' + x_{12}}$ sin δ where $\frac{1}{x_{12}} = \frac{1}{x_1} + \frac{1}{x_2}$

and x_d' is the transient reactance of the generator. Now, if a fault occurs on line 2 for example, then the two circuit breakers on either side will open and disconnect the line 2. Since, $x_1 > x_{12}$ (two lines in parallel), the P - δ curve for one line in operation is given by

$$P_{e1} = \frac{EV}{x_d' + x_1}\;\sin\delta$$

will be below the P - δ curve $P_{e_{12}}$ as shwon in Fig. 8.12. The operating point shifts from a to b on **P-δ curve** P_{e_1} and the rotor accelerates to poin c where $\delta = \delta_1$. Since the rotor speed is not synchronous, the rotor decelerates till point d is reched at $\delta = \delta_2$ so that area A_1 (= area a b c) is equal to area A_2 (= area c d e). The rotor will finally settle down at point c due to damping. At point c

$$P_m = P_{e_1}$$

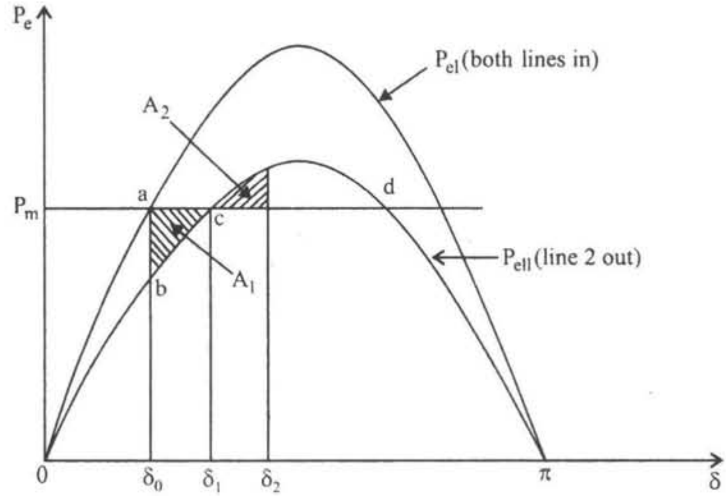


Fig. 8.12

8.13 Transient Stability When Power is Transmitted During the Fault

Consider the case where during the fault period some load power is supplied to the load or to the infinite bus. If the P-δ curve during the fault is represented by curve 3 in Fig. 8.13.

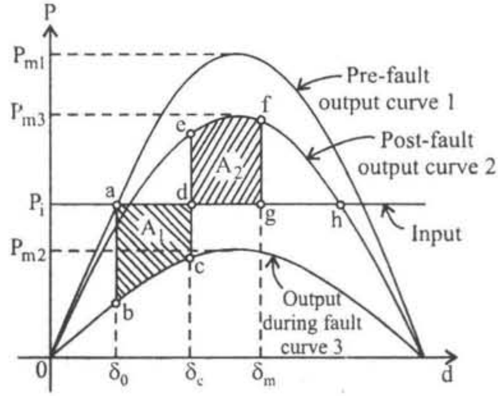


Fig. 8.13

Upon the occurrence of fault, the operating point moves from a to b on the during the fault curve 3. When the fault is cleared at $\delta = \delta_1$, the operating point moves from b to c along the curve P_{e3} and then shifts to point e. If area d e f g e could equal area a b c d ($A_2 = A_1$) then the system will be stable.

If the fault clearance is delayed till $\delta_1 = \delta_c$ as shown in Fig. 8.14 such that area a b c d (A_1) is just equal to and e d f (A_2) then

$$\int_{\delta_0}^{\delta_c} (P_{max} \sin \delta - P_m)\, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max} \sin \delta - P_m)\, d\delta$$

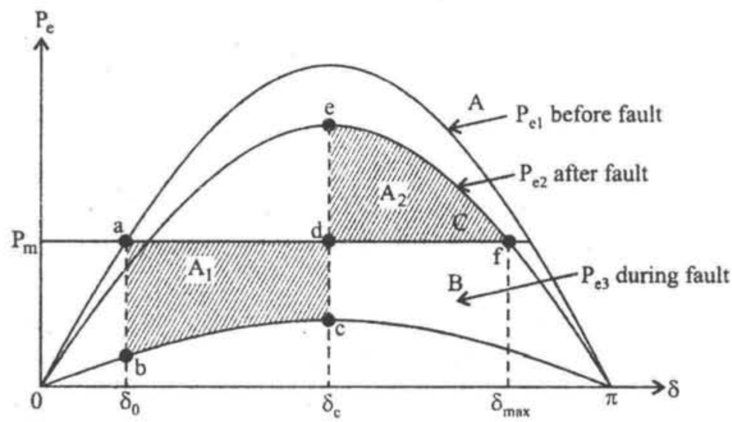


Fig. 8.14 Critical clearing angle-power transmitted during fault.

It is clear from the Fig. 8.14 that $\delta_{max} = \pi - \delta_o = \pi - \sin^{-1} \dfrac{P_m}{P_{max\,2}}$

Integrating

$$(P_m \cdot \delta + P_{max} \cos \delta) \Big|_{\delta_0}^{\delta_c} + (P_{max\,2} \cos \delta - P_m . \delta) \Big|_{\delta_c}^{\delta_{max}} = 0$$

$$P_m (\delta_c - \delta_o) + P_{max}\, 3 (\cos \delta_c - \cos \delta_o) + P_m (\delta_{max} - \delta_c)$$
$$+ P_{max2} (\cos \delta_{max} - \cos \delta_c) = 0$$

$$\cos \delta_c = \frac{P_m (\delta_{max} - \delta_o) - P_{max\,3} \cos \delta_o + P_{max\,2} \cos \delta_{max}}{P_{max\,2} - P_{max\,3}} \qquad .....(8.33)$$

The angles are all in radians.

8.14 Fault Clearance and Reclosure in Double-Circuit System

Consider a double circuit system as in section 8.12. If a fault occurs on one of the lines while supplying a power of P_{mo}; as in the previous case then an area $A_2 = A_1$ will be located and the operating characteristic changes from pre-fault to during the fault . If the faulted line is removed then power transfer will be again shifted to post-fault characteristic where line 1 only is in operation. Subsequently, if the fault is cleared and line 2 is reclosed, the operation once again shifts back to pre-fault characteristic and normalcy will be restored. For stable operation area A_1 (= area abcd) should be equal to area A_2 (= area defghk). The maximum angle the rotor swings is δ_3. For stability δ_2 should be lessthan δ_m. The illustration in Fig. 8.15 assumes fault clearance and instantaness reclosure.

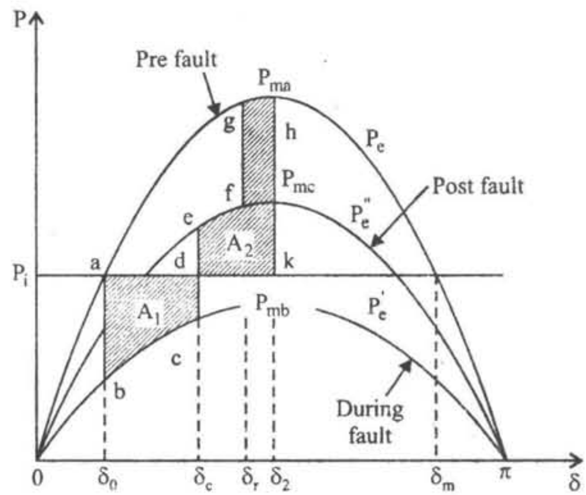


Fig. 8.15 Fault clearance and reclosing.

8.15 Solution to Swing Equation Step-by-Step Method

Solution to swing equation gives the change in δ with time. Uninhibited increase in the value of δ will cause instability. Hence, it is desired to solve the swing equation to see that the value of δ starts decreasing after an initial period of increase, so that at some later point in time, the machine reaches the stable state. Gnerally 8, 5, 3 or 2 cycles are the times suggested for circuit breaker interruption after the fault occurs. A variety of numerical step-by-step methods are available for solution to swing equation. The plot of δ versus t in seconds is called the swing curve. The step-by-step method suggested here is suitable for hand calculation for a single machine connected to system.

Since δ is changing continuously, both the assumption are not true. When Δt is made very, small, the calculated values become more accurate.

Let the time intervals be Δt

Consider, (n – 2), (n – 1) and n^{th} intervals. The accelerating power P_a is computed at the end of these intervals and plotted at circles in Fig. 8.16 (a).

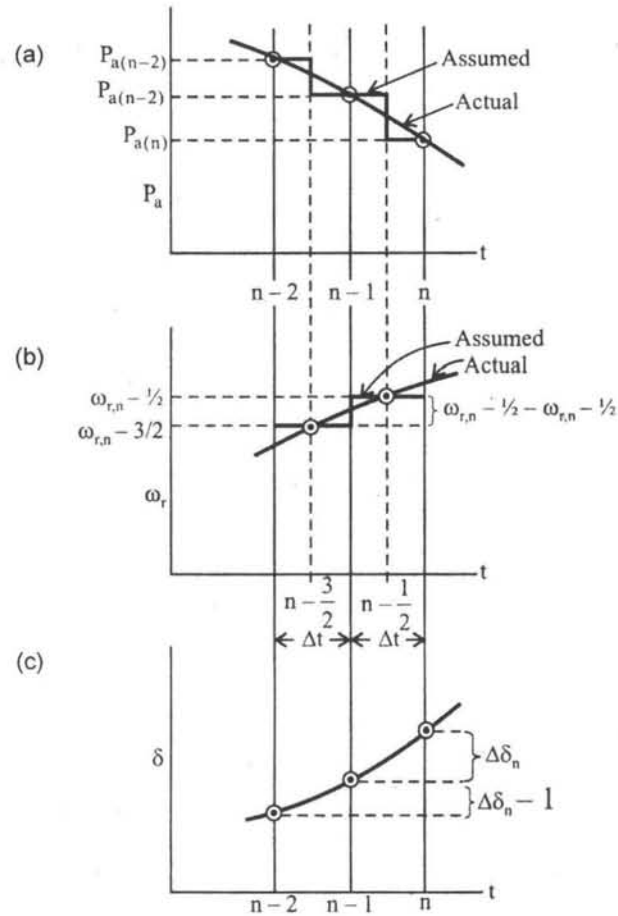


Fig. 8.16 Plotting swing curve.

Note that these are the beginnings for the next intervals viz., (n – 1), n and (n + 1). P_a is kept constant between the mid points of the intervals.

Likewise, w_r, the difference betwen w and w_s is kept constant throughout the interval at the value calculated at the mid point. The angular speed therefore is assumed to change between (n – 3/2) and (n – 1/2) ordinates

we know that $\Delta\omega = \dfrac{d\omega}{dt} \cdot \Delta t$

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Likewise, w_r, the difference betwen w and w_s is kept constant throughout the interval at the value calculated at the mid point. The angular speed therefore is assumed to change between (n – 3/2) and (n – 1/2) ordinates

we know that $\Delta\omega = \frac{d\omega}{dt} . \Delta t$

Hence $$\omega_{r(n-1)} - \omega_{r(n-3/2)} = \frac{d^2\delta}{dt^2}.\Delta t = \frac{180f}{H} P_{a(n-1)}.\Delta t \qquad .....(8.34)$$

Again change in δ

$$\Delta\delta = \frac{d\delta}{dt} . \Delta t$$

i.e., $$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \omega_{r(n-3/2)} . \Delta t \qquad .....(8.35)$$

for (n – 1)th interval

and $$\Delta\delta_n = \delta_n - \delta_{n-1} = \omega_{r(n-1/2)} . \Delta t \qquad .....(8.36)$$

From the two equations (8.16) and (8.15) we obtain

$$\Delta\delta_n = \delta_{n-1} + \left(\frac{180 f}{H}\right) \Delta t^2 . P_{a(n-1)} \qquad .....(8.37)$$

Thus, the plot of δ with time increasing after a transient disturbance has occured or fault takes place can be plotted as shown in Fig. 8.16 (c).

8.16 Factors Affecting Transient Stability

Transient stability is very much affected by the type of the fault. A three phase dead short circuit is the most severe fault; the fault severity decreasing with two phase fault and single line-to ground fault in that order.

If the fault is farther from the generator the severity will be less than in the case of a fault occurring at the terminals of the generator.

Power transferred during fault also plays a major role. When, part of the power generated is transferred to the load, the accelerating power is reduced to that extent. This can easily be understood from the curves of Fig. 8.16.

Theoretically an increase in the value of inertia constant M reduces the angle through which the rotor swings farther during a fault. However, this is not a practical proposition since, increasing M means, increasing the dimensions of the machine, which is uneconomical. The dimensions of the machine are determined by the output desired from the machine and stability cannot be the criterion. Also, increasing M may interfere with speed governing system. Thus looking at the swing equations

$$M \frac{d^2\delta}{dt^2} = P_a = P_m - P_e = P_m - \frac{EV}{X_{12}} \text{Sin}\delta$$

the possible methods that may improve the transient stability are :

(i) Increase of system voltages, and use of automatic voltage regulators.

(ii) Use of quick response excitation systems

(iii) Compensation for transfer reactance X_{12} so that P_e increases and $P_m - P_e = P_a$ reduces.

(iv) Use of high speed circuit breakers which reduce the fault duration time and hence the acclerating power.

When faults occur, the system voltage drops. Support to the system voltages by automatic voltage controllers and fast acting excitation systems will improve the power transfer during the fault and reduce the rotor swing.

Reduction in transfer reactance is possible only when parallel lines are used in place of single line or by use of bundle conductors. Other theoretical methods such as reducing the spacing between the conductors and increasing the size of the conductors are not practicable and are uneconomical.

Quick opening of circuit breakers and single pole reclosing is helpful. Since majority of the faults are line-to-ground faults selective single pole opening and reclosing will ensure transfer of power during the fault and improve stability.

8.17 Dynamic Stability

Consider a synchronous machine with terminal voltage V_t. The voltage due to excitation acting along the quadrature axis is E_q and E_q^1 is the voltage along this axis. The direct axis rotor angle with respect to a synchronously revolving axis is d. If a load change occurs and the field current I_f is not changed then the various quantities mentioned change with the real power delivered P as shown in Fig. 8.17 (a).

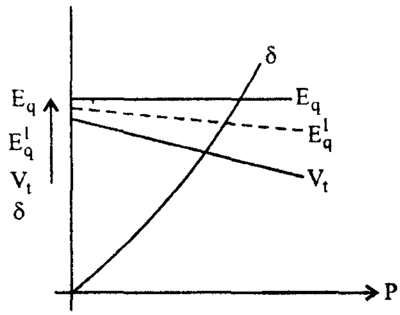


Fig. 8.17 (a)

In case the field current I_f is changed such that the transient flux linkages along the q-axis E_q^1 proportional to the field flux linkages is maintained constant the power transfer could be increased by 30-60% greater than case (a) and the quantities for this case are plotted in Fig. 8.17 (b).

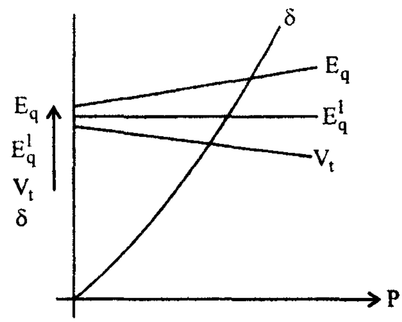


Fig. 8.17 (b)

If the field current I_f is changed alongwith P simultaneously so that V_t is maintained constant, then it is possible to increase power delivery by 50-80% more than case (a). This is shown in Fig. 8.17 (c).

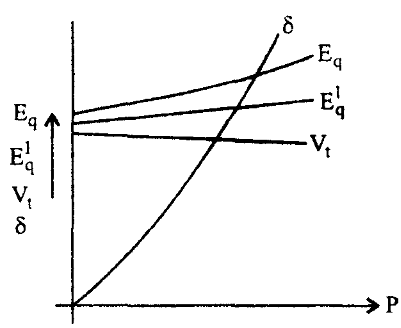


Fig. 8.17 (c)

It can be concluded from the above, that excitation control has a great role to play in power system stability and the speed with which this control is achieved is very important in this context.

Note that $P_{max} = \frac{E.V}{X}$

and increase of E matters in increasing P_{max}.

In Russia and other countries, control signals utilizing the derivatives of output current and terminal voltage deviation have been used for controlling the voltage in addition to propostional control signals. Such a situation is termed 'forced excitation' or 'forced field control'. Not only the first derivatives of ΔI and ΔV are used, but also higher derivatives have been used for voltage control on load changes.

There controller have not much control on the first swing stability, but have effect on the operation subsequent swings.

This way of system control for satisfactory operation under changing load conditions using excitation control comes under the purview of dynamci stability.

Power System Stabilizer

An voltage regulator in the forward path of the exciter-generator system will introduce a damping torque and under heavy load contions this damping torque may become ndegative. This is a situation where dynamic in stability may occur and casue concern. It is also observed that the several time constants in the forward path of excitation control loop introduce large phase lag at low frequencies just baoe the natural frequency of the excitation system.

To overcome there effects and toi improve the damping, compensating networks are introduced to produce torque in phase with the speed.

Such a network is called "Power System Stabilizer" (PSS).

8.18 Node Elimination Methods

In all stability studies, buses which are excited by internal voltages of the machines only are considered. Hence, load buses are eliminated. As an example consider the system shown in Fig. 8.18.

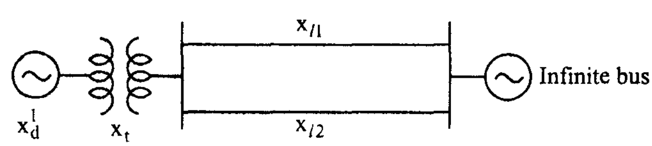


Fig. 8.18

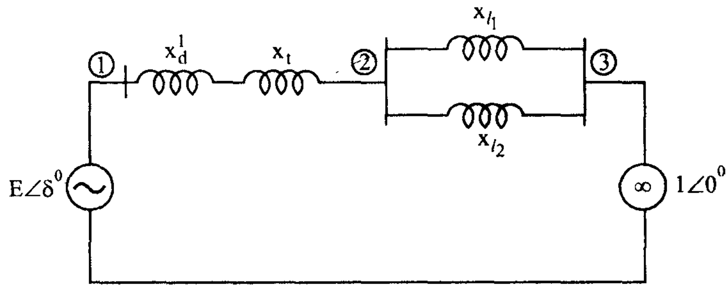


Fig. 8.18 (a)

The transfer reactance between the two buses (1) and (3) is given by

$$X_{13} = j\ x_d^1 + x_t + x_{l1\ l2}$$

Where $$\frac{1}{x_{l_1 l_2}} = \frac{1}{x_{l_1}} + \frac{1}{x_{l_2}}$$

If a fault occurs an all the phases on one of the two parallel lines, say, line 2, then the reactance diagram will become as shown in Fig. 8.18(b).

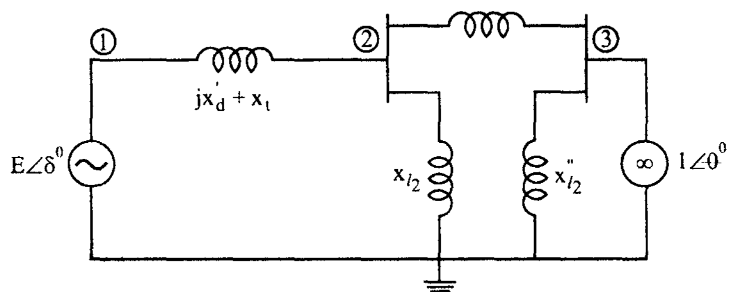


Fig. 8.18 (b)

Since, no source is connected to bus (2), it can be eliminated. The three reactances between buses (1), (2), (3) and (g) become a star network, which can be converted into a delta network using the standard formulas. The network willbe modified into Fig. 8.18 (c).

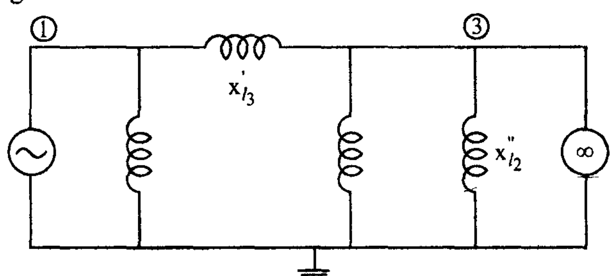


Fig. 8.18 (c)

X_{13}^1 is the transfer reactance between buses (1) and (3).

Consider the same example with delta network reproduced as in Fig. 8.18 (d).

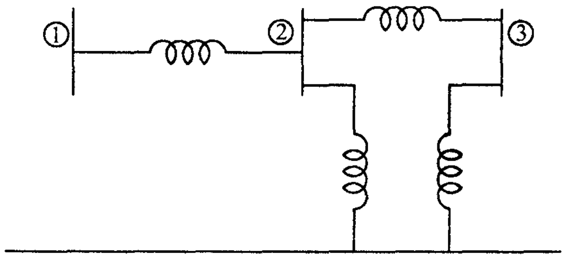


Fig. 8.18 (d)

For a three bus system, the nodal equations are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Since no source is connected to bus (2), it can be eliminated.

i.e., I_2 has to be mode equal to zero

$$Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 = 0$$

Hence $$V_2 = -\frac{Y_{21}}{Y_{22}} V_1 - \frac{Y_{23}}{Y_{22}} V_3$$

This value of V_2 can be substituted in the other two equation of ( ) so that V_2 is eliminated

$$I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3$$

$$= Y_{11} V_1 + Y_{12} \left[\frac{-Y_{21}}{Y_{22}} V_1\right] + Y_{13}\left[\frac{Y_{23}}{Y_{22}} V_3\right] + Y_{13} V_3$$

$$I_3 = Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3$$

$$= Y_{31} V_1 + Y_{32} \left[\frac{-Y_{21}}{Y_{22}} V_1 - \frac{Y_{23}}{Y_{22}} V_3\right] + Y_{33} V_3$$

Thus Y_{BUS} changes to $\begin{bmatrix} Y_{11}^1 & Y_{12}^1 \\ Y_{31}^1 & Y_{33}^1 \end{bmatrix}$

where $$Y_{11}^1 = Y_{11} - Y_{12} \frac{Y_{21}}{Y_{22}} \text{ and } Y_{13}^1 = Y_{31}^1 = Y_{13} - \frac{Y_{23} Y_{12}}{Y_{22}}$$

$$Y_{33}^1 = Y_{33} - \frac{Y_{32}.Y_{23}}{Y_{22}}$$

Worked Examples

E 8.1 A 4-pole, 50 Hz, 11 KV turbo generator is rated 75 MW and 0.86 power factor lagging. The machine rotor has a moment of intertia of 9000 Kg-m^2. Find the inertia constant in MJ / MVA and M constant or momentum in MJs/elec degree

Solution :

$$\omega = 2\pi f = 100\ \pi \text{ rad/sec}$$

$$\text{Kinetic energy} = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 9000 + (100\pi)^2$$

$$= 443.682 \times 10^6 \text{ J}$$

$$= 443.682 \text{ MH}$$

$$\text{MVA rating of the machine} = \frac{75}{0.86} = 87.2093$$

$$H = \frac{MJ}{MVA} = \frac{443.682}{87.2093} = 8.08755$$

$$M = \frac{GH}{180f} = \frac{87.2093 \times 5.08755}{180 \times 50}$$

$$= 0.0492979 \text{ MJS/0 dc}$$

E 8.2 Two generators rated at 4-pole, 50 Hz, 50 Mw 0.85 p.f (lag) with moment of inertia 28,000 kg-m^2 and 2-pole, 50Hz, 75 MW 0.82 p.f (lag) with moment of inertia 15,000 kg-m^2 are connected by a transmission line. Find the inertia constant of each machine and the inertia constant of single equivalent machine connected to infinite bus. Take 100 MVA base.

Solution :

For machine I

$$K.E = \frac{1}{2} \times 28{,}000 \times (100\ \pi)^2 = 1380.344 \times 10^6 \text{ J}$$

$$MVA = \frac{50}{0.85} = 58.8235$$

$$H_1 = \frac{1380.344}{58.8235} = 23.46586 \text{ MJ/MVA}$$

$$M_1 = \frac{58.8235 \times 23.46586}{180 \times 50} = \frac{1380.344}{180 \times 50}$$

$$= 0.15337 \text{ MJS/degree elect.}$$

For the second machine

$$K.E = \frac{1}{2} \times 15{,}000 \ \frac{1}{2} \times (100\ \pi)^2 = 739{,}470{,}000 \text{ J}$$

$$= 739.470 \text{ MJ}$$

$$MVA = \frac{75}{0.82} = 91.4634$$

$$H_2 = \frac{739.470}{91.4634} = 8.0848$$

$$M_2 = \frac{91.4634 \times 8.0848}{180 \times 50} = 0.082163 \text{ MJS/}^o\text{Elc}$$

$$\frac{1}{M} = \frac{1}{M_1} = \frac{1}{M_2}$$

$$M = \frac{M_1 M_2}{M_1 + M_2} = \frac{0.082163 \times 0.15337}{0.082163 + 0.15337}$$

$$= \frac{0.0126}{0.235533} = 0.0535 \text{ MJS/Elec.degree}$$

$$GH = 180 \times 50 \times M = 180 \times 50 \times 0.0535$$

$$= 481.5 \text{ MJ}$$

on 100 MVA base, inertia constant.

$$H = \frac{481.5}{100} = 4.815 \text{ MJ/MVA}$$

E 8.3 A four pole synchronous generator rated 110 MVA 12.5 KV, 50 HZ has an inertia constant of 5.5 MJ/MVA

(i) Determine the stored energy in the rotor at synchronous speed.

(ii) When the generator is supplying a load of 75 MW, the input is increased by 10 MW. Determine the rotor acceleration, neglecting losses.

(iii) If the rotor acceleration in (ii) is maintained for 8 cycles, find the change in the torque angle and the rotor speed in rpm at the end of 8 cycles

Solution :

(i) Stored energy = GH = 110 × 5.5 = 605 MJ where G = Machine rating

(ii) P_a = The acclerating power = 10 MW

$$10 \text{ MW} = M \frac{d^2\delta}{dt^2} = \frac{GH}{180f} \frac{d^2\delta}{dt^2}$$

$$\frac{605}{180 \times 50}\frac{d^2\delta}{dt^2} = 10$$

$$0.0672\ \frac{d^2\delta}{dt^2} = 10 \text{ or } \frac{d^2\delta}{dt^2} = \frac{10}{0.0672} = 148.81$$

α = 148.81 elec degrees/sec^2

(iii) 8 cyles = 0.16 sec.

Change in $\delta = \frac{1}{2} \times 148.81 \times (0.16)^2$

Rotor speed at the end of 8 cycles

$$= \frac{120f}{P}.(\delta) \times t = \frac{120 \times 50}{4} \times 1.90476\ \delta \times 0.16$$

$= 457.144$ r.p.m

E 8.4 Power is supplied by a generator to a motor over a transmission line as shown in Fig. E8.4(a). To the motor bus a capacitor of 0.8 pu reactance per phase is connected through a switch. Determine the steady state power limit with and without the capacitor in the circuit.

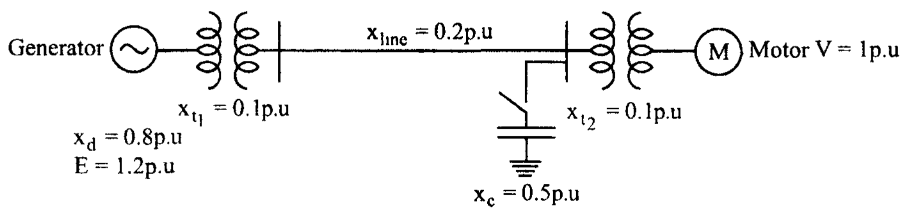


Fig. E.8.4 (a)

Steady state power limit without the capacitor

$$P_{max\ 1} = \frac{1.2 \times 1}{0.8 + 0.1 + 0.2 + 0.8 + 0.1} = \frac{1.2}{2.0} = 0.6 \text{ pu}$$

With the capacitor in the circuit, the following circuit is obtained.

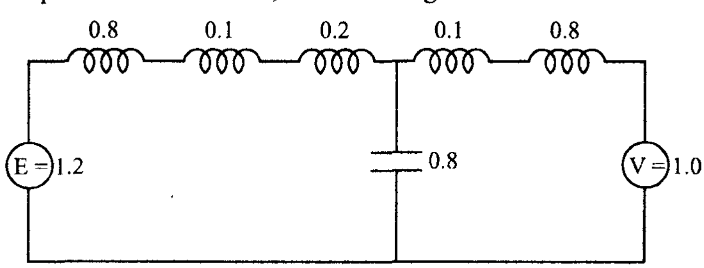


Fig. E.8.4 (b)

Simplifying

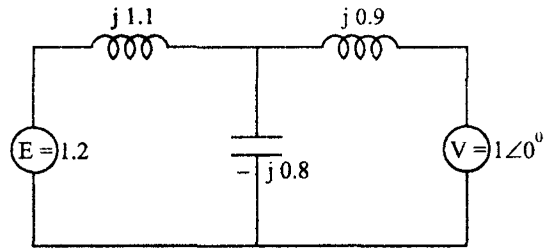


Fig. E.8.4 (c)

Converting the star to delta network, the transfer reactance between the two nodes X_{12}.

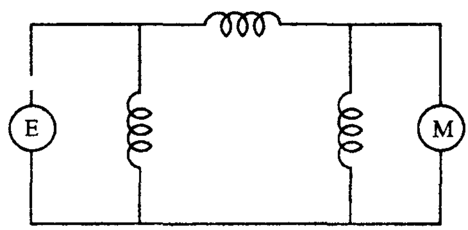


Fig. E.8.4 (d)

$$X_{12} = \frac{(j1.1)(j0.9) + (j0.9)(-j0.8) + (-j0.8 \times j1.1)}{-j0.8}$$

$$= \frac{-0.99 + 0.72 + 0.88}{-j0.8} = \frac{-0.99 + 1.6}{-j0.8} = \frac{j0.61}{0.8}$$

$$= j0.7625 \text{ p.u}$$

Steady state power limit $= \frac{1.2 \times 1}{0.7625} = 1.5738$ pu

E 8.5 A generator rated 75 MVA is delivering 0.8 pu power to a motor through a transmission line of reactance j 0.2 p.u. The terminal voltage of the generator is 1.0 p.u and that of the motor is also 1.0p.u. Determine the generator e.m.f behind transient reactance. Find also the maximum power that can be transferred.

Solution :

When the power transferred is 0.8 p.u

$$0.8 = \frac{1.0 \times 1.0 \sin\theta}{(0.1 + 0.2)} = \frac{1}{0.3} \sin\theta$$

$$\text{Sin } \theta = 0.8 \times 0.3 = 0.24$$

$$\theta = 13.^0 8865$$

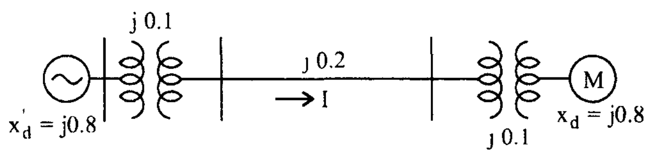


Fig. E.8.5

Current supplied to motor

$$I = \frac{1\angle 13.^0 8865 - 1\angle 0^0}{j0.3} = \frac{(0.9708 + j0.24) - 1}{j0.3}$$

$$= \frac{-0.0292 + j0.24}{j0.3} = j\ 0.0973 + 0.8 = 0.8571 \angle \text{Tan}^{-1}\ 0.1216$$

$$I = 0.8571 \angle 6.^0 934$$

Voltage behind transient reactance

$$= 1\angle 0^0 + j\ 1.2\ (0.8 + j\ 0.0973)$$

$$= 1 + j\ 0.96 - 0.11676$$

$$= 0.88324 + j\ 0.96$$

$$= 1.0496\ \ 47^0.8$$

$$P_{max} = \frac{EV}{X} = \frac{1.0496 \times 1}{1.2} = 0.8747 \text{ p.u}$$

E 8.6 Determine the power angle characteristic for the system shown in Fig. E.8.6(a). The generator is operating at a terminal voltage of 1.05 p.u and the infinite bus is at 1.0 p.u. voltage. The generator is supplying 0.8 p.u power to the infinite bus.

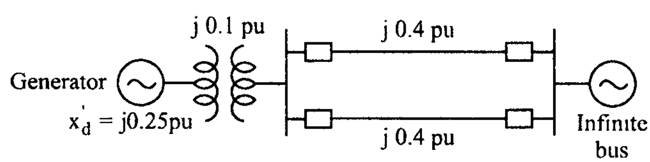


Fig. E.8.6 (a)

Solution :

The reactance diagram is drawn in Fig. E.8.6(b).

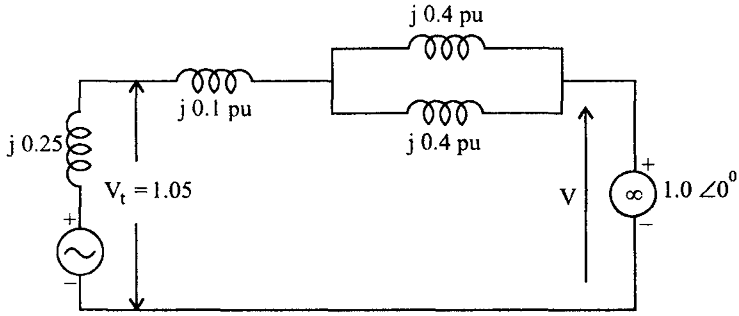


Fig. E.8.6 (b)

The transfer reactance between V_t and V is $= j\,0.1 + \dfrac{j0.4}{2} = j\,0.3$ p.u

we have $\dfrac{V_t V}{X}\sin\delta = \dfrac{(1.05)(1.0)}{0.3}\sin\delta = 0.8$

Solving for δ, $\sin\delta = 0.22857$ and $\delta = 13^0.21$

The terminal voltage is $1.05 \angle 13^0.21$

$1.022216 + j\,0.24$

The current supplied by the generator to the infinite bus

$$I = \frac{1.022216 + j0.24 - (1 + j0)}{j0.3}$$

$$= \frac{(0.022216 + j0.24)}{j0.3} = 0.8 - j\,0.074$$

$$= 1.08977 \angle 5.^0 28482 \text{ p.u}$$

The transient internal voltage in the generator

$$E^1 = (0.8 - j\,0.074)\,j\,0.25 + 1.22216 + j\,0.24$$

$$= j\,0.2 + 0.0185 + 1.02216 + j\,0.24$$

$$= 1.040 + j\,0.44$$

$$= 1.1299 \angle 22^0.932$$

The total transfer reactance between E^1 and V

$$= j\,0.25 + j\,0.1 + \frac{j0.4}{2} = j\,0.55 \text{ p.u}$$

The power angle characteristic is given by

$$P_e = \frac{E^1 V}{X}\sin\delta = \frac{(1.1299).(1.0)}{j0.55}\sin\delta$$

$$\mathbf{P_e = 2.05436\ sin\ \delta}$$

E 8.7 Consider the system in E 8.1 showin in Fig. E.8.7. A three phase fault occurs at point P as shown at the mid point on line 2. Determine the power angle characteristic for the system with the fault persisting.

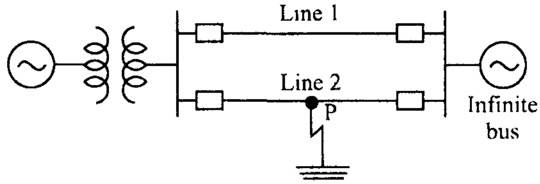


Fig. E.8.7

Solution :

The reactance diagram is shown in Fig. E.8.7(a).

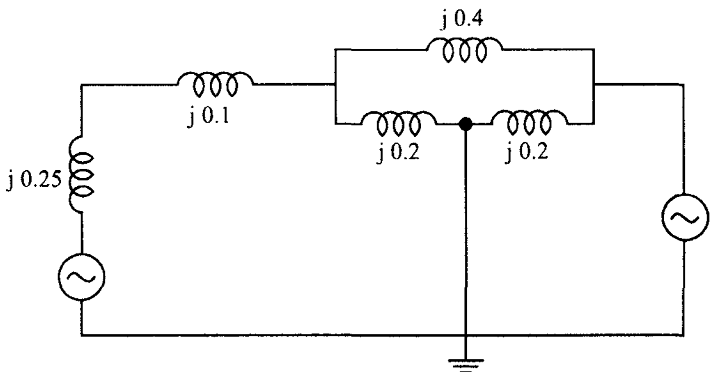


Fig. E.8.7 (a)

The admittance diagram is shown in Fig. E.8.7(b).

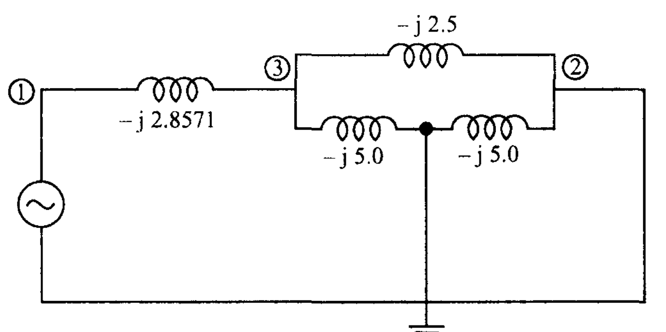


Fig. E.8.7 (b)

The buses are numbered and the bus admittance matrix is obtained.

	①	②	③
①	–j 1.85271	0.0	j 2.85271
②	0.0	–j 7.5	j 2.5
③	j 2.8271	j 2.5	–j 10.3571

Node 3 or bus 3 has no connection to any source directly, it can be eliminated.

$$Y_{11\,(\text{modified})} = Y_{11\,(\text{old})} - \frac{Y_{13}Y_{31}}{Y_{33}}$$

$$= -j\,2.8571 - \frac{(2.8527)(2.85271)}{(-10.3571)}$$

$$= -2.07137$$

$$Y_{12\,(\text{modified})} = 0 - \frac{(2.85271)(2.5)}{(-10.3571)} = 0.6896$$

$$Y_{22\,(\text{modified})} = Y_{22\,(\text{old})} - \frac{Y_{32}Y_{23}}{Y_{33}}$$

$$= -7.5 - \frac{(2.5)(2.5)}{(-10.3571)} = -6.896549$$

The modified bus admittance matrix between the two sources is

	①	②
①	–2.07137	0.06896
②	0.6896	–6.89655

The transfer admittance between the two sources is 0.6896 and the transfer reactance = 1.45

$$P_2 = \frac{1.05 \times 1}{1.45} \sin\delta \text{ p.u}$$

or $\quad P_e = 0.7241 \sin\delta$ p.u

E 8.8 For the system considered in E.8.6 if the H constant is given by 6 MJ/MVA obtain the swing equation

Solution :

The swing equation is $\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$, the acclerating power

If δ is in electrical radians

$$\frac{d^2\delta}{dt^2} = \frac{180 \times f}{H} P_a = \frac{180 \times 50}{6} P_a = 1500\ P_a$$

E 8.9 In E8.7 if the 3-phase fault is cleared on line 2 by operating the circuit breakers on both sides of the line, determine the post fault power angle characteristic.

Solution : The net transfer reactance between E^1 and V_a with only line 1 operating is

$$j\ 0.25 + j\ 0.1 + j\ 0.4 = j\ 0.75 \text{ p.u}$$

$$P_e = \frac{(1.05)(1.0)}{j0.75} \text{ Sin } \delta = 1.4 \text{ Sin } \delta$$

E8.10 Determine the swing equation for the condition in E 8.9 when 0.8 p.u power is delivered.

Given $M = \frac{1}{1500}$

Solution : $\frac{180f}{H} = \frac{180 \times 50}{6} = 1500$

$$\frac{1}{1500}\frac{d^2\delta}{dt^2} = 0.8 - 1.4 \sin\delta \text{ is the swing equation}$$

where δ in electrical-degrees.

E8.11 Consider example E 8.6 with the swing equation

$$P_e = 2.05 \sin\delta$$

If the machine is operating at 28^0 and is subjected to a small transient disturbance, determine the frequency of oscillation and also its period.

Given $H = 5.5$ MJ/MVA

$$P_e = 2.05 \sin 28^0 = 0.9624167$$

Solution :

$$\frac{dP_e}{d\delta} = 2.05 \cos 28^0 = 1.7659$$

The angular frequency of oscillation = ω_n

$$\omega_n = \sqrt{\frac{\omega S^0}{2H}} = \sqrt{\frac{2\pi \times 50 \times 1.7659}{2 \times 5.5}}$$

$$= 7.099888 = 8 \text{ elec rad/sec.}$$

$$f_n = \frac{1}{2\pi} \times 8 = \frac{4}{\pi} = 1.2739 \text{ Hz}$$

Period of oscillation = $T = \frac{1}{f_n} = \frac{1}{1.2739} = T = 0.785$ sec

E8.12 The power angle characteristic for a synchronous generator supplying infinite bus is given by

$$P_e = 1.25 \sin \delta$$

The H constant is 5 sec and initially it is delivering a load of 0.5 p.u. Determine the critical angle.

Solution :

$$\text{Cos } \delta_c = \frac{P_{mo}}{P_{max}}[(\pi - 2\delta_o)] + \text{Cos}(\pi - \delta_o)$$

$$\frac{P_{mo}}{P_{max}} = \frac{0.5}{1.25} = 0.4 = \text{Sind } \delta_o \text{ ; } \delta_o = 23^o.578$$

$\text{Cos } \delta_o = 0.9165$

δ_o in radians = 0.4113

$2\delta_o = 0.8226$

$\pi - 2\delta_o = 2.7287$

$$\frac{P_{mo}}{P_{max}} (\pi - 2\delta_o) = 1.09148$$

$\text{Cos } \delta_c = 1.09148 - 0.9165 = 0.17498$

$\boldsymbol{\delta_c = 79^o.9215}$

E8.13 Consider the system shown in Fig. E.8.13.

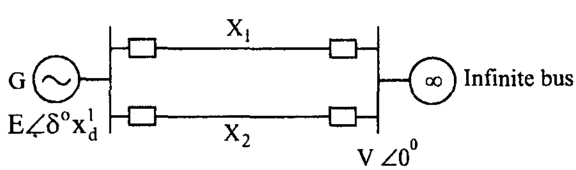


Fig. E.8.13

$x_d^1 = 0.25$ p.u

$|E| = 1.25$ p.u and $|V| = 1.0$ p.u ; $X_1 = X_2 = 0.4$ p.u

Initially the system is operating stable while delivering a load of 1.25 p.u. Determine the stability of the system when one of the lines is switched off due to a fault.

Solution .

When both the lines are working

$$P_{e\,max} = \frac{1.25 \times 1}{0.25 + 0.2} = \frac{1.25}{0.45} = 2.778 \text{ p.u}$$

When one line is switched off

$$P^1_{e\,max} = \frac{1.25 \times 1}{0.25 + 0.4} = \frac{1.25}{0.65} = 1.923 \text{ p.u}$$

$$P_{e0} = 2.778 \text{ Sin } \delta_0 = 1.25 \text{ p.u}$$

$$\text{Sin } \delta_0 = 0.45$$

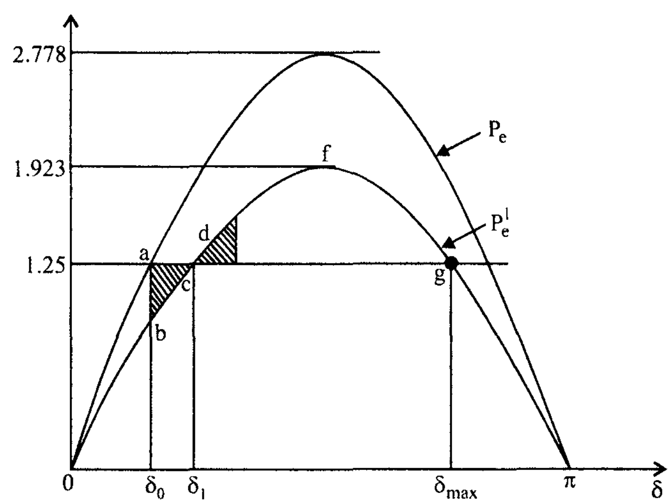
$$\delta_0 = 26^o.7437 = 0.4665 \text{ radinas}$$

At point C $\quad P^1_e = 1.923 \text{ Sin } \delta_1 = 1.25$

$$\text{Sin } \delta_1 = 0.65$$

$$\delta_1 = 40^o.5416$$

$$= 0.7072 \text{ radian}$$



$$A_1 = \text{area abc} = \int_{\delta_0}^{\delta_1}\left(P_2 - P_e^{I}\right)d\delta = \int_{0.4665}^{0.7072}\left(1.25 - 1.923\sin\delta\right)d\delta$$

$$= 1.25 \Big|_{0.4665}^{0.7072} + 1.923 \text{ Cos } \delta \Big|_{26^\circ.7437}^{40^\circ.5416}$$

$$= 0.300625 + (-0.255759) = 0.0450$$

Maximum area available = area c d f g c = $A_{2\,max}$

$$A_{2\,max} = \int_{\delta_1}^{\delta_{max}}\left(P_e^{I} - P_1\right)d\delta = \int_{0.7072}^{\pi-0.7072}\left(1.923 \text{ Sin } \delta - 1.25\right)d\delta$$

$$= -1.923 \text{ Cos } \delta \Big|_{40^\circ.5416}^{139^\circ.46} - 1.25\ (2.4328 - 0.7072)$$

$$= 0.7599 - 1.25 \times 1.7256$$

$$= 0.7599 - 2.157 = -1.3971 >> A_1$$

The system is stable

[Note : area A_1 is below P_2 = 1.25 line and

area A_2 is above P_2 = 1.25 line ; hence the negative sign]

E8.14 Determine the maximum value of the rotor swing in the example E8.13.

Solution :

Maximum value of the rotor swing is given by condition

$$A_1 = A_2$$

$$A_1 = 0.044866$$

$$A_2 = \int_{\delta_1}^{\delta_2}\left(-1.25 + 1.923 \text{ Sin}\delta\right)d\delta$$

$$= (-1.25\ \delta_2 + 1.25 \times 0.7072) - 1.923\ (\text{Cos } \delta_2 - 0.76)$$

i.e., $= +1.923 \text{ Cos } \delta_2 + 1.25\ \delta_2 = 2.34548 - 0.0450$

i.e., $= 1.923 \text{ Cos } \delta_2 + 1.25\ \delta_2 = 2.30048$

By trial and error **$\delta_2 = 55^\circ.5$**

E8.15 The M constant for a power system is 3×10^{-4} S^2/elec. degree

The prefault, during the fault and post fault power angle characteristics are given by

$$P_{e_1} = 2.45 \text{ Sin } \delta$$

$$P_{e_2} = 0.8 \text{ Sin } \delta$$

and $\quad P_{e_3} = 2.00 \text{ Sin } \delta \quad$ respectively

choosing a time interval of 0.05 second obtain the swing curve for a sustained fault on the system. The prefault power transfer is 0.9 p.u.

Solution :

$$P_{e_1} = 0.9 = 2.45 \text{ Sin } \delta_o$$

The initial power angle $\delta_o = \text{Sin}^{-1}\left(\frac{0.9}{2.45}\right)$

$$= 21.55^o$$

At $t = 0_-$ just before the occurrence of fault.

$$P_{max} = 2.45$$

$$\text{Sin } \delta_o = \text{Sin } 21^o.55 = 0.3673$$

$$P_e = P_{max} \text{ Sin } \delta_o = 0.3673 \times 2.45 = 0.9$$

$$P_a = 0$$

At $t = 0_+$, just after the occurrence of fault

$$P_{max} = 0.8; \text{ Sin } \delta_o = 0.6373 \text{ and hence}$$

$$P_e = 0.3673 \times 0.8 = 0.2938$$

P_a, the acclerating power $= 0.9 - P_e$

$$= 0.9 - 0.2938 = 0.606$$

Hence, the average acclerating powr at $t = 0_{ave}$

$$= \frac{0 + 0.606}{2} = 0.303$$

$$\frac{(\Delta t)^2}{M} P_a = \frac{(0.05 \times 0.05)}{3 \times 10^{-4}} = 8.33 \ P_a = 8.33 \times 0.303 = 2^o.524$$

$$\Delta\delta = 2^o.524 \text{ and } \delta^o = 21^o.55.$$

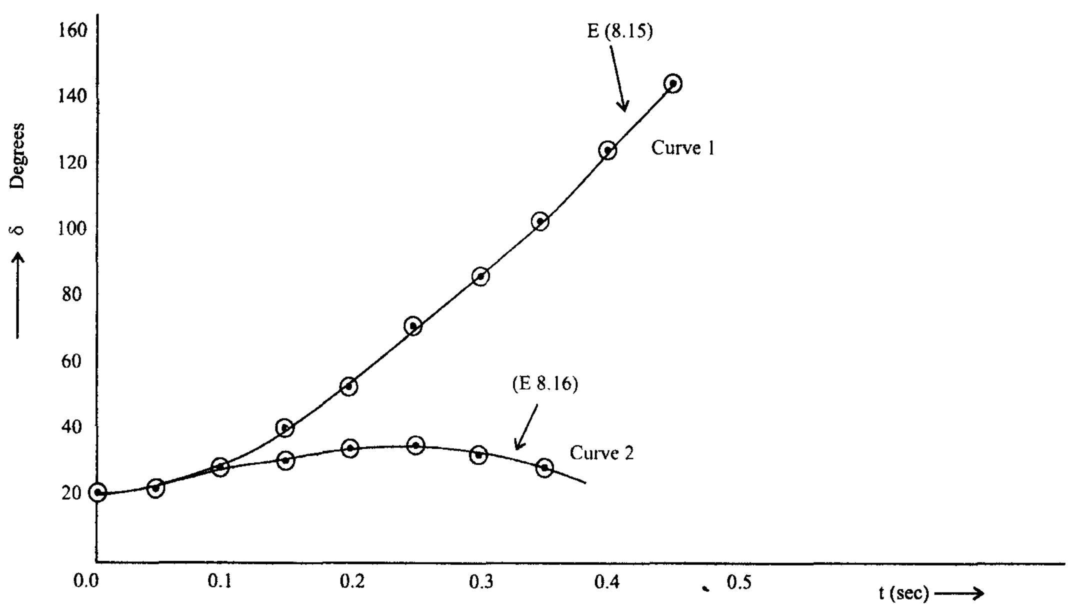
The calculations are tabulated upto t = 0.4 sec.

Table 8.1

S.No	t (sec)	P_{max} (p.u.)	Sin δ	P_e = P_{max} Sin δ	P_a = 0.9 – P_e	$\frac{(\Delta t)^2}{M}$. $P_a = 8.33 \times P_a$	Δδ	δ
1.	0–	2.45	0.3673	0.9	0	–	–	21 55⁰
	0_+	0.8	0.3673	0.2938	0.606	–	–	21.55⁰
	0_{ave}		0.3673	–	0.303	2.524	2°.524	24°.075
2.	0.05	0.8	0.4079	0.3263	0.5737	4.7786	7°.3	24°.075
3.	0.10	0.8	0.5207	0.4166	0.4834	4.027	11°.327°	31.3766
4.	0.15	0.8	0.6782	0.5426	0.3574	2.977	14°.304	42°.7036
5.	0.20	0.8	0 8357	0.6709	0.2290	1.9081	16°.212	57°.00
6.	0.25	0.8	0.9574	0.7659	0.1341	1.1170	17°.329	73°.2121
7.	0.30	0.8	0.9999	0.7999	0.1000	0 8330	18°.1623	90.5411
8.	0.35	0.8	0.9472	0.7578	0.1422	1.1847	19°.347	108.70
9	0.40	0.8	0.7875	0.6300	0.2700	2.2500	21°.596	128.047 149°.097

Table of results for E8.15.

From the table it can be seen that the angle δ increases continuously indicating instability.



E8.16 If the fault in the previous example E.8.14 is cleared at the end of 2.5 cycles determine the swing curve and examine the stability of the system.

Solution :

As before $\frac{(\Delta t^2)}{M} P_a = 8.33\ P_a$

$$\text{Time to clear the fault} = \frac{2.5 \text{ cycles second}}{50 \text{ cycles}}$$

$$= 0.05 \text{ sec.}$$

In this the calculations performed in the previous example E8.14 hold good for O_{ave}. However, since the fault in cleared at 0.05 sec., there will two values for P_{a_1} one for $P_{e_2} = 0.8 \sin \delta$ and another for $P_{e_3} = 2.00 \sin \delta$.

At t = 0.5 – (just before the fault is cleared)

$$P_{max} = 0.5; \ \ \text{Sin } \delta = 0.4079, \text{ and}$$

$$P_e = P_{max} \text{ Sind } \delta = 0.3263, \text{ so that } P_a = 0.9 - P_e = 0.57367$$

giving as before $\delta = 24^o.075$

But, at t = 0.5+ (just after the fault is cleared) P_{max} becomes 2.0 p.u at the same δ and $P_e = P_{max}$ Sin δ = 0.8158. This gives a value for $P_a = 0.9 - 0.815\ \delta = 0.0842$. Then for t = 0.05 are the average accelerating power at the instant of fault clearance becomes

$$P_{a\ ave} = \frac{0.57367 + 0.0842}{2} = 0.8289$$

$$\frac{(\Delta t)^2}{M} \cdot P_a = 8.33 \times 0.3289 = 2^o.74$$

and $\Delta\delta = 5.264$

$$\delta = 5.264 + 24.075 = 29^o.339$$

These calculated results and further calculated results are tabulated in Table 8.2.

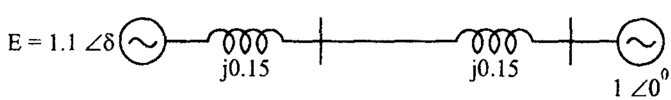
Table 8.2

S.No	t	P_{max}	Sin δ	P_e = P_{max} Sin δ	P_a = 0.9 – P_e	$\frac{(\Delta t)^2}{M}$ $P_a = 8.33 \times P_a$	Δδ	δ
1.	0–	2.45	0.3673	0.9	0	–	–	21.55°
	0_+	0.8	0 3673	0.2938	0.606	–	–	21.55°
	0_{ave}		0.3673	–	0.303	2.524	2.524	24.075
2.	0.05_-	0.8	0.4079	0.3263	0.5737	–	–	–
	0.05_+	2.0	0.4079	0.858	0.0842	–	–	–
	0.05_{ave}		0.4079	–	0.3289	2.740	5.264	29.339
3.	0.10	2.0	0.49	0.98	–0.08	–0.6664	4.5976	33.9367
4.	0.15	2.0	0.558	1.1165	–0.2165	–1.8038	2.7937	36.730
5.	0.20	2.0	0.598	1.1196	–0.296	–2.4664	0.3273	37.05
6.	0.25	2.0	0.6028	1.2056	–0.3056	–2.545	–2.2182	34°.83
7.	0.30	2.0	0.5711	1.1423	–0.2423	–2.018	–4.2366	30°.5933

Table of results for E8.15.

The fact that the increase of angle δ, started decreasing indicates stability of the system.

E8.17 A synchronous generator represented by a voltage source of 1.1 p.u in series with a transient reactance of j0.15 p.u and an inertia constant H = 4 sec is connected to an infinite bus through a transmission line. The line has a series reactance of j0.40 p.u while the infinite bus is represented by a voltage source of 1.0 p.u.



The generator is transmitting an active power of 1.0 p.u when a 3-phase fault occurs at its terminals. Determine the critical clearing time and critical clearing angle. Plot the swing curve for a sustained fault.

Solution :

$$P = \frac{EV}{X} \sin \delta_0 ; 1.0 \frac{1.1 \times 1.0}{(0.45 + 0.15)} \text{Sin } \delta_0 ; \delta_0 = 30^0$$

$$\delta_c = \text{Cos}^{-1}[(\pi - 2\delta_o)\text{Sin}\,\delta_o - \text{Cos}\,\delta_o]$$

$$= \text{Cos}^{-1}[(180^0 - 2 \times 30^0)\text{Sin}\,30^0 - \text{Cos}\,30^0]$$

$$= \text{Cos}^{-1}\left[\frac{\pi}{3} - 0.866\right] = \text{Cos}^{-1}[1.807]$$

$$= 79^0.59$$

Critical clearing angle = $79^0.59$

Critical clearing time = $\sqrt{\frac{2H}{P_m}\frac{(\delta_c - \delta_o)}{\pi f}}$

$$\delta_c - \delta_o = 79^0.59 - 30^0 = 49.59^0 = \frac{49.59 \times 3.14}{180} \text{ rad}$$

$$= 0.86507 \text{ rad}$$

$$t_c = \sqrt{\frac{2 \times 4 \times 0.86507}{1 \times 3.14 \times 50}} = 0.2099 \text{ sec}$$

Calculation for the swing curve

$$\Delta\delta_n = \delta_{n-1} + \left(\frac{180f}{H}\right) \Delta t^2 P_{a(n-1)}$$

Let $\Delta t = 0.05$ sec

$$\delta_{n-1} = 30^0$$

$$\frac{180f}{H} = \frac{180 \times 50}{4} = 2250$$

$$M = \frac{H}{180f} = \frac{1}{2250} = 4.44 \times 10^{-4}$$

$$\frac{(\Delta t)^2}{M} P_a = \frac{(0.05 \times 0.05)}{(4.44 \times 10^{-4})} P_a = 5.63\, P_a$$

Accelerating power before the occurrence of the fault = $P_{a-} = 2\,\text{Sin}\,\delta_o - 1.0 = 0$

Accelerating power immediately after the occrrence of the fault

$$P_{a+} = 2\,\text{Sin}\,\delta_o - 0 = 1 \text{ p.u}$$

Average acclerating powr = $\frac{0+1}{2}$ = 0.5 p.u. Change in the angle during 0.05 sec after fault occurrence.

$$\Delta\delta_1 = 5.63 \times 0.5 = 2^0.81$$

$$\delta_1 = 30^0 + 2^0.81 = 32^0.81$$

The results are plotted in Fig. E8.17.

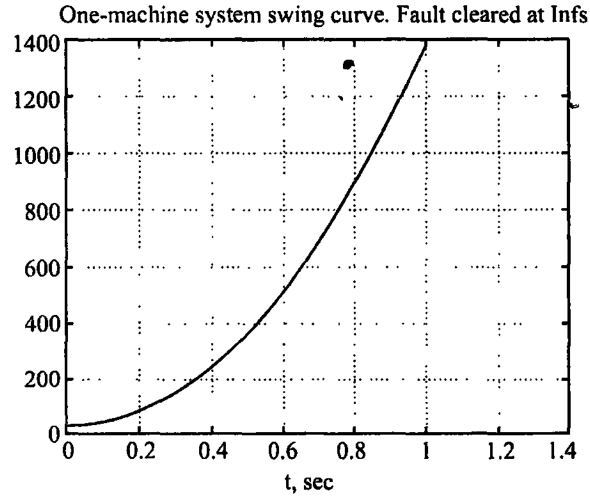


Fig. E.8.17 (a)

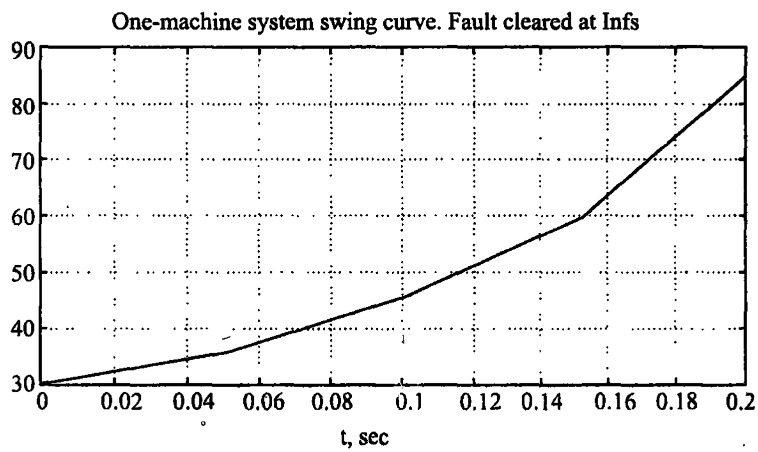


Fig. E.8.17 (b)

The system is unstable.

E8.18 In example no. E8.17, if the fault is cleared in 100 msec, obtain the swing curve.

Solution :

The swing curve is obtained using MATLAB and plotted in Fig. E.8.18.

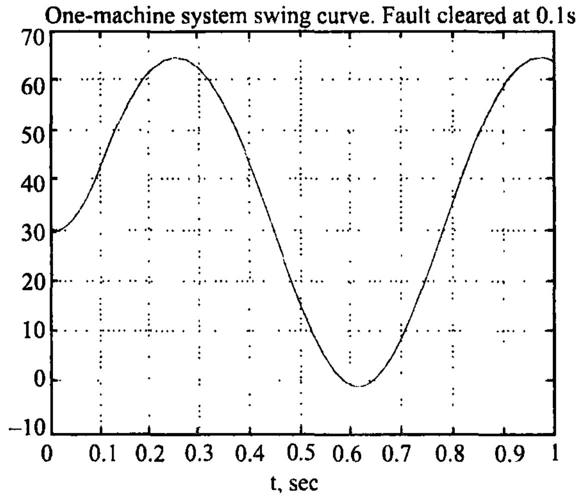


Fig. E8.18

The system is stable.

Problems

P8.1 A 2 pole, 50 Hz, 11 KV synchronous generator with a rating of 120 Mw and 0.87 lagging power factor has a moment of inertia of 12,000 kg-m^2. Calculate the constants H and M.

P8.2 A 4-pole synchronous generator supplies over a short line a load of 60 Mw to a load bus. If the maximum steady stae capacity of the transmission line is 110 Mw, determine the maximum sudden increase in the load that can be tolerated by the system without loosing stability.

P8.3 The prefault power angle chracteristic for a generator infinite bus system is given by

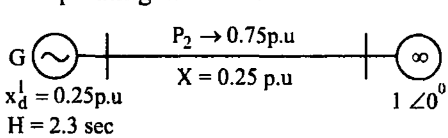
$$P_{e_1} = 1.62 \text{ Sin } \delta$$

and the initial load supplied is 1 p.u. During the fault power angle characteristic is given by

$$P_{e_2} = 0.9 \text{ Sin } \delta$$

Determine the critical clearing angle and the clearing time.

P8.4 Consider the system operating at 50 Hz.



If a 3-phase fault occurs across the generator terminals plot the swing curve.

Plot also the swing curve, if the fault is cleared in 0.05 sec.

Questions

8.1 Explain the terms

(a) Steady state stability (b) Transient stabiltiy (c) Dynamic stability

8.2 Discuss the various methods of improving steady state stability.

8.3 Discuss the various methods of improving transient stability.

8.4 Explain the term (i) critical clearing angle and (ii) critical clearing time

8.5 Derive an expression for the critical clearing angle for a power system consisting of a single machine supplying to an infinite bus, for a sudden load increment.

8.6 A double circuit line feeds an infinite bus from a power station. If a fault occurs on one of the lines and the line is switched off, derive an expression for the critical clearing angle.

8.7 Explain the equal area critrion.

8.8 What are the various applications of equal area criterion ? Explain.

8.9 State and derive the swing equations

8.10 Discuss the method of solution for swing equation.

Ojective Questions

1. Base current in amperes is [ ]

(a) $\dfrac{\text{Base KVA}}{\sqrt{3}\ \text{Base KV(line to line)}}$

(b) $\dfrac{\text{Base KVA}}{\text{Base KV(line to line)}}$

(c) $\dfrac{\text{Base KVA}}{3\ \text{Base KV(line to line)}}$

2. Base impedance in ohms is [ ]

(a) $\dfrac{[\text{Base Voltage in KV (line} - \text{to} - \text{line)}]\text{X}\,1000}{\text{base KVA}}$

(b) $\dfrac{[\text{Base Voltage in KV (line} - \text{to} - \text{line)}]^2\ \text{X}\,1000}{\text{base KVA}}$

(c) $\dfrac{[\text{Base Voltage in KV (line} - \text{to} - \text{line)}]^2\ \text{X}\,3}{\text{base KVA}}$

3. Impedance in ohms is [ ]

(a) $\dfrac{(\text{p.u. impedance})[\text{base KV (line} - \text{to} - \text{line)}]^2}{\text{base KVA} \times 1000}$

(b) $\dfrac{(\text{p.u. impedance})[\text{base KV (line} - \text{to} - \text{line)}]^2 \times 1000}{\sqrt{3}\ \text{base KVA}}$

(c) $\dfrac{(\text{p.u. impedance})[\text{base KV (line} - \text{to} - \text{line)}]^2 \times 1000}{\text{base KVA}}$

4. Per unit impedance on new KVA and KV base is [ ]

(a) $\left(\dfrac{\text{p.u.impedance on}}{\text{given KVA and KV base}}\right)\left(\dfrac{\text{given KVA base}}{\text{New KVA base}}\right)\left(\dfrac{\text{new KV base}}{\text{given KV base}}\right)^2$

(b) $\left(\dfrac{\text{p.u.impedance on}}{\text{given KVA and KV base}}\right)\left(\dfrac{\text{given KVA base}}{\text{New KVA base}}\right)\left(\dfrac{\text{given KV base}}{\text{new KV base}}\right)^2$

(c) $\left(\dfrac{\text{p.u.impedance on}}{\text{given KVA andKV base}}\right)\left(\dfrac{\text{new KVA base}}{\text{given KVA base}}\right)\left(\dfrac{\text{given KV base}}{\text{new KV base}}\right)^2$

5. Which of the following is false? [ ]

(a) An element of a graph is called an edge.

(b) Each line segment is called an element.

(c) Each current source is replaced by a short circuit in a graph.

6. The rank of a graph is [ ]

(a) n (b) n-1 (c) n+1

where n is the number of nodes in the graph.

7. In a graph if there are 4 nodes and 7 elements the number of links is [ ]

(a) 3 (b) 4 (c) 5

8. Which of the following statements is true ? [ ]

(a) n basic cutsets are linearly independent where n is the number of nodes.

(b) The cut set is a minimal set of branches of the graph.

(c) The removal of k branches does not reduce the rank of a graph provided that no proper subset of this set reduces the rank of the graph by one when it is removed from the graph.

9. The dimension of the bus incidence matrix is [ ]

(a) $e \times n$ (b) $e \times (n-1)$ (c) $e \times e$

10. If A_b and A_l are the sub matrices of bus incidence matrix A containing branches and links only and k is the branch path incidence matrix then [ ]

(a) $A_b k^t = U$ (b) $k^t = A_b$ (c) $A_l A_b^{-1} = k^t$

11. Which of the following statements is true ? [ ]
    (a) There is a one-to-one correspondence between links and basic cut sets.
    (b) $A_l A_b^{-1} = B$ where B is the basic cut set incidence matrix.
    (c) $B_l = A_l k^t$ where B_l is the basic cut set incidence matrix containing links only.

12. Identify the correct relation [ ]
    (a) $Y_{BUS} = [B^t] [y] [B]$
    (b) $Y_{loop} = [C^t] [y] [C]$
    (c) $Y_{BR} = [B]^t [y] [B]$

13. Identify the current relations [ ]
    (a) $[A^t] [y] [A] = Y_{loop}$
    (b) $[B^t] [y] [B] = Z_{BR}$
    (c) $[C^t] [z] [C] = Z_{loop}$

14. With the addition of a branch to a partial network with usual notation, the mutual impedance is given by [ ]

    (a) $$Z_{bi} = Z_{ai} + \frac{\bar{y}_{ab-xy}\left(\bar{Z}_{xi} - \bar{Z}_{yi}\right)}{y_{ab-ab}}$$

    (b) $$Z_{bi} = Z_{ai} + \frac{\bar{y}_{ab-xy}\left(\bar{Z}_{ai} - \bar{Z}_{bi}\right)}{y_{ab-ab}}$$

    (c) $$Z_{bi} = Z_{ai} + \frac{\bar{y}_{ab-xy}\left(\bar{Z}_{yi} \bullet \bar{Z}_{ai}\right)}{y_{ab-ab}}$$

15. The self impedance Z_{bb} of a branch ab added to an existing partial network is given by [ ]

    (a) $$Z_{bb} = Z_{ab} + \frac{1 + \bar{y}_{ab-xy}\left(Z_{ab} - Z_{xy}\right)}{y_{ab-ab}}$$

    (b) $$Z_{bb} = Z_{ab} + \frac{1 + \bar{y}_{ab-xy}\left(Z_{xa} - \bar{Z}_{xy}\right)}{y_{ab-ab}}$$

    (c) $$Z_{bb} = Z_{ab} + \frac{1 + \bar{y}_{ab-xy}\left(\bar{y}_{xa} - \bar{y}_{yb}\right)}{y_{ab-ab}}$$

16. If in q.no 18 there is no mutual coupling [ ]

(a) $Z_{bb} = Z_{ab}$ (b) $Z_{bb} = Z_{ab} + z_{ab-ab}$ (c) $Z_{bb} = z_{ab-ab}$

17. If in q. no 18 there is no mutual coupling and if "a" is the reference node [ ]

(a) $Z_{bb} = z_{ab-ab}$ (b) $Z_{bb} = Z_{ab}$ (c) $Z_{bb} = Z_{ab} + z_{ab-ab}$

18. Modified impedances are computed when the fictitious node introduced for the addition of a link is eliminated. Then Z_{ij} (modified) [ ]

(a) Z_{ij} (before elimination) $- \dfrac{Z_{il}\,Z_{lj}}{Z_{ll}}$

(b) Z_{ij} (before elimination) $- \dfrac{Z_{ll}}{Z_{il}\,Z_{lj}}$

(c) Z_{ij} (before elimination) $- \dfrac{(Z_{il} - Z_{lj})}{Z_{ll}}$

19. Identify the correct relation [ ]

(a) $a^2 = -0.5 + j\,0.866$ (b) $a = 1.\,e^{j4\pi/3}$ (c) $1 + a + a^2 = 0 + j0$

20. Which of the following is correct [ ]

(a) $V_R = V_{R0} + a\,V_{R1} + a^2\,V_{R2}$

(b) The sequence compoenets are related to the phase components through the transformation matrix $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$

(c) $I_{R2} = \dfrac{1}{3}\,(I_R + a^2\,I_Y + a\,I_B)$

21. For stationery bi lateral unbalanced network elements [ ]

(a) $Z_{ab}^{RY} = Z_{ab}^{YR}$ and $Z_{ab}^{RR} = Z_{ab}^{YY}$

(b) $Z_{ab}^{RY} = -Z_{ab}^{YR}$ and $Z_{ab}^{RR} = Z_{ab}^{YY}$

(c) $Z_{ab}^{RY} \neq Z_{ab}^{YY}$

22. For balanced rotating 3-phase network elements [ ]

(a) $Z_{ab}^{RY} = Z_{ab}^{YR}$

(b) $Z_{ab}^{RY} \neq Z_{ab}^{YR}$

(c) The admittances are symmetric

23. A balanced three phase element with balanced excitation can be considered as a single phase element [ ]

(a) true

(b) false

(c) some times it is true

24. For a stationary 3-phase network element ab, zero sequence impedance is given by [ ]

(a) $Z_{ab}^{R} + 2Z_{ab}^{m}$ (b) $Z_{ab}^{R} - 2Z_{ab}^{m}$ (c) $Z_{ab}^{R} - Z_{ab}^{m}$

25. For a 3-phase stationary network element the positive impedance is given by [ ]

(a) $Z_{ab}^{R} + 2Z_{ab}^{m}$ (b) $Z_{ab}^{R} - 2Z_{ab}^{m}$ (c) $Z_{ab}^{R} - Z_{ab}^{m}$

26. For a 3-phase stationary network element the negative sequence impedance is given by [ ]

(a) $Z_{ab}^{R} + 2Z_{ab}^{m}$ (b) $Z_{ab}^{R} - 2Z_{ab}^{m}$ (c) $Z_{ab}^{R} - Z_{ab}^{m}$

27. The inertia constant H is of the order of [ ]

(a) H = 4 (b) H = 8 (c) H = 1

28. When a synchronous machine is working with 1.1 p.u excitation and is connected to an infinite bus of voltage 1.0 p.u. delivering power at a load angle of 30^0 the power delivered with $x_d = 0.8$ p.u. and $x_q = 0.6$ p.u. [ ]

(a) P = 0.675 (b) P = 0.6875 (c) P = 1.375

Neglect reluctance power

29. Unit inertia constant H is defined as [ ]

(a) Ws / Pr (b) Pr / ws (c) Ws Pr / rad

30. At a slack-bus the quantities specified are [ ]

(a) P and Q (b) P and |V| (c) |V| and δ (d) P and δ

31. At a load bus the quantities specified are [ ]

(a) P and |V| (b) Q and |V| (c) P and Q (d) |V| and δ

32. At a Generator bus the quantities specified are [ ]

(a) |V| and δ (b) Q and |V| (c) P and Q (d) P and |V|

33. In load flow studies, the state variables are [ ]

(a) P and Q (b) |V| and δ (c) P and |V| (d) P and δ

34. Which one of the following is not correct ? [ ]

(a) $P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$

(b) $V_i = |V_i| (\text{Cos } \delta_i + j \text{ Sin } \delta_i)$

(c) Real power loss = $\sum_{i=1}^{n} P_i = \sum_{i=1}^{N} P_{gi} - \sum_{i=1}^{N} P_{di}$

(Total generation) – (Total load)

(d) $Q_i = \sum_{j=1}^{N} |Y_{ij} V_i V_j| \text{ Cos } (\delta_i - \delta_j - \theta_{ij})$

35. Which of the following is true ? [ ]

(a) Gauss-Seidel method is a direct solution method for power flow

(b) All iterative methods ensure convergence

(c) A generator bus is also called a swing bus

(d) If the reactive generation exceeds the limit then the P, |V| bus will become a P, Q bus

36. The number of iteration required for an n-bus system in Gauss-Seidel method are approximately [ ]

(a) n (b) n^2 (c) 3 (d) $\frac{n(n+1)}{2}$

37. The number of iteration required for an n-bus system in Newton-Raphson method are approximately [ ]

(a) n (b) n^2 (c) 3 (d) $\frac{n(n+1)}{2}$

38. With usual notation which of the following is true for a decoupled model [ ]

(a) $\underline{\Delta P} = [H]\, \underline{\Delta \delta}$ (b) $\Delta Q = [L]\, \Delta\delta$

(c) $\Delta P = [M]\, \Delta|V|$ (d) $\Delta P = [M] \dfrac{\Delta |V|}{|V|}$

39. The speed of fast decoupled load flow method when compared to Newton-Raphson method is [ ]

(a) Veru slow

(b) almost the same

(c) double the N-R method speed per iteration

(d) Five times the N-R method speed per iteration

40. Which of the following is true ? [ ]

(a) Short circuit KVA = $\dfrac{\text{Bore KVA } (\%X)}{100}$

(b) $I_{\text{short circuit}} = I_{\text{full load}} \times \dfrac{100}{(\%X)}$

(c) $\%Z = \dfrac{V}{IZ} \times 100$

41. Which of the following is not true [ ]

(a) In a feeder reactor protection, there is no protection for bus-bar fautls.

(b) In a tie-bar system current fed into a fault has to pass through two reactors in series.

(c) In ring system of reactor connection the voltage drop and power loss are considerable.

42. The operator 'a' is given by [ ]

(a) $\in^{j120^0}$ (b) $\in^{-j120^0}$ (c) $\in^{j60^0}$ (d) $\in^{-j60^0}$

43. $(a^2 - a)$ is given by [ ]

(a) $j\sqrt{3}$ (b) $-j\sqrt{3}$ (c) $\dfrac{1}{j\sqrt{3}}$ (d) $-\dfrac{1}{j\sqrt{3}}$

44. Phase voltages E_a, E_b and E_c are related to symmetrical components V_0, V_1 and V_2; then which of the following is true ? [ ]

(a) $V_{a2} = \dfrac{1}{3}(E_a + aE_b + a^2E_c)$ (b) $V_{a0} = E_a + E_b + E_c$

(c) $V_{a1} = \dfrac{1}{3}(E_a + a^2 E_b + aE_c)$ (d) $I_{a0} = \dfrac{1}{3}(I_a + I_b + I_c)$

45. For the solution of 3-phase star connected unbalanced load problem which method is more suitable [ ]

(a) Symmetrical components (b) Direct analysis

(c) Thevenin's theorem (d) Millman's theorem

46. If I_{s1}, I_{s2} and I_{s0} are star connected network sequence current components and I_{d1}, I_{d2} and I_{d0} are delta connected network sequence currents for the same unbalnced network, then [ ]

(a) $I_{s1} = -j\sqrt{3}\, I_{d1}$ (b) $I_{s2} = j\sqrt{3}\, I_{d2}$

(c) $I_{s0} = 0$ (d) $I_{s1} = \dfrac{1}{j\sqrt{3}} I_{d1}$

47. In case of star-delta connected transformers [ ]

(a) There is only a phase shift of 90^0 between the sequence components on either side

(b) There is only a change in the magnitude

(c) There is change both in phase and magnitude

(d) There is no change

48. The positive sequence impedance component of three unequal impedances Z_a, Z_b and Z_c is [ ]

(a) $\frac{1}{3}(Z_a + aZ_b + a^2Z_c)$ (b) $\frac{1}{3}(Z_a + a^2Z_b + aZ_c)$

(c) $(Z_a + aZ_b + a^2Z_c)$ (d) $(Z_a + a^2Z_b + aZ_c)$

49. For a single line-to-ground fault, the terminal conditions are [ ]

(a) $V_a = 0;\ I_b = I_c = 0$ (b) $I_b = -I_c;\ V_b = V_c$

(c) $V_a = \dfrac{V_b + V_c}{2};\ I_b = I_c = 0$ (d) $I_a = 0;\ V_b - V_c$

50. For a double line fault on phase b and c [ ]

(a) $I_a = \dfrac{3E_a}{Z_1 + Z_2 + Z_0}$ (b) $V_b = -V_c$

(c) $V_b = -\left(\dfrac{Z_2}{Z_1 + Z_2}\right) . E_a$ (c) $I_b = I_c$

51. For an ungrounded neutral, in case of a double-line to ground fault on phases b and c [ ]

(i) $I_{a1} = \dfrac{E_0}{Z_1 + Z_1}$; (ii) $V_b = V_c = 0$

(a) (i) only is correct (b) (ii) only is correct

(c) (i) and (ii) are both correct (d) both (i) and (ii) are false

52. A 15MVA, 6.9 KV generator, star connected has positive, negative and zero sequence reactances of 50%, 50% and 10% respectively. A line-to-line fault occurs at the terminals of the generator operating on no-load. What is the positive sequence component of the fault current in per unit. [ ]

(a) – j p.u (b) – 2j p.u. (c) – 0.5 j p.u. (d) 0.5 j p.u.

53. What is the negative sequence component of fault current in Q.No. (11) [ ]

(a) (– j 0.5 – 0.866) (b) (j 0.5 – j 0.866)

(c) + j p.u (d) (j 0.5 + 0.866)

54. In Q.No. (52) if the fault is a line-to-ground fault on phase a then, the positive sequence component of the fault current in p.u. is [ ]

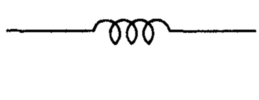
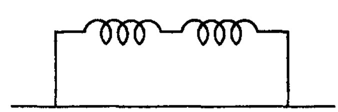
(a) + j 0.9 p.u (b) – j 0.9 p.u. (c) + j 1.0 p.u. (d) – j 0.866 p.u.

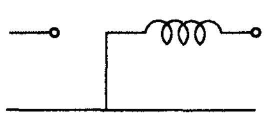
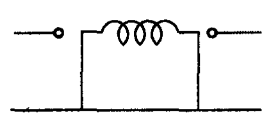
55. The most common type of fault to occur is [ ]

(a) Symmetrical 3-phase fault (b) Single line-to-ground fault

(c) Double line fault (d) Double line-to-ground fault

56. The zero sequence network for the transformer connection delta-star with star point earthed is given by [ ]

(a)  (b) 

(c)  (d) 

57. The negative sequence reactance of a synchronous machine is given by [ ]

(a) $j\left(\dfrac{x_d^{'} + x_q^{1}}{2}\right)$ (b) $j\left(\dfrac{x_d^{''} + x_q^{''}}{2}\right)$

(c) $j\left(\dfrac{x_d^{'} - x_q^{'}}{2}\right)$ (d) $j\left(\dfrac{x_d^{''} - x_q^{''}}{2}\right)$

58. A star connected synchoronous machine with neutral point grounded through a reactance x_n and winding zero sequence reactance x_0 experiences a single line-to ground fault through an impedance x_f. The total zero sequence impedance is [ ]

(a) $x_0 + x_n + x_f$ (b) $x_0 + 3x_n + x_f$

(c) $x_0 + 3x_n + 3x_f$ (d) $3\,(x_0 + x_n + x_f)$

59. In case of a turbo generator the positvie sequence reactnce [ ]

(a) Under subtransient state is more than transient state but less than steady state synchronous reactnace

(b) Under subtransient state is less than transient state and morethan synchronous reactance

(c) The transient state reactance is more than subtransient state reactance but less than synchronous reactance

(d) The transient state reactane is less than subtransient state, but more than synchronous reactance.

60. A synchronous machine having E = 1.2 p.u is supplying power to an infinite bus with voltage 1.0 p.u. If the transfer reactace is 0.6 p.u, the steady stae power limit is

[ ]

(a) 0.6 p.u (b) 1.0 p.u (c) 2 p.u

61. A synchronous generator is feeding as infinite bus through a transmission line. If the middle of the line a shunt reactor gets connected, the steadystate stability limit will [ ]

(a) increase (b) decrease (c) remain unaltered

62. A synchronous generator is supplying power to an infinite bus through a transmission line. If a shunt capacitor is added near the middle of the line, the steady state stability limit will [ ]

(a) increase (b) decrease (c) remain unaltered

63. If the shunt capacitor in q.no. (3) is shifted to the infinite bus, the stability limit will [ ]

(a) increase (b) decrease (c) remain unchanged

64. Which of the following is correct [ ]

(a) In steady state stability excitation response is important

(b) In transient stability studies, excitation response is important

(c) Dynamic stability is independent of excitation system response

65. Coefficient of stiffness is defines as [ ]

(a) $\frac{E.V}{X}\text{Cos}\,\delta$ (b) $\frac{EV}{X}\text{Sin}\,\delta$ (c) $\frac{EV}{X}$

66. For a step load disturbance, the frequency of oscillations is given by [ ]

(a) $f = \frac{1}{2\pi}\frac{\omega\delta^0}{\sqrt{2H}}$ (b) $f = \frac{1}{2\pi}\frac{\sqrt{\omega\delta^0}}{2H}$ (c) $f = \frac{1}{2\pi}\sqrt{\frac{\omega\delta^0}{2H}}$

67. Critical clearing angle for a step load change is [ ]

(a) $\delta_c = \text{Cos}^{-1}\left[\left(\frac{P_m}{P_{max}}(\pi - \delta_o) - \text{Cos}\,\delta_o\right)\right]$

(b) $\delta_c = \text{Cos}^{-1}\left(\frac{P_m}{P_{max}}(\pi - 2\delta_o) - \text{Cos}\,\delta_o\right)$

(c) $\delta_c = \text{Cos}^{-1}\left\{\left(\frac{P_m}{P_{max}}(\pi - 2\delta_o) - \text{Cos}\,\delta_o\right)\right\}$

68. If power is transmitted during the fault period on a double-line circuit with fault as one of the lines the critical clearing angle δ_c is given by [ ]

(a) $\text{Cos}\,\delta_c = \frac{P_m(\delta_{max} - \delta_0) - P_{max3}\text{Cos}\delta_0 + P_{max2}\text{Cos}\delta_{max}}{P_{max2} - P_{max3}}$

(b) $\text{Cos}\,\delta_c = \frac{P_m(\delta_{max} - \delta_0) + P_{max3}\text{Cos}\delta_0 - P_{max2}\text{Cos}\delta_{max}}{P_{max2} - P_{max3}}$

(c) $\text{Cos}\,\delta_c = \frac{P_m(\delta_{max} - \delta_0) - P_{max3}\text{Cos}\delta_0 + P_{max2}\text{Cos}\delta_{max}}{P_{max2} + P_{max3}}$

69. In step by step method of solution to swing equation [ ]

(a) $\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \omega_{r\,(n-3/2)}\,\Delta t$

(b) $\Delta\delta_n = \delta_n - \delta_{n-1} = \omega_{r\,(n-1)} \cdot \Delta t$

(c) $\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \omega_{r\,(n-2)} \cdot \Delta t$

Answers to Objective Questions

1.	(a)	26.	(b)	51.	(c)
2.	(b)	27.	(b)	52.	(a)
3.	(c)	28.	(b)	53.	(a)
4.	(c)	29.	(c)	54.	(b)
5.	(c)	30.	(c)	55.	(b)
6.	(b)	31.	(c)	56.	(c)
7.	(b)	32.	(d)	57.	(b)
8.	(b)	33.	(b)	58.	(c)
9.	(b)	34.	(d)	59.	(c)
10.	(a)	35.	(d)	60.	(c)
11.	(c)	36.	(a)	61.	(b)
12.	(c)	37.	(c)	62.	(a)
13.	(c)	38.	(a)	63.	(c)
14.	(a)	39.	(d)	64.	(a)
15.	(b)	40.	(b)	65.	(a)
16.	(b)	41.	(c)	66.	(c)
17.	(a)	42.	(a)	67.	(b)
18.	(a)	43.	(b)	68.	(a)
19.	(c)	44.	(d)	69.	(a)
20.	(b)	45.	(d)		
21.	(c)	46.	(c)		
22.	(c)	47.	(c)		
23.	(c)	48.	(a)		
24.	(c)	49.	(a)		
25.	(a)	50.	(c)		

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