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Computer Algebra Systems in Engineering Education

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This paper discusses the use of Computer Algebra Systems (CAS) in engineering education. A brief overview of the challenges and problems of computer and network-based lecturing and distance learning is given. From this general point of view, the power and limitations of CAS as systems for doing mathematics and simulations; calculators with infinite precision; teaching-tools for non-trivial examples; and learning-tools for experimental mathematics are shown. Examples tracing the use of CAS from the very first lectures to dissertations are then provided. New skills are necessary in order for students to manipulate algebra systems and to judge the results; the *new skills* are discussed and it is argued that the fear that students will forget their basic mathematical knowledge is unjustified.

INTRODUCTION

Computer Algebra Systems (CAS) are programs designed for the symbolic manipulation of mathematical objects such as polynomials, triangles, integrals and equations. Typical actions are simplification or expansion of expressions, solving (systems of) differential or algebraic equations, and the computation of prime numbers. Most CAS allow the user at least to write sequential programs for complex tasks, and have all features of high-level programming languages available. As well as such features, CAS also have most of the features of numerical systems for visualisation (2D-plots, 3D-plots, animations) and numerical computations (numerical equation solving, numerical integration). However, numerical systems are typically faster in regard to the numerical handling of floats with fixed precision. Some CAS solve this problem by offering embedded links to such numerical software as MATLAB™ (ie Maple V Release 5™).

Besides being a tool for the manipulation of formulae, CAS should be expert systems *knowing* all of the mathematics in a good mathematical handbook. This has not really been achieved yet, but some progress has been made, ie CAS should know all integrals found in, for example, Gradshteyn, Ryzhik and all differential equations from Kamke's famous book [1][2].

There are many commercial and non-commercial products available. The most popular are Mathematica™ [3] and Maple™ [4] which will, in a (hopefully) everlasting contest, continue to evolve. Other systems are AXIOM™ [5], MuPAD™ [6] or REDUCE™ [7]. All systems can be used for high-school to university mathematics, but they differ in comfort and complexity and each has a different *look and feel*.

Throughout this paper, Maple V Release 5™ is used as an exemplary CAS, although for most points discussed it is simply a matter of taste as to which program is used. But Maple has some features (especially the handling of some partial differential equations) that others do not have and, from the author's very personal point of view, the user interface is easier to handle than that of the competitors [8].

There are also some *hybrid* programs that allow symbolic computations as a feature of numeric systems (Symbolic Toolbox for Matlab™ [9], Mathcad™ [10][11], PV Wave™ [12]), and text processors (Scientific Workplace™ [13]) that have embed a full CAS. All these programs contain a *kernel* of the Maple CAS. Other programs merely link to existing CAS using the OLE concept. The problem of such *hybrids* is that they are fixed to a certain release of the underlying kernel or linked CAS and that normally they could not be used across platforms.

MATHEMATICS GOES MULTIMEDIA

Computer Algebra Systems can have a significant impact on the way mathematics is taught and applied.

The situation can in some sense be compared to the advent of the pocket calculator. Today, even in schools, these are simply a tool and it has not meant the decline of mathematics. It is however no longer necessary to memorise the multiplication table up to twenty-five. In teaching mathematics now, it is possible to concentrate on mathematical content and not on counting numbers.

Using CAS it is possible to go one step further. Instead of training integration rules on exotic cases over and over again, for example, it is possible to concentrate on the meaning of integration and its approximation by numbers. We are not limited to trivial examples that *work*. Students are invited to play with mathematics. They learn that *real life* examples normally do not lead to closed formulae. But they can even play with and visualise different approximations and learn to judge the results. They also learn that there are a lot a mathematical tools, each with their own rights and applicability.

Computer programs alone do not make up multimedia education, and we are a long way from having a full integration of tools such as CAS for network-based teaching and learning environments. Distance learning has the advantage that students can learn where and when they like. First steps have been taken: plug-ins for web-browsers, such as Netscape, are available, and allow students to look at and manipulate mathematical content across the web (eg MathView™ [14]). Others for Maple or Mathematica will probably follow. One big step will be made when the standards (XML, MathML) for mathematical typesetting will be available for all browsers and platforms; compare for example IBM's techexplorer [15].

But there should be no illusion about the amount of work that must be done to convert a mathematical example from the blackboard to an interactive, *living* web-based document as computer-based training (CBT). CBTs should then be embedded in a learning environment that manages access to different levels of mastered lectures, collects the credits etc. It would be an advantage if students could work in a similar way (*look and feel*) with all the CBTs offered by a site.

EXAMPLES

Without going into details, some examples that should provide an idea of the possibilities of CAS, especially in contrast to numerical systems, are presented in this section. Throughout this document the formatted Maple *output* is emulated using ordinary postscript symbols, although CAS normally have the possibility to export formulae in a more sophisticated manner. The Maple *input* is denoted by a preceding angle bracket (>).

Infinite precision

Contrary to most numeric systems, CAS have the ability to perform computations with real numbers symbolically without loss of precision. Additionally, CAS are able to represent the results with arbitrary precision. However, it should be noted that results that are achieved symbolically can sometimes turn out to be less accurate than approximations if they are translated to numbers; for example, if the final solution formula contains a lot of (exact, rational or real) terms each producing little rounding errors if turned to floats. Invoking a numerical solver could then actually do a better job.

Real numbers

Often the symbolic representation is more intuitive than the numerical representation, eg the value of an infinite sum could be tackled symbolically:

```
>Sum(1/(1-2*n)*(-1)^n,n=1..infinity): %=values(%);
```

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{-2n+1} = \frac{1}{4}\pi$$

Floats with arbitrary digits

Shanks calculated the value of p to 707 places and published this result in 1873. Unfortunately, he made a little mistake calculating the 528th digit (detected 71 years later by Ferguson) and so spent some years absolutely in vain [16]. CAS are able to compute numbers with precision limited only by memory and computing time. Using Maple we quickly find Shanks' (corrected) result:

```
> evalf(Pi,527);
```

```
3.1415926535897932384626433832795028841971693
  9937510582097494459230781640628620899862803
```

Even on small computer systems the computing time for Shanks' total (corrected) work of years is hardly measurable:

```
> restart:time(evalf(Pi,707));
                              .013
```

A nontrivial example with Maple V Release 5: geometric optics for irregular & gravitational lensing

The optics of the foot of a wineglass or a black-hole in a galaxy can be described in the geometrical optics limit by a very simple two-parameter mapping from the lens-plane (x,y) to the observer/screen-plane (u,v). As is typical for irregular lenses, very bright caustics appear. Below, it is shown how resultant theory is used to classify the occurring morphologies uniquely:

```
> restart: with(linalg):with(plots):with(plottools):
```

The lens mapping connects the plane (x,y) of the lens with the plane (u,v) where the light rays (coming from a distant light source) are observed. The two functions are defined as:

```
> u :=(x, y, a)-> a*x-x/(x^2+y^2):
> v :=(x, y, b)-> b*y-y/(x^2+y^2):
> alias(U = u(x,y,a),V = v(x,y,b)):
```

Then the Jacobian of the mapping reads (as a Maple-function):

```
> j:=unapply((det(jacobian([U,V],[y,x]))),x,y,a,b);
```

$$J := (x,y,a,b) \to \frac{abx^4 + x^2b - x^2a + 2x^2aby^2 + aby^4 - 1 - by^2 + ay^4}{(x^2+y^2)^2}$$

```
> alias(J=j(x,y,a,b)):
```

The Jacobian J=0 of the mapping (also called *critical curve*) and the corresponding caustic (bifurcation set), which are the image of the J=0 under the mapping, are plotted using the transform-function of the plottools, one for each quadrant of the parameter plane (a,b) (ie a=±0.1, b=±1).

```
> j1:=implicitplot(j(x,y, .1, 1),x=-1.3..1.3,y=-3.2..3.2,
      axes=none, labels=["",""]):
> j2:=implicitplot(j(x,y, .1,-1),x=-1.3..1.3,y=-3.2..3.2,
      axes=none,labels=["",""]):
> j3:=implicitplot(j(x,y,-.1,-1),x=-3.2..3.2,y=-1.3..1.3,
      axes=none, labels=["",""]):
> j4:=implicitplot(j(x,y,-.1, 1),x=-3.2..3.2,y=-1.3..1.3,
      axes=none, labels=["",""]):
```

Note, that the following transformations act on *plots*:

```
> M1 := transform((x, y)-> [u(x,y,.1), v(x,y, 1.)]):
> M2 := transform((x, y)-> [u(x,y,.1), v(x,y,-1.)]):
> M3 := transform((x, y)-> [u(x,y,-.1),v(x,y,-1.)]):
> M4 := transform((x, y)-> [u(x,y,-.1),v(x,y, 1.)]):
```

Show all plots together (See Figure 1):

```
> display(array(1..2,1..4,[[j1,j2,j3,j4],[M1(j1),M2(j2),
      M3(j3),M4(j4)]]));
```

Are these shapes (up to scalings) complete? The next step is to try to find areas in the parameter plane where the number of cusps of the caustics are constant. Cusps are given by the vanishing directional derivative of the mapping in the direction of the tangent of the vanishing Jacobian (ie vanishing tangent at the caustic). Up to some non-vanishing factors (we are not interested in the cusps on the axes) the components of the directional derivatives are given by the following two polynomials. (Additionally, the variable x^2 was replaced by X and some term ordering was applied.)

```
> P1 := subs(x = sqrt(X),collect(numer(normal((-diff
      (J,y)*diff(U,x)+diff(J,x)*diff(U,y))/(2*y))),
      x,factor));
```

$$P1 := 3a(-b+a)X^2 + (2a^2y^2 - b + 3a - 2aby^2)X - (-1 + ay^2 - by^2 - 2)$$

```
> P2 := subs(x = sqrt(X),collect(numer(normal((-diff
      (J,y)*diff(V,x)+diff(J,x)*diff(V,y))/
      (2*x))),x,factor));
```

$$P2 := -b(-b+a)X^2 + (2aby^2 + a - 3b - 2b^2y^2)X + ay^2 + 3aby^4 + 2 - 3by^2 - 3b^2y^4$$

The resultant of P1 and P2 wrt X is a fourth order polynomial in y which has to vanish for simultaneous solutions of P1 and P2. For convenience, an unimpor-

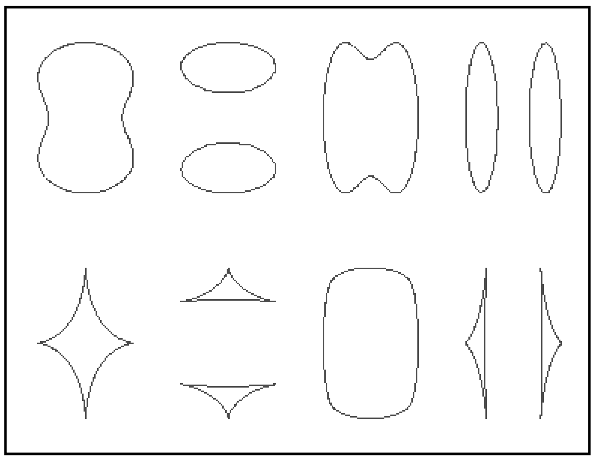

Figure 1: The first row shows the graph of implicit equation J=0 (critical curves) for the values a=±0.1, b=±1 in the plane (x,y). The second row shows the corresponding caustics (images of J=0) in the (u,v) plane.

tant factor $4(b-a)^2$, has been eliminated:

> R:=collect(resultant(P1,P2,X)/(4*(b-a)^2),y,factor);

$$R := 16ab(-b+a)^2 y^4 + 2(-b+a)$$
$$(3a^2 - 14ab - b^2)y^2 + 3(a+b)^2$$

The number of solutions (cusps) changes if the discriminant DIS (resultant of R and diff(R,y) wrt y) vanishes:

> DIS:=factor(discrim(R,y));

$$DIS := 12288(b^2 - 14ab + a^2)^2 (3a-b)^4$$
$$(-b+a)^6 ba(a+b)^2$$

Result

As one could check only the first factor in the third quadrant leads to real solutions for the cusps. $(b^2 - 14ab + a^2)^2$ cuts the third quadrant into three parts. Values near the axes (eg a=-10, b=0.1) lead to *new types of caustics*:

> plot({solve(a^2-14*b*a+b^2,b)},a=-10..0,b=-10..0);
(See Figure 2)

> j5:=implicitplot(j(x,y,-10,-.1),x=-1.2..1.2,y=-3.2..3.2,
axes=none, labels=[“”,””]):
> M5 := transform((x, y)-> [u(x,y,-10), v(x,y,-.1)]):
> display(array(1..2,[j5,M5(j5)]));
(See Figure 3).

It would have been difficult to find all possible shapes of caustics if a pure numerical scheme had been used. However, without a CAS it would be even difficult to build up the appropriate resultants. This is maybe a reason why, before the advent of CAS, elimination- and resultant-theory was rarely taught at high-schools or universities.

However, it should be noted that the complete classification of a general two-lens system is an outstanding problem far beyond the scope of today’s CAS.

FROM A-LEVEL TO PHD

This section provides examples of the use of CAS at the University of Technology of Hamburg-Harburg.

First steps

As the mathematical skills of freshmen can vary considerably, preparatory training in mathematics and in the use of computers is offered in order to equalise starting conditions. In terms of computing knowledge, students are introduced to UNIX and/or Windows, some standard wordprocessing software, such as MSWord (to prepare documents), Netscape (to find more information), mathematical software, such as Maple and Matlab, and a programming language, such as Pascal or C (just up to *hello world*).

The preparatory course in mathematics was, in the past, delivered through lectures and exercises. Through collaboration between the Mathematics Research Department and the Computing Centre, it was possible to have students attempt to solve some of the exercises using Maple; solutions were provided. Typically, students are quite impressed and have fun using Maple for the first time. Curiously, in preparing the solutions it was found that the (applied) mathematicians proposed different examples (computing numbers) to those that would have been selected by our local Maple-experts (applied calculus, visualisation).

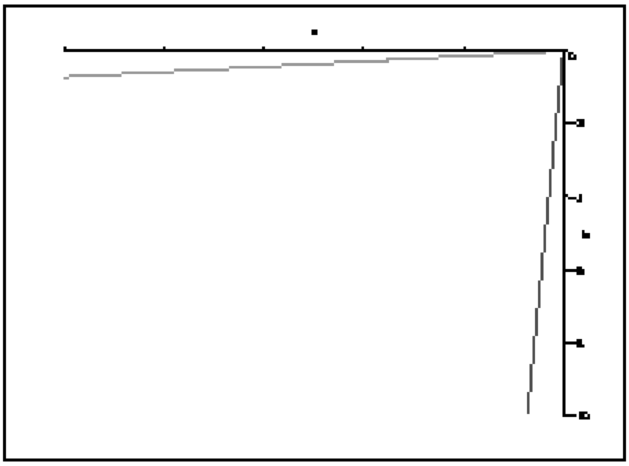

Figure 2: Simultaneous solving and plotting of the important discriminant factor $(b^2 - 14ab + a^2)$ in the parameter plane (a,b).

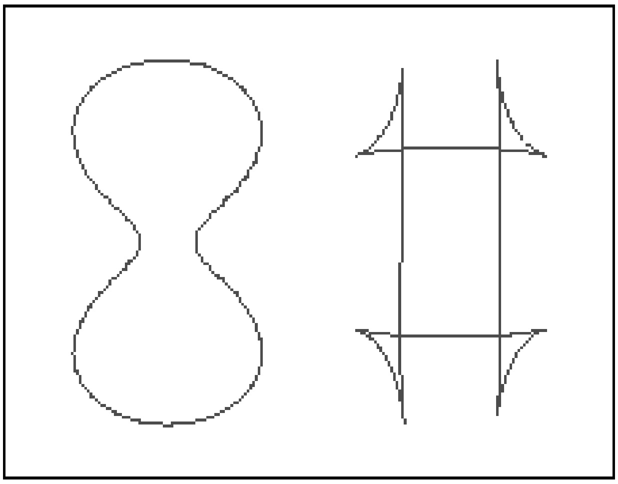

Figure 3: Critical curve (left) and corresponding caustic (right) for the parameters a=-10, b=-0.1.

Playing with differential equations

Students at a university of technology frequently encounter differential equations in their subjects, in several practical exercises and finally in their diploma or PhD theses.

Several ordinary differential equations or systems of them can be solved symbolically or numerically by most CAS. In Maple V Release 5 it is now possible to analytically solve all equations given in Kamke's book [2]. In addition, Lie-symmetry, Laplace transform or series methods are implemented and the solver can switch to several numerical methods if nothing else helps.

Also new, and perhaps unique, is an implementation of some methods to solve *partial* differential equations (pde). Although boundary value problems are not treated in this release, a first step is made towards finding general solutions for several pdes using separation of variables. The following example shows the solution by separation for the wave equation (somewhat uncommon in terms of hyperbolic functions containing arbitrary constants _C1 .. _C4 and separation constant _c [17]). A fit of the solution to boundary conditions must then be done by hand:

```
> pde:=diff(f(x,t),x,x)-1/c^2*diff(f(x,t),t,t)=0;
```

$$pde := \frac{\partial^2}{\partial x^2} f(x,t) - \frac{\frac{\partial^2}{\partial t^2} f(x,t)}{c^2} = 0$$

```
> pdsolve(pde,f(x,t),HINT=‘*‘,INTEGRATE,build);
```

$$\begin{aligned} f(x,t) = {} & \_C3 \sinh\left(c\sqrt{\_c\boxed{1}}\,t\right) \_C1 \sinh\left(\sqrt{\_c\boxed{1}}\,x\right) \\ & + \_C3 \sinh\left(c\sqrt{\_c\boxed{1}}\,t\right) \_C2 \cosh\left(\sqrt{\_c\boxed{1}}\,x\right) \\ & + \_C4 \cosh\left(c\sqrt{\_c\boxed{1}}\,t\right) \_C1 \sinh\left(\sqrt{\_c\boxed{1}}\,x\right) \\ & + \_C4 \cosh\left(c\sqrt{\_c\boxed{1}}\,t\right) \_C2 \cosh\left(c\sqrt{\_c\boxed{1}}\,x\right) \end{aligned}$$

Publishing results: statistics, plots, html and ps

Students should learn very early on to present or publish their results. Maple has several possibilities to export or print entire worksheets or parts as plots in several formats, including postscript and jpg or LATeX for formulae. In Release 5 the feature to export as HTML is added, which is very important if the worksheets are to be published on the web. Plots and formulae are then generated in GIF-format and animations as animated GIFs. (Some examples can be found at our Maple web-page [18].)

Probability paper with Maple

The following is an example that stems from the thesis of Axel Vötter, who investigated the stability of bikes [19]. He required a special kind of plot that was not found in the available numerical or statistical programs, so it was decided to use a feature of Maple to program the basic plot structures to redefine the tick-marks of the plots.

If a normally distributed set of values is plotted in a co-ordinate system that is scaled with the inverse normal distribution, the data will lie on a straight line. Normally this is done by hand using the paper version of the co-ordinate system, similar to the use of *logarithmic paper* for exponential distributed data. Since these kinds of plots are not foreseen in Maple, it is necessary to do some work to simulate this *probability paper*. For this sake, the inverse cumulative distribution function (icdf) of the normal distribution function (from Maple's statistic package) is used:

```
> with(stats[statevalf]):
> invF:=icdf[normald]:
```

First we create the tick-marks of the plot for the values of the cumulative distribution function from 0.1% to 99.0% and glue them together with a list of some additional tick-marks without numbers:

```
> YTicks1:=[seq(evalf(invF(i/100))=convert(i,string),
>       i=[.1,.5,1.0,5.0,10.0,50.0,90.0,95.0,99.0])]:
> YTicks2:=[seq(evalf(invF(i/100))="",
>       i=[.2,.3,.4,2,3,4,20,30,40,60,70,80,96,97,98])]:
> YTicks:=[op(YTicks1),op(YTicks2)]:
```

Now a test example from Bronstein *et al* can be filled in [17]. The upper class limits (XTest) and the related cumulated frequencies in percent (YTest1) are taken from the table, and then the inverse distribution function is applied and the values are zipped together to get a list of co-ordinate pairs (L):

```
> XTest:=[70,90,110,130,150,170,190,210,230,250]:
> YTest1:=[.8,1.6,3.2,10.4,22.4,40.0,64.0,85.6,92.8,97.6]:
> YTest:=map(invF,YTest1/100.):
> L:=zip((x,y)->[x,y],XTest,YTest)[];
```

L := [70, -2.408915546], [90, -2.144410621], [110, -1.852179859], [130, -1.259083980], [150, -.7587535445], [170, -.2533471031], [190, .3584587933], [210, 1.062519302], [230, 1.461056269], [250, 1.977368428]

It is possible to fit a straight line into the data to find the mean and the standard deviation:

```
> with(stats):
> FIT:=unapply(
>       rhs(fit[leastsquare[[x,y]]]([XTest,YTest])),x);
```

$$FIT := x \to -4.485794463 + 0.02565041048\,x$$

```
> mu:=solve(FIT(x)=0): sigma:=solve(FIT(x)=1)-mu:
```

and put everything together in a plot structure. Note that the essential scaling of the axis is achieved by the highlighted part (See Figure 4):

```
> plots[display](
>   plot(FIT,50..250),
>   plot(-1,50..mu-sigma,color=blue),
>   plot([mu-sigma,y,y=-4..-1],color=blue),
>   plot( 0,50..mu,color=blue),
>   plot([mu,y,y=-4..0],color=blue),
>   plot( 1,50..mu+sigma,color=blue),
>   plot([mu+sigma,y,y=-4..1],color=blue),
>   PLOT(POINTS(L),AXESTICKS(DEFAULT,
>   YTicks),
>       VIEW(50..250,-4..4),AXESSTYLE(BOX),
>       TEXT([mu,-3],"m",ALIGNBELOW,FONT
>               (SYMBOL)),
>       TEXT([mu-sigma,-3],"m-s",ALIGNBELOW,
>               FONT(SYMBOL)),
>       TEXT([mu+sigma,-3],"m+s",ALIGNBELOW,
>               FONT(SYMBOL)),
>       TEXT([50,4],"Q%",ALIGNLEFT))
>   );
```

NEW POSSIBILITIES - NEW DIFFICULTIES

Although CAS free the users from boring, standard tasks, it is sometimes hard to bring mathematical expressions exactly to a desired form, even if the necessary transformations are obvious. The reason is often the internal representation of expressions, which is quite different compared to representation on paper. It is therefore quite helpful if a user knows something about how CAS (ie the programmers) think. Another problem is the simplification or expansion rules. It is often hard to see what *simple* means; for example what is simpler log(xy) or log(x)+log(y)?

One way to gain experience is to play around with them after a short (and maybe online) introduction. Another way is to analyse given examples. There is a vast amount of literature about CAS. Often mathematical examples are used to demonstrate CAS features. There are also some textbooks about mathematics that use CAS to solve examples. But there are only a few mathematical handbooks that show simultaneously how the proposed formulae or algorithms are implemented in CAS. One exception is Stöcker's book, which is also available on CD-ROM or online, called *Desktop Mathematik* [20][21]. Here a lot of formulae are translated to PASCAL, Mathematica or Maple. In the next release the Maple, and probably the Matlab, content will be strongly enhanced by about 200 working examples. They are available as pure HTML documents and also as executable worksheets. A preliminary version can be found at the author's Desktop Evaluation Area [22].

Another example is the CRC Interactive, the CD-

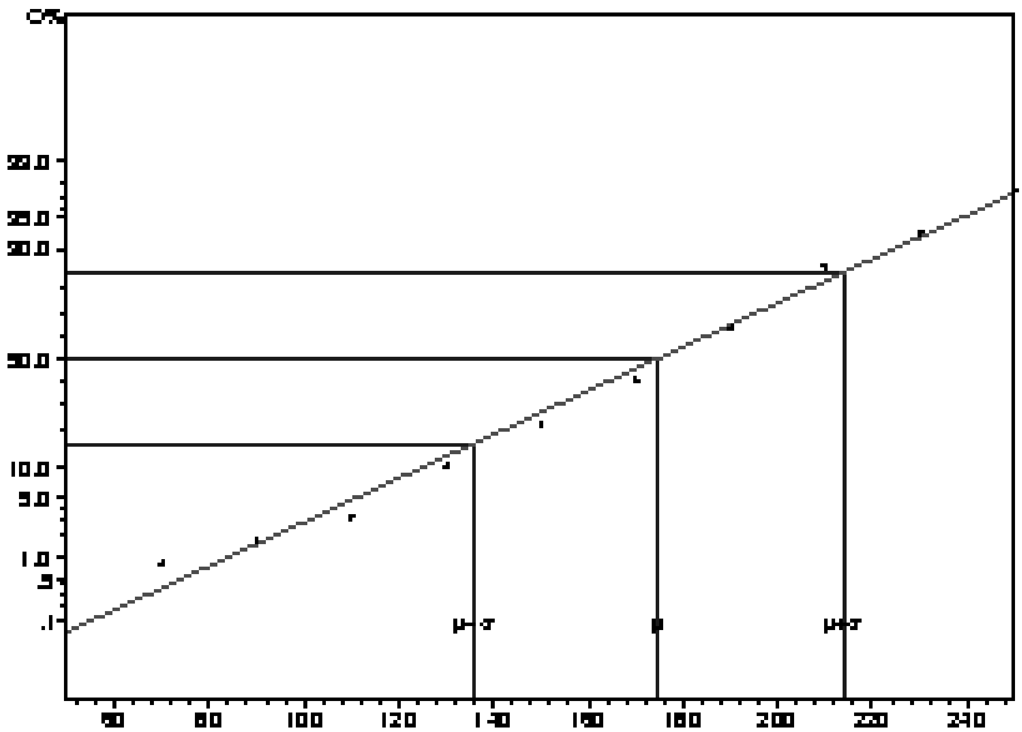

Figure 4: A plot scaled by the inverse cumulative distribution function showing all important information: the data as points, a linear fit, the mean and the standard deviation.

ROM version of the CRC Standard Mathematical Tables and Formulae (English, Windows only) [23]. In this interactive CD-ROM application, the textual information is enriched by *working* formula using the Maple kernel. Using the Windows' interface, it is also possible to use cut and paste methods to bring the formulae to text processors or a full Maple environment.

CONCLUSION

In the near future CAS will be as standard a tool for students and engineers for doing mathematics as the pocket calculator is today. But CAS are not able to perform miracles, since all the symbolic calculations could in principle be done by hand. However, CAS open up a field of computations that are often practically impossible to carry out manually.

Students will use CAS to solve their mathematical everyday problems in exercises or theses. Teachers can use the power of CAS to enrich their lectures with more nontrivial mathematical content. Scientist and engineers could save their work using CAS as a 4th-level computing/documentation environment.

GETTING MORE INFORMATION

A general CAS site is at [24]. Comparisons of different CAS are found at [25][26]. Maple related material can be found at [4][18][27][28]. To subscribe to the worldwide Maple User Group click at [29]. Information about other CAS is at: [3][5-7]. Finally, information in and about this article and more can be found at [30].

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BIOGRAPHY

Thomas Schramm was born in Hamburg in 1956. He received a Diploma in physics in Hamburg in 1986, and his PhD in physics/astrophysics from the University of Hamburg, Hamburg Observatory, in 1988. Until 1995 he worked on the theory and application/observation of gravitational lenses, with topics and areas of interest including: optics, image (re-)construction, relativistic astrophysics, potential theory, inverse problems, partial differential equations, complex calculus, computer algebra.

Since 1995 he has been a consultant for scientific computing at the Technical University of Hamburg-Harburg's Computing Centre, working primarily in the areas of: computer algebra, computing environments, visualisation, numerical libraries and multimedia teaching and learning.

His is also involved in teaching ecology, physics and nuclear physics, and is a consultant for commercial information technology for students at the Fern-Fachhochschule Hamburg (University of Applied Science). His latest project is implementation of Maple/Matlab functionality in a multimedia version of a mathematical Handbook.