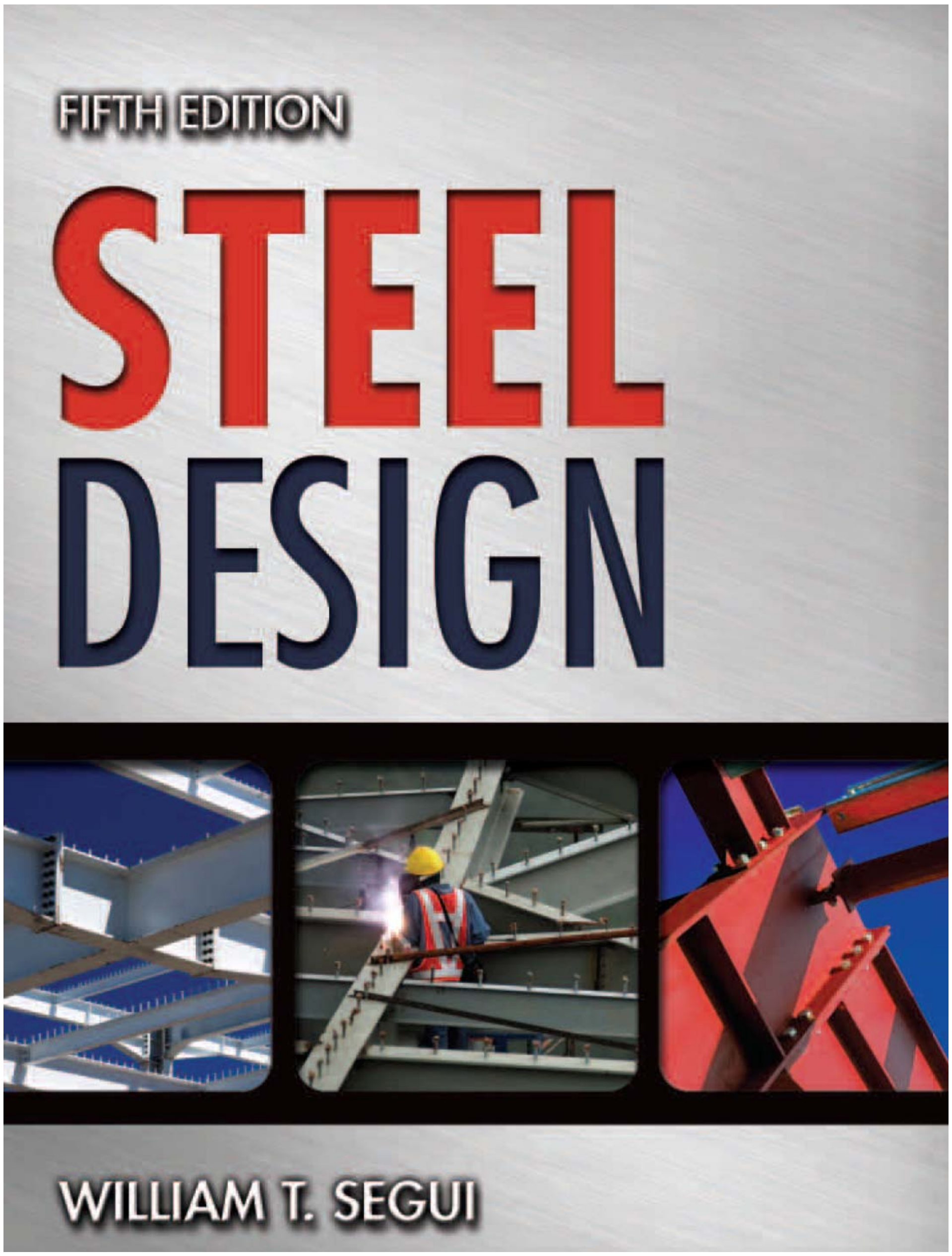
An Instructor's Solutions Manual to Accompany

# STEEL DESIGN, 5th Edition

WILLIAM T. SEGUI

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INSTRUCTOR'S SOLUTIONS MANUAL

TO ACCOMPANY

# STEEL DESIGN

FIFTH EDITION

William T. Segui

# Contents

# PREFACE

This instructor's manual contains solutions to the problems in Chapters 1–10 of *Steel Design, 5th Edition*. Solutions are given for all problems in the Answers to Selected Problems section of the textbook, as well as most of the others.

In general, intermediate results to be used in subsequent calculations were recorded to four significant figures, and final results were rounded to three significant figures. Students following these guidelines should be able to reproduce the numerical results given. However, the precision of the results could depend on the grouping of the computations and on whether intermediate values are retained in the calculator between steps.

In many cases, there will be more than one acceptable solution to a design problem; therefore, the solutions given for design problems should be used only as a guide in grading homework.

I would appreciate learning of any errors in the textbook or solutions manual that you may discover. You can contact me at wsegui@memphis.edu. A list of errors and corrections in the textbook will be maintained at http://www.ce.memphis.edu/segui/errata.html.

*William T. Segui*
*August 15, 2011*

# CHAPTER 1 - INTRODUCTION

**1.5-1**

(a) $P = 20(67) = 1340 \text{ lb}$

$$f = \frac{P}{A} = \frac{1340}{19.7} = 68.02 \text{ psi} \qquad \underline{f = 68.0 \text{ psi}}$$

(b) Since $E = \frac{f}{\epsilon}$,

$$\epsilon = \frac{f}{E} = \frac{68.02}{29{,}000{,}000} = 2.35 \times 10^{-6} \qquad \underline{\epsilon = 2.35 \times 10^{-6}}$$

---

**1.5-2**

(a) $L = 9/\sin 45° = 12.73 \text{ ft}$

$$\Delta L = \epsilon L = 8.9 \times 10^{-4} \times 12.73 \times 12 = 0.136 \text{ in.} \qquad \underline{\Delta L = 0.136 \text{ in.}}$$

(b) $f = \epsilon E = 8.9 \times 10^{-4} \times 29{,}000 = 25.81 \text{ ksi}$

$$P = fA = 25.81(1.31) = 33.8 \text{ kips} \qquad \underline{P = 33.8 \text{ kips}}$$

---

**1.5-3**

(a) $A = \frac{\pi d^2}{4} = \frac{\pi(0.5)^2}{4} = 0.1963 \text{ in.}^2$

$$f = \frac{P}{A} = \frac{5000}{0.1963} = 25{,}470 \text{ psi}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{6.792 \times 10^{-3}}{8} = 8.49 \times 10^{-4}$$

$$E = \frac{f}{\epsilon} = \frac{25{,}470}{8.49 \times 10^{-4}} = 3.0 \times 10^{7} \text{ psi} \qquad \underline{E = 30{,}000 \text{ ksi}}$$

(b) $F_u = \frac{P_u}{A} = \frac{14{,}700}{0.1963} = 74{,}900 \text{ psi} \qquad \underline{F_u = 74.9 \text{ ksi}}$

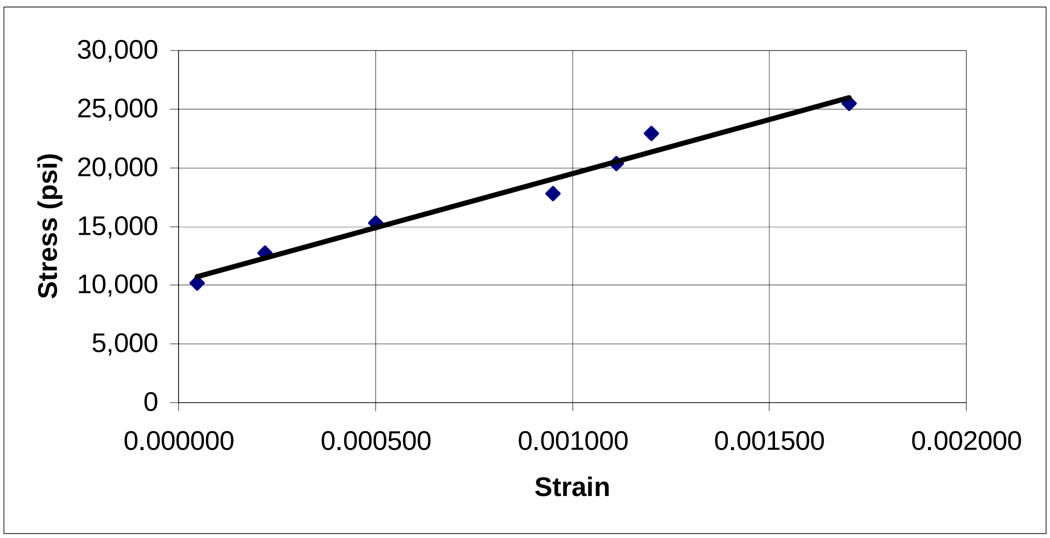
---

**1.5-4**

Spreadsheet results:

(a) (b)

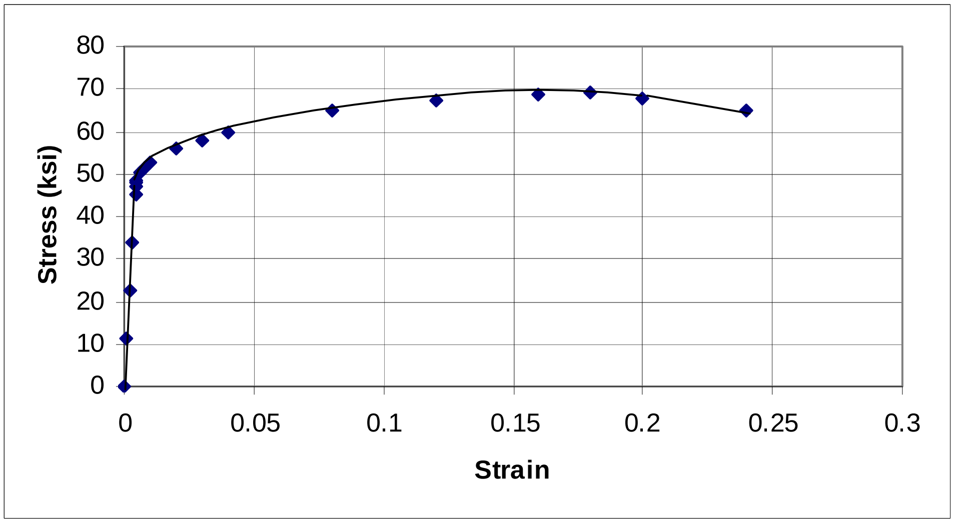
| Load (lb) | Stress (psi) | microstrain |
|---|---|---|
| 2,000 | 10,186 | 47 |
| 2,500 | 12,732 | 220 |
| 3,000 | 15,279 | 500 |
| 3,500 | 17,825 | 950 |
| 4,000 | 20,372 | 1,111 |
| 4,500 | 22,918 | 1,200 |
| 5,000 | 25,465 | 1,702 |

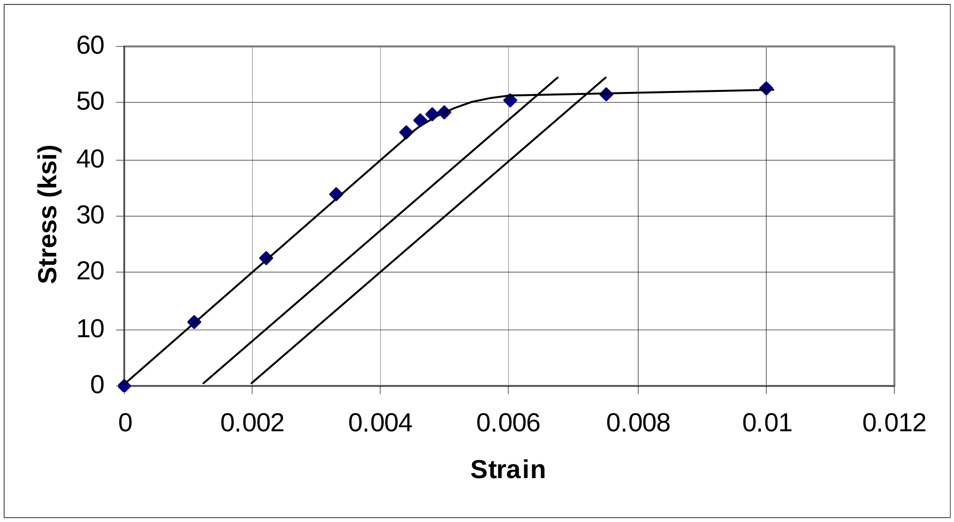



(c) Slope = 9,210,000 psi = modulus of elasticity

**1.5-5** (Note: These results are very approximate and depend on how the curves are drawn.)

(a)




(b) $F_{prop} \approx 47$ ksi

(c) $E \approx 40/0.004 = 10,000$ ksi

(d) $F_y \approx 52$ ksi

(e) $F_u \approx 70$ ksi

(f) $A = \frac{\pi d^2}{4} = \frac{\pi(0.5)^2}{4} = 0.1963 \text{ in.}^2$

$f = \frac{P}{A} = \frac{10}{0.1963} = 50.94 \text{ ksi}$

$\epsilon_r \approx 0.0015, \qquad \delta_r = \epsilon_r L = 0.0015(8) = 0.012 \text{ in.} \qquad \underline{\delta_r = 0.012 \text{ in.}}$

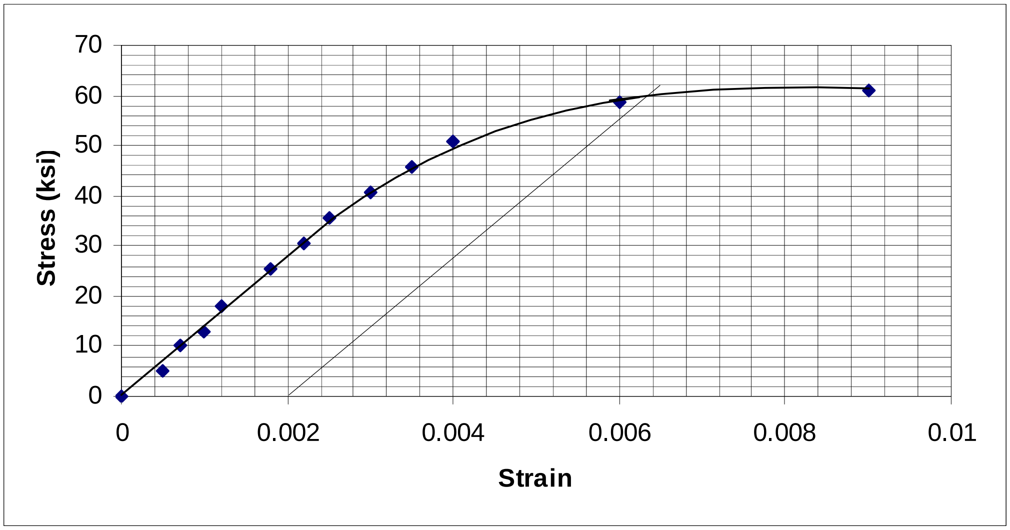
**1.5-6**

Spreadsheet results:

(a)

| Load (kips) | Elongation (in.) | Stress (ksi) | Strain |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1.0 | 0.0010 | 5.094 | 0.0005 |
| 2.0 | 0.0014 | 10.19 | 0.0007 |
| 2.5 | 0.0020 | 12.74 | 0.0010 |
| 3.5 | 0.0024 | 17.83 | 0.0012 |
| 5.0 | 0.0036 | 25.47 | 0.0018 |
| 6.0 | 0.0044 | 30.57 | 0.0022 |
| 7.0 | 0.0050 | 35.66 | 0.0025 |
| 8.0 | 0.0060 | 40.75 | 0.0030 |
| 9.0 | 0.0070 | 45.85 | 0.0035 |
| 10.0 | 0.0080 | 50.94 | 0.0040 |
| 11.5 | 0.0120 | 58.58 | 0.0060 |
| 12.0 | 0.0180 | 61.13 | 0.0090 |

(b)



(c) $E \approx \dfrac{38}{0.0028} = 13,600 \text{ ksi}$ $\underline{E \approx 13,600 \text{ ksi}}$

(d) $\underline{F_{pl} \approx 38 \text{ ksi}}$

(e) $\underline{F_y \approx 60 \text{ ksi}}$

---

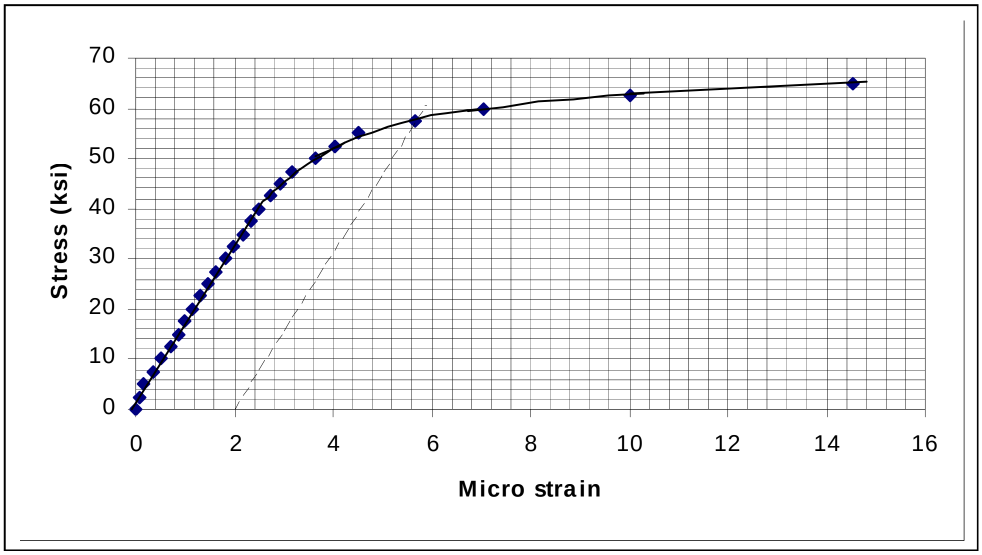
**1.5-7**

Spreadsheet results:

(a)

| **Load (kips)** | **Elongation x $10^3$ (in.)** | **Stress (ksi)** | **Strain x $10^3$ (in./in.)** |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.5 | 0.16 | 2.5 | 0.080 |
| 1 | 0.352 | 5 | 0.176 |
| 1.5 | 0.706 | 7.5 | 0.353 |
| 2 | 1.012 | 10 | 0.506 |
| 2.5 | 1.434 | 12.5 | 0.717 |
| 3 | 1.712 | 15 | 0.856 |
| 3.5 | 1.986 | 17.5 | 0.993 |
| 4 | 2.286 | 20 | 1.143 |
| 4.5 | 2.612 | 22.5 | 1.306 |
| 5 | 2.938 | 25 | 1.469 |
| 5.5 | 3.274 | 27.5 | 1.637 |
| 6 | 3.632 | 30 | 1.816 |
| 6.5 | 3.976 | 32.5 | 1.988 |
| 7 | 4.386 | 35 | 2.193 |
| 7.5 | 4.64 | 37.5 | 2.320 |
| 8 | 4.988 | 40 | 2.494 |
| 8.5 | 5.432 | 42.5 | 2.716 |
| 9 | 5.862 | 45 | 2.931 |
| 9.5 | 6.362 | 47.5 | 3.181 |
| 10 | 7.304 | 50 | 3.652 |
| 10.5 | 8.072 | 52.5 | 4.036 |
| 11 | 9.044 | 55 | 4.522 |
| 11.5 | 11.31 | 57.5 | 5.655 |
| 12 | 14.12 | 60 | 7.060 |
| 12.5 | 20.044 | 62.5 | 10.02 |
| 13 | 29.106 | 65 | 14.55 |

(b)




(c) Using the dashed line, $E \approx \dfrac{56}{(5.6 - 2) \times 10^{-3}} = 15,600$ ksi

$\underline{E \approx 16,000}$ ksi

(d) $\underline{F_{pl} \approx 42 \text{ ksi}}$

(e) $\underline{F_y \approx 58 \text{ ksi}}$

---

# CHAPTER 2 - CONCEPTS IN STRUCTURAL STEEL DESIGN

**2-1**

$D = 30.8$ kips, $L = 1.7$ kips, $L_r = 18.7$ kips, $S = 19.7$ kips

Combination 1: $1.4D = 1.4(30.8) = 43.12$ kips

Combination 2: $1.2D + 1.6L + 0.5S = 1.2(30.8) + 1.6(1.7) + 0.5(19.7)$

$= 49.53$ kips

Combination 3: $1.2D + 1.6S + 0.5L = 1.2(30.8) + 1.6(19.7) + 0.5(1.7)$

$= 69.33$ kips

(a) Combination 3 controls. $\underline{P_u = 69.3 \text{ kips}}$

(b) Since $P_u \leq \phi_c P_n$, $\underline{\phi_c P_n = 69.3 \text{ kips}}$

(c) $P_n = \dfrac{\phi_c P_n}{\phi_c} = \dfrac{69.33}{0.90} = 77.03$ kips $\underline{P_n = 77.0 \text{ kips}}$

(d) Combination 3 controls.

$P_a = D + (L_r \text{ or } S \text{ or } R) = D + S = 30.8 + 19.7 = 50.5$ kips $\underline{P_a = 50.5 \text{ kips}}$

(e) $P_a \leq \dfrac{P_n}{\Omega}$, $P_n = \Omega P_a = 1.67(50.5) = 84.34$ kips $\underline{P_n = 84.3 \text{ kips}}$

---

**2-2**

$D = 26$ kips, $L = 15$ kips, $L_r = 5$ kips, $S = 8$ kips, $R = 5$ kips, $W = 8$ kips

Combination 1: $1.4D = 1.4(26) = 36.4$ kips

Combination 2: $1.2D + 1.6L + 0.5S = 1.2(26) + 1.6(15) + 0.5(8) = 59.2$ kips

Combination 3: $1.2D + 1.6S + 0.5L = 1.2(26) + 1.6(8) + 0.5(15) = 51.5$ kips

Combination4: $1.2D + 1.0W + 0.5L + 0.5S = 1.2(26) + 1.0(8) + 0.5(15) + 0.5(8)$

$= 50.7$ kips

(a) Combination 2 controls. $\underline{P_u = 59.2 \text{ kips}}$

(b) Since $P_u \leq \phi_c P_n$, $\underline{\phi_c P_n = 59.2 \text{ kips}}$

(c) $P_n = \dfrac{\phi_c P_n}{\phi_c} = \dfrac{59.2}{0.90} = 65.78$ $\underline{P_n = 65.8 \text{ kips}}$

(d) Combination 6a controls.

$$P_a = D + 0.75(0.6W) + 0.75L + 0.75S$$

$$= 26 + 0.75(0.6)(8) + 0.75(15) + 0.75(8) = 46.85 \text{ kips} \qquad \underline{P_a = 46.9 \text{ kips}}$$

(e) $P_a \leq \dfrac{P_n}{\Omega}$, $P_n = \Omega P_a = 1.67(46.85) = 78.24$ kips $\underline{P_n = 78.2 \text{ kips}}$

---

**2-3**

$D = 0.2$ kips/ft, $L_r = 0.13$ kips/ft, $S = 0.14$ kips/ft

Combination 1: $1.4D = 1.4(0.2) = 0.28$ kips/ft

Combination 2: $1.2D + 1.6L + 0.5S = 1.2(0.2) + 1.6(0) + 0.5(0.14)$

$= 0.31$ kips/ft

Combination 3: $1.2D + 1.6S = 1.2(0.2) + 1.6(0.14) = 0.464$ kips/ft

(a) Combination 3 controls. $\underline{P_u = 0.464 \text{ kips/ft}}$

(b) Combination 3 controls: $P_a = D + S = 0.2 + 0.14 = 0.34$ kips/ft

$\underline{P_a = 0.34 \text{ kips/ft}}$

---

**2-4**

(a) LRFD

**Roof:**

$D = 30$ psf, $L_r = 20$ psf, $S = 21$ psf, $R = \dfrac{4}{12}(62.4) = 20.8$ psf

Combination 1: $1.4D = 1.4(30) = 42.0$ psf

Combination 2: $1.2D + 1.6L + 0.5S = 1.2(30) + 1.6(0) + 0.5(21) = 46.5$ psf

Combination 3: $1.2D + 1.6S = 1.2(30) + 1.6(21) = 69.6$ psf

Combination 3 controls. $\underline{P_u = 69.6 \text{ psf}}$

**Floor:**

$D = 62$ psf, $L = 80$ psf

Combination 1: $1.4D = 1.4(62) = 86.8$ psf

Combination 2: $1.2D + 1.6L = 1.2(62) + 1.6(80) = 202$ psf

Combination 2 controls. $\underline{P_u = 202 \text{ psf}}$

(b) ASD

**Roof:**

Combination 3 controls: $D + S = 30 + 21 = 51.0$ psf $\underline{P_a = 51.0 \text{ psf}}$

**Floor:**

Combination 2 controls: $D + L = 62 + 80 = 142.0$ psf $\underline{P_a = 142 \text{ psf}}$

---

**<u>2-5</u>**

$D = 13.3$ kips, $L = 6.9$ kips, $L_r = 1.3$ kips, $S = 1.3$ kips, $W = 150.6$ kips,

$E = 161.1$ kips

(a) LRFD

Combination 1: $1.4D = 1.4(13.3) = 18.62$ kips

Combination 2: $1.2D + 1.6L + 0.5L_r = 1.2(13.3) + 1.6(6.9) + 0.5(1.3)$

$= 27.65$ kips

Combination 3: $1.2D + 1.6S + 0.5W = 1.2(13.3) + 1.6(1.3) + 0.5(150.6)$

$= 93.34$ kips

Combination 4: $1.2D + 1.0W + 0.5L + 0.5L_r$

$= 1.2(13.3) + 1.0(150.6) + 0.5(6.9) + 0.5(1.3) = 170.7$ kips

Combination 5: $1.2D \pm 1.0E + 0.5L + 0.2S$

$= 1.2(13.3) + 1.0(161.1) + 0.5(6.9) + 0.2(1.3) = 180.8$ kips

Combination 5 controls. $\underline{P_u = 181 \text{ kips}}$

(b) ASD

Combination 5 controls: $D \pm (0.6W \text{ or } 0.7E) = 13.3 + 0.7(161.1) = 126.1$ kips

$\underline{P_a = 126 \text{ kips}}$

---

# CHAPTER 3 - TENSION MEMBERS

## 3.2-1

For yielding of the gross section,

$$A_g = 7(3/8) = 2.625 \text{ in.}^2, \qquad P_n = F_y A_g = 36(2.625) = 94.5 \text{ kips}$$

For fracture of the net section,

$$A_e = (3/8)\left[7 - \left(1 + \frac{1}{8}\right)\right] = 2.203 \text{ in.}^2$$

$$P_n = F_u A_e = 58(2.203) = 127.8 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(94.5) = 85.05 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(127.8) = 95.85 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_t P_n = 85.1 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{94.5}{1.67} = 56.59 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{127.8}{2.00} = 63.9 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 56.6 \text{ kips}}$

Alternate solution using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is $F_t A_g = 21.6(2.625) = 56.7$ kips

For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is $F_tA_e = 29.0(2.203) = 63.89 = 63.89$ kips

The allowable service load is the smaller value = $56.7$ kips

---

**3.2-2**

For A242 steel and $t$ = ½ in., $F_y = 50$ ksi and $F_u = 70$ ksi. For yielding of the gross section,

$$A_g = 8(1/2) = 4 \text{ in.}^2$$

$$P_n = F_yA_g = 50(4) = 200 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 4 - (1/2)\left(1 + \frac{1}{8}\right) \times 2 \text{ holes} = 2.875 \text{ in.}^2$$

$$A_e = A_n = 2.875 \text{ in.}^2$$

$$P_n = F_uA_e = 70(2.875) = 201.3 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_tP_n = 0.90(200) = 180 \text{ kips}$$

The design strength based on fracture is

$$\phi_tP_n = 0.75(201.3) = 151 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_tP_n = 151 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{200}{1.67} = 120 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{201.3}{2.00} = 101 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 101 \text{ kips}}$

Alternate solution using allowable *stress***:** For yielding,

$$F_t = 0.6F_y = 0.6(50) = 30 \text{ ksi}$$

and the allowable load is

$$F_tA_g = 30(4) = 120 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(70) = 35 \text{ ksi}$$

and the allowable load is

$$F_tA_e = 35(2.875) = 101 \text{ kips}$$

The allowable service load is the smaller value = 101 kips

---

**3.2-3**

Gross section: $P_n = F_yA_g = 50(8.81) = 440.5 \text{ kips}$

Net section: Hole diameter = $1 + \frac{1}{8} = 1\frac{1}{8}$ in.

$$A_n = A_g - 2t_fd_h = 8.81 - 2(0.501)(1.125) = 7.683 \text{ in.}^2$$

$$A_e = 0.9A_n = 0.9(7.683) = 6.915 \text{ in.}^2$$

$$P_n = F_uA_e = 65(6.915) = 449.5 \text{ kips}$$

a. Gross: $\phi_tP_n = 0.90(440.5) = 396 \text{ kips}$

Net: $\phi_tP_n = 0.75(449.5) = 337 \text{ kips}$

Net section controls: $\underline{\phi_tP_n = 337 \text{ kips}}$

b. Gross: $\frac{P_n}{\Omega_t} = \frac{440.5}{1.67} = 264 \text{ kips}$

Net: $\dfrac{P_n}{\Omega_t} = \dfrac{449.5}{2.00} = 225 \text{ kips}$

Net section controls: $\underline{P_n/\Omega_t = 225 \text{ kips}}$

---

**3.2-4**

For yielding of the gross section,

$$A_g = 6(3/8) = 2.25 \text{ in.}^2$$

$$P_n = F_y A_g = 36(2.25) = 81.0 \text{ kips}$$

For fracture of the net section,

$$A_e = A_g = 2.25 \text{ in.}^2$$

$$P_n = F_u A_e = 58(2.25) = 130.5 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(81.0) = 72.9 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(130.5) = 97.88 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_t P_n = 72.9 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{81.0}{1.67} = 48.5 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{130.5}{2.00} = 65.25 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 48.5 \text{ kips}}$

Alternate solution using allowable *stress***:** For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.25) = 48.6 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(2.25) = 65.25 \text{ kips}$$

The allowable service load is the smaller value = 48.6 kips

---

**3.2-5**

Gross section:

$$A_g = 8(1/2) = 4.0 \text{ in.}^2, \qquad P_n = F_y A_g = 36(4.0) = 144.0 \text{ kips}$$

Net section: Hole diameter = $1\frac{1}{8} + \frac{1}{8} = 1\frac{1}{4}$ in.

$$A_e = A_n = w_n t = (8 - 1.25)(1/2) = 3.375 \text{ in.}^2$$

$$P_n = F_u A_e = 58(3.375) = 195.8 \text{ kips}$$

a. Gross: $\phi_t P_n = 0.90(144.0) = 130$ kips

Net: $\phi_t P_n = 0.75(195.8) = 147$ kips

Gross section controls; $\phi_t P_n = 130$ kips

Factored load:

$$P_u = 1.4(95) = 133 \text{ kips or}$$

$$P_u = 1.2(95) + 1.6(9) = 128 \text{ kips (133 kips controls)}$$

Since 133 kips > 130 kips, member does not have enough strength.

b. Gross: $\frac{P_n}{\Omega_t} = \frac{144.0}{1.67} = 86.2$ kips

Net: $\dfrac{P_n}{\Omega_t} = \dfrac{195.8}{2.00} = 97.9$ kips

Gross section controls; $P_n/\Omega_t = 86.2$ kips

Load:

$$P_a = D + L = 95 + 9 = 104 \text{ kips}$$

Since 104 kips > 86.2 kips, member does not have enough strength.

---

**3.2-6**

Compute the strength for one angle, then double it. For the gross section,

$$P_n = F_y A_g = 36(1.20) = 43.2 \text{ kips}$$

For two angles, $P_n = 2(43.2) = 86.4$ kips

Net section:

$$A_n = 1.20 - \left(\frac{1}{4}\right)\left(\frac{3}{4} + \frac{1}{8}\right) = 0.9813 \text{ in.}^2$$

$$A_e = 0.85 A_n = 0.85(0.9813) = 0.8341 \text{ in.}^2$$

$$P_n = F_u A_e = 58(0.8341) = 48.38 \text{ kips}$$

For two angles, $P_n = 2(48.38) = 96.76$ kips

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(86.4) = 77.76 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(96.76) = 72.57 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 72.6$ kips

$$P_u = 1.2D + 1.6L = 1.2(12) + 1.6(36) = 72.0 \text{ kips} < 72.6 \text{ kips} \qquad \text{(OK)}$$

The member has enough strength.

b) For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{86.4}{1.67} = 51.74 \text{ kips}$$

Alternately, the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable strength is $F_tA_g = 21.6(2 \times 1.20) = 51.84$ kips

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{96.76}{2.00} = 48.38 \text{ kips}$$

Alternately, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

and the allowable strength is $F_tA_e = 29(2 \times 0.8341) = 48.38$ kips

The net setion strength controls; the allowable strength is 48.4 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 12 + 36 = 48 \text{ kips} < 48.4 \text{ kips} \qquad \text{(OK)}$$

<u>The member has enough strength.</u>

---

**<u>3.2-7</u>**

Gross section:

$$A_g = 3.37 \text{ in.}^2, \qquad P_n = F_yA_g = 50(3.37) = 168.5 \text{ kips}$$

Net section: Hole diameter $= \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_n = 3.37 - 0.22(2)(1.0) = 2.93 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.93) = 2.491 \text{ in.}^2$$

$$P_n = F_uA_e = 65(2.491) = 161.9 \text{ kips}$$

a. Gross: $\phi_t P_n = 0.90(168.5) = 151.7$ kips

Net: $\phi_t P_n = 0.75(161.9) = 121.4$ kips

Net section controls; $\phi_t P_n = 121.4$ kips

Let $P_u = \phi_t P_n$:

$1.2D + 1.6L = 1.2D + 1.6(3D) = 121.4$ kips

$D = 20.23$ kips, $P = D + L = 20.23 + 3(20.23) = 80.9$ kips

$\underline{P = 80.9 \text{ kips}}$

b. Gross: $\dfrac{P_n}{\Omega_t} = \dfrac{168.5}{1.67} = 100.9$ kips

Net: $\dfrac{P_n}{\Omega_t} = \dfrac{161.9}{2.00} = 80.95$ kips

Gross section controls; $P_n/\Omega_t = 80.95$ kips

Let $P_a = \dfrac{P_n}{\Omega_t}$ :

$D + L = D + 3D = 80.95$ kips

$D = 20.24$ kips, $P = D + L = 20.24 + 3(20.24) = 81.0$ kips

$\underline{P = 81.0 \text{ kips}}$

---

**<u>3.3-1</u>**

(a) $U = 1 - \dfrac{\bar{x}}{\ell} = 1 - \dfrac{1.47}{5} = 0.7060$

$A_e = A_g U = 5.90(0.7060) = 4.165$ in.$^2$

$\underline{A_e = 4.17 \text{ in.}^2}$

(b) Plate with longitudinal welds only:

$\dfrac{\ell}{w} = \dfrac{5}{4} = 1.25, \qquad U = 0.75$ in.$^2$

$$A_e = A_g U = \left(\frac{3}{8} \times 4\right)(0.75) = 1.125 \text{ in.}^2 \qquad \underline{A_e = 1.13 \text{ in.}^2}$$

(c) $U = 1.0$

$$A_e = A_g U = \left(\frac{5}{8} \times 5\right)(1.0) = 3.13 \text{ in.}^2 \qquad \underline{A_e = 3.13 \text{ in.}^2}$$

(d) $U = 1.0$

$$A_g = 0.5(5.5) = 2.750 \text{ in.}^2$$

$$A_n = A_g - A_{holes} = 2.750 - \frac{1}{2}\left(\frac{3}{4} + \frac{1}{8}\right) = 2.313 \text{ in.}^2$$

$$A_e = A_n U = 2.313(1.0) = 2.313 \text{ in.}^2 \qquad \underline{A_e = 2.31 \text{ in.}^2}$$

(e) $U = 1.0$

$$A_g = \frac{5}{8} \times 6 = 3.750 \text{ in.}^2$$

$$A_n = A_g - A_{holes} = 3.750 - \frac{5}{8}\left(\frac{7}{8} + \frac{1}{8}\right) = 3.125 \text{ in.}^2$$

$$A_e = A_n U = 3.125(1.0) = 3.125 \text{in.}^2$$

$$\underline{A_e = 3.13 \text{ in.}^2}$$

---

**<u>3.3-2</u>**

Gross section:

$$A_g = 8\left(\frac{1}{2}\right) = 4.0 \text{ in.}^2, \qquad P_n = F_y A_g = 36(4.0) = 144.0 \text{ kips}$$

Net section:

$$\frac{\ell}{w} = \frac{12}{8} = 1.5 \qquad \therefore\ U = 0.87$$

$$A_e = A_g U = 4.0(0.87) = 3.48 \text{ in.}^2$$

$$P_n = F_u A_e = 58(3.48) = 201.8 \text{ kips}$$

(a) Yielding: $\phi_t P_n = 0.90(144.0) = 129.6$ kips

Rupture: $\phi_t P_n = 0.75(201.8) = 151.4$ kips

Yielding controls: $\underline{\phi_t P_n = 130 \text{ kips}}$

(b) Yielding: $\dfrac{P_n}{\Omega_t} = \dfrac{144.0}{1.67} = 86.23$ kips

Rupture: $\dfrac{P_n}{\Omega_t} = \dfrac{201.8}{2.00} = 100.9$ kips

Yielding controls: $\underline{P_n/\Omega_t = 86.2 \text{ kips}}$

---

**3.3-3**

$$A_n = A_g - \Sigma t d_h = 3.88 - 4\left(\frac{1}{4}\right)\left(\frac{7}{8} + \frac{1}{8}\right) = 2.88 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.648}{3} = 0.784$$

$$A_e = A_g U = 2.88(0.784) = 2.258 \text{ in.}^2$$

$P_n = F_u A_e = 70(2.258) = 158.1$ kips $\underline{P_n = 158 \text{ kips}}$

---

**3.3-4**

Gross section:

$A_g = 6(1/4) = 1.5 \text{ in.}^2$, $P_n = F_y A_g = 036(1.5) = 54.0$ kips

Net section:

$A_e = A_g = 1.5 \text{ in.}^2$, $P_n = F_u A_e = 58(1.5) = 87.0$ kips

(a) Yielding: $\phi_t P_n = 0.90(54.0) = 48.6$ kips

Rupture: $\phi_t P_n = 0.75(87.0) = 65.25$ kips

Yielding controls: $\underline{\phi_t P_n = 48.6 \text{ kips}}$

(b) Yielding: $\dfrac{P_n}{\Omega_t} = \dfrac{54.0}{1.67} = 32.34$ kips

Rupture: $\dfrac{P_n}{\Omega_t} = \dfrac{87.0}{2.00} = 43.5 \text{ kips}$

Yielding controls: $\underline{P_n/\Omega_t = 32.3 \text{ kips}}$

---

**3.3-5**

Net section: Hole diameter = $\frac{7}{8} + \frac{1}{8} = 1.0$ in.

$$A_n = A_g - 4t_f d_h = 13.3 - 4(0.565)(1.0) = 11.04 \text{ in.}^2$$

a. From the properties of a WT8 × 22.5, $\bar{x} = 1.86$ in.

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.86}{(3+3+3)} = 0.7933 < 0.9$$

$$A_e = A_n U = 11.04(0.7933) = 8.758 \text{ in.}^2$$

$$P_n = F_u A_e = 65(8.758) = 569 \text{ kips} \qquad \underline{P_n = 569 \text{ kips}}$$

b. Check with alternative *U* value.

$$\frac{b_f}{d} = \frac{7.04}{16.1} = 0.437 < \frac{2}{3} \;\therefore \text{ this shape does not qualify for } U = 0.9.$$

Since there are more than 3 bolts per line, $U = 0.85$.

$$A_e = A_n U = 11.04(0.85) = 9.384 \text{ in.}^2$$

$$P_n = F_u A_e = 65(9.384) = 610 \text{ kips} \qquad \underline{P_n = 610 \text{ kips}}$$

---

**3.3-6**

Gross section: $P_n = F_y A_g = 50(6.08) = 304.0$ kips

Net section:

$$A_n = A_g - \Sigma t_w d_h = 6.08 - 3(0.282)\left(\frac{7}{8} + \frac{1}{8}\right) = 5.234 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.698}{3(2.5)} = 0.9069$$

$A_e = A_nU = 5.234(0.9069) = 4.747\text{ in.}^2$

$P_n = F_uA_e = 65(4.747) = 308.6\text{ kips}$

a) The design strength based on yielding is

$\phi_tP_n = 0.90(304.0) = 273.6\text{ kips}$

The design strength based on fracture is

$\phi_tP_n = 0.75(308.6) = 231.5\text{ kips}$

The design strength is the smaller value: $\phi_tP_n = 232\text{ kips}$

$P_u = 1.2D + 1.6L = 1.2(60) + 1.6(125) = 272\text{ kips} > 232\text{ kips}$ (N.G.)

The member is not adequate.

b) For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{304.0}{1.67} = 182.0\text{ kips}$$

Alternately, the allowable stress is

$F_t = 0.6F_y = 0.6(50) = 30.0\text{ ksi}$

and the allowable strength is $F_tA_g = 30(6.08) = 182.4\text{ kips}$

For the net section, the allowable strength is $\dfrac{P_n}{\Omega_t} = \dfrac{308.6}{2.00} = 154.3\text{ kips}$

Alternately, the allowable stress is $F_t = 0.5F_u = 0.5(65) = 32.5\text{ ksi}$

and the allowable strength is $F_tA_e = 32.5(4.747) = 154.3\text{ kips}$

The net setion strength controls; the allowable strength is 154 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$P_a = D + L = 60 + 125 = 185\text{ kips} > 154\text{ kips}$ (N.G.)

The member is not adequate.

**3.3-7**

Gross section: $A_g = 2 \times 1.69 = 3.38 \text{ in.}^2$

$$P_n = F_y A_g = 36(3.38) = 121.7 \text{ kips}$$

Net section:

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.725}{8} = 0.9094 > 0.9 \quad \therefore \text{ use } U = 0.9$$

$$A_e = A_g U = 3.38(0.9) = 3.042 \text{ in.}^2$$

$$P_n = F_u A_e = 58(3.042) = 176.4 \text{ kips}$$

(a) Gross: $\phi_t P_n = 0.90(121.7) = 110 \text{ kips}$

Net: $\phi_t P_n = 0.75(176.4) = 132 \text{ kips}$ Gross section controls:

$\underline{\phi_t P_n = 110 \text{ kips}}$

(b) Gross: $\dfrac{P_n}{\Omega_t} = \dfrac{121.7}{1.67} = 72.9 \text{ kips}$

Net: $\dfrac{P_n}{\Omega_t} = \dfrac{176.4}{2.00} = 88.2 \text{ kips}$ Gross section controls:

$\underline{P_n/\Omega_t = 72.9 \text{ kips}}$

---

**3.3-8**

For A242 steel, $F_y = 50$ ksi and $F_u = 70$ ksi (based on flange thickness)

For yielding of the gross section,

$$P_n = F_y A_g = 50(4.79) = 239.5 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 4.79 - \frac{1}{2}\left(\frac{3}{4} + \frac{1}{8}\right) = 4.353 \text{ in.}^2$$

From AISC Table D3.1, Case 8, $U = 0.80$

$A_e = A_nU = 4.353(0.80) = 3.482 \text{ in.}^2$

$P_n = F_eA_e = 70(3.482) = 243.7$ kips

a) The design strength based on yielding is

$\phi_tP_n = 0.90(239.5) = 215.6$ kips

The design strength based on fracture is

$\phi_tP_n = 0.75(243.7) = 182.8$ kips

The design strength is the smaller value: $\phi_tP_n = 182.8$ kips

Let $P_u = \phi_tP_n$

$1.2D + 1.6(2D) = 182.8$, Solution is: $\{D = 41.55\}$

$P = D + L = 41.55 + 2(41.55) = 124.7$ kips $\underline{P = 125 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{239.5}{1.67} = 143.4 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{243.7}{2.00} = 121.9 \text{ kips}$$

The allowable load is the smaller value = 121.9 kips $\underline{P = 122 \text{ kips}}$

Alternate computation of allowable load using allowable stress**:** For yielding,

$F_t = 0.6F_y = 0.6(50) = 30.0$ ksi

and the allowable load is $F_tA_g = 30.0(4.79) = 143.7$ kips

For fracture, $F_t = 0.5F_u = 0.5(70) = 35$ ksi

and the allowable load is

$F_tA_e = 35(3.482) = 121.9 \text{ kips} \therefore P = 122 \text{ kips}$

**3.4-1**

a. Gross section: $P_n = F_y A_g = 36(5/8 \times 12) = 270$ kips $\underline{P_n = 270 \text{ kips}}$

b. Net section: Hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

Possibilities for net width:

$$w_n = 12 - 2(7/8) = 10.25 \text{ in.}$$

$$w_n = 12 - 3(7/8) + \frac{(5)^2}{4(4)}(2) = 12.5 \text{ in.}$$

$$w_n = \left[12 - 3(7/8) + \frac{(3)^2}{4(4)}(2)\right] \times \frac{11}{9} = 12.83 \text{ in.}$$

$$w_n = \left[12 - 2(7/8) + \frac{(5)^2}{4(4)}\right] \times \frac{11}{10} = 12.99 \text{ in}$$

$$w_n = \left[12 - 3(7/8) + \frac{(3)^2}{4(4)} + \frac{(5)^2}{4(4)}\right] \times \frac{11}{10} = 12.65 \text{ in}$$

The effective net area is

$$A_e = A_n = tw_n = (5/8)(10.25) = 6.406 \text{ in.}^2$$

$$P_n = F_u A_e = 58(6.406) = 372 \text{ kips}$$ $\underline{P_n = 372 \text{ kips}}$

---

**3.4-2**

Gross section: $P_n = F_y A_g = 36(5/8 \times 10) = 225.0$ kips

Net section: Hole diameter $= \frac{7}{8} + \frac{1}{8} = 1.0$ in.

Possibilities for net width:

$$w_n = 10 - 1.0 = 9.0 \text{ in.}$$

$$w_n = 10 - 3(1.0) + \frac{(3)^2}{4(3)}(2) = 8.5 \text{ in.}$$

$$w_n = [10 - 2(1.0)] \times \frac{7}{6} = 9.333 \text{ in.}$$

$$w_n = [10 - 3(1.0)] \times \frac{7}{4} = 12.25 \text{ in}$$

The effective net area is

$$A_e = A_n = tw_n = (5/8)(8.5) = 5.313 \text{ in.}^2$$

$$P_n = F_u A_e = 58(5.313) = 308.2 \text{ kips}$$

a) The design strength based on yielding is $\phi_t P_n = 0.90(225.0) = 202.5$ kips

The design strength based on fracture is $\phi_t P_n = 0.75(308.2) = 231.2$ kips

The design strength is the smaller value: $\underline{\phi_t P_n = 203 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{225.0}{1.67} = 134.7 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{308.2}{2.00} = 154.1 \text{ kips}$$

The allowable load is the smaller value : $\underline{\frac{P_n}{\Omega_t} = 135 \text{ kips}}$

Alternate computation of allowable load using allowable stress**:** For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(10 \times 5/8) = 135.0 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(5.313) = 154.1 \text{ kips} \therefore P = 135 \text{ kips}$$

---

**3.4-3**

Gross section: $P_n = F_y A_g = 50(7.02) = 351.0$ kips

Net section: Hole diameter = $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_n = 7.02 - 2(7/8)(0.400) = 6.320 \text{ in.}^2$$

or $$[7.02 - 3(7/8)(0.400)] \times \frac{7}{5} = 8.358 \text{ in}^2$$

or $$7.02 - 0.4(7/8) - 0.400\left[\frac{7}{8} - \frac{(2.5)^2}{4(2.5)}\right] \times 2 = 6.47 \text{ in.}^2$$

or $$\left(7.02 - 2(0.4)(7/8) - 0.400\left[\frac{7}{8} - \frac{(2.5)^2}{4(2.5)}\right]\right) \times \frac{7}{6} = 7.256 \text{ in.}^2$$

Use $A_n = 6.320 \text{ in.}^2$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.981}{5.5} = 0.8216$$

The effective net area is

$$A_e = A_n U = 6.320(0.8216) = 5.193 \text{ in.}^2$$

$$P_n = F_u A_e = 65(5.193) = 337.6 \text{ kips}$$

a. Gross: $\phi_t P_n = 0.90(351.0) = 316$ kips

Net: $\phi_t P_n = 0.75(337.6) = 253$ kips

Net section controls: $\underline{\phi_t P_n = 253 \text{ kips}}$

b. Gross: $\frac{P_n}{\Omega_t} = \frac{351.0}{1.67} = 210$ kips

Net: $\frac{P_n}{\Omega_t} = \frac{337.6}{2.00} = 169$ kips Net section controls:

$\underline{P_n/\Omega_t = 169 \text{ kips}}$

---

**3.4-4**

(a) Gross section: $P_n = F_y A_g = 50(11.8) = \underline{590 \text{ kips}}$

(b) Net section: Hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_n = 11.8 - 0.305(7/8)(2) = 11.27 \text{ in.}^2$$

or $$11.8 - 0.305(7/8) - 0.305\left[\frac{7}{8} - \frac{(3)^2}{4(3)}\right]$$

$$- 0.305(7/8) - 0.305\left[\frac{7}{8} - \frac{(3)^2}{4(3)}\right] = 11.19 \text{ in.}^2$$

Use $A_n = 11.19 \text{ in.}^2$

$$\bar{x} = t_{PL} + \frac{t_w}{2} = \frac{3}{8} + \frac{0.305}{2} = 0.5275 \text{ in.}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.5275}{3.5 + 3} = 0.9188$$

$$A_e = A_n U = 11.19(0.9188) = 10.28 \text{ in.}^2$$

$P_n = F_u A_e = 65(10.28) = 668$ kips $\qquad P_n = 668$ kips

---

**3.4-5**

Gross section: $P_n = F_y A_g = 36(2.89) = 104.0$ kips

Net section: Hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_n = 2.89 - 2(7/8)(5/16) = 2.343 \text{ in.}^2$$

or use $s = 1.5$ in., $g = 2.25 + 2 - \frac{5}{16} = 3.938$ in. and

$$A_n = 2.89 - (5/16)(7/8) - (5/16)\left[\frac{7}{8} - \frac{(1.5)^2}{4(3.938)}\right] - (5/16)(7/8)$$

$$= 2.114 \text{ in.}^2 \text{(controls)}$$

The effective net area is

$A_e = A_n = 2.114 \text{ in.}^2$

$P_n = F_u A_e = 58(2.114) = 122.6 \text{ kips}$

a. Gross: $\phi_t P_n = 0.90(104.0) = 93.6 \text{ kips}$

Net: $\phi_t P_n = 0.75(122.6) = 92.0 \text{ kips}$

Net section controls; $\phi_t P_n = 92.0 \text{ kips}$

Factored load:

$P_u = 1.2(31) + 1.6(31) = 86.8 \text{ kips}$

Since 86.8 kips < 92.0 kips, member is adequate.

b. Gross: $\dfrac{P_n}{\Omega_t} = \dfrac{104.0}{1.67} = 62.3 \text{ kips}$

Net: $\dfrac{P_n}{\Omega_t} = \dfrac{122.6}{2.00} = 61.3 \text{ kips}$

Net section controls; $P_n/\Omega_t = 61.3 \text{ kips}$

Load:

$P_a = D + L = 31 + 31 = 62 \text{ kips}$

Since 62 kips > 61.3 kips, member is not adequate.

---

**3.4-6**

Compute the strength of one channel and double it.

Gross section:

$P_n = F_y A_g = 50(5.87) = 293.5 \text{ kips for one channel.}$

Net section: Hole diameter $= \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$ in.

$A_n = 5.87 - 0.379(5/8) = 5.633 \text{ in.}^2$

or $$5.87 - (0.379)(5/8) - 0.379\left[\frac{5}{8} - \frac{(4)^2}{4(5)}\right] = 5.699 \text{ in.}^2$$

Use $A_n = 5.633 \text{ in.}^2$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.606}{(4 \times 4)} = 0.9621$$

The effective net area is

$$A_e = A_n U = 5.633(0.9621) = 5.420 \text{ in.}^2$$

$$P_n = F_u A_e = 65(5.420) = 352.3 \text{ kips for one channel}$$

(a) Gross: $\phi_t P_n = 0.90(293.5) = 264.2$ kips

Net: $\phi_t P_n = 0.75(352.3) = 264.2$ kips

For two channels, $\phi_t P_n = 264.2(2) = 528.4$ kips

Let $P_u = \phi_t P_n$:

$$1.2D + 1.6L = 1.2D + 1.6(3D) = 528.4 \text{ kips}$$

$$D = 88.07 \text{ kips}, P = D + L = 88.07 + 3(88.07) = 352.3 \text{ kips}$$

$\underline{P = 352 \text{ kips}}$

(b) Gross: $\frac{P_n}{\Omega_t} = \frac{293.5}{1.67} = 175.7$ kips

Net: $\frac{P_n}{\Omega_t} = \frac{352.3}{2.00} = 176.2$ kips

Gross section controls. For two channels, $P_n/\Omega_t = 175.7(2) = 351.4$ kips

Let $P_a = \frac{P_n}{\Omega_t}$ :

$$D + L = D + 3D = 351.4 \text{ kips}$$

$\underline{P = 351 \text{ kips}}$

---

**3.5-1**

the shear areas are

$$A_{gv} = (3/8)(7.5) \times 2 = 5.625 \text{ in.}^2$$

and, since there are 2.5 hole diameters in each line of bolts,

$$A_{nv} = (3/8)[7.5 - 2.5(3/4 + 1/8)] \times 2 = 3.984 \text{ in.}^2$$

The tension area is

$$A_{nt} = (3/8)[2.5 - 1(7/8)] = 0.6094 \text{ in.}^2$$

$$F_y = 50 \text{ ksi}, \ F_u = 70 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(70)(3.984) + 1.0(70)(0.6094) = 210 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(5.625) + 1.0(70)(0.6094)$$

$$= 211 \text{ kips} > 210 \text{ kips}$$

$$\underline{R_n = 210 \text{ kips}}$$

---

**3.5-2**

The shear areas are $A_{gv} = \frac{5}{8}(7)(2) = 8.75 \text{ in.}^2 = A_{nv}$

The tension area is $A_{nt} = A_{gt} = \frac{5}{8}(6) = 3.75 \text{ in.}^2$

$$F_y = 36 \text{ ksi}, \ F_u = 58 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(8.75) + 1.0(58)(3.75) = 522 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(8.75) + 1.0(58)(3.75) = 407 \text{ kips} < 522 \text{ kips}$$

$$\underline{R_n = 407 \text{ kips}}$$

**3.5-3**

the shear areas are

$$A_{gv} = 0.250(13.5) = 3.375 \text{ in.}^2$$

and, since there are 4.5 hole diameters,

$$A_{nv} = 0.250[13.5 - 4.5(7/8 + 1/8)] = 2.25 \text{ in.}^2$$

The tension areas are

$$A_{gt} = 0.25(7.85 - 4.5) = 0.8375 \text{ in.}^2$$

$$A_{nt} = 0.25[7.85 - 4.5 - 0.5(1.0)] = 0.7125 \text{ in.}^2$$

$$F_y = 50 \text{ ksi}, \;\; F_u = 65 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(65)(2.25) + 1.0(65)(0.7125) = 134 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(3.375) + 1.0(65)(0.7125)$$

$$= 148 \text{ kips} > 134 \text{ kips}$$

$$\underline{R_n = 134 \text{ kips}}$$

**3.5-4**

The shear areas are $A_{gv} = \frac{1}{2}(7.5)(2) = 7.5 \text{ in.}^2 = A_{nv}$

The tension area is $A_{nt} = A_{gt} = \frac{1}{2}(6) = 3.0 \text{ in.}^2$

$$F_y = 36 \text{ ksi}, \;\; F_u = 58 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(7.5) + 1.0(58)(3.0) = 435.0 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(7.5) + 1.0(58)(3.0) = 336.0 \text{ kips} < 435 \text{ kips}$$

$$\therefore\ R_n = 336 \text{ kips}$$

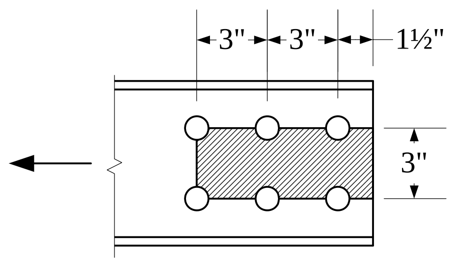
(a) LRFD: $\phi R_n = 0.75(336) = \underline{252 \text{ kips}}$

(b) ASD: $R_n/\Omega = 336/2.0 = \underline{168 \text{ kips}}$

---

**3.5-5**

(a)




The shear areas are

$$A_{gv} = 0.210(7.5)(2) = 3.15 \text{ in.}^2$$

and since there are 2.5 hole diameters,

$$A_{nv} = 0.210[7.5 - 2.5(7/8)](2) = 2.231 \text{ in.}^2$$

The tension areas are

$$A_{gt} = 0.210(3) = 0.63 \text{ in.}^2, \qquad A_{nt} = 0.210[3 - 1.0(7/8)] = 0.4463 \text{ in.}^2$$

$$F_y = 36 \text{ ksi}, \ F_u = 58 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(2.231) + 1.0(58)(0.4463) = 103.5 \text{ kips}$$

Check upper limit:

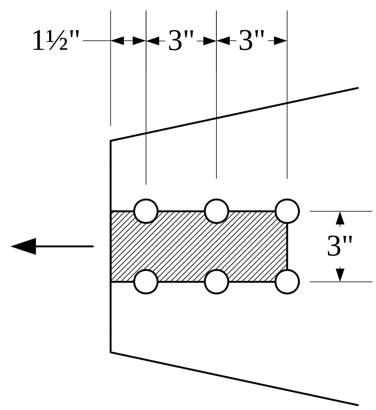
$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.15) + 1.0(58)(0.4463)$$

$$= 93.92 \text{ kips} < 103.5 \text{ kips}$$

LRFD: $\phi R_n = 0.75(93.92) = \underline{70.4 \text{ kips}}$

ASD: $R_n/\Omega = 93.92/2.00 = \underline{47.0 \text{ kips}}$

(b)




The spacings and edge distance are the same as in Part a, but the thickness is different. The shear areas are

$$A_{gv} = (3/8)(7.5)(2) = 5.625 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = (3/8)[7.5 - 2.5(7/8)](2) = 3.984 \text{ in.}^2$$

The tension areas are

$$A_{gt} = (3/8)(3) = 1.125 \text{ in.}^2, \qquad A_{nt} = (3/8)[3 - 1.0(7/8)] = 0.7969 \text{ in.}^2$$

$$F_y = 36 \text{ ksi}, \; F_u = 58 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(3.984) + 1.0(58)(0.7969) = 184.9 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.625) + 1.0(58)(0.7969)$$

$$= 167.7 \text{ kips} < 184.9 \text{ kips}$$

LRFD: $\phi R_n = 0.75(167.7) = \underline{126 \text{ kips}}$

ASD: $R_n/\Omega = 167.7/2.00 = \underline{83.8 \text{ kips}}$

---

**3.5-6**

Find the strength of one channel and double it. $A_g = 5.51 \text{ in.}^2$, $t_w = 0.487$ in.

Gross section:

$$\phi_t P_n = 0.9F_yA_g = 0.9(50)(5.51) = 248.0 \text{ kips; } 248.0 \times 2 = 496 \text{ kips}$$

Net section: Hole diameter = $\frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_n = 5.51 - 2(1.0)(0.487) = 4.536 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.565}{6} = 0.9058$$

The effective net area is

$$A_e = A_nU = 4.536(0.9058) = 4.109 \text{ in.}^2$$

$$\phi_t P_n = 0.75F_uA_e = 0.75(65)(4.109) = 200.3 \text{ kips; } 200.3 \times 2 = 401 \text{ kips}$$

Check block shear. The gusset plate controls; its thickness is 3/8 in., whereas the channels have a total thickness of $2(0.487) = 0.974$ inch. Also, the gusset plate is of a lower strength steel, and its shear length is larger.

The shear areas are

$$A_{gv} = (3/8)(9)(2) = 6.75 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = (3/8)[9 - 2.5(1.0)](2) = 4.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = (3/8)[4 - 1.0] = 1.125 \text{ in.}^2$$

$$F_y = 36 \text{ ksi}, \ F_u = 58 \text{ ksi}$$

$$\phi R_n = 0.75[0.6F_uA_{nv} + U_{bs}F_uA_{nt}] = 0.75[0.6(58)(4.875) + 1.0(58)(1.125)]$$

$$= 176 \text{ kips}$$

Check upper limit:

$$0.75[0.6F_yA_{gv} + U_{bs}F_uA_{nt}] = 0.75[0.6(36)(6.75) + 1.0(58)(1.125)]$$

$$= 158 \text{ kips} < 176 \text{ kips}$$

The upper limit controls; $\phi R_n = 158$ kips

Block shear controls. $\underline{\phi R_n = 158 \text{ kips}}$

---

**3.6-1**

(a) $P_u = 1.2D + 1.6L = 1.2(50) + 1.6(100) = 220.0$ kips

or $P_u = 1.2D + 1.0W + 0.5L = 1.2(50) + 1.0(45) + 0.5(100) = 155.0$ kips

Use $P_u = 220$ kips.

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{220}{0.9(36)} = 6.79 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{220}{0.75(58)} = 5.06 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try L8 × 6 × 9/16

$A_g = 7.61 \text{ in.}^2 > 6.79 \text{ in.}^2$ (OK) $r_{\min} = r_z = 1.30 \text{ in.} > 0.80 \text{ in.}$ (OK)

$$A_n = 7.61 - 2(1.125)(9/16) = 6.344 \text{ in.}^2$$

From Case 8 in AISC Table D3.1, use $U = 0.80$

$A_e = A_n U = 6.344(0.80) = 5.075 \text{ in.}^2 > 5.06 \text{ in.}^2$ (OK) Use L8 × 6 × $\frac{9}{16}$

(b) $P_a = D + L = 50 + 100 = 150$ kips

or $D + 0.75L + 0.75(0.6W) = 50 + 0.75(100) + 0.75(0.6)(45) = 145.3$ kips

Use $P_a = 150$ kips

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{150}{0.6(36)} = 6.94 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{150}{0.5(58)} = 5.17 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20(12)}{300} = 0.8 \text{ in.}$$

Try L8 × 8 × 1/2

$A_g = 7.84 \text{ in.}^2 > 6.94 \text{ in.}^2$ (OK)

$r_{\min} = r_z = 1.59 \text{ in.} > 0.8 \text{ in.}$ (OK)

$A_n = 7.84 - 2(1.125)(1/2) = 6.715 \text{ in.}^2$

$A_e = A_n U = 6.715(0.80) = 5.37 \text{ in.}^2 > 5.17 \text{ in.}^2$ (OK)

Use an L8 × 8× ½

---

**3.6-2**

(a) $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0$ kips

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{120}{0.9(36)} = 3.70 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{120}{0.75(58)} = 2.76 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15 \times 12}{300} = 0.6 \text{ in.}$$

Try 2L5 × $3\frac{1}{2} \times \frac{1}{4}$, long legs back-to-back:

$A_g = 2.07 \times 2 = 4.14 \text{ in.}^2 > 3.70 \text{ in.}^2 \quad \text{(OK)}$

$r_x = 0.853 \text{ in.}, \ r_y = 1.43 \text{ in.}, \quad \therefore \ r_{\min} = 0.853 \text{ in.} > 0.6 \text{ in.} \quad \text{(OK)}$

$A_n = 4.14 - 1.0(1/4) = 3.89 \text{ in.}^2$

From Case 8 in AISC Table D3.1, use $U = 0.80$.

$A_e = A_n U = 3.89(0.80) = 3.11 \text{ in.}^2 > 2.76 \text{ in.}^2 \ \text{(OK)}$

<u>$2L5 \times 3\frac{1}{2} \times \frac{1}{4}$ LLBB</u>

(b) $P_a = D + L = 20 + 60 = 80 \text{ kips}$

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{80}{0.6(36)} = 3.70 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{80}{0.5(58)} = 2.76 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

Try $2L5 \times 3\frac{1}{2} \times \frac{1}{4}$, long legs back-to-back:

$A_g = 2.07 \times 2 = 4.14 \text{ in.}^2 > 3.70 \text{ in.}^2 \quad \text{(OK)}$

$r_x = 0.853 \text{ in.}, \ r_y = 1.43 \text{ in.}, \quad \therefore \ r_{\min} = 0.853 \text{ in.} > 0.6 \text{ in.} \quad \text{(OK)}$

$A_n = 4.14 - 1.0(1/4) = 3.89 \text{ in.}^2$

From Case 8 in AISC Table D3.1, use $U = 0.80$.

$A_e = A_n U = 3.89(0.80) = 3.11 \text{ in.}^2 > 2.76 \text{ in.}^2 \ \text{(OK)}$

<u>$2L5 \times 3\frac{1}{2} \times \frac{1}{4}$ LLBB</u>

---

**<u>3.6-3</u>**

(a) $P_u = 1.2D + 1.6L + 0.5S = 1.2(38) + 1.6(115) + 0.5(75) = 267.1 \text{ kips}$

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{267.1}{0.9(50)} = 5.94 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{267.1}{0.75(65)} = 5.48 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try ST10 × 33

$$A_g = 9.70 \text{ in.}^2 > 5.94 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = 1.19 \text{ in.} > 0.80 \text{ in.} \quad \text{(OK)}$$

$$A_n = 9.70 - 2(7/8)(0.795) = 8.299 \text{ in.}^2$$

Use Case 7 of AISC Table D3.1 to determine $U$.

$$b_f = 6.26 < 2/3d = 2/3(20.0) = 13.33 \text{ (}d\text{ is for an S20} \times 66\text{)}$$

Use $U = 0.85$

$$A_e = A_n U = 8.299(0.85) = 7.05 \text{ in.}^2 > 5.48 \text{ in.}^2 \text{ (OK)}$$

Check an ST9 × 27.35.

$$A_g = 8.02 \text{ in.}^2 > 5.94 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = 1.00 \text{ in.} > 0.80 \text{ in.} \quad \text{(OK)}$$

$$A_n = 8.02 - 2(7/8)(0.691) = 6.811 \text{ in.}^2$$

Use Case 7 of AISC Table D3.1 to determine $U$.

$$b_f = 6.00 < 2/3d = 2/3(18) \text{ (}d\text{ is for an S18} \times 54.7\text{)}$$

Use $U = 0.85$

$$A_e = A_n U = 6.811(0.85) = 5.79 \text{ in.}^2 > 5.48 \text{ in.}^2 \text{ (OK)} \qquad \underline{\text{Use an ST9} \times 27.35}$$

(b) $P_a = D + L = 38 + 115 = 153$ kips

or $D + 0.75L + 0.75S = 38 + 0.75(115) + 0.75(75) = 180.5$ kips

Use $P_a = 180.5$ kips

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{180.5}{0.6(50)} = 6.02 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{180.5}{0.5(65)} = 5.55 \text{ in.}^2$$

Required $r_{\min} = \dfrac{L}{300} = \dfrac{20(12)}{300} = 0.8$ in.

Try an ST9 × 27.35.

$A_g = 8.02 \text{ in.}^2 > 6.02 \text{ in.}^2$ (OK) $r_{\min} = 1.00 \text{ in.} > 0.80 \text{ in.}$ (OK)

$A_n = 8.02 - 2(7/8)(0.691) = 6.811 \text{ in.}^2$

Use Case 7 of AISC Table D3.1 to determine $U$.

$b_f = 6.00 < 2/3d = 2/3(18)$ ($d$ is for an S18 × 54.7)

Use $U = 0.85$

$A_e = A_nU = 6.811(0.85) = 5.79 \text{ in.}^2 > 5.55 \text{ in.}^2$ (OK) <u>Use an ST9 × 27.35</u>

---

**<u>3.6-4</u>**

(a) $\dfrac{D}{L} = \dfrac{216}{25} = 8.64 > 8 \;\therefore$ load combination 1 controls.

$P_u = 1.4D = 1.4(216) = 302.4$ kips

Required $A_g = \dfrac{P_u}{0.9F_y} = \dfrac{302.4}{0.9(36)} = 9.33 \text{ in.}^2$

Required $A_e = \dfrac{P_u}{0.75F_u} = \dfrac{302.4}{0.75(58)} = 6.95 \text{ in.}^2$

Required $r_{\min} = \dfrac{L}{300} = \dfrac{22 \times 12}{300} = 0.88$ in.

Try S12 × 40.8

$A_g = 11.9 \text{ in.}^2 > 9.33 \text{ in.}^2$ (OK) $r_{\min} = 1.06 \text{ in.} > 0.88 \text{ in.}$ (OK)

$A_n = 11.9 - 2(1)(0.462) = 10.98 \text{ in.}^2$

Use $U = 0.70$ (Case 7, AISC Table D3.1)

$A_e = A_nU = 10.98(0.70) = 7.69 \text{ in.}^2 > 6.95 \text{ in.}^2$ (OK) <u>Use S12 × 40.8</u>

(b) $P_a = D + L = 216 + 25 = 241.0$ kips

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{241}{0.6(36)} = 11.2 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{241}{0.5(58)} = 8.31 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{22(12)}{300} = 0.88 \text{ in.}$$

Try S15 × 50

$A_g = 14.6 \text{ in.}^2 > 11.2 \text{ in.}^2$ (OK) $r_{\min} = 1.03 \text{ in.} > 0.88 \text{ in.}$ (OK)

$A_n = 14.6 - 2(1)(0.550) = 13.5 \text{ in.}^2$

Use $U = 0.70$ (Case 7, AISC Table D3.1)

$A_e = A_n U = 13.5(0.70) = 9.45 \text{ in.}^2 > 8.31 \text{ in.}^2$ (OK) Use S15 × 50

---

**3.6-5**

(a) $P_u = 1.2D + 1.6L = 1.2(10) + 1.6(25) = 52.0$ kips

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{52.0}{0.9(35)} = 1.65 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{52.0}{0.75(60)} = 1.16 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{8 \times 12}{300} = 0.32 \text{ in.}$$

Try Pipe 3 Std.

$A_g = 2.07 \text{ in.}^2 > 1.65 \text{ in.}^2$ (OK) $r_{\min} = 1.17 \text{ in.} > 0.32 \text{ in.}$ (OK)

$A_e = A_g = 2.07 \text{ in.}^2 > 1.16 \text{ in.}^2$ (OK) Use Pipe 3 Std.

(b) $P_a = D + L = 10 + 25 = 35$ kips

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{35}{0.6(35)} = 1.67 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{35}{0.5(60)} = 1.17 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{8(12)}{300} = 0.32 \text{ in.}$$

Try Pipe 3 Std.

$A_g = 2.07 \text{ in.}^2 > 1.67 \text{ in.}^2$ (OK) $r_{\min} = 1.17 \text{ in.} > 0.32 \text{ in.}$ (OK)

$A_e = A_g = 2.07 \text{ in.}^2 > 1.17 \text{ in.}^2$ (OK) Use Pipe 3 Std.

---

**3.6-6**

$$P_u = 1.2D + 1.6L = 1.2(54) + 1.6(80) = 192.8 \text{ kips}$$

or $P_u = 1.2D + 1.0W + 0.5L = 1.2(54) + 1.0(75) + 0.5(80) = 179.8 \text{ kips}$

Use $P_u = 192.8$ kips

$$\text{Required } A_g = \frac{P_u}{0.90F_y} = \frac{192.8}{0.90(50)} = 4.28 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{192.8}{0.75(65)} = 3.96 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{17.5 \times 12}{300} = 0.7 \text{ in.}$$

Try C10 × 20:

$A_g = 5.87 \text{ in.}^2 > 4.28 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.690 \text{ in.} \approx 0.7 \text{ in.}$ (say OK)

$A_e = A_gU = 5.87(0.85) = 4.99 \text{ in.}^2 > 3.96 \text{ in.}^2$ (OK)

Compute $U$ with Equation 3.1.

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.606}{9} = 0.9327$$

The next lighter shape that meets slenderness requirements is a C10 × 15.3 with

$A_g = 4.48 \text{ in.}^2$, $r_{\min} = 0.711$ in., and $\bar{x} = 0.634$ in.

From Equation 3.1,

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.634}{9} = 0.9296$$

$$A_e = A_g U = 4.48(0.9296) = 4.16 \text{ in.}^2 > 3.96 \text{ in.}^2 \qquad \text{(OK)}$$

Use a C10 × 15.3

---

**3.7-1**

(a) LRFD: Load combination 1 controls: $P_u = 1.4(43) = 60.2$ kips

$$\text{Required } A_b = \frac{P_u}{0.75(0.75F_u)} = \frac{60.2}{0.75(0.75)(58)} = 1.845 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 1.845, \ d = 1.53 \text{ in.}$$

Required $d = 1.53$ in. Use $1^5/8$ in.

(b) ASD: Load combination 2 controls: $P_a = D + L = 43 + 4 = 47$ kips

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{P_a}{F_t} = \frac{47}{21.75} = 2.161 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 2.161, \ d = 1.66 \text{ in.}$$

Required $d = 1.66$ in. Use $d = 1¾$ in.

---

**3.7-2**

(a) Dead load = beam weight = 0.036 kips/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.036) = 0.0432 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.6(30) = 48.0 \text{ kips}$$

Because of symmetry, the tension is the same in both rods.

$$T_u = \frac{1}{2}[0.0432(30) + 48] = 24.65 \text{ kips}$$

$$\text{Required Area} = A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{24.65}{0.75(0.75)(58)} = 0.7556 \text{ in.}^2$$

From $A_b = \dfrac{\pi d^2}{4}$, required $d = \sqrt{\dfrac{4(0.7556)}{\pi}} = 0.981$ in.

Required $d = 0.981$ in., use $d = 1$ in.

(b) Maximum force in rod occurs when live load is at $A$ or $D$. The entire live load is taken by one rod.

$$T_u = \frac{0.0432(30)}{2} + 48 = 48.65 \text{ kips}$$

$$\text{Required } A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{48.65}{0.75(0.75)(58)} = 1.491 \text{ in.}^2$$

Let $\dfrac{\pi d^2}{4} = 1.491$, $d = 1.38$ in., Required $d = 1.38$ in., use $d = 1\frac{7}{16}$ in.

---

**3.7-3**

(a) Dead load = beam weight = 0.036 kips/ft

Because of symmetry, the tension is the same in both rods.

$$T_a = \frac{1}{2}[0.036(30) + 30] = 15.54 \text{ kips}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{15.54}{21.75} = 0.7145 \text{ in.}^2$$

Let $\dfrac{\pi d^2}{4} = 0.7145$, $d = 0.954$ in.

Required $d = 0.954$ in., use $d = 1$ in.

(b) Maximum force in rod occurs when live load is a $A$ or $D$. Entire live load is taken by one rod.

$$T_a = \frac{0.036(30)}{2} + 30 = 30.54 \text{ kips}$$

$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{30.54}{21.75} = 1.404 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 1.404, \qquad d = 1.34 \text{ in.}$$

Required $d = 1.34$ in., use $d = 1^3/8$ in.

---

**3.7-4**

All members are pin-connected, and all loads are applied at the joints; therefore, all members are two-force members (either tension members or compression members). Load combination 4 controls.

$$1.0W = 1.0(10) = 10 \text{ kips}$$

Slope of brace = 20/35. Angle with horizontal = $\tan^{-1}(20/35) = 29.74°$

Equilibrium at joint $C$:

$$\sum F_x = 10 - T_u \cos 29.74° = 0 \qquad \Rightarrow \qquad T_u = 11.52 \text{ kips}$$

$$\text{Required } A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{11.52}{0.75(0.75)(58)} = 0.3531 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 0.3531, \qquad d = 0.670 \text{ in.}$$

Required $d = 0.670$ in., use $d = 11/16$ in.

---

**3.7-5**

(a) LRFD: $P_u = 1.2D + 1.6L = 1.6(30) = 48.0$ kips

Slope of member $AB = \tan^{-1}(9/15) = 30.96°$. Equilibrium of member $CB$:

$$\sum M_C = 48(15) - [T_u \sin(30.96°)](15) = 0 \qquad \Rightarrow \qquad T_u = 93.31 \text{ kips}$$

$$\text{Required } A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{93.31}{0.75(0.75)(58)} = 2.86 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 2.86, \qquad d = 1.91 \text{ in.}$$

<u>Required $d = 1.91$ in. Use 2 in.</u>

(b) ASD: $P_a = D + L = 30$ kips

Slope of member $AB = \tan^{-1}(9/15) = 30.96°$. Equilibrium of member $CB$:

$$\sum M_C = 30(15) - [T_a \sin(30.96°)](15) = 0, \qquad T_a = 58.32 \text{ kips}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{58.32}{21.75} = 2.681 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 2.681, \qquad d = 1.85 \text{ in.}$$

<u>Required $d = 1.85$ in. Use $1^7/_8$ in.</u>

---

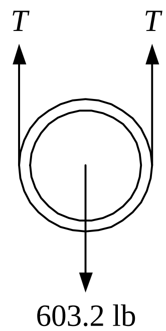
**<u>3.7-6</u>**

From Part 1 of the *Manual*, pipe weight = 28.6 lb/ft

$$\text{Water weight} = \left[\frac{\pi(7.98)^2/4}{144}\right](62.4) = 21.67 \text{ lb/ft}$$

$$\text{Total} = 28.6 + 21.67 = 50.27 \text{ lb/ft}$$

$$\text{Load at each support} = 50.27(12) = 603.2 \text{ lb}, \quad \text{Load on rod} = \frac{603.2}{2} = 301.6 \text{ lb}$$




$$\Sigma F_y = 2T - 603.2 = 0$$

$$T = 301.6 \text{ lb}$$

(a) LRFD

$$T_u = 1.4D = 1.4(301.6) = 422.2 \text{ lb} = 0.4222 \text{ kips}$$

$$\text{Required } A_g = \frac{T_u}{\phi_t(0.75F_u)} = \frac{0.4222}{0.75(0.75)(58)} = 0.01294 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 0.01294, \qquad d = \sqrt{\frac{4(0.01294)}{\pi}} = 0.128 \text{ in.}$$

<u>Required $d = 0.128$ in., Use $\frac{5}{8}$ in. minimum</u>

(b) ASD

$$T_a = 301.6 \text{ kips}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{0.3016}{21.75} = 1.387 \times 10^{-2} \text{ in.}^2$$

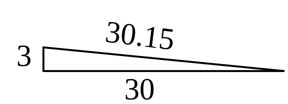
$$\text{Let } \frac{\pi d^2}{4} = 0.01387, \qquad d = 0.133 \text{ in.}$$

<u>Required $d = 0.133$ in. Use $^5/_8$ in.minimum</u>

---

**<u>3.8-1</u>**

Interior joint load:



Snow: $20(10)(15) = 3000$ lb

Roofing: $12(10)(30.15/30)(15) = 1809$ lb

Purlins: $8.5(15) = 127.5$ lb

Truss weight: $1000/3 = 333.3$ lb

(The assumption that the truss weight is distributed equally to the joints is approximate but is consistent with the approximate nature of the estimate of total truss weight.)

(a) Load combination 3 controls:

$$1.2D + 1.6S = 1.2(1.809 + 0.1275 + 0.3333) + 1.6(3.0) = 7.524 \text{ kips}$$

Exterior joint load. Use half of the above loads except for the purlin weight, which is the same:

$$1.2D + 1.6S = 1.2\left(\frac{1.809}{2} + 0.1275 + \frac{0.3333}{2}\right) + 1.6\left(\frac{3.0}{2}\right) = 3.838 \text{ kips}$$

$$\sum M_A = 7.524(10) + 7.524(20) + 3.838(30) - R_{Bx}(3) = 0$$

$$R_{Bx} = 113.6 \text{ kips} \leftarrow$$

Equilibrium of joint $B$:

$$\sum F_x = -113.6 + \frac{30}{30.15}F_{BC} = 0 \qquad \Rightarrow \qquad F_{BC} = 114.2 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{BC}}{0.9F_y} = \frac{114.2}{0.9(50)} = 2.54 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{BC}}{0.75F_u} = \frac{114.2}{0.75(65)} = 2.34 \text{ in.}^2$$

$$L = 10\left(\frac{30.15}{30}\right) = 10.05 \text{ ft}$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10.05 \times 12}{300} = 0.402 \text{ in.}$$

Try WT5 × 11

$$A_g = 3.24 \text{ in.}^2 > 2.54 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = 1.33 \text{ in.} > 0.402 \text{ in.} \quad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.07}{12} = 0.9108$$

$$A_e = A_g U = 3.24(0.9027) = 2.92 \text{ in.}^2 > 2.34 \text{ in.}^2 \quad \text{(OK)}$$

Try WT5 × 9.5

$$A_g = 2.81 \text{ in.}^2 > 2.54 \text{ in.}^2 \qquad \text{(OK)} \qquad r_{\min} = 0.874 \text{ in.} > 0.402 \text{ in.} \qquad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.28}{12} = 0.8933$$

$$A_e = A_g U = 2.81(0.8933) = 2.51 \text{ in.}^2 > 2.34\text{in.}^2 \quad \text{(OK)}$$

Try WT4 × 9

$$A_g = 2.63 \text{ in.}^2 > 2.54 \text{ in.}^2 \qquad \text{(OK)} \qquad r_{\min} = 1.14 \text{ in.} > 0.402 \text{ in.} \qquad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.834}{12} = 0.9305$$

$$A_e = A_g U = 2.63(0.9305) = 2.45 \text{ in.}^2 > 2.34\text{in.}^2 \quad \text{(OK)}$$

<u>Use WT4 × 9</u>

(b) Load combination 2 controls:

$$D + S = 1.809 + 0.1275 + 0.3333 + 3.0 = 5.270 \text{ kips}$$

Exterior joint load: use half of the above loads except for the purlin weight, which is the same:

$$D + S = \frac{1.809}{2} + 0.1275 + \frac{0.3333}{2} + \frac{3.0}{2} = 2.699 \text{ kips}$$

For a free-body diagram of the entire truss,

$$\sum M_A = 5.270(10) + 5.270(20) + 2.699(30) - R_{Bx}(3) = 0$$

$$R_{Bx} = 79.69 \text{ kips } \leftarrow$$

For a free body of joint *B*:

$$\sum F_x = -79.69 + \frac{30}{30.15} F_{BC} = 0, \qquad F_{BC} = 80.09 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{BC}}{0.6F_y} = \frac{80.09}{0.6(50)} = 2.67 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{BC}}{0.5F_u} = \frac{80.09}{0.5(65)} = 2.46 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10.05 \times 12}{300} = 0.402 \text{ in.}$$

Try WT5 × 9.5

$$A_g = 2.81 \text{ in.}^2 > 2.67 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = 0.874 \text{ in.} > 0.402 \text{ in.} \quad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.28}{12} = 0.8933$$

$$A_e = A_g U = 2.81(0.8933) = 2.51 \text{ in.}^2 > 2.46 \text{in.}^2 \text{ (OK)} \qquad \underline{\text{Use WT5} \times 9.5}$$

---

**3.8-2**

The diagonal web members are the tension members, and member *AL* has the largest force.

Using the method of sections and considering the force in member *AL* to act at *L*,

$$\sum M_G = 45(F_{AL} \sin 45°) - 8(45 + 36 + 27 + 18 + 9) = 0$$

$$F_{AL} = 33.94 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{AL}}{0.6F_y} = \frac{33.94}{0.6(36)} = 1.57 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{AL}}{0.75F_u} = \frac{33.94}{0.5(58)} = 1.17 \text{ in.}^2$$

$$L = \sqrt{(9)^2 + (9)^2} = 12.73 \text{ ft}$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{12.73 \times 12}{300} = 0.509 \text{ in.}$$

Try L3$\frac{1}{2}$ × 3 × $\frac{1}{4}$

$$A_g = 1.58 \text{ in.}^2 > 1.57 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = 0.628 \text{ in.} > 0.509 \text{ in.} \quad \text{(OK)}$$

$$A_n = A_g = 1.58 \text{ in.}^2$$

$$A_e = A_n U = 1.58(0.85) = 1.34 \text{ in.}^2 > 1.17 \text{ in.}^2 \quad \text{(OK)}$$

<u>Use L3$\frac{1}{2}$ × 3 × $\frac{1}{4}$ for member *AL*</u>

This shape can be used for all of the web tension members. Although each member could be a different size, this would not usually be practical.

---

**3.8-3**

Use load combination 3: $1.2D + 1.6S$.

Tributary surface area per joint $= 18\sqrt{(9)^2 + (9/6)^2} = 164.2\text{ ft}^2$

Roofing: $1.2(8)(164.2) = 1576\text{ lb}$

Snow: $1.6(20)(9 \times 18) = 5184\text{ lb}$

Purlin weight: $1.2(33 \times 18) = 713\text{ lb}$

Truss weight: $1.2(5000)/12 = 500\text{ lb}$

Interior joint: $1576 + 5184 + 500 + 713 = 7973\text{ lb} = 7.97\text{ kips}$

At peak: $1576 + 5184 + 500 + 2(713) = 8686\text{ lb} = 8.69\text{ kips}$

Load = 8.69 kips at peak, 7.97 kips elsewhere

---

**3.8-4**

Dead load per truss $= (4 + 12 + 3)(40.79 \times 2)(30) + 5(80)(30) = 5.85 \times 10^4\text{ lb}$

Snow load per truss $= 20(80)(30) = 48{,}000\text{ lb}$

$D = 58500/8 = 7313\text{ lb}, \qquad S = 48{,}000/8 = 6000\text{ lb}$

Load combination 3 controls:

Factored joint load $= 1.2D + 1.6S = 1.2(7.313) + 1.6(6) = 18.38\text{ kips}$

Bottom chord: Member *FE* (member adjacent to the support) has the largest tension force.

Use a free body of joint $E$ (at right support):

$$R = \text{Reaction at right support} = 7(18.38)/2 = 64.33 \text{ kips}$$

$$\sum F_y = 64.33 - \frac{8}{40.79}F_{DE} = 0, \qquad F_{DE} = 328.0 \text{ kips}$$

$$\sum F_x = 328.0\left(\frac{40}{40.79}\right) - F_{FE} = 0, \qquad F_{FE} = 321.6 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{FE}}{0.9F_y} = \frac{321.6}{0.9(36)} = 9.93 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{FE}}{0.75F_u} = \frac{321.6}{0.75(58)} = 7.39 \text{ in.}^2$$

$$\text{Required } A_n = \frac{\text{Required } A_e}{U} = \frac{7.393}{0.80} = 9.24 \text{ in.}^2$$

$$r_{\min} = \frac{L}{300} = \frac{10(12)}{300} = 0.4 \text{ in.}$$

(The required $A_g$ of $9.93 \text{ in.}^2$ will satisfy both area requirements)

Try 2L $6 \times 6 \times \frac{7}{16}$

$$A_g = A_n = 10.2 \text{ in.}^2 > 9.93 \text{ in.}^2 \qquad \text{(OK)}$$

$$r_x = 2.62 \text{ in.},\ r_y = 1.86 \text{ in.}, \quad \therefore\ r_{\min} = 1.86 \text{ in.} > 0.4 \text{ in.} \qquad \text{(OK)}$$

Use 2L $6 \times 6 \times \frac{7}{16}$ for bottom chord

Web members: Design for the maximum tensile force, which occurs in member *AH*, and use one shape for all tension web members (the diagonal web members). Using the method of sections (see figure), consider the force in member *AH* to act at *H*.

$$\text{Length} = \sqrt{(8)^2 + (10)^2} = 12.81 \text{ ft.}$$

$$\sum M_E = \frac{8}{12.81}F_{AH}(30) - 18.38(10 + 20 + 30) = 0, \qquad F_{AH} = 58.86 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{AH}}{0.9F_y} = \frac{58.86}{0.9(36)} = 1.82 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{AH}}{0.75F_u} = \frac{58.86}{0.75(58)} = 1.353 \text{ in.}^2$$

$$\text{Required } A_n = \frac{\text{Required } A_e}{U} = \frac{1.353}{0.80} = 1.69 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{12.81 \times 12}{300} = 0.512 \text{ in.}$$

Try 2L $3 \times 2 \times \frac{3}{16}$

$$A_g = A_n = 1.83 \text{ in.}^2 > 1.82 \text{ in.}^2 \qquad \text{(OK)}$$

$$r_x = 0.869 \text{ in.},\ r_y = 0.577 \text{ in.},\quad \therefore\ r_{\min} = 0.577 \text{ in.} > 0.512 \text{ in.} \qquad \text{(OK)}$$

Use 2L $3 \times 2 \times \frac{3}{16}$ for web members

---

**3.8-5**

Use sag rods at midspan of purlins.

$$\text{Top Chord length} = \sqrt{(40)^2 + (8)^2} = 40.79 \text{ ft}$$

$$\text{Tributary area} = 40.79(30/2) = 611.9 \text{ ft}^2$$

$$\text{Total vertical load} = 3(611.9) = 1836 \text{ lb}$$

$$\text{Component parallel to roof} = 1836\left(\frac{8}{40.79}\right) = 360.1 \text{ lb}$$

(a) Since the design is for dead load only, use load combination 1:

$$P_u = 1.4D = 1.4(360.1) = 504.1 \text{ lb}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t(0.75F_u)} = \frac{0.5041}{0.75(0.75)(58)} = 0.01545 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 0.01545, \qquad d = \sqrt{\frac{4(0.01545)}{\pi}} = 0.140 \text{ in.}$$

Required $d = 0.140$ in., Use $\frac{5}{8}$ in. minimum

(b) $P_a = 360.1$ lb

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{P_a}{F_t} = \frac{0.3601}{21.75} = 0.01656 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 0.01656, \qquad d = 0.145 \text{ in.}$$

$$\underline{\text{Required } d = 0.145 \text{ in., Use } \tfrac{5}{8} \text{ in. minimum}}$$

---

# CHAPTER 4 - COMPRESSION MEMBERS

**4.3-1**

(a) $\dfrac{KL}{r} = \dfrac{1.0(15 \times 12)}{1.94} = 92.784$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(92.78)^2} = 33.250 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 92.78 < 113.4,$ use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/33.25)}(50) = 26.646 \text{ ksi}$$

$$P_n = F_{cr}A_g = 26.65(9.71) = 258.77 \text{ kips}$$

<u>$P_n = 259$ kips</u>

(b) $\dfrac{KL}{r} = \dfrac{1.0(20 \times 12)}{1.94} = 123.71$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(123.7)^2} = 18.705 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 123.7 > 113.4,$ use AISC Eq. E3-3:

$$F_{cr} = 0.877F_e = 0.877(18.71) = 16.409 \text{ ksi}$$

$$P_n = F_{cr}A_g = 16.41(9.71) = 159.34 \text{ kips}$$

<u>$P_n = 159$ kips</u>

---

**4.3-2**

$$\frac{KL}{r} = \frac{1.0(15 \times 12)}{2.20} = 81.82$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(81.82)^2} = 42.75 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{35}} = 135.6$$

Since $KL/r = 81.82 < 135.6,$ use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(35/42.75)}(35) = 24.85 \text{ ksi}$$

$$P_n = F_{cr}A_g = 24.85(7.83) = 195 \text{ kips}$$ $\underline{P_n = 195 \text{ kips}}$

---

**4.3-3**

$$\frac{KL}{r} = \frac{2.1(16 \times 12)}{2.86} = 140.98$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(141.0)^2} = 14.397 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 141.0 > 113.4,$ use AISC Eq. E3-3:

$$F_{cr} = 0.877F_e = 0.877(14.40) = 12.629 \text{ ksi}$$

$$P_n = F_{cr}A_g = 12.63(15.5) = 195.77 \text{ kips}$$ $\underline{P_n = 196 \text{ kips}}$

---

**4.3-4**

(a) $$\frac{KL}{r} = \frac{0.65(15 \times 12)}{3.15} = 37.14$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(37.14)^2} = 207.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{46}} = 118.3$$

Since $KL/r = 37.14 < 118.3,$ use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/207.5)}(46) = 41.92 \text{ ksi}$$

$$P_n = F_{cr}A_g = 41.92(7.10) = 297.6 \text{ kips}$$

Design strength $= \phi_c P_n = 0.90(297.6) = 268$ kips $\underline{\phi_c P_n = 268 \text{ kips}}$

Allowable strength $= \dfrac{P_n}{\Omega_c} = \dfrac{297.6}{1.67} = 178$ kips $\underline{\dfrac{P_n}{\Omega_c} = 178 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 37.14$ and $F_y = 46$ ksi,

$\phi_c F_{cr} = 37.77$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 37.77(7.10) = 268$ kips $\underline{\phi_c P_n = 268 \text{ kips}}$

$\dfrac{F_{cr}}{\Omega_c} = 25.09$ ksi (by interpolation)

$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}A_g}{\Omega_c} = 25.09(7.10) = 178$ kips $\underline{\dfrac{P_n}{\Omega_c} = 178 \text{ kips}}$

---

**4.3-5**

a) $$\frac{KL}{r} = \frac{1.2(12 \times 12)}{2.69} = 64.24$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(64.24)^2} = 69.36 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 64.24 < 113.4,$ use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/69.36)}(50) = 36.98 \text{ ksi}$$

$$P_n = F_{cr}A_g = 36.98(35.1) = 1298 \text{ kips}$$

Design strength $= \phi_c P_n = 0.90(1298) = 1168$ kips $\underline{\phi_c P_n = 1170 \text{ kips}}$

Allowable strength = $\frac{P_n}{\Omega_c} = \frac{1298}{1.67} = 777.2$ kips $\qquad$ $\underline{\frac{P_n}{\Omega_c} = 777 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 64.24$ and $F_y = 50$ ksi,

$\phi_c F_{cr} = 33.30$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 33.30(35.1) = 1169$ kips $\qquad$ $\underline{\phi_c P_n = 1170 \text{ kips}}$

$\frac{F_{cr}}{\Omega_c} = 22.15$ ksi (by interpolation)

$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = 22.15(35.1) = 777.5$ kips $\qquad$ $\underline{\frac{P_n}{\Omega_c} = 777 \text{ kips}}$

---

**4.3-6**

$$\frac{KL}{r} = \frac{0.8(13 \times 12)}{3.70} = 33.73$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(33.73)^2} = 251.6 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 33.73 < 113.4$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/251.6)}(50) = 46.01 \text{ ksi}$$

$$P_n = F_{cr} A_g = 46.01(26.5) = 1219 \text{ kips}$$

(a) Design strength = $\phi_c P_n = 0.90(1219) = 1100$ kips

$P_u = 1.2D + 1.6L = 1.2(180) + 1.6(540) = 1080 \text{ kips} < 1100 \text{ kips}$ $\qquad$ (OK)

<u>Column has enough available strength.</u>

(b) Allowable strength = $\frac{P_n}{\Omega_c} = \frac{1219}{1.67} = 730$ kips

$P_a = D + L = 180 + 540 = 720 < 730$ kips $\qquad$ (OK)

<u>Column has enough available strength.</u>

**4.3-7**

$$\frac{KL}{r} = \frac{0.8(20 \times 12)}{3.05} = 62.95$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(62.95)^2} = 72.23 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 62.95$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/72.23)}(50) = 37.42 \text{ ksi}$$

$$P_n = F_{cr}A_g = 37.42(23.2) = 868.1 \text{ kips}$$

(a) $\phi_c P_n = 0.90(868.1) = 781$ kips

$P_u = 1.4D = 1.4(560) = 784$ kips > 781 kips (N.G.)

A W12 × 79 is not adequate

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{868.1}{1.67} = 520$ kips

$P_a = D + L = 560 + 68 = 628$ kips > 520 kips (N.G.)

A W12 × 79 is not adequate

---

**4.3-8**

$$\frac{KL}{r} = \frac{0.8(10 \times 12)}{2.26} = 42.48$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(42.48)^2} = 158.6 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{42}} = 123.8 > 42.48$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/158.6)}(42) = 37.20 \text{ ksi}$$

$$P_n = F_{cr}A_g = 37.20(4.68) = 174.1 \text{ kips}$$

(a) Let $P_u = \phi_c P_n$

$$1.2D + 1.6(2D) = 0.90(174.1), \text{ Solution is: } \{D = 35.61\}$$

$$P = D + L = 35.61 + 2(35.61) = 107 \text{ kips} \qquad \underline{P = 107 \text{ kips}}$$

(b) Let $P_a = P_n/\Omega_c$

$$D + L = 174.1/1.67 = 104 \text{ kips} \qquad \underline{P = 104 \text{ kips}}$$

---

**4.4-1**

Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{2.0(12 \times 12)}{3.28} = 87.80 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118$$

Since 87.80 < 118, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(87.80)^2} = 37.13 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(46/37.13)}(46) = 27.39 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 27.39(6.06) = 166.0 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio for the larger overall dimension is

$$\frac{h}{t} = 54.5$$

The ratio for the smaller dimension is

$$\frac{b}{t} = 43.0$$

From AISC Table B4.1, case 12 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29,000}{46}} = 35.15$$

Since both $\frac{h}{t}$ and $\frac{b}{t}$ are > $1.40\sqrt{\frac{E}{F_y}}$, both elements are slender and the local buckling strength must be computed. (Although the limiting width-thickness ratio is labeled *b/t* in the table, that is a generic notation, and it applies to *h*/*t* as well.)

Because these cross-sectional elements are stiffened elements, $Q_s = 1.0$, and $Q_a$ must be computed from AISC Section E7.2. The shape is a rectangular section of uniform thickness, so AISC E7.2(b) applies, provided that

$$\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}}$$

where

$$f = \frac{P_n}{A_e}$$

and $A_e$ is the reduced effective area. The Specification User Note for square and rectangular sections permits a value of $f = F_y$ to be used in lieu of determining $f$ by iteration. From AISC Equation E7-18, the effective width of the slender element is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \qquad \text{(AISC Equation E7-18)}$$

For the 10-inch side, using $f = F_y$ and the *design* thickness from the dimensions and properties table,

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{46}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29000}{46}}\right] = 6.920 \text{ in.}$$

From AISC B4.2(d) and the discussion in Part 1 of the Manual, the unreduced length of the 10-inch side between the corner radii can be taken as

$$b = 10 - 2(1.5t) = 10 - 2(1.5)(0.174) = 9.478 \text{ in.}$$

where the corner radius is taken as 1.5 times the design thickness. The loss in area for the two10-inch sides is therefore

$$2(b - b_e)t = 2(9.478 - 6.920)(0.174) = 0.8902 \text{ in.}^2$$

For the 8-inch sides, the unreduced length between the corner radii can be taken as

$$b = 8 - 2(1.5t) = 8 - 2(1.5)(0.174) = 7.478 \text{ in.}$$

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{46}}\left[1 - \frac{0.38}{(43.0)}\sqrt{\frac{29000}{46}}\right] = 6.527 \text{ in.}$$

The loss in area for the two 8-inch sides is

$$2(b - b_e)t = 2(7.478 - 6.527)(0.174) = 0.3309 \text{ in.}^2$$

The reduced area is

$$A_e = 6.06 - 0.8902 - 0.3309 = 4.839 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A} = \frac{4.893}{6.06} = 0.8074$$

$$Q = Q_s Q_a = 1.0(0.8074) = 0.8074$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.8074(46)}} = 131.6$$

$$\frac{KL}{r} = 87.80 < 131.6 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.8074\left(0.658^{\frac{0.8074(46)}{37.13}}\right)46 = 24.44 \text{ ksi}$$

$$P_n = F_{cr}A_g = 24.44(6.06) = 148 \text{ kips} \qquad \underline{P_n = 148 \text{ kips}}$$

---

**Iterative solution for $f$:**

As an initial trial value use

$$f = F_{cr} = 24.44 \text{ ksi}$$

(the value obtained above after using an initial value of $f = F_y$)

For the 10-inch side, $b = 9.478$ in., and

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{24.44}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29000}{24.44}}\right] = 8.744 \text{ in..}$$

The loss in area for the two10-inch sides is therefore

$$2(b - b_e)t = 2(9.478 - 8.744)(0.174) = 0.2554 \text{ in.}^2$$

For the 8-inch sides, $b = 7.478$ in., and

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{24.44}}\left[1 - \frac{0.38}{(43.0)}\sqrt{\frac{29000}{24.44}}\right] = 8.005 \text{ in.} > b$$

Therefore, there is no reduction for the 8-inch sides, and the reduced area is

$$A_e = 6.06 - 0.2554 = 5.805 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A} = \frac{5.805}{6.06} = 0.9579$$

$$Q = Q_s Q_a = 1.0(0.9579) = 0.9579$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.9579(46)}} = 120.8$$

$$\frac{KL}{r} = 87.80 < 120.8 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9579\left(0.658^{\frac{0.9579(46)}{37.13}}\right)46 = 26.81 \text{ ksi}$$

$$\neq 24.44 \text{ ksi (the assumed value)}$$

Try $f = 26.81$ ksi:

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{26.81}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29000}{26.81}}\right] = 8.468 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(9.478 - 8.468)(0.174) = 0.3515 \text{ in.}^2$$

and the reduced area is

$$A_e = 6.06 - 0.3515 = 5.709 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A} = \frac{5.709}{6.06} = 0.9421$$

$$Q = Q_s Q_a = 1.0(0.9421) = 0.9421$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.9421(46)}} = 121.8$$

$$\frac{KL}{r} = 87.80 < 121.8 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9421\left(0.658^{\frac{0.9421(46)}{37.13}}\right)46 = 26.59 \text{ ksi} \neq 26.81 \text{ ksi}$$

Try $f = 26.59$ ksi

$$b_e = 1.92(0.174)\sqrt{\frac{29000}{26.59}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29000}{26.59}}\right] = 8.492 \text{ in..}$$

Loss in area $= 2(b - b_e)t = 2(9.478 - 8.492)(0.174) = 0.3431 \text{ in.}^2$

Reduced area $= A_e = 6.06 - 0.3431 = 5.717 \text{ in.}^2$

$$Q_a = \frac{A_e}{A} = \frac{5.717}{6.06} = 0.9434$$

$$Q = Q_s Q_a = 1.0(0.9434) = 0.9434$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.9434(46)}} = 121.8$$

$$\frac{KL}{r} = 87.80 < 121.8 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9434\left(0.658^{\frac{0.9434(46)}{37.13}}\right)46 = 26.61 \text{ ksi} \approx 26.59 \text{ ksi}$$

Recall that AISC Equation E7-18 for $b_e$ applies when $b/t \geq 1.40\sqrt{E/f}$. In the present case,

$$1.40\sqrt{\frac{E}{f}} = 1.40\sqrt{\frac{29000}{26.61}} = 46.2$$

Since $54.5 > 46.2$, AISC Equation E7-18 does apply.

$$P_n = F_{cr}A_g = 26.61(6.06) = 161.3 \text{ kips} \qquad \therefore \text{ local buckling controls.}$$

$$\underline{P_n = 161 \text{ kips}}$$

---

**4.4-2**

Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{2.10(10 \times 12)}{2.89} = 87.20 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since 87.20 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(87.20)^2} = 37.64 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/37.64)}(50) = 28.68 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 28.68(29.8) = 855 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 37.5$$

From AISC Table B4.1, case 10 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29000}{50}} = 35.88$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{F_y}}$, the web is slender.

For the flange,

$$\frac{b_f}{2t_f} = 7.68 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29000}{50}} = 13.49 \therefore \text{ flange is not slender}$$

Because the web is a stiffened element, $Q_s = 1.0$, and $Q_a$ must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where $b/t$ is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 28.68$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29000}{28.68}} = 47.38$$

Since $\frac{h}{t_w} < 1.49\sqrt{\frac{E}{f}}$, local buckling does not control. $\underline{P_n = 855 \text{ kips}}$

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**4.6-1**

$KL = 1.0(18) = 18$ ft

(a-1) $P_u = 1.2D + 1.6L = 1.2(265) + 1.6(130) = 526.0$ kips

From the column load tables for $KL = 18$ ft, a W12 × 65 has a design strength of 591 kips.

Use a W12 × 65

(a-2) $P_a = D + L = 265 + 130 = 395$ kips

From the column load tables for $KL = 18$ ft, a W12 × 72 has an allowable strength of 437 kips.

Use a W12 × 72

(b-1) Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{526.0}{0.90(25)} = 23.4 \text{ in.}^2$$

Try W18 × 86 (a nonslender shape), $A_g = 25.3$ in.$^2$, $r_y = 2.63$ in.

$$\frac{KL}{r_y} = \frac{(18 \times 12)}{2.63} = 82.13 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(82.13)^2} = 42.43 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 82.13$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/42.43)}(50) = 30.53 \text{ ksi}$$

$$P_n = F_{cr}A_g = 30.53(25.3) = 772.4 \text{ kips}$$

$$\phi_c P_n = 0.90(772.4) = 695.2 \text{ kips} > P_u = 526 \text{ kips} \qquad \text{(OK)}$$

Try the next lighter nonslender shape. Try a W18 × 71.

$A_g = 20.9$ in.$^2$, $r_y = 1.70$ in.

$$\frac{KL}{r_y} = \frac{(18 \times 12)}{1.70} = 127.1 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(127.1)^2} = 17.72 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 < 127.1$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(17.72) = 15.54 \text{ ksi}$$

$$P_n = F_{cr}A_g = 15.54(20.9) = 324.8 \text{ kips}$$

$$\phi_c P_n = 0.90(324.8) = 292 \text{ kips} < P_u = 526 \text{ kips} \qquad \text{(N.G.)}$$ <u>Use a W18 ×86</u>

(b-2) Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_a}{0.6F_{cr}} = \frac{395}{0.6(25)} = 26.3 \text{ in.}^2$$

Try W18 × 86 (a nonslender shape), $A_g = 25.3$ in.$^2$, $r_y = 2.63$ in.

$$\frac{KL}{r_y} = \frac{(18 \times 12)}{2.63} = 82.13 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(82.13)^2} = 42.43 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 82.13$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/82.13)}(50) = 38.75 \text{ ksi}$$

$$P_n = F_{cr}A_g = 38.75(25.3) = 980.4 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{980.4}{1.67} = 587 > P_a = 395 \text{ kips} \qquad \text{(OK)}$$ <u>Use a W18 ×86</u>

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**<u>4.6-2</u>**

$$KL = 2.0(15) = 30 \text{ ft}$$

(a-1) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(100) = 280$ kips

From the column load tables for $KL = 30$ ft, a W12 × 65 has a design strength of 304 kips.

<u>Use a W12 × 65</u>

(a-2) $P_a = D + L = 100 + 100 = 200$ kips

From the column load tables for $KL = 30$ ft, a W12 × 65 has an allowable strength of 202 kips.

<u>Use a W12 × 65</u>

(b-1) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(100) = 280$ kips

Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{280}{0.90(25)} = 12.44 \text{ in.}^2$$

Try W16 × 57 (a nonslender shape), $A_g = 16.8$ in.$^2$, $r_y = 1.60$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{1.60} = 225 > 200$$

Try W16 × 77 (a nonslender shape), $A_g = 22.6$ in.$^2$, $r_y = 2.47$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{2.47} = 145.7 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(145.7)^2} = 13.48 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 < 145.7$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(13.48) = 11.82 \text{ ksi}$$

$$P_n = F_{cr}A_g = 11.82(22.6) = 267.1 \text{ kips}$$

$$\phi_c P_n = 0.90(267.1) = 240 \text{ kips} < P_u = 280 \text{ kips} \qquad \text{(N.G.)}$$

Try W16 × 89 (a nonslender shape), $A_g = 26.2$ in.$^2$, $r_y = 2.49$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{2.49} = 144.6 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(144.6)^2} = 13.69 \text{ ksi}$$

$$\frac{KL}{r_y} > 4.71\sqrt{\frac{E}{F_y}} \qquad \therefore\ F_{cr} = 0.877F_e = 0.877(13.69) = 12.01 \text{ ksi}$$

$$P_n = F_{cr}A_g = 12.01(26.2) = 314.7 \text{ kips}$$

$$\phi_c P_n = 0.90(314.7) = 283 \text{ kips} > P_u = 280 \text{ kips} \qquad \text{(OK)} \qquad \underline{\text{Use a W16} \times 89}$$

(b-2) $P_a = D + L = 100 + 100 = 200$ kips

Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_a}{0.6F_{cr}} = \frac{200}{0.6(25)} = 13.33 \text{ in.}^2$$

Try W16 × 57 (a nonslender shape), $A_g = 16.8 \text{ in.}^2$, $r_y = 1.60$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{1.60} = 225.0 > 200$$

Try W16 × 77 (a nonslender shape), $A_g = 22.6 \text{ in.}^2$, $r_y = 2.47$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{2.47} = 145.7 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(145.7)^2} = 13.48 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 < 145.7$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(13.48) = 11.82 \text{ ksi}$$

$$P_n = F_{cr}A_g = 11.82(22.6) = 267.1 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{267.1}{1.67} = 159.9 < P_a = 200 \text{ kips} \qquad \text{(N.G.)}$$

Try W16 × 89 (a nonslender shape), $A_g = 26.2 \text{ in.}^2$, $r_y = 2.49$ in.

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{2.49} = 144.6 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(144.6)^2} = 13.69 \text{ ksi}$$

$$\frac{KL}{r_y} > 4.71\sqrt{\frac{E}{F_y}} \qquad \therefore \; F_{cr} = 0.877F_e = 0.877(13.69) = 12.01 \text{ ksi}$$

$$P_n = F_{cr}A_g = 12.01(26.2) = 314.7 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{314.7}{1.67} = 188.4 < P_a = 200 \text{ kips} \qquad \text{(N.G.)}$$

Try W16 × 100 (a non-slender shape), $A_g = 29.4$, $r_y = 2.51$

$$\frac{KL}{r_y} = \frac{2.0(15 \times 12)}{2.51} = 143.4 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(143.4)^2} = 13.92 \text{ ksi}$$

$$\frac{KL}{r_y} > 4.71\sqrt{\frac{E}{F_y}} \qquad \therefore \; F_{cr} = 0.877F_e = 0.877(13.92) = 12.21 \text{ ksi}$$

$$P_n = F_{cr}A_g = 12.21(29.4) = 359.0 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{359.0}{1.67} = 215 > P_a = 200 \text{ kips} \qquad \text{(OK)}$$

Use a W16 × 100

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**4.6-3**

$$KL = 2.1(12) = 25.2 \text{ ft}$$

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips

From the column load tables:

HSS 12 × 12 × $\frac{1}{2}$ : $\phi_c P_n = 653$ kips > 600 kips, $w = 76.1$ lb/ft

HSS 14 × 14 × $\frac{3}{8}$ : $\phi_c P_n = 634$ kips > 600 kips, $w = 68.3$ lb/ft

Use HSS 14 × 14 × $\frac{3}{8}$

(b) $P_a = D + L = 100 + 300 = 400$ kips

From the column load tables:

HSS $14 \times 14 \times \frac{3}{8}$ : $P_n/\Omega_c = 422$ kips $> 400$ kips, $w = 68.3$ lb/ft

Use HSS $14 \times 14 \times \frac{3}{8}$

---

**4.6-4**

$KL = 0.8(15) = 12$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(122) + 1.6(242) = 534$ kips

Pipe 8-XXS: $\phi_c P_n = 549$ kips $> 534$ kips; $w = 72.5$ lb/ft Use a Pipe 8-XXS.

(b) $P_a = D + L = 122 + 242 = 364$ kips

Pipe 8-XXS: $P_n/\Omega_c = 88.7$ kips $> 84$ kips; $w = 19.0$ lb/ft

Use a Pipe 8-XXS

---

**4.6-5**

$KL = 0.8(15) = 12$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(122) + 1.6(242) = 534$ kips

HP10 × 57: $\phi_c P_n = 584$ kips

HP12 × 53: $\phi_c P_n = 579$ kips Use HP12 × 53

(b) $P_a = D + L = 122 + 242 = 364$ kips

HP10 × 57: $P_n/\Omega_c = 388$ kips

HP12 × 53: $P_n/\Omega_c = 386$ kips Use HP12 × 53

---

**4.6-6**

$KL = 2.1(12) = 25.2$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips

From the column load tables:

HSS $20 \times 12 \times \frac{3}{8}$ : $\phi_c P_n = 629$ kips $>$ 600 kips, $w = 78.5$ lb/ft

Use HSS $20 \times 12 \times \frac{3}{8}$

(b) $P_a = D + L = 100 + 300 = 400$ kips

From the column load tables:

HSS $20 \times 12 \times \frac{3}{8}$ : $P_n/\Omega_c = 419$ kips $>$ 400 kips, $w = 78.5$ lb/ft

Use HSS $20 \times 12 \times \frac{3}{8}$

---

**4.6-7**

$P_u = 1.2D + 1.6L = 1.2(100) + 1.6(250) = 520$ kips

$KL = 0.65(16) = 10.4$ ft

(a) Use a W10 × 49 ($\phi_c P_n = 543$ kips)

(b) Use an HSS $9 \times 9 \times \frac{1}{2}$ ($w = 55.7$ lb/ft, $\phi_c P_n = 580$ kips)

(c) Use an HSS $12 \times 10 \times \frac{3}{8}$ ($w = 53.0$ lb/ft, $\phi_c P_n = 566$ kips)

(d) Use an HSS $16 \times 0.312$ ($w = 52.3$ lb/ft, $\phi_c P_n = 528$ kips)

---

**4.6-8**

$P_a = D + L = 100 + 250 = 350$ kips

$KL = 0.65(16) = 10.4$ ft

(a) Use a W10 × 49 ($P_n/\Omega_c = 361$ kips)

(b) Use an HSS 9 × 9 × $\frac{1}{2}$ ($w = 55.5$ lb/ft, $P_n/\Omega_c = 386$ kips)

(c) Use an HSS 12 ×10 × $\frac{3}{8}$ ($w = 53.0$ lb/ft, $P_n/\Omega_c = 377$ kips)

(d) Use an HSS 16 × 0.312 ($w = 52.3$ lb/ft, $P_n/\Omega_c = 351$ kips)

---

**4.6-9**

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(250) = 520$ kips

Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{520}{0.90(25)} = 23.11 \text{ in.}^2$$

Try W21 × 83 (a slender shape), $A_g = 24.4$ in.$^2$, $r_y = 1.83$ in.

First, compute the strength without regard to local buckling. If the selection is adequate, then adjust for local buckling.

$$\frac{KL}{r_y} = \frac{0.65(16 \times 12)}{1.83} = 68.20 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(68.20)^2} = 61.54 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 68.20$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/61.54)}(50) = 35.59 \text{ ksi}$$

$$P_n = F_{cr} A_g = 35.59(24.4) = 868.4 \text{ kips}$$

$$\phi_c P_n = 0.90(868.4) = 782 \text{ kips} > 520 \text{ kips} \qquad \text{(OK, but too conservative)}$$

Assume $F_{cr} = 35$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{520}{0.90(35)} = 16.5 \text{ in.}^2$$

Try W21 × 62 (a slender shape), $A_g = 18.3$ in.$^2$, $r_y = 1.77$ in.

$$\frac{KL}{r_y} = \frac{0.65(16 \times 12)}{1.77} = 70.51 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(70.51)^2} = 57.57 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 70.51$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/57.57)}(50) = 34.76 \text{ ksi}$$

$$P_n = F_{cr}A_g = 34.76(18.3) = 636.1 \text{ kips}$$

$$\phi_c P_n = 0.90(636.1) = 572.5 \text{ kips} > 524 \text{ kips} \qquad \text{(OK)}$$

Check flange local buckling. The flange is an unstiffened element. From the dimensions and properties table in the *Manual*, the width-to-thickness ratio of the flange is

$$\frac{b_f}{2t_f} = 6.70 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29000}{50}} = 13.49$$

Therefore, $Q_s = 1.0$

Check web local buckling. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 46.9$$

Because this cross-sectional element is a stiffened element, $Q_a$ must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where $b/t$ is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 34.76$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29000}{34.76}} = 43.04$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{f}}$, local buckling strength must be checked. The unreduced width of the web is $b = d - 2k_{des} = 21.0 - 2(1.12) = 18.76$ in. From AISC Eq. E7-17, The reduced effective width is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

$$= 1.92(0.400)\sqrt{\frac{29000}{34.76}}\left[1 - \frac{0.34}{(46.9)}\sqrt{\frac{29000}{34.76}}\right] = 17.54 \text{ in.} < 18.76 \text{ in.}$$

The reduced area is

$$A_e = A - t_w(b - b_e) = 18.3 - 0.400(18.76 - 17.54) = 17.81 \text{ in.}^2$$

$$Q_a = Q = \frac{A_e}{A} = \frac{17.81}{18.3} = 0.9732$$

Determine which critical stress equation to use:

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.9732(50)}} = 115.0 > \frac{KL}{r} = 70.51$$

$$\therefore\ F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.9732\left[0.658^{\frac{0.9732(50)}{57.57}}\right](50)$$

$$= 34.16 \text{ ksi}$$

$$P_n = F_{cr}A_g = 34.16(18.3) = 625.1 \text{ kips}$$

$\phi_c P_n = 0.90(625.1) = 563 \text{ kips} > 524 \text{ kips}$ (OK) Use a W21 × 62

---

**4.7-1**

$$\frac{K_xL}{r_x} = \frac{2.2(13 \times 12)}{7.82} = 43.89, \qquad \frac{K_yL}{r_y} = \frac{1.0(13 \times 12)}{2.65} = 58.87 \text{ (controls)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(58.87)^2} = 82.59 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{60}} = 103.5 > 58.87$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(60/82.59)}(60) = 44.27 \text{ ksi}$$

$$P_n = F_{cr}A_g = 44.27(28.5) = 1262 \text{ kips}$$

Check for slender compression elements for $F_y = 60$ ksi.

Flange: $\lambda = \dfrac{b_f}{2t_f} = 6.41, \qquad \lambda_r = 0.56\sqrt{\dfrac{E}{F_y}} = 0.56\sqrt{\dfrac{29000}{60}} = 12.3$

Since $\lambda < \lambda_r$, flange is nonslender.

Web: $\lambda = \dfrac{h}{t_w} = 30.8, \qquad \lambda_r = 1.49\sqrt{\dfrac{E}{F_y}} = 1.49\sqrt{\dfrac{29000}{60}} = 32.8$

Since $\lambda < \lambda_r$, web is nonslender. Therefore, the nominal compressive strength is

$$\underline{P_n = 1260 \text{ kips}}$$

---

**4.7-2**

$$\frac{K_xL}{r_x} = \frac{16 \times 12}{3.85} = 49.87, \qquad \frac{K_yL}{r_y} = \frac{10 \times 12}{3.25} = 36.92$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(49.87)^2} = 115.1 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{46}} = 118.3$$

Since $KL/r = 49.87 < 118.3$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/115.1)}(46) = 38.91 \text{ ksi}$$

$$P_n = F_{cr}A_g = 38.91(8.03) = 312.4 \text{ kips}$$

(a) $\phi_c P_n = 0.90(312.4) = 281$ kips $\qquad \underline{\phi_c P_n = 281 \text{ kips}}$

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{312.4}{1.67} = 187$ kips $\qquad \underline{\dfrac{P_n}{\Omega_c} = 187 \text{ kips}}$

---

**4.7-3**

For a W12 × 65, $A_g = 19.1 \text{ in.}^2,\ r_x = 5.28,\ \text{in.},\ r_y = 3.02 \text{ in.}$

$$\frac{K_x L}{r_x} = \frac{26 \times 12}{5.28} = 59.09, \qquad \frac{K_y L}{r_y} = \frac{14 \times 12}{3.02} = 55.63$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29000)}{(59.09)^2} = 81.97 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{60}} = 103.5$$

Since $KL/r = 59.09 < 103.5,$ use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(60/81.97)}(60) = 44.17 \text{ ksi}$$

$$P_n = F_{cr} A_g = 44.17(19.1) = 843.6 \text{ kips}$$

Check for slender compression elements for $F_y = 60$ ksi.

Flange: $\lambda = \dfrac{b}{2t_f} = 9.92, \qquad \lambda_r = 0.56\sqrt{\dfrac{E}{F_y}} = 0.56\sqrt{\dfrac{29000}{60}} = 12.3$

Since $\lambda < \lambda_r,$ flange is nonslender.

Web: $\lambda = \dfrac{h}{t_w} = 24.9, \qquad \lambda_r = 1.49\sqrt{\dfrac{E}{F_y}} = 1.49\sqrt{\dfrac{29000}{60}} = 32.8$

Since $\lambda < \lambda_r,$ web is nonslender. Therefore, the nominal compressive strength is

$$P_n = 843.6 \text{ kips.}$$

(a) LRFD Solution:

$$\phi_c P_n = 0.90(843.6) = 759 \text{ kips}$$

$$P_u = 1.2D + 1.6L = 1.2(180) + 1.6(320) = 728 \text{ kips} < 759 \text{ kips} \qquad \text{(OK)}$$

Yes; member is satisfactory.

(b) ASD Solution:

$$\frac{P_n}{\Omega_c} = \frac{843.6}{1.67} = 505 \text{ kips}$$

$P_a = D + L = 180 + 320 = 500 \text{ kips} < 505 \text{ kips}$ (OK)

Yes; member is satisfactory.

---

**4.7-4**

$K_xL = 24 \text{ ft}, \quad K_yL = 24 - 10 = 14 \text{ ft}$

(a) $P_u = 1.2D + 1.6L = 1.2(142) + 1.6(356) = 740.0 \text{ kips}$

From the column load tables for $KL = 14$ ft, try a W12 × 72

$\phi_c P_n = 759 \text{ kips for } KL = 14 \text{ ft.}$

$$\frac{K_xL}{r_x/r_y} = \frac{24}{1.75} = 13.71 \text{ ft} < 14 \text{ ft}$$

Use a W12 × 72

(b) $P_a = D + L = 142 + 356 = 498 \text{ kips}$

From the column load tables for $KL = 14$ ft, try a W12 × 72

$\frac{P_n}{\Omega_c} = 505 \text{ kips for } KL = 14 \text{ ft.}$

$$\frac{K_xL}{r_x/r_y} = \frac{24}{1.75} = 13.71 \text{ ft} < 14 \text{ ft}$$

Use a W12 × 72

---

**4.7-5**

$K_xL = 35 \text{ ft}, \quad K_yL = 15 \text{ ft}$

(a) $P_u = 1.2D + 1.6L = 1.2(340) + 1.6(670) = 1480 \text{ kips}$

From the column load tables for $KL = 15$ ft, there are no W8 or W10 shapes with enough strength. Try a W12 × 152 :

$\phi_c P_n = 1590 \text{ kips for } KL = 15 \text{ ft}$

$$\frac{K_xL}{r_x/r_y} = \frac{35}{1.77} = 19.77 \text{ ft} > 15 \text{ ft}$$

For $KL = 19$ ft, $\phi_c P_n = 1380$ kips < 1480 kips (N.G.)

Try a W12 × 170:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.78} = 19.66 \text{ ft} > 15 \text{ ft}$$

For $KL = 20$ ft, $\phi_c P_n = 1500$ kips > 1480 kips (OK)

Investigate W14 shapes: Try a W14 × 132. $\phi_c P_n = 1480$ kips for $KL = 15$ ft

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.67} = 20.96 \text{ ft.} > 15 \text{ ft}$$

For $KL = 20$ ft, $\phi_c P_n = 1300$ kips < 1480 kips (N.G.)

Try a W14 × 145:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.59} = 22.01 \text{ ft} > 15 \text{ ft}$$

For $KL = 22$ ft, $\phi_c P_n = 1390$ kips < 1480 kips (N.G.)

Try a W14 × 159:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.60} = 21.88 \text{ ft} > 15 \text{ ft}$$

For $KL = 22$ ft, $\phi_c P_n = 1530$ kips > 1480 kips (OK)

The W14 × 159 is the lightest W shape (in the column load tables) that will work.

Use a W14 × 159

(b) $P_a = D + L = 340 + 670 = 1010$ kips

From the column load tables for $KL = 15$ ft, there are no W8 or W10 shapes with enough strength.

Try a W12 × 152 :

$$\frac{P_n}{\Omega_c} = 1060 \text{ kips for } KL = 15 \text{ ft}$$

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.77} = 19.77 \text{ ft} > 15 \text{ ft}$$

For $KL = 20$ ft, $\dfrac{P_n}{\Omega_c} = 885$ kips < 1010 kips (N.G.)

Try a W12 × 190:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.79} = 19.55 \text{ ft} > 15 \text{ ft}$$

For $KL = 20$ ft, $\dfrac{P_n}{\Omega_c} = 1130$ kips > 1010 kips (OK)

Investigate W14 shapes: Try a W14 × 145. $\dfrac{P_n}{\Omega_c} = 1100$ kips for $KL = 15$ ft

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.59} = 22.01 \text{ ft.} > 15 \text{ ft}$$

For $KL = 22$ ft, $\dfrac{P_n}{\Omega_c} = 927$ kips < 1010 kips (N.G.)

Try a W14 × 159:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.60} = 21.88 \text{ ft} > 15 \text{ ft}$$

For $KL = 22$ ft, $\dfrac{P_n}{\Omega_c} = 1020$ kips > 1010 kips (OK)

The W14 × 159 is the lightest W shape (in the column load tables) that will work.

Use a W14 × 159

---

**4.7-6**

$K_x L = 15$ ft, $K_y L = 7.5$ ft

Since $r_x/r_y = 1.0$ for a square shape, use $KL = 15$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(35) + 1.6(80) = 170.0$ kips

Use an HSS 8 × 8 × 3/16

$\phi_c P_n = 170 \text{ kips} = P_u, \ w = 19.6$ lb/ft

Use an HSS 8 × 8 × 3/16

(b) $P_a = D + L = 35 + 80 = 115$ kips

Use an HSS 9 × 9 × 3/16

$\dfrac{P_n}{\Omega_c} = 122 \text{ kips} > 115 \text{ kips}, \; w = 22.2 \text{ lb/ft}$

Use an HSS 9 × 9 × 3/16

---

**4.7-7**

$K_xL = 22 \text{ ft}, \qquad K_yL = 12 \text{ ft}$

$K_xL$ will control when $\dfrac{K_xL}{r_x/r_y} > K_yL$, or $\dfrac{K_xL}{K_yL} > r_x/r_y$

For this column, $\dfrac{K_xL}{K_yL} = \dfrac{22}{12} = 1.83$

$r_x/r_y$ is < 1.8 for all rectangular HSS, so $\dfrac{K_xL}{r_x/r_y}$ will control for this column.

(a) $P_u = 1.2D + 1.6L = 1.2(30) + 1.6(90) = 180$ kips

Check within each range of $r_x/r_y$ for possible choices.

For $r_x/r_y \approx 1.2$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{22}{1.2} = 18.33$ ft. Try an HSS 8 × 6 × $\frac{5}{16}$, $w = 27.6$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.25} = 17.6 \text{ ft}, \; \phi_c P_n > 182 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.3$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{22}{1.3} = 16.92$ ft. Try an HSS 7 × 5 × $\frac{1}{2}$, $w = 35.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.31} = 16.79 \text{ ft}, \; \phi_c P_n > 187 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.4$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{22}{1.4} = 15.71$ ft.

Try an HSS 12 × 8 × $\frac{3}{16}$, $w = 24.7$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.36} = 16.18 \text{ ft}, \; \phi_c P_n > 183 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.6$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{22}{1.6} = 13.75$ ft. Try an HSS $9 \times 5 \times \frac{5}{16}$, $w = 26.6$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.58} = 13.92 \text{ ft}, \quad \phi_c P_n > 200 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.7$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{22}{1.7} = 12.94$ ft. Try an HSS $8 \times 4 \times \frac{1}{2}$, $w = 35.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.74} = 12.64 \text{ ft}, \quad \phi_c P_n > 206 \text{ kips} \qquad \text{(OK)}$$

<u>Use an HSS $12 \times 8 \times \frac{3}{16}$, $w = 24.7$ lb/ft</u>

(b) $P_a = D + L = 30 + 90 = 120$ kips

Try an HSS $12 \times 8 \times \frac{3}{16}$, $w = 24.7$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{22}{1.36} = 16.18 \text{ ft}, \quad \frac{P_n}{\Omega_c} > 120 \text{ kips} \qquad \text{(OK)}$$

<u>Use an HSS $12 \times 8 \times \frac{3}{16}$, $w = 24.7$ lb/ft</u>

---

**<u>4.7-8</u>**

(a) Column *AB*: $G_A = 10$, $\quad G_B = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{2(475)/13}{2(518)/20} = 1.41$

From the alignment chart,
$K_x \approx 1.98$ <u>$K_x = 1.98$</u>

(b) Column *BC*: $G_C = G_B = 1.41$ and <u>$K_x = 1.42$</u>

(c) Column *AB*:

$$\frac{K_xL}{r_x} = \frac{1.98(12 \times 12)}{5.28} = 54.0$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\dfrac{K_xL}{r_x} < 4.71\sqrt{\dfrac{E}{F_y}}$, the column is inelastic. Since $K_x$ for column *BC* is smaller,

$K_xL/r_x$ is smaller, so column $BC$ is also inelastic.

$\tau_b$ is applicable to both columns.

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**4.7-9**

$$G_A = 1.0, \qquad G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{999/13 + 1110/13}{800/25} = 5.07$$

From the alignment chart, $K_x \approx 1.68$

$$\frac{K_xL}{r_x} = \frac{1.68(13 \times 12)}{6.14} = 42.68$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic.

(a) $P_u = 1.2D + 1.6L = 1.2(155) + 1.6(460) = 922.0$ kips

$$\frac{P_u}{A_g} = \frac{922.0}{26.5} = 34.79 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_b = 0.846$ by interpolation.

Use $G_B = 0.846(5.07) = 4.29$

From the alignment chart,
$K_x \approx 1.65$ $\underline{K_x = 1.65}$

(b) $P_a = D + L = 155 + 460 = 615.0$ kips

$$\frac{P_a}{A_g} = \frac{615}{26.5} = 23.21 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_b = 1.00$

Use $G_B = 1.00(5.07) = 5.07$

From the alignment chart,
$K_x \approx 1.68$ $\underline{K_x = 1.68}$

**4.7-10**

$$G_A = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{623/15}{758/18 + 758/20} = 0.519, \qquad G_B = 10$$

From the alignment chart, $K_x \approx 1.80$

$$\frac{K_x L}{r_x} = \frac{1.80(15 \times 12)}{4.60} = 70.43$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic.

(a) $P_u = 1.2D + 1.6L = 1.2(90) + 1.6(110) = 284.0$ kips

$$\frac{P_u}{A_g} = \frac{284.0}{29.3} = 9.693 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_b = 1.00$ ∴ no modification is necessary.

$$\frac{K_y L}{r_y} = \frac{1.0(15 \times 12)}{2.65} = 67.92 < 70.43 \therefore \frac{K_x L}{r_x} \text{ controls}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(70.43)^2} = 57.70 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/57.70)}(50) = 34.79 \text{ ksi}$$

$$P_n = F_{cr} A_g = 34.79(29.3) = 1019 \text{ kips}$$

$$\phi_c P_n = 0.90(1019) = 917.1 \text{ kips}$$

$\underline{\phi_c P_n = 917 \text{ kips}}$

(b) $P_a = D + L = 90 + 110 = 200$ kips

$$\frac{P_a}{A_g} = \frac{200}{29.3} = 6.826 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_b = 1.00$ ∴ no modification is necessary.

$$\frac{K_y L}{r_y} = \frac{1.0(15 \times 12)}{2.65} = 67.92 < 70.43 \therefore \frac{K_x L}{r_x} \text{ controls}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(70.43)^2} = 57.70 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/57.70)}(50) = 34.79 \text{ ksi}$$

$P_n = F_{cr}A_g = 34.79(29.3) = 1019$ kips The allowable strength is

$$\frac{P_n}{\Omega_c} = \frac{1019}{1.67} = 610 \text{ kips}$$

$\underline{\frac{P_n}{\Omega_c} = 610 \text{ kips}}$

---

**4.7-11**

(a) Member *AB*: $G_A = 10$, $G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2I/13}{3I/30} = 1.54$

From the alignment chart,

$K_x \approx 2.00$ $\underline{K_x = 2.00}$

(b) Member *BC*: From part (a), $G_B = 1.54$

$G_C = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{I/13}{2I/30} = 1.15$ $\underline{K_x = 1.40}$

(c) Member *DE*: $G_D = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{I/13}{2(2I/30)} = 0.577$

$G_E = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2I/13}{2(3I/30)} = 0.769$ $\underline{K_x = 1.2}$

(d) Member *EF*: From part (c), $G_E = 0.769$. Use $G_F = 1.0$ $\underline{K_x = 1.28}$

---

**4.7-12**

(a) $G_A = 1.0$, $G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2 \times (833)/13}{712/30} = 5.40$

From the alignment chart, $K_x \approx 1.7$, $\frac{K_xL}{r_x/r_y} = \frac{1.7(13)}{1.76} = 12.56 \text{ ft} < K_yL = 13 \text{ ft}$

For $KL = 13$ ft, $\phi_c P_n = 1050$ kips > 750 kips (OK) <u>Member is adequate.</u>

(b) $G_N = 10,\quad G_M = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{2 \times (833)/13}{2 \times (712)/30} = 2.70$

From the alignment chart, $K_x \approx 2.3,\quad \dfrac{K_x L}{r_x/r_y} = \dfrac{2.3(13)}{1.76}$

$= 16.99 \text{ ft} > K_y L = 13 \text{ ft}$

For $KL = 17$ ft, $\phi_c P_n = 923$ kips < 1000 kips (N.G.)

Check for inelastic behavior:

$$\frac{K_x L}{r_x} = \frac{2.3(13 \times 12)}{5.44} = 65.96$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\dfrac{K_x L}{r_x} < 4.71\sqrt{\dfrac{E}{F_y}}$, the column is inelastic.

$$\frac{P_u}{A_g} = \frac{1000}{28.2} = 35.46 \text{ ksi.}$$

From Table 4-1 in the *Manual*, $\tau_b = 0.824$ by interpolation.

Use $G_M = 0.824(2.70) = 2.22$

From the alignment chart, $K_x \approx 2.14$

$$\frac{K_x L}{r_x/r_y} = \frac{2.14(13)}{1.76} = 15.81 \text{ ft} > K_y L = 13 \text{ ft}$$

For $KL = 16$ ft, $\phi_c P_n = 957$ kips < 1000 kips (N.G.) <u>Member not adequate.</u>

(c) $G_B = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{2 \times (833)/13}{712/30} = 5.40,\quad G_C = \dfrac{662/13 + 833/13}{712/30} = 4.85$

From the alignment chart, $K_x \approx 2.22,\quad \dfrac{K_x L}{r_x/r_y} = \dfrac{2.22(13)}{1.76}$

$= 16.40 \text{ ft} > K_y L = 13 \text{ ft}$

For $KL = 17$ ft, $\phi_c P_n = 923$ kips $> 600$ kips (OK)

(There is no need to check for inelastic behavior since the member has enough strength as it is.)

Member is adequate.

---

(d) $G_L = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{662/13 + 833/13}{2 \times 712/30} = 2.42$, From Part b, $G_M = 2.70$

From the alignment chart, $K_x \approx 1.72$, $\dfrac{K_x L}{r_x/r_y} = \dfrac{1.72(13)}{1.76}$

$= 12.7 \text{ ft} < K_y L = 13 \text{ ft}$

For $KL = 13$ ft, $\phi_c P_n = 1050$ kips $<1200$ kips (N.G.)

Since $K_y L$ controls, $\tau_b$ cannot help. Member is not adequate.

---

(e) $G_F = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{2 \times 272/13}{612/30} = 2.05$, $G_G = \dfrac{272/13}{301/30} = 2.09$

From the alignment chart, $K_x \approx 1.6$, $\dfrac{K_x L}{r_x/r_y} = \dfrac{1.6(13)}{1.71} = 12.2 \text{ ft} < K_y L = 13 \text{ ft}$

For $KL = 13$ ft, $\phi_c P_n = 492$ kips $> 240$ kips (OK) Member is adequate.

---

(f) $G_H = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{272/13}{2 \times 301/30} = 1.04$, $G_I = \dfrac{2 \times 272/13}{2 \times 612/30} = 1.03$

From the alignment chart, $K_x \approx 1.32$, $\dfrac{K_x L}{r_x/r_y} = \dfrac{1.32(13)}{1.71}$

$= 10.0 \text{ ft} < K_y L = 13 \text{ ft}$

For $KL = 13$ ft, $\phi_c P_n = 492$ kips $> 480$ kips (OK)

Member is adequate.

**4.7-13**

$$P_u = 1.2D + 1.6L = 1.2(48) + 1.6(72) = 172.8 \text{ kips}$$

For purposes of determining *G*, assume that *y*-axis buckling controls and select a shape for *AB*. For $KL = 1.0(14) = 14$ ft, select a W8 × 31 with $\phi_c P_n = 248$ kips.

$$G_A = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14}{2(245/20)} = 0.3207$$

$$G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14 + 391/14}{2(518/20)} = 0.6908$$

From the alignment chart, $K_x \approx 1.16$

$$\frac{K_x L}{r_x} = \frac{1.16(14 \times 12)}{3.47} = 56.16$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic, and the stiffness reduction factor can be used. But

$$\frac{K_y L}{r_y} = \frac{1.0(14 \times 12)}{2.02} = 83.17 > \frac{K_x L}{r_x}$$

so *y*-axis buckling controls, and the stiffness reduction factor is not needed.

Use a W8 × 31

**4.8-1**

Compute the flexural buckling strength for the *x*-axis:

$$\frac{K_x L}{r_x} = \frac{16 \times 12}{3.06} = 62.75$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(62.75)^2} = 72.69 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-2 applies.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/72.69)}(50) = 37.49 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 37.49(19.4) = 727 \text{ kips}$$

Compute the flexural-torsional buckling strength about the y-axis (the axis of symmetry):

$$\frac{K_yL}{r_y} = \frac{16 \times 12}{2.93} = 65.53$$

From the *AISC Shapes Database*, $\bar{r}_o = 4.65$ in. and $H = 0.845$

Compute $F_{cry}$ using AISC E3. From AISC Equation E3-4,

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 E}{(K_yL/r_y)^2} = \frac{\pi^2(29000)}{(65.53)^2} = 66.65 \text{ ksi}$$

Since $K_yL/r_y < 4.71\sqrt{\frac{E}{F_y}} = 113$

$$F_{cry} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/66.65)}(50) = 36.53 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g\bar{r}_o^2} = \frac{11200(5.62)}{19.4(4.65)^2} = 150.1 \text{ ksi}$$

$$F_{cry} + F_{crz} = 36.53 + 150.1 = 186.6 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{186.6}{2(0.845)}\left[1 - \sqrt{1 - \frac{4(36.53)(150.1)(0.845)}{(186.6)^2}}\right] = 34.9 \text{ ksi}$$

$$P_n = F_{cr}A_g = 34.9(19.4) = 677 \text{ kips}$$

The flexural-torsional buckling strength controls. $\underline{P_n = 677 \text{ kips}}$

**4.8-2**

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. Check flexural buckling strength about the *y* axis (this is the axis of no symmetry for a channel):

$$\frac{K_yL}{r_y} = \frac{0.65(10 \times 12)}{0.797} = 97.87 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(97.87)^2} = 29.88 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 133.7 > 97.87$$

$$\therefore\ F_{cry} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/29.88)}(36) = 21.74 \text{ ksi}$$

$$P_n = F_{cr}A_g = 21.74(6.08) = 132.2 \text{ kips}$$

Flexural-torsional buckling strength about the *x* axis (this is the axis of symmetry for a channel):

$$\frac{K_xL}{r_x} = \frac{0.65(10 \times 12)}{4.61} = 16.92$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(16.92)^2} = 999.8 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 EC_W}{(K_zL)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29000)(112)}{(0.65 \times 10 \times 12)^2} + 11200(0.369)\right]\frac{1}{6.08(4.93)^2} = 63.62 \text{ ksi}$$

$$F_{ey} + F_{ez} = 999.8 + 63.62 = 1063 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{1063}{2(0.899)}\left[1 - \sqrt{1 - \frac{4(999.8)(63.62)(0.899)}{(1063)^2}}\right] = 63.22 \text{ ksi}$$

Determine which compressive strength equation to use.

$$\frac{F_y}{F_e} = \frac{36}{63.22} = 0.5694 < 2.25$$

$\therefore$ use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(0.5694)}(36) = 28.37 \text{ ksi}$$

$$P_n = F_{cr}A_g = 28.36(6.08) = 172.4 \text{ kips}$$

The flexural buckling strength controls. $\underline{P_n = 132 \text{ kips}}$

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**4.8-3**

$$KL = 0.8(20) = 16.0 \text{ ft}$$

For a live load-to-dead load ratio of 2,

$$D + 2D = 180, \qquad D = 60 \text{ kips}, \ L = 2(60) = 120 \text{ kips}$$

(a) $P_u = 1.2D + 1.6L = 1.2(60) + 1.6(120) = 264.0 \text{ kips}$

From the column load tables,

a WT7 × 54.5 has a design strength of 277 kips based on buckling about the *x* axis.

a WT8 × 38.5 has a design strength of 295 kips based on buckling about the *x* axis.

a WT9 × 32.5 has a design strength of 273 kips based on buckling about the *x* axis.

Use a WT9 × 32.5

(b) $P_a = 180 \text{ kips}$

From the column load tables, a WT9 × 38 has an allowable strength of 189 kips based on buckling about the *y* axis.

Use a WT9 × 38

---

**4.8-4**

(a) $P_u = 1.2D + 1.6L = 1.2(30) + 1.6(70) = 148$ kips

Try a C15 × 33.9

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. Check flexural buckling strength about the *y* axis (this is the axis of no symmetry for a channel):

$$\frac{K_y L}{r_y} = \frac{0.65(10 \times 12)}{0.901} = 86.57 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(86.57)^2} = 38.19 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 86.57$$

$$\therefore\ F_{cry} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/38.19)}(50) = 28.91 \text{ ksi}$$

$$P_n = F_{cr} A_g = 28.91(10.0) = 289.1 \text{kips}$$

$$\phi_c P_n = 0.90(289.1) = 260.2 \text{ kips}$$

This shape may be too conservative. Try a C12 ×30.

$$\frac{K_y L}{r_y} = \frac{0.65(10 \times 12)}{0.762} = 102.4 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(102.4)^2} = 27.30 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 102.4$$

$$\therefore\ F_{cry} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/27.30)}(50) = 23.23 \text{ ksi}$$

$$P_n = F_{cr} A_g = 23.23(8.81) = 204.7 \text{kips}$$

$$\phi_c P_n = 0.90(204.7) = 184.2 \text{ kips}$$

Flexural-torsional buckling strength about the *x* axis (this is the axis of symmetry for a channel):

$$\frac{K_xL}{r_x} = \frac{0.65(10 \times 12)}{4.29} = 18.18$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(18.18)^2} = 866.0 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 EC_W}{(K_zL)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29000)(151)}{(0.65 \times 10 \times 12)^2} + 11200(0.861)\right]\frac{1}{8.81(4.54)^2} = 92.22 \text{ ksi}$$

$$F_{ey} + F_{ez} = 866.0 + 92.22 = 958.2 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{958.2}{2(0.919)}\left[1 - \sqrt{1 - \frac{4(866.0)(92.22)(0.919)}{(958.2)^2}}\right] = 91.35 \text{ ksi}$$

Determine which compressive strength equation to use.

$$\frac{F_y}{F_e} = \frac{50}{91.35} = 0.5473 < 2.25$$

$\therefore$ use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(0.5473)}(50) = 39.76 \text{ ksi}$$

$$P_n = F_{cr}A_g = 39.76(8.81) = 350.3 \text{ kips}$$

$$\phi_c P_n = 0.90(350.3) = 315.3 \text{ kips}$$

The flexural buckling strength controls. Try a C12 × 25. Check flexural buckling strength about the $y$ axis:

$$\frac{K_yL}{r_y} = \frac{0.65(10 \times 12)}{0.779} = 100.1 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(100.1)^2} = 28.56 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 100.1$$

$$\therefore\ F_{cry} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/28.56)}(50) = 24.03 \text{ ksi}$$

$$P_n = F_{cr}A_g = 24.03(7.34) = 176.4 \text{ kips}$$

$$\phi_c P_n = 0.90(176.4) = 158.8 \text{ kips} > 148 \text{ kips} \qquad \text{(OK)}$$

Try a C12 × 20.7.

$$\frac{K_y L}{r_y} = \frac{0.65(10 \times 12)}{0.797} = 97.87 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(97.87)^2} = 29.88 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 97.87$$

$$\therefore\ F_{cry} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/29.88)}(50) = 24.82 \text{ ksi}$$

$$P_n = F_{cr}A_g = 24.82(6.08) = 150.9 \text{ kips}$$

$$\phi_c P_n = 0.90(150.9) = 135.8 \text{ kips} < 148 \text{ kips} \qquad \text{(N.G.)}$$

Return to the C12 × 25. Check flexural-torsional buckling strength about the *x* axis:

$$\frac{K_x L}{r_x} = \frac{0.65(10 \times 12)}{4.43} = 17.61$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(17.61)^2} = 923.0 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 E C_W}{(K_z L)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29000)(130)}{(0.65 \times 10 \times 12)^2} + 11200(0.538)\right]\frac{1}{7.34(4.72)^2} = 74.25 \text{ ksi}$$

$$F_{ey} + F_{ez} = 923.0 + 74.25 = 997.3 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{997.3}{2(0.909)}\left[1 - \sqrt{1 - \frac{4(923.0)(74.25)(0.909)}{(997.3)^2}}\right] = 73.66 \text{ ksi}$$

Determine which compressive strength equation to use.

$$\frac{F_y}{F_e} = \frac{50}{73.66} = 0.6788 < 2.25$$

$\therefore$ use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(0.6788)}(50) = 37.63 \text{ ksi}$$

$$P_n = F_{cr} A_g = 37.63(7.34) = 276.2 \text{ kips}$$

$$\phi_c P_n = 0.90(276.2) = 248.6 \text{ kips}$$

Flexural Buckling controls. $\phi_c P_n = 159 \text{ kips} > 148 \text{ kips}$ (OK) <u>Use a C12 × 25</u>

(b) $P_a = D + L = 30 + 70 = 100 \text{ kips}$

Try a C12 × 25

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. First, check the flexural buckling strength about the *y* axis (this is the axis of no symmetry for a channel). From the LRFD solution in Part (a),

$$P_n = 176.4 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{176.4}{1.67} = 106 \text{ kips} > 100 \text{ kips} \quad \text{(OK)}$$

Next, check the flexural-torsional buckling strength about the *x* axis (this is the axis of symmetry for a channel). From the LRFD solution in Part (a),

$$P_n = 276.2 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{276.2}{1.67} = 165 \text{ kips} > 100 \text{ kips} \quad \text{(OK)}$$

<u>Use a C12 × 25</u>

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**<u>4.9-1</u>**

With the short leg vertical, the needed properties of a single angle are

$$I_x = 2.89 \text{ in.}^4, \quad \bar{y} = 1.14 \text{ in.}, \quad A = 1.82 \text{ in.}^2$$

For the two angles,

$$I_y = \left[2.89 + 1.82\left(1.14 + \frac{3}{16}\right)^2\right] \times 2 = 12.19 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.19}{2(1.82)}} = 1.83 \text{ in.} \qquad \underline{r_y = 1.83 \text{ in.}}$$

---

**4.9-2**

*x*-axis:

| Segment | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| C8 × 11.5 | 3.37 | 0.572 | 1.928 | 1.31 | 4.147 | 59.27 |
| S12 × 31.8 | 9.31 | 6.220 | 57.91 | 217 | 1.501 | 238.0 |
| $\sum$ | 12.68 | | 59.84 | | | 297.3 |

$$\bar{y} = y_2 = \frac{\sum Ay}{\sum A} = \frac{59.84}{12.68} = 4.719 \text{ in.} \qquad I_x = 297.3 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{297.3}{12.68}} = 4.84 \text{ in.}$$

*y*-axis: $I_y = 9.33 + 32.5 = 41.83 \text{ in.}^4$, $\quad r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{41.83}{12.68}} = 1.82$ in

$$\underline{y_2 = 4.72 \text{ in.}, r_x = 4.84 \text{ in.}, \ r_y = 1.82 \text{ in.}}$$

---

**4.9-3**

For one angle, $A = 6.98 \text{ in.}^2$, $I_x = I_y = 15.7 \text{ in.}^4$, $\bar{x} = \bar{y} = 1.52$ in.

$$I = [15.7 + 6.98(1.52 + 5/2)^2] \times 4 = 514.0 \text{ in.}^4, \qquad A = 6.98 \times 4 = 27.92 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{514.0}{27.92}} = 4.291 \text{ in.} \qquad \underline{r_x = r_y = 4.29 \text{ in.}}$$

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**4.9-4**

*x*-axis:

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{6(0.25) + 5(0.5 + 8) + 3(0.5 + 16 + 0.25)}{6 + 5 + 3} = 6.732 \text{ in.}$$

$$I_x = \sum(\bar{I} + Ad^2) = \frac{1}{12}(12)(0.5)^3 + 6(6.732 - 0.25)^2$$

$$+ \frac{1}{12}(5/16)(16)^3 + 5(6.732 - 8.5)^2$$

$$+ \frac{1}{12}(6)(0.5)^3 + 3(6.732 - 16.75)^2 = 675.7 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{675.7}{14}} = 6.947 \text{ in.}$$

*y*-axis:

$$I_y = \frac{1}{12}(0.5)(12)^3 + \frac{1}{12}(16)(5/16)^3 + \frac{1}{12}(0.5)(6)^3 = 81.04 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{81.04}{14}} = 2.406 \text{ in.}$$

$\underline{r_x = 6.95 \text{ in.},\ r_y = 2.41 \text{ in.}}$

---

**4.9-5**

$$I_x = \frac{1}{12}(36)(40)^3 - \frac{1}{12}(28)(32)^3 = 1.155 \times 10^5 \text{ in.}^4$$

$$I_y = \frac{1}{12}(40)(36)^3 - \frac{1}{12}(32)(28)^3 = 9.698 \times 10^4 \text{ in.}^4$$

$$A = 36(40) - 28(32) = 544.0 \text{ in.}^2$$

$$r_{\text{min}} = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{96980}{544.0}} = 13.35 \text{ in.}$$

$$\frac{KL}{r} = \frac{0.8(40 \times 12)}{13.35} = 28.76 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(28.76)^2} = 346.0 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 28.76$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/346.0)}(50) = 47.07 \text{ ksi}$$

$$P_n = F_{cr}A_g = 47.07(544.0) = 2.561 \times 10^4 \text{ kips} \qquad \underline{P_n = 25{,}600 \text{ kips}}$$

---

**4.9-6**

*x*-axis:

| Segment | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| $1 \times 5$ | 5 | 0.5 | 2.5 | 0.4167 | 0.429 | 1.337 |
| $1 \times 2$ | 2 | 2.0 | 4.0 | 0.6667 | 1.071 | 2.961 |
| $\sum$ | 7 | | 6.5 | | | 4.298 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{6.5}{7} = 0.9286 \text{ in.} \qquad I_x = 4.298 \text{ in.}^4$$

*y*-axis:

$$I_y = \frac{1}{12}[1(5)^3 + 2(1)^3] = 10.58 \text{ in.}^4$$

*x* axis controls. $\quad r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.298}{7}} = 0.7836 \text{ in.}$

$$\frac{KL}{r} = \frac{0.8(15 \times 12)}{0.7836} = 183.8 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(183.8)^2} = 8.472 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134 < 183.8$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(8.472) = 7.430 \text{ ksi}$$

$$\phi_c P_n = 0.90A_gF_{cr} = 0.90(7)(7.430) = 46.81 \text{ kips}$$

$$\underline{\phi_c P_n = 46.8 \text{ kips}}$$

**4.9-7**

(a) *y*-axis:

| Segment | $A$ | $\bar{I}$ | $d$ | $\bar{I}+Ad^2$ |
|---|---|---|---|---|
| Channel | 2.64 | 0.624 | 0.6655 | 1.793 |
| Channel | 2.64 | 0.624 | 0.6655 | 1.793 |
| $\sum$ | 5.28 | | | 3.586 |

$$I_y = 3.586 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3.586}{5.28}} = 0.8241 \text{ in.} \qquad \underline{r_y = 0.824 \text{ in.}}$$

*x*-axis: $r_x = 1.84$ in.

*y*-axis controls.

$$\frac{KL}{r} = \frac{14 \times 12}{0.8241} = 203.9 > 200 \qquad \text{(not recommended but can be used)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(203.9)^2} = 6.884 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 < 203.9$$

$$\therefore F_{cr} = 0.877F_e = 0.877(6.884) = 6.037 \text{ ksi}$$

$$\frac{P_n}{\Omega} = \frac{A_g F_{cr}}{1.67} = \frac{5.28(6.037)}{1.67} = 19.09 \text{ kips} \qquad \underline{P_n/\Omega = 19.1 \text{ kips}}$$

**4.9-8**

*y*-axis:

| Segment | $A$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|
| C6 × 13 | 3.82 | 1.05 | 7.746 | 230.3 |
| W12 × 50 | 14.6 | 391 | 0 | 391 |
| C6 × 13 | 3.82 | 1.05 | 7.746 | 230.3 |
| $\sum$ | 22.24 | | | 851.6 |

$$I_y = 851.6 \text{ in.}^4$$

*x*-axis: $I_x = 2(17.3) + 56.3 = 90.9 \text{ in.}^4$ (controls)

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{90.9}{22.24}} = 2.022 \text{ in.}$$

$$\frac{KL}{r} = \frac{16 \times 12}{2.022} = 94.96 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(94.96)^2} = 31.74 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 94.96$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/31.74)}(50) = 25.86 \text{ ksi}$$

(a) $\phi_c P_n = 0.90 F_{cr} A_g = 0.90(25.86)(22.24) = 517.6 \text{ kips}$ $\underline{\phi_c P_n = 518 \text{ kips}}$

From the column load tables, the design strength of a W12 × 50 for $KL = 16$ ft is $\phi_c P_n = 326$ kips. Therefore, the reinforcement increases the strength by

$$\frac{518 - 326}{326} \times 100 = 58.90\%$$

<u>Increase = 58.9%</u>

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}A_g}{1.67} = \dfrac{25.86(22.24)}{1.67} = 344.4$ $\underline{\dfrac{P_n}{\Omega_c} = 344 \text{ kips}}$

From the column load tables, the allowable strength of a W12 × 50 for $KL = 16$ ft is $P_n/\Omega_c = 217$ kips. Therefore, the reinforcement increases the strength by

$$\frac{344 - 217}{217} \times 100 = 58.53\%$$

<u>Increase = 58.5%</u>

**4.9-9**

(a) *x*-axis:

$$I_x = 999 + 93.4 = 1092 \text{ in.}^4$$

$$A = 26.5 + 14.4 = 40.9 \text{ in.}^2$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1092}{40.9}} = 5.167 \text{ in.}$$

y-axis:

| Segment | $A$ | $x$ | $Ax$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| W14 × 90 | 26.5 | 0 | 0 | 362 | 1.834 | 451.1 |
| W10 × 49 | 14.4 | 5.21 | 75.02 | 272 | 3.376 | 436.1 |
| $\sum$ | 40.9 | | 75.02 | | | 887.2 |

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{75.02}{40.9} = 1.834 \text{ in.} \qquad I_y = 887.2 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{887.2}{40.9}} = 4.657 \text{ in.}$$

$\underline{r_x = 5.17 \text{ in.}, \ r_y = 4.66 \text{ in.}}$

(b) $$\frac{KL}{r} = \frac{(30 \times 12)}{4.657} = 77.3 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29000)}{(77.3)^2} = 47.9 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 77.3$$

$$\therefore \ F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/47.9)}(50) = 32.3 \text{ ksi}$$

$$P_n = F_{cr} A_g = 32.3(40.9) = 1321 \text{ kips}$$

LRFD:

$$\phi_c P_n = 0.90(1321) = 1190 \text{ kips}$$

$\underline{\phi_c P_n = 1190 \text{ kips}}$

ASD:

$$P_n/\Omega_c = 1321/1.67 = 791 \text{ kips} \qquad \underline{P_n/\Omega_c = 791 \text{ kips}}$$

---

**4.9-10**

Check width-thickness ratio. From AISC Table B4.1a,

$$0.45\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{29,000}{36}} = 12.77$$

$$\frac{b}{t} = \frac{8}{0.75} = 10.67 < 12.77 \qquad \text{(OK)}$$

Flexural buckling strength about the *x* axis (this is the axis of no symmetry):

$$\frac{KL}{r_x} = \frac{20 \times 12}{2.55} = 94.12 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(94.12)^2} = 32.31 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4 > 94.96$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/32.31)}(36) = 22.58 \text{ ksi}$$

$$A_g = 2 \times 8.49 = 16.98 \text{ in.}^2$$

$$P_n = F_{cr}A_g = 22.58(16.98) = 383.4 \text{ kips}$$

Flexural-torsional buckling strength about the *y* axis (the axis of symmetry):

$$a = \frac{20 \times 12}{3 \text{ spaces}} = 80 \text{ in.}$$

$$\frac{a}{r_i} = \frac{80}{0.850} = 94.12 < \frac{3}{4}(154.8) = 116.1 \qquad \text{(OK)}$$

Since $a/r_i > 40$,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_0^2 + \left(\frac{K_i a}{r_i}\right)^2}$$

$$\left(\frac{KL}{r}\right)_0 = \frac{KL}{r_y} = \frac{20\times 12}{1.55} = 154.8$$

$$\frac{K_i a}{r_i} = \frac{0.5(80)}{0.850} = 47.06$$

$$\left(\frac{KL}{r}\right)_m = \sqrt{(154.8)^2 + (47.06)^2} = 161.8$$

Compute $F_{cry}$ using AISC E3. From AISC Equation E3-4,

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 E}{(K_y L/r_y)^2} = \frac{\pi^2(29000)}{(161.8)^2} = 10.93 \text{ ksi}$$

Since $K_y L/r_y > 4.71\sqrt{\frac{E}{F_y}} = 113$,

$$F_{cry} = 0.877F_e = 0.877(10.93) = 9.586 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} = \frac{11200(2\times 1.61)}{16.98(3.93)^2} = 137.5 \text{ ksi}$$

$$F_{cry} + F_{crz} = 9.586 + 137.5 = 147.1 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry}+F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry}+F_{crz})^2}}\right]$$

$$= \frac{147.1}{2(0.575)}\left[1 - \sqrt{1 - \frac{4(9.586)(137.5)(0.575)}{(147.1)^2}}\right] = 9.298 \text{ ksi}$$

$$P_n = F_{cr}A_g = 9.298(16.98) = 157.9 \text{ kips}$$

The flexural-torsional buckling strength controls; $P_n = 157.9$ kips

Design strength: $\phi_c P_n = 0.90(157.9) = \underline{142 \text{ kips}}$

Allowable strength: $P_n/\Omega_c = 157.9/1.67 = \underline{94.6 \text{ kips}}$

---

**4.9-11**

(a) $P_u = 1.2D + 1.6L = 1.2(90) + 1.6(260) = 524$ kips

$$K_x L = K_y L = 0.65(15.33) = 9.965 \text{ ft}$$

From the column load tables, for $KL = 10$ ft,

Try $2L8 \times 6 \times \frac{3}{4}$, $\phi_c P_n = 527$ kips ($y$ axis controls), $w = 67.6$ lb/ft

Determine the number of intermediate connectors. To obtain the tabulated strength for the $y$ axis, 2 intermediate connectors must be used. For the $x$ axis, from AISC E4,

$$\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$$

where $KL/r$ is the controlling slenderness ratio for the member.

$$a = \text{spacing} = \frac{15.33 \times 12}{n+1}$$

$$r_i = r_z = 1.29 \text{ in.}, \qquad \frac{Ka}{r_i} = \frac{15.33 \times 12}{(n+1)(1.29)}$$

The larger slenderness ratio for the member is

$$\frac{K_y L}{r_y} = \frac{0.65(15.33 \times 12)}{2.47} = 48.41$$

For $\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$,

$$\frac{15.33 \times 12}{(n+1)(1.29)} \le \frac{3}{4}(48.41) \qquad \Rightarrow \qquad n \ge 2.93, \qquad \text{use 3.}$$

<u>Use $2L8 \times 6 \times \frac{3}{4}$ with 3 intermediate connectors</u>

(b) $P_a = D + L = 90 + 260 = 350$ kips

$$K_x L = K_y L = 0.65(15.33) = 9.965 \text{ ft}$$

From the column load tables, for $KL = 10$ ft,

Try $2L8 \times 6 \times \frac{3}{4}$, $P_n/\Omega_c = 351$ kips ($y$ axis controls), $w = 67.6$ lb/ft

Determine the number of intermediate connectors. To obtain the tabulated strength for the $y$ axis, 2 intermediate connectors must be used. For the $x$ axis, from AISC E4,

$$\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$$

where $KL/r$ is the controlling slenderness ratio for the member.

$$a = \text{spacing} = \frac{15.33 \times 12}{n+1}$$

$$r_i = r_z = 1.29 \text{ in.}, \qquad \frac{Ka}{r_i} = \frac{15.33 \times 12}{(n+1)(1.29)}$$

The larger slenderness ratio for the member is

$$\frac{K_y L}{r_y} = \frac{0.65(15.33 \times 12)}{2.47} = 48.41$$

For $\frac{Ka}{r_i} \leq \frac{3}{4}\frac{KL}{r}$,

$$\frac{15.33 \times 12}{(n+1)(1.29)} \leq \frac{3}{4}(48.41) \qquad \Rightarrow \qquad n \geq 2.93, \qquad \text{use 3.}$$

Use 2L8 × 6 × $\frac{3}{4}$ with 3 intermediate connectors

---

**4.9-12**

$$P_a = 280 \text{ kips}$$

$$K_x L = K_y L = 0.8(21) = 16.8 \text{ ft}$$

From the column load tables, for $KL = 16.8$ ft,

Try WT9 × 53

$$\frac{P_n}{\Omega_n} = 297 \text{ kips} > 280 \text{ kips} \qquad \text{(OK - } y \text{ axis controls)}$$

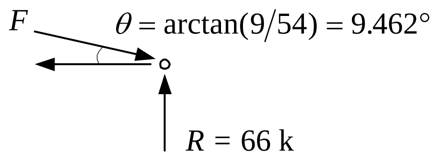
Use a WT9 × 53

---

**4.9-13**

$$\text{Reaction} = \frac{\sum P}{2} = \frac{11 \times 8}{2} = 44 \text{ kips}$$

Consider the joint at the right support:




$$\sum F_y = 44 - F\sin(9.462°) = 0 \qquad \Rightarrow \qquad F = 268 \text{ kips}$$

(This is the maximum force in the top chord.)

$$K_xL = K_yL = \frac{9}{\cos(9.462°)} = 9.124 \text{ ft.}$$

From the column load tables, for $KL = 9.124$ ft,

Try 2L8 × 6 × $\frac{5}{8}$ LLBB, $\quad P_n/\Omega_c > 286$ kips ($y$ axis controls), $w = 57.0$ lb/ft

Determine the number of intermediate connectors. To obtain the tabulated strength for the $y$ axis, 2 connectors must be used. For the $x$ axis, from AISC E4,

$$\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$$

where $KL/r$ is the controlling slenderness ratio for the member.

$$a = \text{spacing} = \frac{9.124 \times 12}{n+1}$$

$$r_i = r_z = 1.29 \text{ in.,} \qquad \frac{Ka}{r_i} = \frac{9.124 \times 12}{(n+1)(1.29)}$$

The larger slenderness ratio for the member is

$$\frac{K_xL}{r_x} = \frac{1.0(9.124 \times 12)}{2.45} = 44.69$$

For $\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$,

$$\frac{9.124 \times 12}{(n+1)(1.29)} \le \frac{3}{4}(44.69) \qquad \Rightarrow \qquad n \ge 1.53, \qquad \text{2 required for the } x \text{ axis.}$$

<u>Use 2L8 × 6 × $\frac{5}{8}$ LLBB with 2 intermediate connectors</u>

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# CHAPTER 5 - BEAMS

## 5.2-1

(a) Flange area = $0.5(16) = 8.0$ in.$^2$ Half web area = $(1/4)(10) = 2.5$ in.$^2$

From mid-depth of the cross section,

$$\bar{y} = \frac{8.0(10 + 0.25) + 2.5(10/2)}{8.0 + 2.5} = 9.0 \text{ in.}$$

$$Z = \frac{A}{2} \cdot a = (8.0 + 2.5)(2 \times 9.0) = 189.0 \text{ in.}^3$$

$$M_p = F_y Z = 50(189) = 9450 \text{ in-kips} = 788 \text{ ft-kips}$$

$Z = 189$ in.$^3$, $M_p = 788$ ft-kips

(b) Moment of inertia:

| Component | $A$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|
| top fl | 8.00 | 0.1667 | 10.25 | 840.7 |
| web | 5.00 | 167 | 0.000 | 167.0 |
| bot fl | 8.00 | 0.1667 | 10.25 | 840.7 |
| Sum | 21.00 | | | 1848 |

$$S = \frac{I}{c} = \frac{1848}{10 + 0.5} = 176.0 \text{ in.}^3$$

$$M_y = F_y S = 50(176.0) = 8800 \text{ in.-kips} = 733 \text{ ft-kips}$$

$S = 176$ in.$^3$, $M_y = 733$ ft-kips

---

## 5.2-2

(a) Area above PNA = Area below PNA

$$3(22) + (\bar{y} - 3)(1/2) = (3 + 66 - \bar{y})(1/2) + 3(16) \quad \Rightarrow \quad \bar{y} = 18$$

$\bar{y} = 18$ in.

(b) To locate centroids of areas, take moments about plastic neutral axis. For area above PNA,

| Component | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| top fl | 66.00 | 16.50 | 1089 |
| web | 7.50 | 7.50 | 56.25 |
| Sum | 73.50 | | 1145 |

$$\bar{y}_1 = \frac{\sum Ay}{\sum A} = \frac{1145}{73.50} = 15.58 \text{ in.}$$

For area below PNA,

| Component | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| bot fl | 48.00 | 52.50 | 2520 |
| web | 25.50 | 25.50 | 650.25 |
| Sum | 73.50 | | 3170 |

$$\bar{y}_2 = \frac{\sum Ay}{\sum A} = \frac{3170}{73.50} = 43.13 \text{ in.}$$

Plastic moment:

$$M_p = C \times \text{moment arm} = T \times \text{moment arm}$$

$$= AF_y(\bar{y}_1 + \bar{y}_2) = 73.5(50)(15.58 + 43.13)$$

$$= 3675(58.71) = 2.158 \times 10^5 \text{ in.-kips} = 17{,}900 \text{ ft-kips}$$

$$\underline{M_p = 180 \text{ ft-kips}}$$

(c) Because of symmetry, $\bar{x}_t = \bar{x}_c$

| Component | $A$ | $x$ | $Ax$ |
|---|---|---|---|
| top fl | 33.00 | 5.50 | 181.5 |
| web | 16.50 | 0.13 | 2.06 |
| bot fl | 24.00 | 4.00 | 96.00 |
| Sum | 73.50 | | 279.6 |

$$\bar{x}_t = \bar{x}_c = \frac{\sum Ax}{\sum A} = \frac{279.6}{73.50} = 3.804 \text{ in.}$$

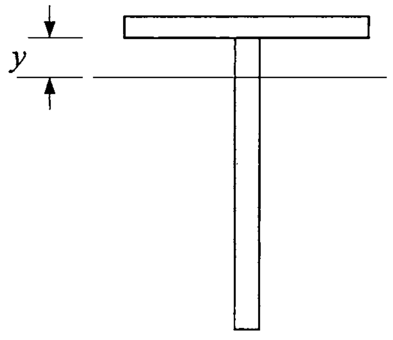
$$Z_y = \frac{A}{2}(\bar{x}_t + \bar{x}_c) = 73.50(3.804 + 3.804) = 559.2 \text{ in.}^3 \qquad \underline{Z_y = 5.59 \text{ in.}^3}$$

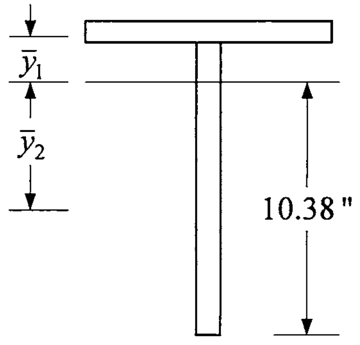
---

**5.2-3**

Locate the plastic neutral axis. Let area above = area below:

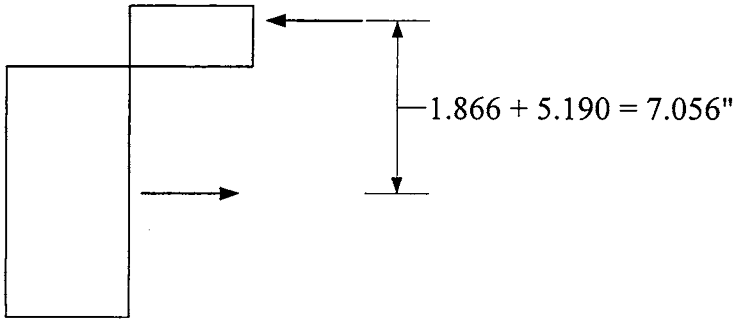
$$10(7/8) + 1.0\,y = 1.0(12 - y), \qquad y = 1.625 \text{ in.}, \qquad 12 - 1.625 = 10.38 \text{ in.}$$




| Segment | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| Flange | 8.75 | 2.062 | 18.05 |
| Web | 1.625 | 0.8125 | 1.32 |
| $\Sigma$ | 10.375 | | 19.37 |

$$\bar{y}_1 = \frac{\sum Ay}{\sum A} = \frac{19.37}{10.38} = 1.866 \text{ in.}, \qquad \bar{y}_2 = \frac{10.38}{2} = 5.190 \text{ in.}$$



Total area = 10(7/8) + 1.0(12) = 20.75 in.$^2$

$$M_p = \left(\frac{20.75}{2}\right)(50)(7.056) = 3660 \text{ in.-kips} = 305 \text{ ft-kips}$$

$\underline{M_p = 305 \text{ ft-kips}}$

---

**5.2-4**

For a WT9 × 23, $y = 2.33$ in.

Moment arm = $d$ (for W18 × 46) - $2y$ = $18.1 - 2(2.33) = 13.44$ in.

$Z_x$ = Area × moment arm = $(13.5/2)(13.44) = 90.72$ in.$^3$ $\underline{Z_x = 90.7 \text{ in.}^3}$

(Tabulated value is 90.7 in.$^3$)

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**5.4-1**

| Element | $\lambda$ | $\lambda_p$ | $\lambda_r$ | Classification |
|---|---|---|---|---|
| Flange | 10.2 | 8.35 | 20.0 | noncompact |
| Web | 25.9 | 82.7 | 125 | compact |

Shape is noncompact

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**5.4-2**

| Element | $\lambda$ | $\lambda_p$ | $\lambda_r$ | Classification |
|---|---|---|---|---|
| Flange | 11.5 | 8.35 | 20.0 | noncompact |
| Web | 21.6 | 82.7 | 125 | compact |

Shape is noncompact

---

**5.4-3**

Flange: maximum $\frac{b_f}{2t_f} = 11.0$ for an M4 × 6

$$\text{Let } 11.9 = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29000}{F_y}} \quad \Rightarrow \quad F_y = 205 \text{ ksi}$$

Web: maximum $\frac{h}{t_w}$ = 74.8 for both an M12.5 × 12.4 and an M12.5 × 11.6

$$\text{Let } 74.8 = 5.7\sqrt{\frac{E}{F_y}} = 5.7\sqrt{\frac{29000}{F_y}} \quad \Rightarrow \quad F_y = 168 \text{ ksi (controls)}$$

$\underline{F_y = 168 \text{ ksi}}$

<u>M12.5 × 12.4 and M12.5 × 11.6</u>

Conclusion: No W, M, or S shape in the *Manual* is slender for any of the steels listed in Table 2-3 of the *Manual.*

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**<u>5.5-1</u>**

Check for compactness. From Part 1 of the Manual,

$$\frac{b_f}{2t_f} = 7.75$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15 > 5.86 \qquad \therefore \text{ the flange is compact.}$$

(This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise.)

$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact.

∴ a W14 × 61 is compact for $F_y = 50$ ksi.

Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(102) = 5100 \text{ in.-kips} = 425 \text{ ft kips}$$

(a) Let $M_u = \phi_b M_n$ :

$$\frac{1.2(0.061)(30)^2}{8} + \frac{1.6P(30)}{4} = 0.90(425), \text{ Solution is: } \{P = 31.19\}$$

$\underline{P = 31.2 \text{ kips}}$

(b) Let $M_a = \dfrac{M_n}{\Omega_b}$ :

$$\frac{0.061(30)^2}{8} + \frac{P(30)}{4} = \frac{425}{1.67}, \text{ Solution is: } \{P = 33.017\} \qquad \underline{P = 33.0 \text{ kips}}$$

---

**5.5-2**

Check for compactness. From Part 1 of the Manual,

$$\frac{b_f}{2t_f} = 6.01$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15 > 6.01 \qquad \therefore \text{ the flange is compact.}$$

(This shape can also be identified as compact for flexure because there is no footnote in the dimensions and properties tables to indicate otherwise.)

$\dfrac{h}{t_w} < 3.76\sqrt{\dfrac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact.

$\therefore$ a W33 × 141 is compact for $F_y = 50$ ksi.

Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(514) = 2.57 \times 10^4 \text{ in.-kips} = 2142 \text{ ft kips}$$

(a) Let $M_u = \phi_b M_n$ : $\dfrac{w_u(25)^2}{8} = 0.90(2142), \qquad w_u = 24.68$

$$1.2w_D + 1.6(2w_D) = 24.68, \text{ Solution is: } \{w_D = 5.609\}$$

$$w_D + w_L = 5.609 + 2(5.609) = 16.83 \text{ kips/ft}$$

If the beam weight is deducted, the maximum service load is

$$16.83 - 0.141 = 16.69 \text{ kips/ft}$$

$\underline{w = 16.7 \text{ kips/ft in addition to the beam weight}}$

(b) Let $M_a = \dfrac{M_n}{\Omega_b}$ :  $\dfrac{w_a(25)^2}{8} = \dfrac{2142}{1.67}$, Solution is: $\{w_a = 16.42\}$

If the beam weight is deducted, the maximum service load is

$$16.42 - 0.141 = 16.28 \text{ kips/ft}$$

$w = 16.3$ kips/ft in addition to the beam weight

---

## 5.5-3

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.17 < \lambda_p \;\therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\dfrac{29{,}000}{50}} = 90.6, \qquad \lambda = \dfrac{h}{t_w} = 47.8 < \lambda_p \;\therefore$ web is compact,

and the shape is compact.

$$M_n = F_y Z_x = \frac{50(378)}{12} = 1575 \text{ ft-kips}$$

(a) Design strength $= \phi_b M_n = 0.90(1575) = 1420$ ft-kips

Load combination 2 controls:

Concentrated Load: $P_u = 1.2(45) = 54.0$ kips

Uniform load: $w_u = 1.2(1.0) + 1.6(2.5) = 5.2$ kips/ft

Left Reaction: $R_{left} = 54.0\left(\dfrac{25}{40}\right) + \dfrac{5.2(40)}{2} = 137.8$ kips

Shear left of conc. load: $V_{left} = 137.8 - 5.2(15) = 59.8$ kips

Shear right of conc. load: $V_{right} = 59.8 - 54.0 = 5.8$ kips

$\therefore$ shear is zero when $137.8 - 5.2x - 54.0 = 0$, Solution is: $\{x = 16.12\}$

$$M_u = R_{left}x - w_u x^2/2 - P_u(x - 15)$$

$$= 137.8(16.12) - 5.2(16.12)^2/2 - 54.0(16.12 - 15)$$

$$= 1480 \text{ ft-kips} > 1420 \text{ ft-kips} \qquad \text{(N.G.)}$$

A W30 × 116 is not adequate.

(b) Allowable strength = $\dfrac{M_n}{\Omega_b} = \dfrac{1575}{1.67} = 943$ ft-kips

ASD load combination 2 controls:

Concentrated Load: $P_a = 45$ kips

Uniform load: $w_a = 1.0 + 2.5 = 3.5$ kips/ft

Left Reaction: $R_{left} = 45\left(\dfrac{25}{40}\right) + \dfrac{3.5(40)}{2} = 98.13$ kips

Shear left of conc. load: $V_{left} = 98.13 - 3.5(15) = 45.63$ kips

Shear right of conc. load: $V_{right} = 45.63 - 45 = 0.63$ kips

$\therefore$ shear is zero when $98.13 - 3.5x - 45 = 0$, Solution is: $\{x = 15.18\}$

$M_u = R_{left}x - w_a x^2/2 - P_a(x - 15)$

$= 98.13(15.18) - 3.5(15.18)^2/2 - 45(15.18 - 15)$

$= 1080$ ft-kips $> 943$ ft-kips (N.G.) A W30 × 116 is not adequate.

---

**5.5-4**

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.28 < \lambda_p \;\therefore \text{ flange is compact}$$

$\dfrac{h}{t_w} < 3.76\sqrt{\dfrac{E}{F_y}}$ (for all shapes in the Manual for $F_y \le 65$ ksi), so the web is compact, and the shape is compact.

(a) $w_u = 1.2D + 1.6L = 1.2(5.5/2) + 1.6(5.5/2) = 7.7$ kips/ft

Because of symmetry, the reactions are equal: $R = 7.7(30)/2 = 115.5$ kips

Maximum negative $M_u = -7.7(6)^2/2 = -139$ ft-kips

Because of symmetry, the maximum positive moment occurs at midspan:

Maximum positive $M_u = -7.7(15)^2/2 + 115.5(9) = 173$ ft-kips (controls)

Compute the design strength: $\phi_b M_n = \phi_b F_y Z_x = 0.9(50)(54.0)/12 = 202.5$ ft-kips

Since $M_u = 173$ ft-kips $< \phi_b M_n = 202$ ft-kips, <u>a W16 × 31 is adequate.</u>

(b) $w_a = D + L = 5.5$ kips/ft

Because of symmetry, the reactions are equal: $R = 5.5(30)/2 = 82.5$ kips

Maximum negative $M_a = -5.5(6)^2/2 = -99.0$ ft-kips

Because of symmetry, the maximum positive moment occurs at midspan:

Maximum positive $M_a = -5.5(15)^2/2 + 82.5(9) = 124$ ft-kips (controls)

Compute the allowable strength: $\dfrac{M_n}{\Omega_b} = \dfrac{F_y Z_x}{\Omega_b} = \dfrac{50(54.0)}{1.67(12)} = 135$ ft-kips

Since $M_a = 124$ ft-kips $< \dfrac{M_n}{\Omega_b} = 135$ ft-kips, <u>a W16 × 31 is adequate.</u>

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<u>**5.5-5**</u>

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 8.22 < \lambda_p \;\therefore\; \text{flange is compact}$$

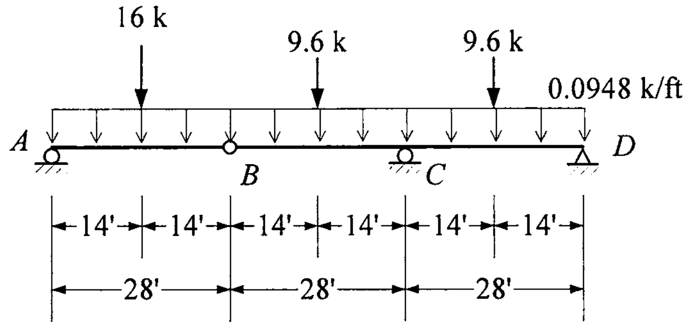
$\dfrac{h}{t_w} < 3.76\sqrt{\dfrac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact, and the shape is compact.

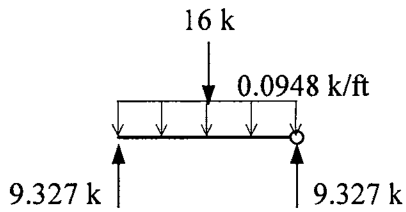
$$M_n = F_y Z_x = \frac{50(119)}{12} = 495.8 \text{ ft-kips}$$

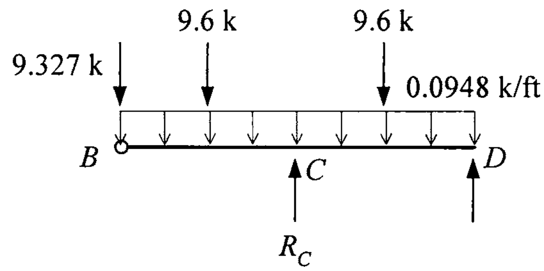
(a) Factored loads, including beam weight:

$w_u = 1.2(0.079) = 0.0948$ kips/ft, $P1_u = 1.6(10) = 16.0$ kips,

$P2_u = 1.6(6) = 9.6$ kips



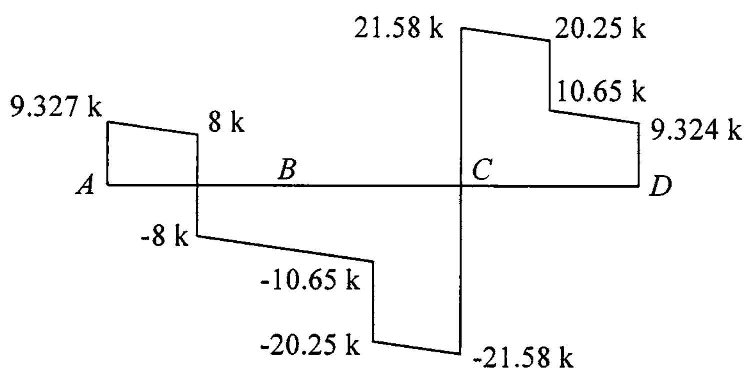




Reaction at $C$ : Using the free-body diagram of $BCD$,

$$\sum M_D = 9.327(56) + 9.6(42 + 14) + 0.0948(56)^2/2 - R_C(28) = 0$$

$$R_C = 43.16 \text{ kips}$$

Shear diagram:



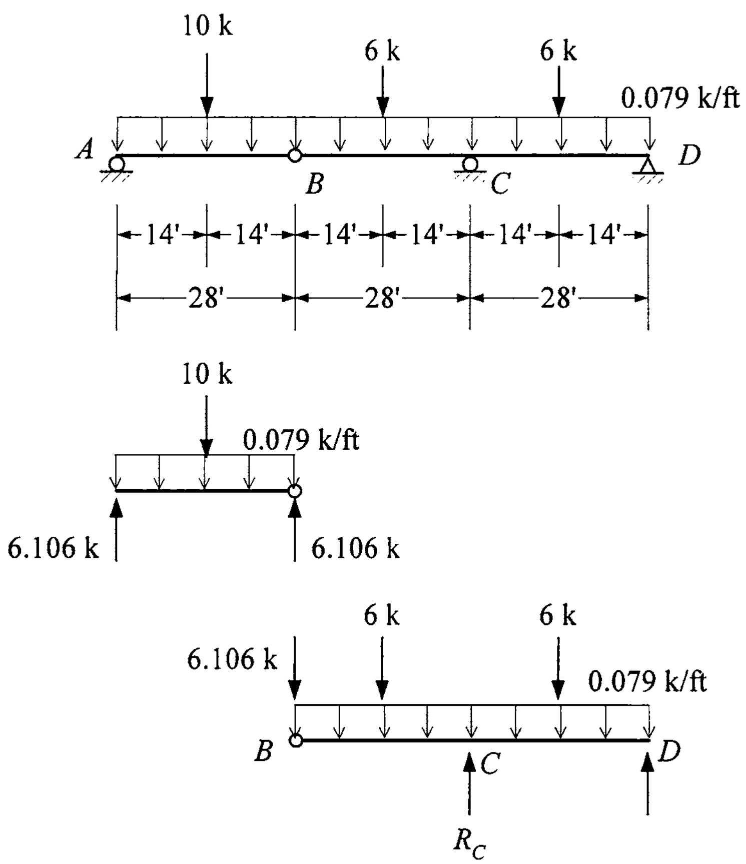

Maximum moment occurs at $C$. Using the free-body diagram of $BCD$,

$M_C = -9.327(28) - 9.6(14) - 0.0948(28)^2/2 = -433$ ft-kips

Design strength = $\phi_b M_n = 0.90(495.8) = 446$ ft-kips

$M_u = 433$ ft-kips < $\phi_b M_n = 446$ ft-kips (OK)  A W18 × 60 is adequate.

(b) Service loads:

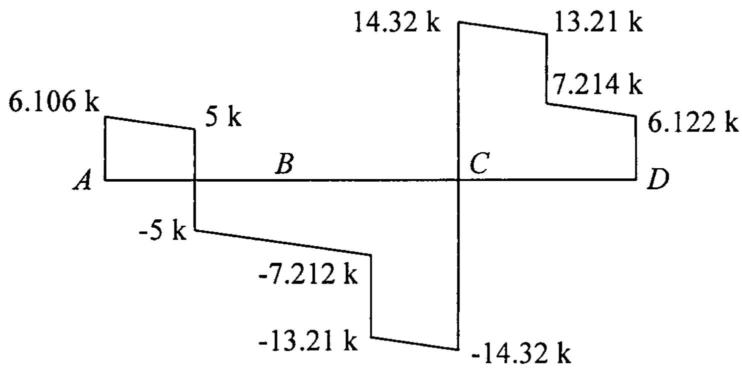



Reaction at $C$ :

$$\sum M_D = -6.106(56) - 6(42) - 6(14) + R_C(28) - 0.079(56)^2/2 = 0,$$

Solution is: $\{R_C = 28.64\}$ kips

Shear diagram:




Maximum moment occurs at $C$. Refer to the free-body diagram of $BCD$.

$$M_a = M_C = -6.106(28) - 6(14) - 0.079(28)^2/2 = -286 \text{ ft-kips}$$

$$\text{Allowable strength} = \frac{M_n}{\Omega_b} = \frac{495.8}{1.67} = 297 \text{ ft-kips}$$

$M_a = 286$ ft-kips < 297 ft-kips (OK) A W18 × 60 is adequate.

---

**5.5-6**

(a) For a W16 × 26, $A = 7.68$ in.$^2$, $d = 15.7$ in., $t_f = 0.345$ in., $S_x = 38.4$ in.$^3$, $I_y = 9.59$ in.$^4$, $r_y = 1.12$ in., $J = 0.262$ in.$^4$, $C_W = 565$ in.$^6$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(1.12)\sqrt{\frac{29000}{50}} = 47.47 \text{ in.} = 3.96 \text{ ft.}$$

The following terms will be needed in the computation of $L_r$ :

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{9.59(565)}}{38.4} = 1.917 \text{ in.}^2$$

$$r_{ts} = \sqrt{1.917} = 1.385 \text{ in.}$$

($r_{ts}$ can also be found in the dimensions and properties tables. For a W16 × 26, it is given as 1.38 in.)

$$h_o = d - t_f = 15.7 - 0.345 = 15.36 \text{ in.}$$

($h_o$ can also be found in the dimensions and properties tables. For a W16 × 26, it is given as 15.4 in.)

For a doubly-symmetric I-shape, $c$ = 1.0. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(1.385)\frac{29000}{0.7(50)} \sqrt{\frac{0.262(1.0)}{38.4(15.36)} + \sqrt{\left(\frac{0.262(1.0)}{38.4(15.36)}\right)^2 + 6.76\left(\frac{0.7(50)}{29000}\right)^2}}$$

$$= 134.5 \text{ in.} = 11.21 \text{ ft}$$

$\underline{L_p = 3.96 \text{ ft}, L_r = 11.2 \text{ ft}}$(b)

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 7.97 < \lambda_p \quad \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29{,}000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 56.8 < \lambda_p \quad \therefore$ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For $L_b$ = 8 ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 50(44.2) = 2210 \text{ in.-kips}$$

$$M_n = 1.0\left[2210 - (2210 - 0.7 \times 50 \times 38.4)\left(\frac{8 - 3.96}{11.2 - 3.96}\right)\right]$$

$$= 1727 \text{ in.-kips} < M_p$$

$$\phi_b M_n = 0.9(1727) = 1554 \text{ in.-kips} = 130 \text{ ft-kips}$$

$\underline{\phi_b M_n = 130 \text{ ft-kips}}$

(c) $\frac{M_n}{\Omega_b} = \frac{1727/12}{1.67} = 86.2$ ft-kips

$\underline{\frac{M_n}{\Omega_b} = 86.2 \text{ ft-kips}}$

---

**5.5-7**

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 5.44 < \lambda_p \;\therefore\; \text{flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 38.7 < \lambda_p \;\therefore$ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For a W18 × 60, $A = 17.6 \text{ in.}^2$, $d = 18.2$ in., $t_f = 0.695$ in., $S_x = 108 \text{ in.}^3$, $I_y = 50.1 \text{ in.}^4$, $r_y = 1.68$ in., $J = 2.17 \text{ in.}^4$, $C_w = 3850 \text{ in.}^6$

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.68)\sqrt{\frac{29000}{50}} = 71.21 \text{ in.} = 5.934 \text{ ft.}$$

The following terms will be needed in the computation of $L_r$ :

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{50.1(3850)}}{108} = 4.067 \text{ in.}^2$$

$$r_{ts} = \sqrt{4.067} = 2.017 \text{ in.}$$

($r_{ts}$ can also be found in the dimensions and properties tables. For a W18 × 60, it is given as 2.02 in.)

$$h_o = d - t_f = 18.2 - 0.695 = 17.51 \text{ in.}$$

($h_o$ can also be found in the dimensions and properties tables. For a W18 × 60, it is given as 17.5 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(2.017)\frac{29000}{0.7(50)}\sqrt{\frac{2.17(1.0)}{108(17.51)} + \sqrt{\left(\frac{2.17(1.0)}{108(17.51)}\right)^2 + 6.76\left(\frac{0.7(50)}{29000}\right)^2}}$$

$$= 218.3 \text{ in.} = 18.19 \text{ ft}$$

For $L_b = 25$ ft, $L_b > L_r$, so

$$M_n = F_{cr}S_x \leq M_p \qquad \text{(elastic LTB)}$$

where

$$F_{cr} = \frac{C_b\pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.0\pi^2(29000)}{\left(\frac{25 \times 12}{2.017}\right)^2}\sqrt{1 + 0.078\frac{2.17(1.0)}{108(17.51)}\left(\frac{25 \times 12}{2.017}\right)^2} = 22.33 \text{ ksi}$$

So $M_n = F_{cr}S_x = 22.33(108) = 2412$ in.-kips = 201 ft-kips

$$M_p = F_y Z_x = 50(123) = 6150 \text{ in.-kips} = 513 \text{ ft-kips}$$

$M_n < M_p$ (OK) $\qquad\qquad$ $\underline{M_n = 201 \text{ ft-kips}}$

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## 5.5-8

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29000}{65}} = 8.03, \qquad \lambda = \frac{b_f}{2t_f} = 6.77 < \lambda_p \quad \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29000}{65}} = 79.4$, $\quad \lambda = \frac{h}{t_w} = 31.2 < \lambda_p$ $\therefore$ web is compact, and the shape is compact.

For a W16 × 77, $A = 22.6$ in.$^2$, $d = 16.5$ in., $t_f = 0.760$ in., $S_x = 134$ in.$^3$, $Z_x = 150$ in.$^3$, $I_y = 138$ in.$^4$, $r_y = 2.47$ in., $r_{ts} = 2.85$ in., $h_o = 15.7$ in., $J = 3.57$ in.$^4$, $C_w = 8590$ in.$^6$

$$L_p = 1.76 r_y\sqrt{\frac{E}{F_y}} = 1.76(2.47)\sqrt{\frac{29000}{65}} = 91.82 \text{ in.} = 7.652 \text{ ft.}$$

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(2.85)\frac{29000}{0.7(65)}\sqrt{\frac{3.57(1.0)}{134(15.7)}+\sqrt{\left(\frac{3.57(1.0)}{134(15.7)}\right)^2+6.76\left(\frac{0.7(65)}{29000}\right)^2}}$$

$$= 277.0 \text{ in.} = 23.08 \text{ ft}$$

For $L_b = 15$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 65(150) = 9750 \text{ in.-kips} = 812.5 \text{ ft-kips}$$

$$M_n = 1.0\left[9750 - (9750 - 0.7\times 65\times 134)\left(\frac{15 - 7.652}{23.08 - 7.652}\right)\right]$$

$$= 8010 \text{ in.-kips} = 668 \text{ ft-kips} < M_p \qquad \underline{M_n = 668 \text{ ft-kips}}$$

---

**5.5-9**

(a) $R_A = R_B = \dfrac{250}{2} = 125.0$ kips

Use lower-case subscripts to identify moments at the quarter points of the unbraced length:

$$M_a = M_c = 125(5) = 625 \text{ ft-kips} \qquad M_b = 125(10) = 1250 \text{ ft-kips} = M_{max}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1250)}{2.5(1250) + 3(625) + 4(1250) + 3(625)} = 1.32 \qquad \underline{C_b = 1.32}$$

(b) $R_A = R_B = \dfrac{250 + 0.210}{2} = 125.1$ ft-kips

$$M_a = M_c = 125.1(5) = 625.5 \text{ ft-kips}$$

$$M_b = 125.1(10) = 1251 \text{ ft-kips} = M_{max}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1251)}{2.5(1251) + 3(625.5) + 4(1251) + 3(625.5)} = 1.32 \qquad \underline{C_b = 1.32}$$

---

**5.5-10**

(a) Using service loads,

$$R_A = R_B = \frac{250}{2} = 125 \text{ ft-kips}$$

Use lower-case letters to identify moments:

$$M_a = 125(2.5) = 312.5 \text{ ft-kips}$$

$$M_b = 125(5) = 625.0 \text{ ft-kips}$$

$$M_c = 125(7.5) = 937.5 \text{ ft-kips}$$

$$M_{\max} = 125(10) = 1250 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1250)}{2.5(1250) + 3(312.5) + 4(625) + 3(937.5)} = 1.667 \qquad \underline{C_b = 1.67}$$

(b) Using factored loads,

$$P_u = 1.6(250) = 400.0 \text{ kips}$$

$$R_A = R_B = \frac{400}{2} = 200 \text{ ft-kips}$$

Use lower-case letters to identify moments:

$$M_a = 200(2.5) = 500 \text{ ft-kips}$$

$$M_b = 200(5) = 1000 \text{ ft-kips}$$

$$M_c = 200(7.5) = 1500 \text{ ft-kips}$$

$$M_{\max} = 200(10) = 2000 \text{ ft-kips}$$

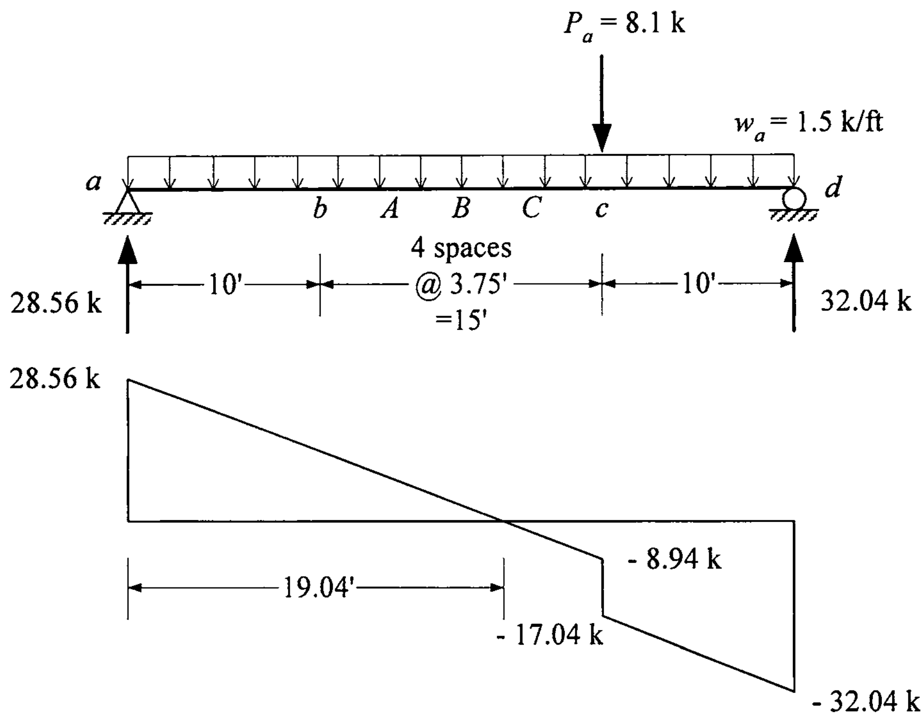
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(2000)}{2.5(2000) + 3(500) + 4(1000) + 3(1500)} = 1.667 \qquad \underline{C_b = 1.67}$$

**5.5-11**

$P_D = 2.7$ kips, $P_L = 5.4$ kips. $w_D = 0.5$ kips/ft, $w_L = 1.0$ kips/ft

(a) $P_a = 2.7 + 5.4 = 8.1$ kips. $w_a = 0.5 + 1.0 = 1.5$ kips/ft




$M_{max} = 28.56(19.04) - 1.5(19.04)^2/2 = 271.9$ ft-kips

$M_A = 28.56(13.75) - 1.5(13.75)^2/2 = 250.9$ ft-kips

$M_B = 28.56(17.5) - 1.5(17.5)^2/2 = 270.1$ ft-kips

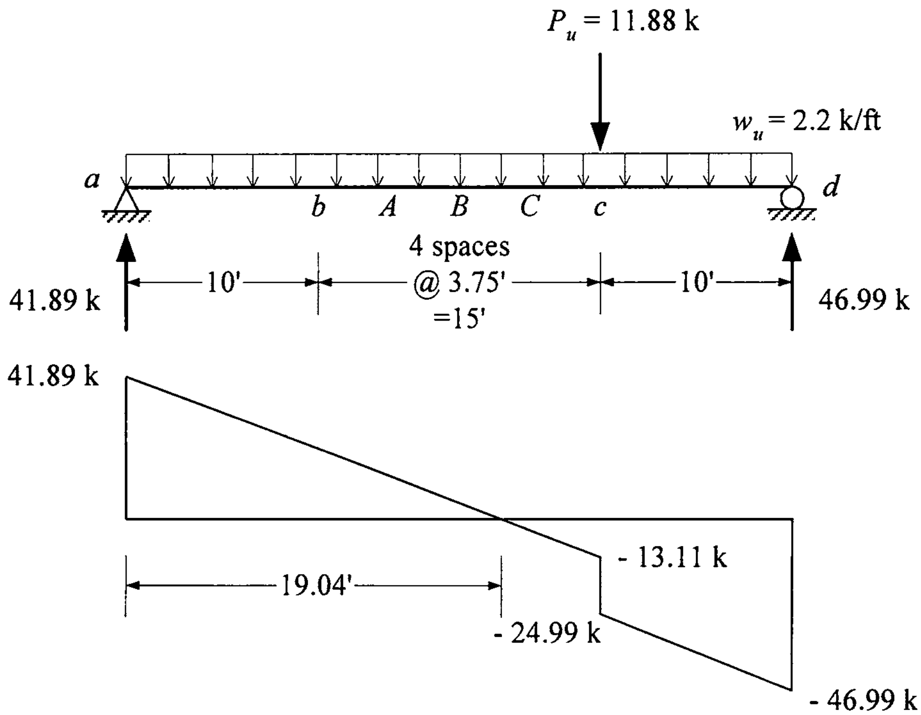
$M_C = 28.56(21.25) - 1.5(21.25)^2/2 = 268.2$ ft-kips

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(271.9)}{2.5(271.9) + 3(250.9) + 4(270.1) + 3(268.2)} = 1.02 \qquad \underline{C_b = 1.02}$$

(b) $P_u = 1.2(2.7) + 1.6(5.4) = 11.88$ kips

$w_u = 1.2(0.5) + 1.6(1.0) = 2.2$ kips/ft




$M_{max} = 41.89(19.04) - 2.2(19.04)^2/2 = 398.8$ ft-kips

$M_A = 41.89(13.75) - 2.2(13.75)^2/2 = 368.0$ ft-kips

$M_B = 41.89(17.5) - 2.2(17.5)^2/2 = 396.2$ ft-kips

$M_C = 41.89(21.25) - 2.2(21.25)^2/2 = 393.4$ ft-kips

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(398.8)}{2.5(398.8) + 3(368.0) + 4(396.2) + 3(393.4)} = 1.02$$

$C_b = 1.02$

---

**5.5-12**

First, determine the nominal flexural strength. From the $Z_x$ table,

$L_p = 6.78$ ft, $L_r = 19.5$ ft

For $L_b = 12$ ft, $L_p < L_b < L_r$

This shape is compact. (There is no footnote in the dimensions and properties table to indicate otherwise.)

Lateral support is provided at intervals of $48/12 = 4.0$ ft, (at the quarater points). From Table 3-1 in Part 3 of the *Manual*, $C_b = 1.06$. Since $L_p < L_b < L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 50(200) = 10,000 \text{ in.-kips}$$

$$M_n = 1.06\left[10000 - (10000 - 0.7 \times 50 \times 176)\left(\frac{12 - 6.78}{19.5 - 6.78}\right)\right]$$

$$= 8930 \text{ in.-kips} = 744.2 \text{ ft-kips}$$

(a) Let $M_u = \phi_b M_n$ :

$$\frac{w_u(48)^2}{8} = 0.90(744.2), \text{ Solution is: } \{w_u = 2.326\}$$

$$1.2w_D + 1.6w_L = 1.2(0.076) + 1.6w_L = 2.326, \text{ Solution is: } \{w_L = 1.397\}$$

$\underline{w_L = 1.40 \text{ kips/ft}}$

(b) Let $M_a = \dfrac{M_n}{\Omega_b}$ :

$$\frac{w_a(48)^2}{8} = \frac{744.2}{1.67}, \text{ Solution is: } \{w_a = 1.547\}$$

If the beam weight is deducted, the maximum service live load is

$w_a - w_D = 1.547 - 0.076 = 1.471$ kips/ft $\qquad$ $\underline{w_L = 1.47 \text{ kips/ft}}$

---

**5.5-13**

First, determine the nominal flexural strength. From the $Z_x$ table,

$L_p = 6.92$ ft, $\qquad L_r = 23.8$ ft

For $L_b = 14$ ft, $L_p < L_b < L_r$

This shape is compact. (There is no footnote in the dimensions and properties table to indicate otherwise.)

From Figure 5.15(c) in the textbook, $C_b = 1.32$. Since $L_p < L_b < L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 50(71.9) = 3595 \text{ in.-kips}$$

$$M_n = 1.32\left[3595 - (3595 - 0.7 \times 50 \times 64.2)\left(\frac{14 - 6.92}{23.8 - 6.92}\right)\right]$$

$$= 3999 \text{ in.-kips} > M_p \qquad \therefore \text{ use } M_n = M_p = 3595 \text{ in.-kips}$$

(a) LRFD solution:

$$\phi_b M_n = 0.90(3595)/12 = 270 \text{ ft-kips}$$

$$M_u = \frac{1}{8}w_uL^2 + \frac{P_uL}{4} = \frac{1}{8}(1.2 \times 0.050)(14)^2 + \frac{1.6(40)(14)}{4}$$

$$= 226 \text{ ft-kips} < 270 \text{ ft-kips} \qquad \text{(OK)}$$

W12 × 50 is adequate.

(b) ASD solution:

$$\frac{M_n}{\Omega_b} = \frac{3595/12}{1.67} = 179 \text{ ft-kips}$$

$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(0.050)(14)^2 + \frac{40(14)}{4}$$

$$= 141 \text{ ft-kips} < 179 \text{ ft-kips} \qquad \text{(OK)}$$

W12 × 50 is adequate.

---

**5.5-14**

All channel shapes in the Manual are compact (there are no footnotes to indicate otherwise).

For an MC18 × 51.9, $A = 15.3$ in.$^2$, $d = 18.0$ in., $t_f = 0.625$ in., $r_{ts} = 1.35$ in., $h_o = 17.4$ in., $S_x = 69.6$ in.$^3$, $Z_x = 87.3$ in.$^3$, $I_y = 16.3$ in.$^4$, $r_y = 1.03$ in., $J = 2.03$ in.$^4$, $C_w = 985$ in.$^6$

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.03)\sqrt{\frac{29000}{36}} = 51.45 \text{ in.} = 4.288 \text{ ft.}$$

For channels, from AISC Equation F2-8b,

$$c = \frac{h_o}{2}\sqrt{\frac{I_y}{C_w}} = \frac{17.4}{2}\sqrt{\frac{16.3}{985}} = 1.119$$

From AISC Equation F2-6,

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(1.35)\frac{29000}{0.7(36)}\sqrt{\frac{2.03(1.119)}{69.6(17.4)} + \sqrt{\frac{2.03(1.119)}{69.6(17.4)}^2 + 6.76\left(\frac{0.7(36)}{29000}\right)^2}}$$

$= 210.2$ in. $= 17.52$ ft. For $L_b = 14$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

From Figure 5.15, $C_b = 1.32$

$$M_p = F_y Z_x = 36(87.3) = 3143 \text{ in.-kips} = 261.9 \text{ ft-kips}$$

$$M_n = 1.32\left[3143 - (3143 - 0.7 \times 36 \times 69.6)\left(\frac{14 - 4.288}{17.52 - 4.288}\right)\right]$$

$$= 2803 \text{ in.-kips} = 233.6 \text{ ft-kips} < M_p$$

(a) LRFD solution:

$$\phi_b M_n = 0.90(233.6) = 210 \text{ ft-kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.2 \times 0.0519)(14)^2 + \frac{1.6(40)(14)}{4}$$

$$= 226 \text{ ft-kips} > 210 \text{ ft-kips} \qquad \text{(N.G.)}$$

An MC18 × 51.9 is not adequate.

(b) ASD solution:

$$\frac{M_n}{\Omega_b} = \frac{233.6}{1.67} = 140 \text{ ft-kips}$$

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(0.0519)(14)^2 + \frac{40(14)}{4}$$

$$= 141 \text{ ft-kips} > 140 \text{ ft-kips} \qquad \text{(N.G.)}$$ An MC18 × 51.9 is not adequate

**5.5-15**

First, determine the nominal flexural strength. From the $Z_x$ table,

$$L_p = 7.42 \text{ ft}, \qquad L_r = 21.3 \text{ ft}. \qquad \text{For } L_b = 20 \text{ ft}, \; L_p < L_b < L_r$$

This shape is compact. (There is no footnote in the dimensions and properties table to indicate otherwise.) Since $L_p < L_b < L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

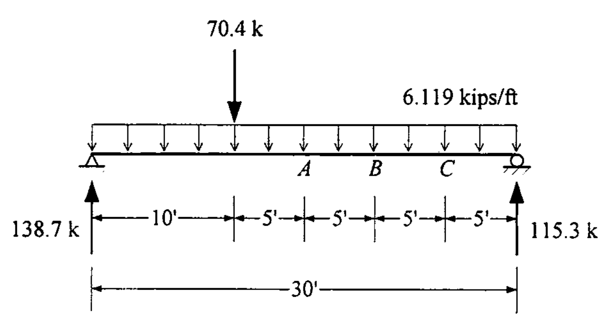
$$M_p = F_yZ_x = 50(312) = 15{,}600 \text{ in.-kips}$$

$$M_n = C_b\left[15600 - (15600 - 0.7 \times 50 \times 269)\left(\frac{20 - 7.42}{21.3 - 7.42}\right)\right]$$

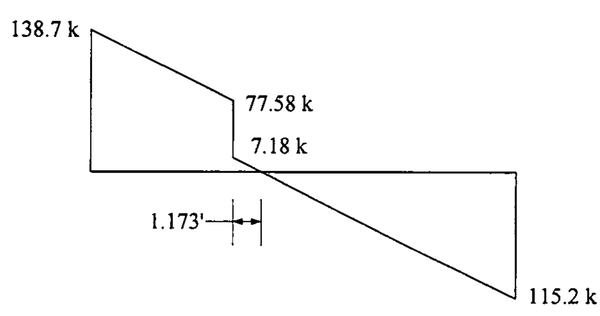
$$= C_b(9994) \text{ in.-kips}$$

(a) LRFD solution: $P_u = 1.2P_D + 1.6P_L = 1.2(16) + 1.6(32) = 70.4$ kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(1 + 0.099) + 1.6(3) = 6.119 \text{ kips/ft}$$




(Note that points have been relabeled for use in the $C_b$ formula.)




$$M_u = 138.7(11.173) - 6.119(11.173)^2/2 - 70.4(1.173) = 1085 \text{ ft-kips}$$

Compute $C_b$ : $M_{\max} = M_u = 1085$ ft-kips

$$M_A = 138.7(15) - 6.119(15)^2/2 - 70.4(5) = 1040 \text{ ft-kips}$$

$$M_B = 138.7(20) - 6.119(20)^2/2 - 70.4(10) = 846.2 \text{ ft-kips}$$

$$M_C = 138.7(25) - 6.119(25)^2/2 - 70.4(15) = 499.3 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1085)}{2.5(1085) + 3(1040) + 4(846.2) + 3(499.3)} = 1.266$$

$$M_n = C_b(9994) = 1.266(9994) = 12{,}650 \text{ in.-kips} < M_p = 15{,}600 \text{ in.-kips}$$

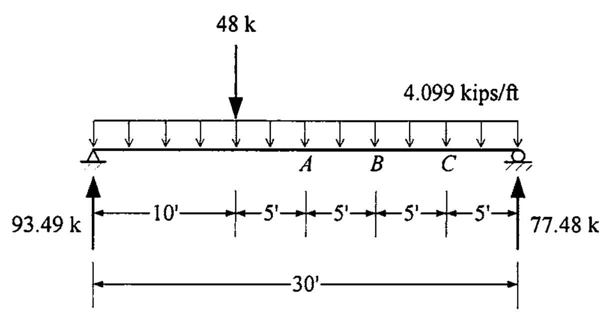
$$\phi_b M_n = 0.9(12650) = 11390 \text{ in.-kips} = 949 \text{ ft-kips}$$

Since $M_u = 1085$ ft-kips $> \phi_b M_n = 949$ ft-kips, W30 × 99 is not adequate.

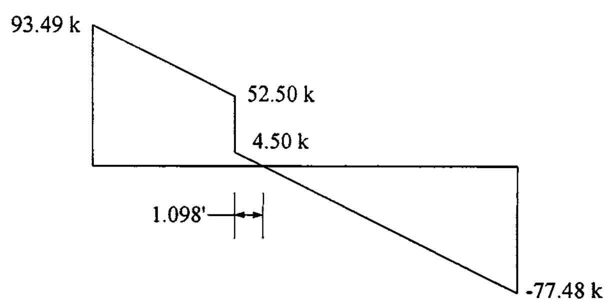
(b) ASD solution:

$$P_a = P_D + P_L = 16 + 32 = 48 \text{ kips}, \; w_a = w_D + w_L = (1 + 0.099) + 3$$

$$= 4.099 \text{ kips/ft}$$




(Note that points have been relabeled for use in the $C_b$ formula.)



From the LRFD solution, the shape is compact. Also from the LRFD solution,

$$L_p = 7.42 \text{ ft}, \ L_r = 21.3 \text{ ft}$$

For $L_b = 20$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(inelastic LTB)}$$

Compute $C_b$ :

$$M_{\max} = 93.49(11.098) - 4.099(11.098)^2/2 - 48(1.098) = 732.4 \text{ ft-kips}$$

$$M_A = 77.48(15) - 4.099(15)^2/2 = 701.1 \text{ ft-kips}$$

$$M_B = 77.48(10) - 4.099(10)^2/2 = 569.9 \text{ ft-kips}$$

$$M_C = 77.48(5) - 4.099(5)^2/2 = 336.2 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(732.4)}{2.5(732.4) + 3(701.1) + 4(569.9) + 3(336.2)} = 1.268$$

(Note that this is essentially the same as the value found in Part (a) with factored loads.)

$$M_n = C_b(9994) = 1.268(9994) = 12{,}670 \text{ in.-kips} < M_p = 15{,}600 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{12670}{1.67} = 7587 \text{ in.-kips} = 632 \text{ ft-kips}$$

Since $M_a = M_{\max} = 732.4$ ft-kips > 632 ft-kips,

W30 × 99 is not adequate.

---

**5.5-16**

This shape is compact. (There is no footnote in the dimensions and properties table to indicate otherwise.) First, determine the nominal flexural strength. From the $Z_x$ table,

$$L_p = 9.50 \text{ ft}, \qquad L_r = 34.3 \text{ ft}$$

Unbraced segment $BC$. For $L_b = 9$ ft, $L_b < L_p$ and

$$M_n = M_p = F_y Z_x = 50(262) = 13,100 \text{ in.-kips} = 1092 \text{ ft-kips}$$

Unbraced segment $CD$ : For $L_b = 18$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$= C_b\left[13100 - (13100 - 0.7(50)(231))\left(\frac{18 - 9.50}{34.3 - 9.50}\right)\right] = C_b(11,380) \text{ in.-kips}$$

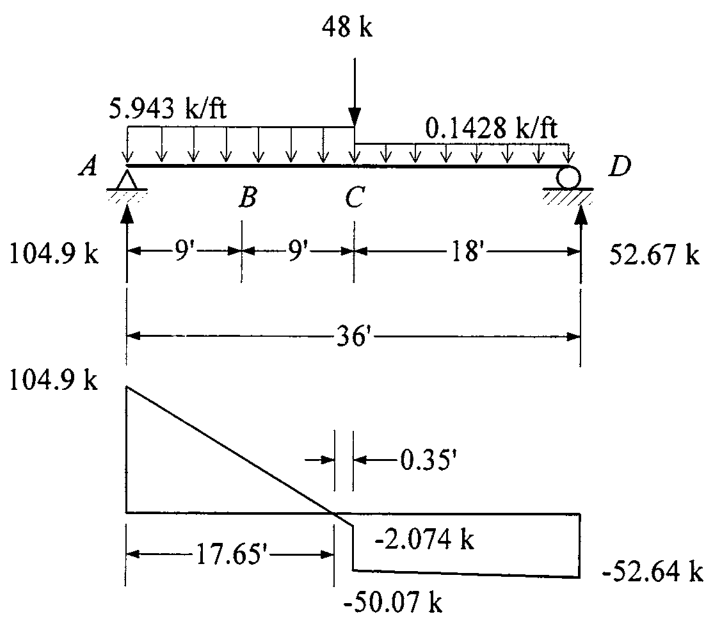
(a) LRFD Solution

For segment $ABC$, when beam weight is included, $w_D = 3.5 + 0.119 = 3.619$ kips/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(3.619) + 1.6(1.0) = 5.943 \text{ kips/ft}$$

For segment $CD$, $w_u = 1.2(0.119) = 0.1428$ kips/ft

$$P_u = 1.6P_L = 1.6(30) = 48 \text{ kips}$$




Check unbraced segment $BC$.

$$\text{Maximum moment} = \frac{1}{2}(104.9)(17.65) = 925.7 \text{ ft-kips}$$

$$\phi_b M_n = 0.9(1092) = 983 \text{ ft-kips} > 925.7 \text{ ft-kips} \qquad \text{(OK)}$$

Check unbraced segment $CD$.

Compute $C_b$. Dividing $CD$ into 4 equal segments of length $18/4 = 4.5$ ft and labeling the 3 interior points $a, b$, and $c$, we obtain

$$M_a = 52.67(13.5) - 0.1428(13.5)^2/2 = 698.0 \text{ ft-kips}$$

$$M_b = 52.67(9) - 0.1428(9)^2/2 = 468.2 \text{ ft-kips}$$

$$M_c = 52.67(4.5) - 0.1428(4.5)^2/2 = 235.6 \text{ ft-kips}$$

$$M_{\max} = 52.67(18) - 0.1428(18)^2/2 = 924.9 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(924.9)}{2.5(924.9) + 3(698.0) + 4(468.2) + 3(235.6)} = 1.655$$

$$M_n = C_b(11380) = 1.655(11380)$$

$$= 1.883 \times 10^4 \text{ in.-kips} > M_p = 13,100 \text{ in.-kips}$$

$$\therefore \ \phi_b M_n = 0.9(13100)/12 = 982.5 \text{ ft-kips} > 924.9 \text{ ft-kips} \qquad \text{(OK)}$$

W14 × 132 is adequate.

(b) ASD Solution

For segment $ABC$, when beam weight is included, $w_D = 3.5 + 0.119 = 3.619$ kips/ft

Check unbraced segment $BC$.

$$\frac{M_n}{\Omega_b} = \frac{1092}{1.67} = 654 \text{ ft-kips}$$

$$M_a = M_{\max} = \frac{1}{2}(64.39)(17.79) = 572.7 \text{ ft-kips} < 654 \text{ ft-kips} \qquad \text{(OK)}$$

Check unbraced segment $CD$.

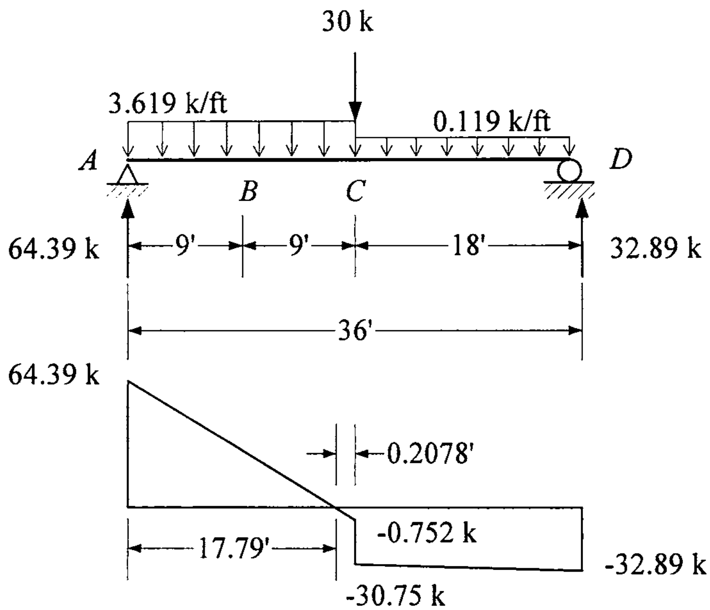
Compute $C_b$. Dividing $CD$ into 4 equal segments of length $18/4 = 4.5$ ft and labeling the 3 interior points $a, b$, and $c$, we obtain

$$M_a = 32.89(13.5) - 0.119(13.5)^2/2 = 433.2 \text{ ft-kips}$$

$$M_b = 32.89(9) - 0.119(9)^2/2 = 291.2 \text{ ft-kips}$$

$M_c = 32.89(4.5) - 0.119(4.5)^2/2 = 146.8$ ft-kips

$M_{max} = 32.89(18) - 0.119(18)^2/2 = 572.7$ ft-kips




$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(572.7)}{2.5(572.7) + 3(433.2) + 4(291.2) + 3(146.8)} = 1.651$$

$M_n = C_b(11380) = 1.651(11380)$

$= 1.879 \times 10^4$ in.-kips $> M_p = 13,100$ in.-kips

$\frac{M_n}{\Omega_b} = \frac{13100/12}{1.67} = 653.7$ ft-kips $> 572.7$ ft-kips (OK)

<u>W14 × 132 is adequate.</u>

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**<u>5.6-1</u>**

Check for compactness. For the flange,

$\lambda_p = 0.38\sqrt{\frac{29000}{60}} = 8.354 \qquad \lambda_r = 1.0\sqrt{\frac{29000}{60}} = 21.98$

$$\lambda = \frac{b_f}{2t_f} = 9.47, \qquad \lambda_p < \lambda < \lambda_r \ \therefore \text{ flange is noncompact}$$

(In the Dimensions and Properties tables, there is a footnote indicating that the flange of a W21 x 48 is noncompact for $F_y \geq 50$ ksi.) The web is compact for all shapes in Part 1 of the *Manual* for $F_y \leq 65$ ksi.

Compute the strength based on the limit state of flange local buckling.

$$M_p = F_y Z_x = 60(107) = 6420 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 6420 - (6420 - 0.7 \times 60 \times 93.0)\left(\frac{9.47 - 8.354}{21.98 - 8.354}\right) = 6214.1$$

$$= 6214 \text{ in.-kips} = 517.8 \text{ ft-kips}$$

(a) Available strength = design strength = $\phi_b M_n = 0.90(517.8) = 466.0$ ft-kips

Factored load moment:

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}w_u(50)^2 = 466.0 \qquad \Rightarrow \qquad w_u = 1.491 \text{ kips/ft}$$

$$1.2w_D + 1.6w_L = 1.2w_D + 1.6(3w_D) = 1.491 \qquad \Rightarrow \qquad w_D = 0.2485 \text{ kips/ft}$$

$$w = w_D + w_L = 0.2485 + 3(0.2485) = 0.994 \text{ kips/ft} \qquad \underline{w = 0.994 \text{ kips/ft}}$$

(b) Available strength = allowable strength = $\dfrac{M_n}{\Omega_b} = \dfrac{517.8}{1.67} = 310.1$ ft-kips

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}w_a(50)^2 = 310.1 \qquad \Rightarrow \qquad w_a = 0.992 \text{ kips/ft}$$

$$\underline{w = 0.992 \text{ kips/ft}}$$

---

**5.6-2**

Check for compactness. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29000}{60}} = 8.354 \qquad \lambda_r = 1.0\sqrt{\frac{29000}{60}} = 21.98$$

$$\lambda = \frac{b_f}{2t_f} = 10.2, \qquad \lambda_p < \lambda < \lambda_r \ \therefore \text{ flange is noncompact}$$

(In the Dimensions and Properties tables, there is a footnote indicating that the flange of a W14 × 90 is noncompact.) The web is compact for all shapes in Part 1 of the *Manual* for $F_y \leq 65$ ksi.

Compute the strength based on the limit state of flange local buckling.

$$M_p = F_y Z_x = 60(157) = 9420 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 9420 - (9420 - 0.7 \times 60 \times 143)\left(\frac{10.2 - 8.354}{21.98 - 8.354}\right)$$

$$= 8957 \text{ in.-kips} = 746.4 \text{ ft-kips}$$

Check for lateral-torsional buckling. From the $Z_x$ table, $L_p = 15.1$ ft.

Since $L_b = 10 \text{ ft} < L_p$, lateral-torsional buckling does not have to be investigated.

$\underline{M_n = 746 \text{ ft-kips}}$

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**5.6-3**

Verify that the shape is noncompact: For the flange, $\lambda = \dfrac{b_f}{2t_f} = \dfrac{18}{2(3/4)} = 12.0$

$$\lambda_p = 0.38\sqrt{\frac{29,000}{65}} = 8.026$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}}, \qquad k_c = \frac{4}{\sqrt{h/t_w}} \qquad (0.35 \leq k_c \leq 0.763)$$

$$k_c = \frac{4}{\sqrt{52/0.75}} = 0.4804, \qquad F_L = 0.7F_y$$

(see footnote b in AISC Table B4.1b)

$$\lambda_r = 0.95\sqrt{\frac{0.4804(29,000)}{0.7(65)}} = 16.62$$

$$\lambda_p < \lambda < \lambda_r \ \therefore \text{ flange is noncompact}$$

For the web, $\lambda = \dfrac{h}{t_w} = \dfrac{52}{3/4} = 69.3$

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{65}} = 79.42$$

$\lambda < \lambda_p$ $\therefore$ web is compact.

Compute properties of the cross section.

Flange area = $0.75(18) = 13.5$ in.$^2$,

Half-web area = $0.75(52/2) = 19.5$ in.$^2$

Taking moments about the PNA (located at mid-depth of the cross section),

$$\bar{y} = \frac{19.5(13) + 13.5(26.38)}{19.5 + 13.5} = 18.47 \text{ in.}$$

$$Z_x = \frac{A}{2} \cdot a = (19.5 + 13.5)(2 \times 18.47) = 1219 \text{ in.}^3$$

$$M_p = F_y Z_x = 65(1219) = 79{,}230 \text{ in.-kips}$$

$$I_x = \frac{1}{12}(0.75)(52)^3 + 2\left[\frac{1}{12}(18)(0.75)^3 + 13.5(26.38)^2\right] = 27{,}580 \text{ in.}^3$$

$$S_x = \frac{I_x}{c} = \frac{27{,}580}{52/2 + 3/4} = 1031 \text{ in.}^3$$

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 79{,}230 - (79{,}230 - 0.7 \times 65 \times 1031)\left(\frac{12.0 - 8.026}{16.62 - 8.026}\right)$$

$= 64{,}280$ in.-kips = 5360 ft-kips $\qquad$ $\underline{M_n = 5360 \text{ ft-kips}}$

---

**5.6-4**

Verify that the shape is noncompact: For the flange,

$$\lambda = \frac{b_f}{2t_f} = \frac{16}{2(1)} = 8.0, \qquad \lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$\lambda < \lambda_p$ $\therefore$ flange is compact.

For the web, $\lambda = \frac{h}{t_w} = \frac{40}{5/16} = 128$

$$\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.55, \qquad \lambda_r = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

$\lambda_p < \lambda < \lambda_r$ $\therefore$ web is noncompact.

Beams with compact flanges and noncompact webs are covered in AISC F4 (this case is not covered in the textbook).

Compression flange yielding:

$$M_n = R_{pc}M_{yc} = R_{pc}F_yS_{xc}$$

$$I_{yc} = \frac{1}{12}(1)(16)^3 = 341.3 \text{ in.}^4$$

$$I_y = 2(1)(16)^3/12 + 40(5/16)^3/12 = 682.8 \text{ in.}^2$$

$$\frac{I_{yc}}{I_y} = \frac{341.3}{682.8} = 0.4999 > 2.3, \qquad \frac{h_c}{t_w} = \frac{h}{t_w} = 128 > \lambda_p$$

$\therefore$ the plastification factor is

$$R_{pc} = \left[\frac{M_p}{M_{yc}} - \left(\frac{M_p}{M_{yc}} - 1\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \le \frac{M_p}{M_{yc}}$$

Other properties of the cross section:

$$I_x = \frac{1}{12}(5/16)(40)^3 + 2\left[\frac{1}{12}(16)(1)^3 + 16(20 + 1/2)^2\right] = 15120 \text{ in.}^3$$

$$S_x = \frac{I_x}{c} = \frac{15120}{20 + 1} = 720.0 \text{ in.}^3$$

$$M_{yc} = F_yS_{xc} = 50(720) = 3.6 \times 10^4 \text{ in.-kips}$$

Flange area = $1.0(16) = 16.0 \text{ in.}^2$ Half-web area = $0.5(5/16 \times 40) = 6.25 \text{ in.}^2$

Taking moments about the PNA (located at mid-depth of the cross section),

$$\bar{y} = \frac{16(20 + 1/2) + 6.25(10)}{16 + 6.25} = 17.55 \text{ in.}$$

$$Z_x = \frac{A}{2} \cdot a = (16 + 6.25)(2 \times 17.55) = 781.0 \text{ in.}^3$$

$$M_p = F_yZ_x = 50(781.0) = 39,050 \text{ in.-kips}$$

$$\leq 1.6F_yS_{xc} = 1.6(50)(720) = 57{,}600 \text{ in.-kips}$$

$$\frac{M_p}{M_{yc}} = \frac{39050}{36000} = 1.085$$

The plastification factor is

$$R_{pc} = \left[1.085 - (1.085 - 1)\left(\frac{128 - 90.55}{137.3 - 90.55}\right)\right] = 1.017 < 1.085$$

$$M_n = R_{pc}M_{yc} = 1.017(36000)/12 = 3051 \text{ ft-kips}$$

No other limit states apply to this beam. $\underline{M_n = 3050 \text{ ft-kips}}$

---

**5.8-1**

$$\frac{h}{t_w} = 25.9, \qquad 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29000}{65}} = 47.31$$

Since $\frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}}$, AISC Section G2.1(a) applies, and $C_v = 1.0$

$$V_n = 0.6F_yA_wC_v = 0.6(65)(24.5 \times 0.800)(1.0) = 764.4 \text{ kips}$$

$\underline{V_n = 764 \text{ kips}}$

---

**5.8-2**

For an M10 × 9 ($t_f = 0.206$ in.) of A242 steel, $F_y = 50$ ksi

$$\frac{h}{t_w} = 58.4, \qquad 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29000}{50}} = 53.95$$

Since $\frac{h}{t_w} > 2.24\sqrt{\frac{E}{F_y}}$, AISC Section G2.1(b) applies.

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5(29000)}{50}} = 59.24$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5(29000)}{50}} = 73.78$$

$\frac{h}{t_w} < 59.24, \therefore C_v = 1.0$

$V_n = 0.6F_yA_wC_v = 0.6(50)(10.0 \times 0.157)(1.0) = 47.1$ kips

$\underline{V_n = 47.1 \text{ kips}}$

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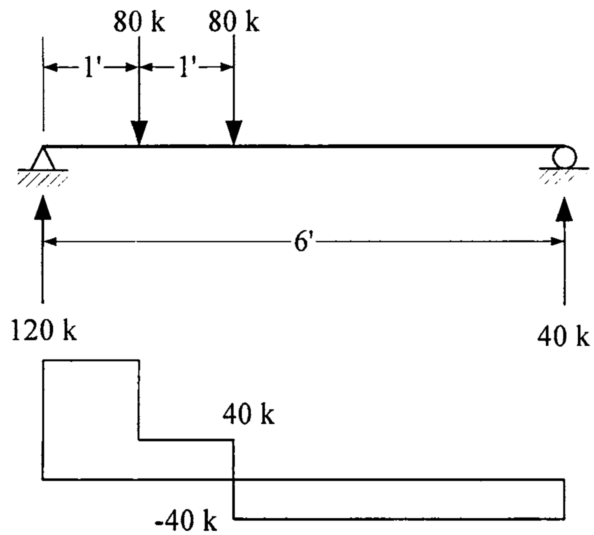
**5.8-3**

(a) $P_u = 1.6P_L = 1.6(50) = 80.0$ kips




Left Reaction = 80(4/6) + 80(5/6) = 120.0 kips

$M_u = 120(1) + 40(1) = 160$ ft-kip

From the $Z_x$ table, $\phi_b M_n = \phi_b M_p = 203$ ft-kips > 160 ft-kips (OK)

Check shear:

$$\frac{h}{t_w} = 51.6 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29{,}000}{50}} = 53.95$$

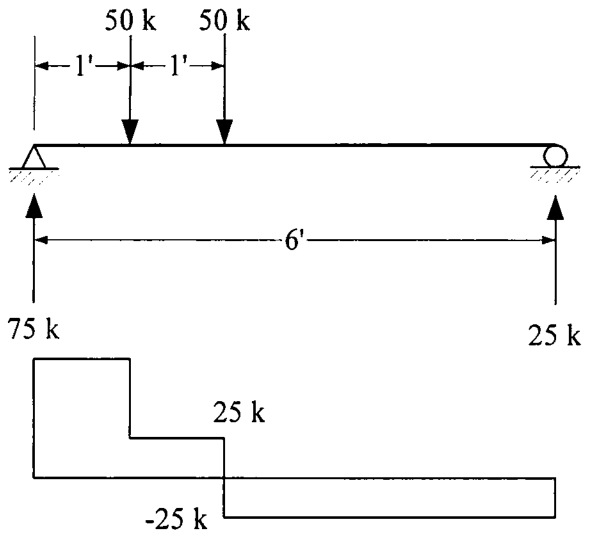
$\therefore V_n = 0.6F_yA_w = 0.6(50)(15.9 \times 0.275) = 131.2$ kips

$\phi_v V_n = 1.0(131.2) = 131$ ksi

$V_u = 120$ kips < 131 kips (OK) <u>Beam is adequate</u>

($\phi_v V_n$ can also be found in the $Z_x$ table.)

(b) $P_a = 50$ kips




$M_a = 75(2) - 50(1) = 100$ ft-kip

From the $Z_x$ table, $M_n/\Omega_b = M_p/\Omega_b = 135$ ft-kips > 100 ft-kips (OK)

Check shear:

$$\frac{h}{t_w} = 51.6 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29{,}000}{50}} = 53.95$$

$$\therefore V_n = 0.6F_yA_w = 0.6(50)(15.9 \times 0.275) = 131.2 \text{ kips}$$

$$\frac{V_n}{\Omega_b} = \frac{131.2}{1.50} = 87.5 \text{ ksi}$$

$V_a = 75$ kips < 87.5 kips (OK) <u>Beam is adequate</u>

($V_n/\Omega_b$ can also be found in the $Z_x$ table.)

---

**5.8-4**

Nominal shear strength: $\frac{h}{t_w} = 14.8 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29{,}000}{50}} = 53.95$

$$\therefore V_n = 0.6F_yA_w = 0.6(50)(10.6 \times 0.530) = 168.5 \text{ kips}$$

$\phi_v = 1.00, \ \Omega_v = 1.50$

Nominal flexural strength: The unbraced length is $L_b = 10$ ft. From the $Z_x$ table, a W10 × 77 is compact, $L_p = 9.18$ ft, and $L_r = 45.3$ ft.

Since $L_p < L_b < L_r$, lateral-torsional buckling must be checked.

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

For cantilevers, use $C_b = 1.0$ (AISC F1).

$$M_p = F_yZ_x = 50(97.6) = 4880 \text{ in.-kips}$$

$$M_n = 1.0\left[4880 - (4880 - 0.7 \times 50 \times 85.9)\left(\frac{10 - 9.18}{45.3 - 9.18}\right)\right]$$

$$= 4837 \text{ in.-kips} = 403.1 \text{ ft-kips}$$

(a) $P_u = 1.6P_L = 1.6(90) = 144.0$ kips,

$$w_u = 1.2w_D = 1.2(0.150) = 0.18 \text{ kips/ft}$$

$$\phi_v V_n = 1.00(168.5) = 168.5 \text{ kips}$$

$$V_u = 144 + 0.18(10) = 145.8 \text{ kips} < 168.5 \text{ kips} \qquad \text{(OK)}$$

$$\phi_b M_n = 0.90(403.1) = 363 \text{ ft-kips}$$

$$M_u = 144(1) + 0.15(10)(10/2) = 152 \text{ ft-kip} < 363 \text{ ft-kips} \qquad \text{(OK)}$$

<u>W10 × 77 is adequate.</u>

(b) $\dfrac{V_n}{\Omega_v} = \dfrac{168.5}{1.50} = 112$ kips

$$V_a = 90 + 0.150(10) = 91.5 \text{ kips} < 112 \text{ kips} \qquad \text{(OK)}$$

$$\frac{M_n}{\Omega_b} = \frac{403.1}{1.67} = 241 \text{ ft-kips}$$

$$M_a = 90(1) + 0.150(10)(10/2) = 97.5 \text{ ft-kip} < 241 \text{ ft-kips} \qquad \text{(OK)}$$

<u>W10 × 77 is adequate.</u>

---

**5.10-1**

(a) Neglect beam weight and check it later.

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) = 1.2 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.6(35) = 56.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.2)(25)^2 + \frac{56.0(25)}{4} = 444 \text{ ft-kips}$$

From the $Z_x$ table, Try a W21 × 55. $\phi_b M_n = 473$ ft-kips > 444 ft-kips (OK)

Check beam weight. $w_D = 1.0 + 0.055 = 1.055$ kips/ft

$$w_u = 1.2(1.055) = 1.266 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.266)(25)^2 + \frac{56.0(25)}{4}$$

$$= 449 \text{ ft-kips} < 473 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table,

$$\phi_v V_n = 234 \text{ kips}$$

$$V_u = \frac{P_u + w_u L}{2} = \frac{56.0 + 1.266(25)}{2} = 43.8 \text{ kips} < 234 \text{ kips} \quad \text{(OK)}$$

Use a W21 × 55

(b) Neglect beam weight and check it later.

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(1.0)(25)^2 + \frac{35(25)}{4} = 297 \text{ ft-kips}$$

From the $Z_x$ table, Try a W21 × 55. $\frac{M_n}{\Omega_b} = 314$ ft-kips > 297 ft-kips (OK)

Check beam weight. $w_D = 1.0 + 0.055 = 1.055$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(1.055)(25)^2 + \frac{35.0(25)}{4} = 301$$

$$= 301 \text{ ft-kips} < 314 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table,

$$\frac{V_n}{\Omega_v} = 156 \text{ kips}$$

$$V_a = \frac{P_a + w_a L}{2} = \frac{35 + 1.055(25)}{2} = 30.7 \text{ kips} < 156 \text{ kips} \qquad \text{(OK)}$$

Use a W21 × 55

---

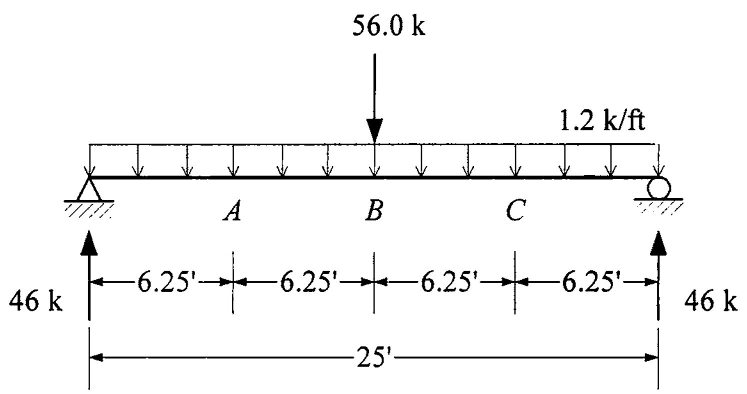
**5.10-2**

(a) Neglect beam weight and check it later.

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) = 1.2 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.6(35) = 56.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.2)(25)^2 + \frac{56.0(25)}{4} = 443.8 \text{ ft-kips}$$




Compute $C_b$ : $\qquad M_{\max} = M_B = 443.8$ ft-kips

$$M_A = M_C = 46(6.25) - 1.2(6.25)^2/2 = 264.1 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(443.8)}{2.5(443.8) + 3(264.1) + 4(443.8) + 3(264.1)} = 1.241$$

Enter the beam design charts with $L_b = 25$ ft and $\frac{M_u}{C_b} = \frac{443.8}{1.241} = 358$ ft-kips

Try a W18 × 76

$\phi_b M_n = 409$ ft-kips for $C_b = 1.0$. For $C_b = 1.241$,

$\phi_b M_n = 1.241(409) = 508$ ft-kips > 443.8 ft-kips (OK)

$\phi_b M_p = 611$ ft-kips > 508 ft-kips (OK)

Check beam weight. $w_D = 1.0 + 0.076 = 1.076$ kips/ft

$w_u = 1.2(1.076) = 1.291$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.291)(25)^2 + \frac{56(25)}{4}$$

$$= 451 \text{ ft-kips} < 508 \text{ ft-kips} \quad \text{(OK)}$$

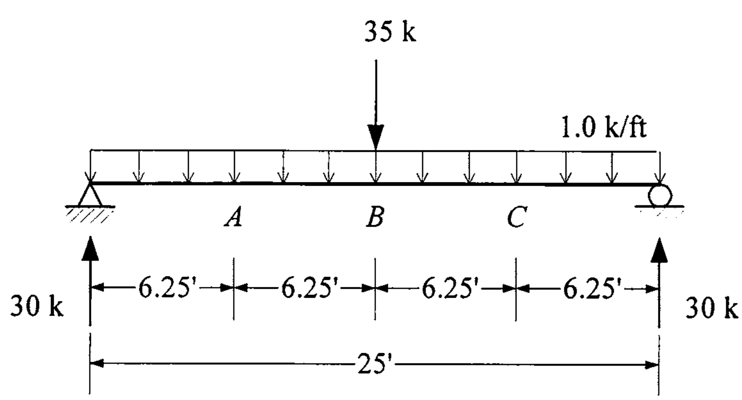
The inclusion of the beam weight changes the total load only slightly, and $C_b$ does not need to be recomputed.

Check shear: From the $Z_x$ table,

$\phi_v V_n = 232$ kips > $V_u \approx 46$ kips (OK) Use a W18 × 76

(b) Neglect beam weight and check it later.

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(1.0)(25)^2 + \frac{35(25)}{4} = 296.9 \text{ ft-kips}$$




Compute $C_b$ : $M_{\max} = M_B = 296.9$ ft-kips

$M_A = M_C = 30(6.25) - 1.0(6.25)^2/2 = 168.0$ ft-kips

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(296.9)}{2.5(296.9) + 3(168.0) + 4(296.9) + 3(168.0)} = 1.263$$

Enter the beam design charts with $L_b = 25$ ft and $\frac{M_a}{C_b} = \frac{296.9}{1.263} = 235$ ft-kips

Try a W18 × 76

$$\frac{M_n}{\Omega_b} = 408 \text{ ft-kips for } C_b = 1.0. \text{ For } C_b = 1.263,$$

$$\frac{M_n}{\Omega_b} = 1.263(408) = 515 \text{ ft-kips} > 296.9 \text{ ft-kips} \quad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 407 \text{ ft-kips} < 515 \text{ ft-kips} \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 407 \text{ ft-kips}$$

Check beam weight. $w_D = 1.0 + 0.076 = 1.076$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(1.076)(25)^2 + \frac{35(25)}{4}$$

$$= 303 \text{ ft-kips} < 296.9 \text{ ft-kips} \quad \text{(OK)}$$

The inclusion of the beam weight changes the total load only slightly, and $C_b$ does not need to be recomputed.

Check shear: From the $Z_x$ table,

$$\frac{V_n}{\Omega_v} = 155 \text{ kips} > V_a \approx 30 \text{ kips} \quad \text{(OK)}$$

Use a W18 × 76

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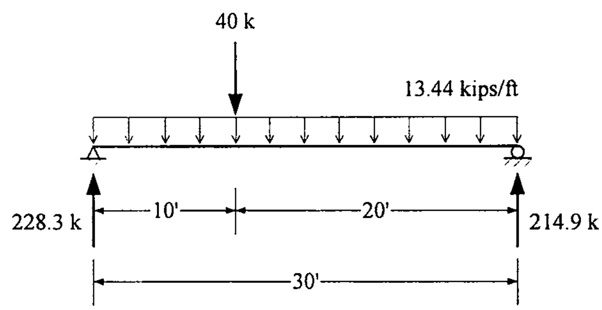
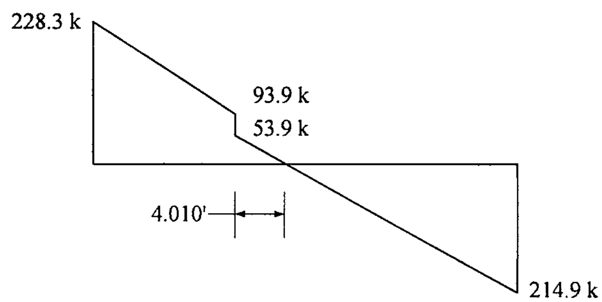
**5.10-3**

(a) Assume a beam weight of 200 lb/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(7 + 0.2) + 1.6(3) = 13.44 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(20) + 1.6(10) = 40.0 \text{ kips}$$




$$M_u = 214.9(20 - 4.010) - 13.44(20 - 4.010)^2/2 = 1718 \text{ ft-kips}$$

From the $Z_x$ table, Try a W33 × 130, $\phi_b M_n = \phi_b M_p = 1750$ ft-kips > 1713 ft-kips (OK)

Beam weight = 130 lb/ft < assumed value of 200 lb/ft (OK)

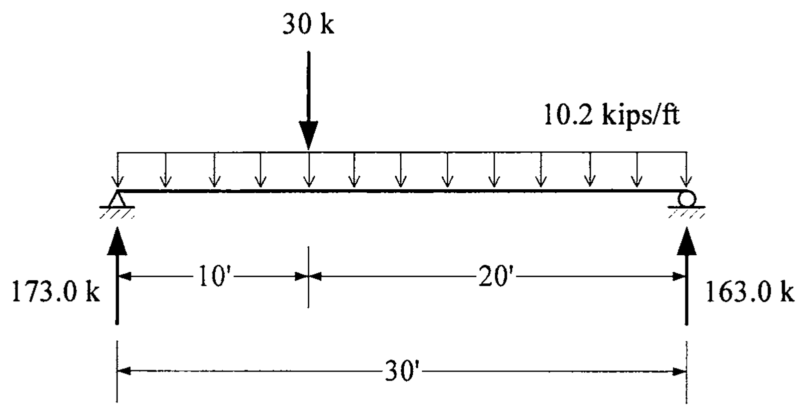
Check shear: From $Z_x$ table,

$\phi_v V_n = 576$ kips > $V_u \approx 228.3$ kips (OK) <u>Use a W33 × 130</u>

(b) Assume a beam weight of 200 lb/ft.

$$P_a = P_D + P_L = 10 + 20 = 30 \text{ kips}$$

$$w_a = w_D + w_L = (3.33 + 0.200) + 6.67 = 10.2 \text{ kips/ft}$$




Take moments about right end to determine left reaction, $R_L$ :

$$\sum M_{right} = R_L(30) - 30(20) - 10.2(30)(30/2) = 0, \qquad R_L = 173.0 \text{ kips}$$

Shear is zero when $173.0 - 30 - 10.2x = 0, \qquad x = 14.02$ ft

Maximum moment is $M_a = 173(14.02) - 30(14.02 - 10) - 10.2(14.02)^2/2 = 1302$ ft-kips

Try W40 × 149, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 1490$ ft-kips > 1302 ft-kips (OK)

Beam weight = 149 lb/ft < assumed value of 200 lb/ft (OK)

Check shear: From $Z_x$ table,

$\dfrac{V_n}{\Omega_v} = 432$ kips > 173 kips (OK) Use a W40 × 149

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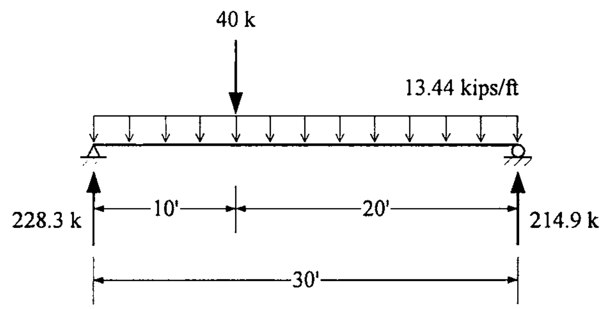
**5.10-4**

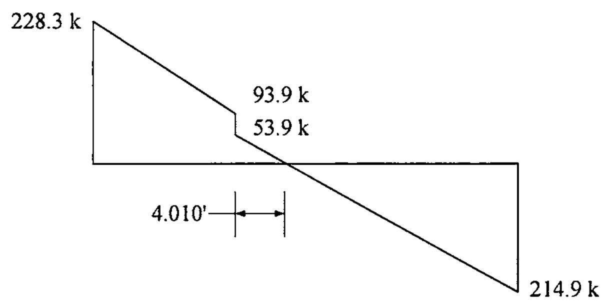
(a) Assume a beam weight of 200 lb/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(7 + 0.2) + 1.6(3) = 13.44 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(20) + 1.6(10) = 40.0 \text{ kips}$$

The 20-ft unbraced length will control. Compute $C_b$ :




$$M_{\max} = 214.9(20 - 4.010) - 13.44(20 - 4.010)^2/2 = 1718 \text{ ft-kips}$$

Divide the 20-ft segment into 4 equal spaces of 5 ft each. This places $A, B,$ and $C$ at distances of 15, 10, and 5 feet from the right end.

$$M_A = 214.9(15) - 13.44(15)^2/2 = 1712 \text{ ft-kips}$$

$$M_B = 214.9(10) - 13.44(10)^2/2 = 1477 \text{ ft-kips}$$

$$M_A = 214.9(5) - 13.44(5)^2/2 = 906.5 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1718)}{2.5(1718) + 3(1712) + 4(1477) + 3(906.5)} = 1.189$$

Enter the beam design charts with $L_b = 20$ ft and $\frac{M_u}{C_b} = \frac{1718}{1.189} = 1445$ ft-kips

Try a W27 × 146

$\phi_b M_n = 1482$ ft-kips for $C_b = 1.0$. For $C_b = 1.189$,

$\phi_b M_n = 1.189(1482) = 1762 \text{ ft-kips} > 1718 \text{ ft-kips}$ (OK)

$\phi_b M_p = 1740$ ft-kips < 1762 ft-kips $\quad \therefore$ use $\phi_b M_n = \phi_b M_p = 1740$ ft-kips

Beam weight = 146 lb/ft < assumed value of 200 lb/ft (OK)

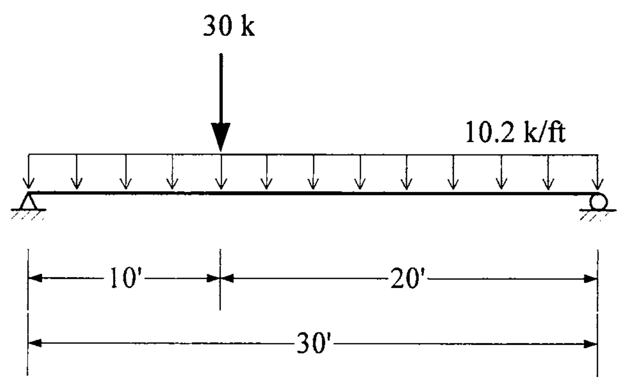
Check shear: From $Z_x$ table,

$\phi_v V_n = 497$ kips > 228.3 kips (OK) <u>Use a W27 × 146</u>

(b) Assume a beam weight of 200 lb/ft.

$$P_a = P_D + P_L = 20 + 10 = 30 \text{ kips}$$

$$w_a = w_D + w_L = (7 + 0.2) + 3 = 10.2 \text{ kips/ft}$$




Take moments about right end to determine left reaction, $R_L$ :

$$\sum M_{right} = R_L(30) - 30(20) - 10.2(30)(30/2) = 0, \qquad R_L = 173.0 \text{ kips}$$

Shear is zero when $173.0 - 30 - 10.2x = 0, \qquad x = 14.02$ ft

Maximum moment is $M_a = 173(14.02) - 30(14.02 - 10) - 10.2(14.02)^2/2 = 1302$ ft-kips

The 20-ft unbraced length will control. Compute $C_b$ :

Divide the 20-foot unbraced length into four equal segments of length $20/4 = 5$ ft. This places $A, B,$ and $C$ at distances of 15, 20, and 25 feet from the left end.

$$M_A = 173(15) - 10.2(15)^2/2 - 30(5) = 1298 \text{ ft-kips}$$

$$M_B = 173(20) - 10.2(20)^2/2 - 30(10) = 1120 \text{ ft-kips}$$

$$M_C = 173(25) - 10.2(25)^2/2 - 30(15) = 687.5 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1302)}{2.5(1302) + 3(1298) + 4(1120) + 3(687.5)} = 1.189$$

Enter the beam design charts with $L_b = 20$ ft and $\frac{M_a}{C_b} = \frac{1302}{1.189} = 1095$ ft-kips

Try W36 × 160

$\frac{M_n}{\Omega_b} = 1156$ ft-kips for $C_b = 1.0$. For $C_b = 1.189$,

$\frac{M_n}{\Omega_b} = 1.189(1156) = 1374$ ft-kips > 1302 ft-kips (OK)

$\frac{M_p}{\Omega_b} = 1560$ ft-kips > 1374 ft-kips

Beam weight = 160 lb/ft < assumed value of 200 lb/ft (OK)

Check shear: From the $Z_x$ table,

$\frac{V_n}{\Omega_v} = 468$ kips > 173 kips (OK) Use a W36 × 160

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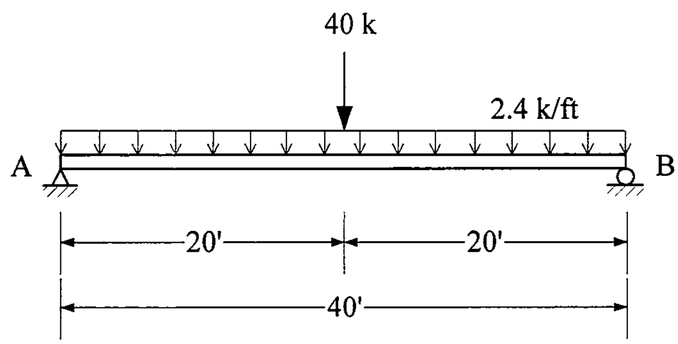
**5.10-5**

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(2) = 2.4$ kips/ft (neglect beam wt. and check it later.)

$P_u = 1.2P_D + 1.6P_L = 1.6(25) = 40.0$ kips




Left reaction $= \frac{40 + 2.4(40)}{2} = 68.0$ kips

For $L_b$ = 40 ft, Compute $C_b$ : Divide the 40-foot unbraced length into four equal segments of length 10 ft. This places $A, B,$ and $C$ at distances of 10, 20, and 30 feet from the left end.

$$M_A = M_C = 68(10) - 2.4(10)^2/2 = 560.0 \text{ ft-kips}$$

$$M_B = M_{\max} = 68(20) - 2.4(20)^2/2 = 880.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(880)}{2.5(880) + 3(560) + 4(880) + 3(560)} = 1.211$$

Enter the beam design charts with $L_b$ = 40 ft and $\frac{M_u}{C_b} = \frac{880}{1.211} = 726.7$ ft-kips

From Manual Table 3-10, p. 3-119, Try a W18 × 143, with $L_r$ = 39.6 ft.

Since $L_b$ = 40 ft > $L_r$,

$$M_n = F_{cr}S_x \le M_p \qquad \text{(elastic LTB)}$$

where

$$F_{cr} = \frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.211\pi^2(29000)}{\left(\frac{40 \times 12}{3.17}\right)^2}\sqrt{1 + 0.078\frac{19.2(1.0)}{282(18.2)}\left(\frac{40 \times 12}{3.17}\right)^2} = 41.92 \text{ ksi}$$

So $M_n = F_{cr}S_x = 41.92(282) = 11{,}820$ in.-kips = 985 ft-kips

$$\phi_bM_n = 0.9(985) = 887 \text{ ft-kips} > 880 \text{ ft-kips} \qquad \text{(OK)}$$

$$\phi_bM_p = 1210 \text{ ft-kips} > 887 \text{ ft-kips} \qquad \text{(OK)}$$

Check beam weight:

$$M_u = 880 + \frac{1}{8}(1.2 \times 0.143)(40)^2 = 914 \text{ ft-kips} > 887 \text{ ft-kips} \qquad \text{(N.G.)}$$

Try a W24 × 146, with $L_r$ = 33.7 ft. < $L_b$ = 40 ft.

$$F_{cr} = \frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.211\pi^2(29000)}{\left(\frac{40 \times 12}{3.53}\right)^2}\sqrt{1 + 0.078\frac{13.4(1.0)}{371(23.6)}\left(\frac{40 \times 12}{3.53}\right)^2} = 33.57 \text{ ksi}$$

So $M_n = F_{cr}S_x = 33.57(371) = 12,450$ in.-kips = 1038 ft-kips

$\phi_b M_n = 0.9(1038) = 934$ ft-kips > 880 ft-kips (OK)

$\phi_b M_p = 1570$ ft-kips > 934 ft-kips (OK)

Check beam weight:

$$M_u = 880 + \frac{1}{8}(1.2 \times 0.146)(40)^2 = 915 \text{ ft-kips} < 1570 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: $V_u = \frac{40}{2} + \frac{1.2(2.4 + 0.146)(40)}{2} = 81.1$ kips

From the $Z_x$ table, $\phi_v V_n = 482$ kips > 81.1 kips (OK)

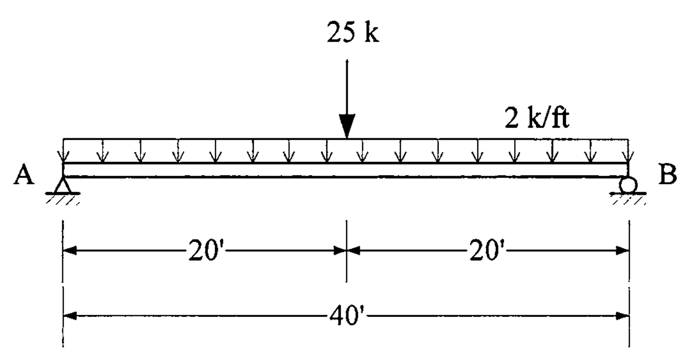
Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{40(12)}{360} = 1.33$ in.

$$\Delta_L = \frac{P_L L^3}{48EI} = \frac{25(40 \times 12)^3}{48(29000)(4580)} = 0.434 \text{ in.} < 1.33 \text{ in.} \quad \text{(OK)}$$

Use a W24 × 146

(b) $w_a = w_D + w_L = 2$ kips/ft (neglect beam wt. and check it later.)

$P_a = P_D + P_L = 25$ kips




Left reaction $= \frac{25 + 2(40)}{2} = 52.5$ kips

For $L_b = 40$ ft, Compute $C_b$ : Divide the 40-foot unbraced length into four equal segments of length 10 ft. This places $A, B,$ and $C$ at distances of 10, 20, and 30 feet from the left end.

$$M_A = M_C = 52.5(10) - 2(10)^2/2 = 425.0 \text{ ft-kips}$$

$$M_B = M_{\max} = 52.5(20) - 2(20)^2/2 = 650.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(650)}{2.5(650) + 3(425) + 4(650) + 3(425)} = 1.199$$

Enter the beam design charts with $L_b = 40$ ft and $\dfrac{M_a}{C_b} = \dfrac{650}{1.199} = 542.1$ ft-kips

From Manual Table 3-10, p. 3-119, Try a W24 × 146, with

$$L_r = 33.7 \text{ ft.} < L_b = 40 \text{ ft.}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.199\pi^2(29000)}{\left(\frac{40 \times 12}{3.53}\right)^2} \sqrt{1 + 0.078 \frac{13.4(1.0)}{371(23.6)} \left(\frac{40 \times 12}{3.53}\right)^2} = 33.24 \text{ ksi}$$

So $M_n = F_{cr} S_x = 33.24(371)12{,}330$ in.-kips = 1028 ft-kips

$$\frac{M_n}{\Omega_b} = \frac{1028}{1.67} = 616 \text{ ft-kips} < 650 \text{ ft-kips} \qquad \text{(N.G.)}$$

Try a W27 × 161, with $L_r = 34.7$ ft. $< L_b = 40$ ft.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.199\pi^2(29000)}{\left(\frac{40 \times 12}{3.79}\right)^2} \sqrt{1 + 0.078 \frac{15.1(1.0)}{458(26.5)} \left(\frac{40 \times 12}{3.79}\right)^2} = 34.21 \text{ ksi}$$

So $M_n = F_{cr} S_x = 34.21(458) = 15{,}670$ in.-kips = 1306 ft-kips

$$\frac{M_n}{\Omega_b} = \frac{1306}{1.67} = 782 \text{ ft-kips} > 650 \text{ ft-kips} \qquad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 1280 \text{ ft-kips} > 782 \text{ ft-kips}$$

Check beam weight:

$$M_a = 650 + \frac{1}{8}(0.161)(40)^2 = 682 \text{ ft-kips} < 782 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table,

$$\frac{V_n}{\Omega_v} = 364 \text{ kips} > 52.5 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \dfrac{L}{360} = \dfrac{40(12)}{360} = 1.33$ in.

$$\Delta_L = \frac{P_L L^3}{48EI} = \frac{25(40 \times 12)^3}{48(29000)(6310)} = 0.315 \text{ in.} < 1.33 \text{ in.} \qquad \text{(OK)}$$

<u>Use a W27 × 161</u>

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**<u>5.10-6</u>**

(a) $P_u = 1.2P_D + 1.6P_L = 1.6(18) = 28.8$ kips (neglect beam wt. and check it later.)

Left Reaction $= 28.8/2 = 14.4$ kips

Segment 2-3 is critical. Divide this 10-foot unbraced length into four equal segments of length $10/4 = 2.5$ ft. This places $A, B,$ and $C$ at distances of 12.5, 15, and 17.5 feet from the left end.

$$M_A = M_C = 14.4(12.5) = 180.0 \text{ ft-kips}$$

$$M_B = M_{\max} = 14.4(15) = 216.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(216)}{2.5(216) + 3(180) + 4(216) + 3(180)} = 1.087$$

Enter the beam design charts with $L_b = 10$ ft and $\dfrac{M_u}{C_b} = \dfrac{216}{1.087} = 199$ ft-kips

Try a W16 × 40

$\phi_b M_n = 229$ ft-kips for $C_b = 1.0$. For $C_b = 1.087$,

$\phi_b M_n = 1.087(229) = 249$ ft-kips > 216 ft-kips (OK)

$\phi_b M_p = 274$ ft-kips > 249 ft-kips (OK)

Check beam weight:

$$M_u = 216 + \frac{1}{8}(1.2 \times 0.040)(30)^2 = 221 \text{ ft-kips} < 249 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: $V_u = \dfrac{28.8}{2} + \dfrac{1.2(0.040)(30)}{2} = 15.1$ kips

From the $Z_x$ table, $\phi_v V_n = 146$ kips > 15.1 kips (OK)

Check deflection:

$$\Delta_{\max} = \frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

$$\Delta_L = \frac{PL^3}{48EI} + \frac{5wL^2}{384EI} = \frac{18(30 \times 12)^3}{48(29000)(518)} + \frac{5(0.040)(30 \times 12)^2}{384(29000)(518)}$$

$$= 1.17 \text{ in.} < 1.5 \text{ in.} \quad \text{(OK)}$$

<u>Use a W16 × 40</u>

(b) $P_a = P_D + P_L = 18$ kips (neglect beam wt. and check it later.)

Left reaction $= \dfrac{18}{2} = 9$ kips

Segment 2-3 is critical. Divide this 10-foot unbraced length into four equal segments of length 10/4 = 2.5 ft. This places $A, B,$ and $C$ at distances of 12.5, 15, and 17.5 feet from the left end.

$$M_A = M_C = 9(12.5) = 112.5 \text{ ft-kips}$$

$$M_B = M_{\max} = 9(15) = 135.0 \text{ ft-kips}$$

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135.0)}{2.5(135.0) + 3(112.5) + 4(135.0) + 3(112.5)} = 1.087$$

Enter the beam design charts with $L_b = 10$ ft and $\frac{M_a}{C_b} = \frac{135.0}{1.087} = 124$ ft-kips

Try a W16 × 36

$$\frac{M_n}{\Omega_b} = 131 \text{ ft-kips for } C_b = 1.0. \text{ For } C_b = 1.087,$$

$$\frac{M_n}{\Omega_b} = 1.087(131) = 142 \text{ ft-kips} > 135 \text{ ft-kips} \quad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 160 \text{ ft-kips} > 142 \text{ ft-kips}$$

Check beam weight:

$$M_a = 135.0 + \frac{1}{8}(0.036)(30)^2 = 139 \text{ ft-kips} < 142 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table,

$$\frac{V_n}{\Omega_v} = 93.8 \text{ kips} > 9 \text{ kips} \quad \text{(OK)}$$

Check deflection:

$$\Delta_{\max} = \frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

$$\Delta_L = \frac{PL^3}{48EI} + \frac{5wL^2}{384EI} = \frac{18(30 \times 12)^3}{48(29000)(448)} + \frac{5(0.036)(30 \times 12)^2}{384(29000)(448)}$$

$$= 1.35 \text{ in.} < 1.5 \text{ in.} \quad \text{(OK)}$$

Use a W16 × 36

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**5.10-7**

(a) $P_u = 1.2P_D + 1.6P_L = 1.6(20) = 32.0$ kips (neglect beam wt. and check it later.)

$$\text{Left reaction} = 32\left(\frac{18}{28}\right) + 32\left(\frac{23}{28}\right) = 46.86 \text{ kips}$$

The maximum moment occurs at the second load from the left:
$M_u = 46.86(10) - 32(5) = 308.6$ ft-kips

Compute $C_b$ :

Divide the 28-foot unbraced length into four equal segments of length $28/4 = 7.0$ ft. This places $A, B,$ and $C$ at distances of 7, 14, and 21 feet from the left end.

$$M_A = 46.86(7) - 32(7-5) = 264.0 \text{ ft-kips}$$

$$M_B = 46.86(14) - 32(14-5) - 32(14-10) = 240.0 \text{ ft-kips}$$

$$M_A = 46.86(21) - 32(21-5) - 32(21-10) = 120.1 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(308.6)}{2.5(308.6) + 3(264.0) + 4(240.0) + 3(120.1)} = 1.338$$

Enter the beam design charts with $L_b = 28$ ft and $\frac{M_u}{C_b} = \frac{308.6}{1.338} = 230.6$ ft-kips

Try a W14 × 61

$\phi_b M_n = 236$ ft-kips for $C_b = 1.0$. For $C_b = 1.338$,

$\phi_b M_n = 1.338(236) = 316$ ft-kips > 308.6 ft-kips (OK)

$\phi_b M_p = 383$ ft-kips > 316 ft-kips (OK)

Check beam weight. $w_u = 1.2(0.061) = 0.0732$ kips/ft

Conservatively, we can use $M_u = 308.6 + 0.0732(28)^2/8 = 315.8$ ft-kips $= \phi_b M_n$ (OK)

Check shear: From the $Z_x$ table, $\phi_v V_n = 156$ kips $> R_L = 46.86$ kips (OK)

<u>Use a W14 × 61</u>

(b) Neglect beam weight and check it later.

$$\text{Left reaction} = 20\left(\frac{18}{28}\right) + 20\left(\frac{23}{28}\right) = 29.29 \text{ kips}$$

The maximum moment occurs at the second load from the left:

$$M_a = 29.29(10) - 20(5) = 193 \text{ ft-kips}$$

Compute $C_b$ :

Divide the 28-foot unbraced length into four equal segments of length $28/4 = 7.0$ ft. This places $A, B,$ and $C$ at distances of 7, 14, and 21 feet from the left end.

$$M_A = 29.29(7) - 20(7-5) = 165.0 \text{ ft-kips}$$

$M_B = 29.29(14) - 20(14 - 5) - 20(14 - 10) = 150.1$ ft-kips

$M_C = 29.29(21) - 20(21 - 5) - 20(21 - 10) = 75.09$ ft-kips

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(193)}{2.5(193) + 3(165.0) + 4(150.1) + 3(75.09)} = 1.338$$

Enter the beam design charts with $L_b = 28$ ft and $\dfrac{M_a}{C_b} = \dfrac{193}{1.338} = 144.2$ ft-kips

Try a W14 × 61

$\dfrac{M_n}{\Omega_b} = 157$ ft-kips for $C_b = 1.0$. For $C_b = 1.338$,

$\dfrac{M_n}{\Omega_b} = 1.338(157) = 210$ ft-kips > 193 ft-kips (OK)

$\dfrac{M_p}{\Omega_b} = 254$ ft-kips > 210 ft-kips (OK)

Check beam weight. Conservatively, we can use

$M_a = 193 + 0.061(28)^2/8 = 199$ ft-kips < 210 ft-kips (OK)

Check shear: From the $Z_x$ table, $\dfrac{V_n}{\Omega_v} = 104$ kips > 29.29 kips (OK)

<u>Use a W14 × 61</u>

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**<u>5.10-8</u>**

(a) Assume a beam weight of 40 lb/ft: $w_D = 180 + 40 = 220$ lb/ft

Load combination 3 controls: $w_u = 1.2D + 1.6S = 1.2(220) + 1.6(275) = 704.0$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(0.704)(26)^2 = 59.49 \text{ ft-kips}$$

For lateral support at the ends and at midspan, $C_b = 1.30$ (Fig. 5.15, textbook)

Enter the beam design charts with $L_b = 26/2 = 13$ ft and

$\dfrac{M_u}{C_b} = \dfrac{59.49}{1.30} = 45.76$ ft-kips:

Try a W8 × 21 :

For $C_b = 1.0$, $\phi_b M_n = 52.5$ ft-kips. For $C_b = 1.30$,

$\phi_b M_n = 1.30(52.5) = 68.25$ ft-kips > 59.49 ft-kips (OK)

$\phi_b M_p = 76.5$ ft-kips > 68.25 ft-kips

Beam weight = 21 lb/ft < assumed value of 40 lb/ft (OK)

Check shear: From the $Z_x$ table, $\phi_v V_n = 62.1$ kips

$$V_u = \frac{w_u L}{2} \approx \frac{0.704(26)}{2} = 9.15 \text{ kips} < 62.1 \text{ kips} \quad \text{(OK)}$$

Check deflection: $\Delta_{\text{max}} = \frac{L}{180} = \frac{25(12)}{180} = 1.67$ in.

$$\Delta = \frac{5(w_D + w_S)L^4}{384EI} = \frac{5[(0.180 + 0.021 + 0.275)/12](26 \times 12)^4}{384(29000)(75.3)}$$

$$= 2.24 \text{ in.} > 1.67 \text{ in.} \quad \text{(N.G.)}$$

$$\text{Required } I_x = \frac{5(w_D + w_S)L^4}{384E\Delta_{\text{max}}} = \frac{5[(0.180 + 0.021 + 0.275)/12](26 \times 12)^4}{384(29000)(1.67)}$$

$$= 101.1 \text{ in.}^4$$

From $I_x$ table and beam design charts, try a W10 × 22:

$\phi_b M_n = 64.5$ ft-kips (for $C_b = 1$) > 59.49 ft-kips (OK)

$I_x = 118$ in.$^4$ > 101.1 in.$^4$ (OK)

$\phi_v V_n = 73.4$ kips > 9.15 kips (OK) Use a W10 × 22

(b) Assume a beam weight of 40 lb/ft: $w_D = 180 + 40 = 220$ lb/ft

Load combination 3 controls: $w_a = D + S = 220 + 275 = 495$ lb/ft

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(0.495)(26)^2 = 41.83 \text{ ft-kips}$$

For lateral support at the ends and at midspan, $C_b = 1.30$ (Fig. 5.15, textbook)

Enter the beam design charts with $L_b = 26/2 = 13$ ft and $\frac{M_a}{C_b} = \frac{41.83}{1.30} = 32.18$ ft-kips:

To avoid deflection issues (see LRFD solution), Try a W10 × 22:

For $C_b = 1.0$, $M_n/\Omega_b = 43$ ft-kips > 41.83 ft-kips. The strength will be higher for $C_b = 1.30$ and need not be investigated.

Beam weight = 22 lb/ft < assumed value of 40 lb/ft (OK)

Check shear: From the $Z_x$ table, $V_n/\Omega_v = 49.0$ kips

$$V_a = \frac{w_a L}{2} \approx \frac{0.495(26)}{2} = 6.44 \text{ kips} < 48.8 \text{ kips} \quad \text{(OK)}$$

Check deflection: $\Delta_{max} = \frac{L}{180} = \frac{25(12)}{180} = 1.67$ in.

$$\Delta = \frac{5(w_D + w_S)L^4}{384EI} = \frac{5[(0.180 + 0.022 + 0.275)/12](26 \times 12)^4}{384(29000)(118)}$$

$$= 1.43 \text{ in.} < 1.67 \text{ in.} \quad \text{(OK)}$$

Use a W10 × 22

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**5.11-1**

Dead loads: Slab weight $= \frac{5}{12}(150) = 62.5$ psf

$$w_D = (62.5 + 10)(6) = 435.0 \text{ lb/ft}$$ (neglect beam wt. and check it later.)

Live load: $w_L = (65 + 20)(6) = 510$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(435) + 1.6(510) = 1338$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.338)(30)^2 = 150.5 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W16 × 26, $\phi_b M_n = \phi_b M_p = 166$ ft-kips > 150.5 ft-kips (OK)

Check beam weight:

$$M_u = 150.5 + \frac{1}{8}(1.2 \times 0.026)(30)^2 = 154 \text{ ft-kips} < 166 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 106$ kips

$$V_u \approx \frac{1.290(30)}{2} = 19.4 \text{ kips} < 106 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{180} = \frac{30(12)}{180} = 2.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.510/12)(30 \times 12)^4}{384(29000)(301)} = 1.07 \text{ in.} < 2.0 \text{ in.} \quad \text{(OK)}$$

<u>Use a W16 × 26</u>

(b) $w_a = w_D + w_L = 435 + 510 = 945$ kips/ft

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(0.945)(30)^2 = 106 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W16 × 26, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 110$ ft-kips > 106 ft-kips (OK)

Check beam weight:

$$M_a = 106 + \frac{1}{8}(0.026)(30)^2 = 109 \text{ ft-kips} < 110 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 70.5$ kips

$$V_a \approx \frac{0.945(30)}{2} = 14.2 \text{ kips} < 70.5 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{180} = \frac{30(12)}{180} = 2.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.510/12)(30 \times 12)^4}{384(29000)(301)} = 1.07 \text{ in.} < 2.0 \text{ in.} \quad \text{(OK)}$$

<u>Use a W16 × 26</u>

---

**<u>5.11-2</u>**

Dead loads: Slab weight $= \frac{5}{12}(150) = 62.5$ psf

$w_D = (62.5 + 5)(5.5) = 371.3$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (100 + 20)(5.5) = 660$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(371.3) + 1.6(660) = 1502$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.502)(30)^2 = 169.0 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W14 ×30, $\phi_b M_n = \phi_b M_p = 177$ ft-kips > 169 ft-kips (OK)

Check beam weight:

$$M_u = 169 + \frac{1}{8}(1.2 \times 0.030)(30)^2 = 173 \text{ ft-kips} < 177 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 112$ kips

$$V_u \approx \frac{1.502(30)}{2} = 22.5 \text{ kips} < 112 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.660/12)(30 \times 12)^4}{384(29000)(291)} = 1.43 \text{ in.} > 1.0 \text{ in.} \quad \text{(N.G.)}$$

$$\text{Required } I = \frac{5w_L L^4}{384E\Delta_{\text{max}}} = \frac{5(0.660/12)(30 \times 12)^4}{384(29000)(1.0)} = 415 \text{ in.}^4$$

From the $I_x$ table, try a W18 × 35, $I_x = 510 \text{ in.}^4 > 415 \text{ in.}^4$ (OK)

$\phi_b M_n = 249$ ft-kips > 173 ft-kips (weight is OK)

$\phi_v V_n = 159$ kips > 22.5 kips (OK) <u>Use a W18 × 35</u>

(b) $w_a = w_D + w_L = 371.3 + 660.0 = 1031$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.031)(30)^2 = 116 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W16 × 31, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 135$ ft-kips > 116 ft-kips (OK)

Check beam weight:

$$M_a = 116 + \frac{1}{8}(0.031)(30)^2 = 120 \text{ ft-kips} < 135 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 87.5$ kips

$$V_a \approx \frac{1.031(30)}{2} = 15.5 \text{ kips} < 87.5 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.660/12)(30 \times 12)^4}{384(29000)(375)} = 1.11 \text{ in.} > 1.0 \text{ in.} \qquad \text{(N.G.)}$$

$$\text{Required } I = \frac{5w_L L^4}{384E\Delta_{\max}} = \frac{5(0.660/12)(30 \times 12)^4}{384(29000)(1.0)} = 415 \text{ in.}^4$$

From the $I_x$ table, try a W18 × 35, $I_x = 510 \text{ in.}^4 > 415 \text{ in.}^4$ (OK)

$$\frac{M_n}{\Omega_b} = 166 \text{ ft-kips} > 120 \text{ ft-kips} \qquad \text{(weight is OK)}$$

$$\frac{V_n}{\Omega_n} = 106 \text{ kips} > 15.5 \text{ kips} \qquad \text{(OK)}$$

<u>Use a W18 × 35</u>

---

**<u>5.11-3</u>**

Dead loads: Slab and deck weight = 51 psf

$$w_D = (51 + 10)(12) = 732 \text{ lb/ft} \quad \text{(neglect beam wt. and check it later.)}$$

Live load: $w_L = (80 + 20)(12) = 1200$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(732) + 1.6(1200) = 2798$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.798)(25)^2 = 219 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W18 × 35, $\phi_b M_n = \phi_b M_p = 249 \text{ ft-kips} > 219 \text{ ft-kips}$ (OK)

Check beam weight:

$$M_u = 219 + \frac{1}{8}(1.2 \times 0.035)(25)^2 = 222.3 \text{ ft-kips} < 249 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 159$ kips

$$V_u \approx \frac{2.798(25)}{2} = 35.0 \text{ kips} < 159 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{25(12)}{360} = 0.833$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(1.200/12)(25 \times 12)^4}{384(29000)(510)} = 0.713 \text{ in.} < 0.833 \text{ in.} \quad \text{(OK)}$$

Use a W18 × 35

(b) $w_a = w_D + w_L = 732 + 1200 = 1932$ kips/ft

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.932)(25)^2 = 151 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W18 × 35, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 166$ ft-kips > 151 ft-kips (OK)

Check beam weight:

$$M_a = 150 + \frac{1}{8}(0.035)(25)^2 = 153 \text{ ft-kips} < 166 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 106$ kips

$$V_a \approx \frac{1.932(25)}{2} = 24.2 \text{ kips} < 106 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{25(12)}{360} = 0.833$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(1.200/12)(25 \times 12)^4}{384(29000)(510)} = 0.713 \text{ in.} < 0.833 \text{ in.} \quad \text{(OK)}$$

Use a W18 × 35

---

**5.11-4**

Dead loads: Slab and deck weight = 43 psf

$w_D = (43 + 5 + 2)(10) = 500$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (160 + 20)(10) = 1800$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(500) + 1.6(1800) = 3480$ lb/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(3.480)(20)^2 = 174 \text{ ft-kips}$$

Because of the large live load, compute $I_x$ required for deflection control.

$$\text{Maximum } \Delta_L = \frac{L}{360} = \frac{20(12)}{360} = 0.667 \text{ in.}$$

$$\text{Required } I_x = \frac{5w_LL^4}{384E\Delta_{\text{max}}} = \frac{5(1.800/12)(20 \times 12)^4}{384(29000)(0.667)} = 335 \text{ in.}^4$$

From the $Z_x$ table,

Try a W16 × 31, $\phi_bM_n = \phi_bM_p = 203$ ft-kips > 174 ft-kips (OK)

$I_x = 375 \text{ in.}^4 > 335 \text{ in.}^2$ (OK)

Check beam weight:

$$M_u = 174 + \frac{1}{8}(1.2 \times 0.031)(20)^2 = 176 \text{ ft-kips} < 203 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_vV_n = 131$ kips

$$V_u \approx \frac{3.480(20)}{2} = 34.8 \text{ kips} < 131 \text{ kips} \quad \text{(OK)}$$

Use a W16 × 31

(b) $w_a = w_D + w_L = 500 + 1800 = 2300$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(2.300)(20)^2 = 115 \text{ ft-kips}$$

Because of the large live load, compute $I_x$ required for deflection control.

$$\text{Maximum } \Delta_L = \frac{L}{360} = \frac{20(12)}{360} = 0.667 \text{ in.}$$

$$\text{Required } I_x = \frac{5w_LL^4}{384E\Delta_{\text{max}}} = \frac{5(1.800/12)(20 \times 12)^4}{384(29000)(0.667)} = 335 \text{ in.}^4$$

From the $Z_x$ table,

Try a W16 × 31, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 135$ ft-kips > 115 ft-kips (OK)

$I_x = 375 \text{ in.}^4 > 335 \text{ in.}^2$ (OK)

Check beam weight:

$$M_a = 115 + \frac{1}{8}(0.031)(20)^2 = 117 \text{ ft-kips} < 135 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 87.5$ kips

$$V_a \approx \frac{2.300(20)}{2} = 23.0 \text{ kips} < 87.5 \text{ kips} \qquad \text{(OK)}$$

Use a W16 × 31

---

## 5.11-5

Dead loads: Slab weight $= \frac{4.5}{12}(150) = 56.25$ psf

$w_D = 56.25(5) = 281.3$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (95)(5) = 475$ lb/ft

Compute $I_x$ required for deflection control. (This applies to both parts a and b.)

$$\text{Maximum } \Delta_{Total} = \frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

$$\text{Required } I_x = \frac{5w_L L^4}{384 E \Delta_{\max}} = \frac{5[(0.2813 + 0.475)/12](30 \times 12)^4}{384(29000)(1.5)} = 317 \text{ in.}^4$$

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(281.3) + 1.6(475) = 1098$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.098)(30)^2 = 124 \text{ ft-kips}$$

From the $Z_x$ table,

Try a W16 × 31, $\phi_b M_n = \phi_b M_p = 203$ ft-kips $> 124$ ft-kips (OK)

$I_x = 375$ in.$^4 > 317$ in.$^4$ (OK)

Check beam weight:

$$M_u = 124 + \frac{1}{8}(1.2 \times 0.031)(30)^2 = 128 \text{ ft-kips} < 203 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 131$ kips

$$V_u \approx \frac{1.098(30)}{2} = 16.5 \text{ kips} < 131 \text{ kips} \qquad \text{(OK)}$$

Use a W16 × 31

(b) $w_a = w_D + w_L = 281.3 + 475 = 756.3$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.7563)(30)^2 = 85.1 \text{ ft-kips}$$

Check the $Z_x$ table for $M_n/\Omega_b > 85.1$ ft-kips and $I_x > 317$ in.$^4$

Try a W16 × 31, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 135$ ft-kips > 85.1 ft-kips (OK)

$I_x = 375 \text{ in.}^4 > 317 \text{ in.}^4$ (OK)

Check moment for beam weight:

$M_a = 85.1 + \frac{1}{8}(0.031)(30)^2 = 88.6$ ft-kips < 135 ft-kips (OK)

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 87.5$ kips

$V_a \approx \frac{0.7563(30)}{2} = 11.3$ kips < 87.5 kips (OK)

Use a W16 × 31

---

**5.11-6**

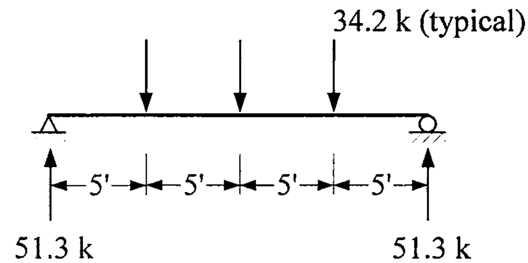
Slab weight = $\frac{4.5}{12}(150) = 56.25$ psf. For beams, use 35 lb/ft

Loads on beams:

$w_D = 56.25(5) + 35 = 316.3$ lb/ft, $w_L = (95)(5) = 475$ lb/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(316.3) + 1.6(475) = 1140$ lb/ft

Beam reaction = $\frac{1.140(30)}{2} \times 2 = 34.2$ kips




Girder reaction = $\frac{34.2(3)}{2} = 51.3$ kips

$M_u = 51.3(10) - 34.2(5) = 342$ ft-kips

Try a W21 × 44, $\phi_b M_n = 358$ ft-kips > 342 ft-kips (OK)

Check weight:

$M_u = 342 + \frac{1}{8}(1.2 \times 0.044)(20)^2 = 345$ ft-kips < 358 ft-kips (OK)

Check shear: From the $Z_x$ table, $\phi_v V_n = 217$ kips

$V_u = 51.3 + \frac{1.2(0.044)(20)}{2} = 51.8$ kips < 217 kips (OK)

Use a W21 × 44

---

**5.11-7**

Slab weight = $\frac{4.5}{12}(150) = 56.25$ psf. For beams, use 35 lb/ft × 30 ft = 1050 lb/beam

Expressed as a distributed load, the weight of the beams is $\frac{1050 \times 3}{20} = 157.5$ lb/ft

$w_D = 56.25(30) + 157.5 = 1845$ lb/ft, $w_L = (95)(30) = 2850$ lb/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(1.845) + 1.6(2.850) = 6.774$ kips/ft

$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(6.774)(20)^2 = 339$ ft-kips

Try a W21 × 44, $\phi_b M_n = 358$ ft-kips > 339 ft-kips (OK)

Check weight:

$$M_u = 339 + \frac{1}{8}(1.2 \times 0.044)(20)^2 = 342 \text{ ft-kips} < 358 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 217$ kips

$$V_u \approx \frac{6.774(20)}{2} = 67.7 \text{ kips} < 217 \text{ kips} \qquad \text{(OK)}$$

Use a W21 × 44

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**5.12-1**

(a) For a W16 × 40, $b_f = 7.00$ in., $t_f = 0.505$ in., and $S_x = 64.7$ in.$^3$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.505(7.00) = 3.535 \text{ in.}^2$$

The effective hole diameter is

$$d_h = \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \sum d_h = 3.535 - 0.505(2 \times 7/8) = 2.651 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.651) = 172.3 \text{ kips}$$

Determine $Y_t$. For A992 steel, the maximum $F_y/F_u$ ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(3.535) = 194.4 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{172.3}{3.535}(64.7) = 3154 \text{ in.-kips} = 262.8 \text{ ft-kips.}$$

$M_n = 263$ ft-kips

(b) Unreduced strength $= M_p = F_y Z_x = 50(73.0) = 3650$ in.-kips (compact shape)

$$\text{Change} = \frac{3154 - 3650}{3650} \times 100 = -13.6\%$$

Reduction = 13.6%

**5.12-2**

(a) For a W14 × 90, $b_f = 14.5$ in., $t_f = 0.710$ in., and $S_x = 143$ in.$^3$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.710(14.5) = 10.30 \text{ in.}^2$$

The effective hole diameter is

$$d_h = \frac{7}{8} + \frac{1}{8} = 1 \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \sum d_h = 10.30 - 0.710(2 \times 1) = 8.88 \text{ in.}^2$$

$$F_u A_{fn} = 65(8.88) = 577.2 \text{ kips}$$

Determine $Y_t$. For A992 steel, the maximum $F_y/F_u$ ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(10.30) = 566.5 \text{ kips}$$

Since $F_u A_{fn} > Y_t F_y A_{fg}$ , the holes need not be accounted for.

From the $Z_x$ table,

$$M_n = \frac{\phi_b M_n}{\phi_b} = \frac{574}{0.90} = 638 \text{ ft-kips}$$

Note that this is a noncompact shape, and 638 ft-kips is the nominal strength based on FLB.

$\underline{M_n = 638 \text{ ft-kips}}$

(b) Reduction = 0%

**5.12-3**

(a) For a W21 × 55, $b_f = 8.22$ in., $t_f = 0.522$ in., and $S_x = 110$ in.$^3$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$A_{fg} = t_f b_f = 0.522(8.22) = 4.291 \text{ in.}^2$

The effective hole diameter is

$$d_h = \frac{7}{8} + \frac{1}{8} = 1 \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \sum d_h = 4.291 - 0.522(2 \times 1) = 3.247 \text{ in.}^2$$

$$F_u A_{fn} = 65(3.247) = 211.1 \text{ kips}$$

Determine $Y_t$. For A992 steel, the maximum $F_y/F_u$ ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(4.291) = 236.0 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{211.1}{4.291}(110) = 5412 \text{ in.-kips} = 451 \text{ ft-kips.}$$ $\underline{M_n = 451 \text{ ft-kips}}$

(b) Unreduced strength $= M_p = F_y Z_x = 50(126) = 6300$ in.-kips (compact shape)

$$\text{Change} = \frac{5412 - 6300}{6300} \times 100 = -14.1\%$$ $\underline{\text{Reduction} = 14.1\%}$

---

**5.13-1**

Dead loads: Slab weight $= \frac{4}{12}(150) = 50.0$ psf

Assuming a joist weight of 3 psf, $w_D = (50 + 3 + 5)(3) = 174$ lb/ft

$$w_L = 80(3) = 240 \text{ lb/ft}$$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(174) + 1.6(240) = 593 \text{ lb/ft}$$

From Figure 5.35, for a span length of 25 ft, a 16K7 has a capacity of 642 lb/ft >593 lb/ft, and the approximate weight is $8.6/3 = 2.87$ psf < 3 psf estimated.

Use a 16K7

---

**5.13-2**

Assuming a joist weight = 4 psf, $w_D = (4 + 30 + 5)(4) = 156$ lb/ft

$$w_L = (50 + 20)(4) = 280 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(156) + 1.6(280) = 635 \text{ lb/ft}$$

From Figure 5.35, for a span length of 22 ft, a 16K5 has a capacity of 687 lb/ft > 635 lb/ft, and the approximate weight is $7.5/4 = 1.88$ psf < 4 psf estimated.

To limit the live-load deflection to $L/360$, the service live load must not exceed 323 lb/ft.

$$w_L = 280 \text{ lb/ft} < 323 \text{ lb/ft} \qquad \text{(OK)}$$

Use a 16K5

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**5.14-1**

(a) Factored load = $R_u = 1.6(150) = 240$ kips

Determine the length of bearing $\ell_b$ required to prevent web yielding. From AISC Equation J10-2, the design strength for this limit state is

$$\phi R_n = \phi F_y t_w (5k + \ell_b)$$

For $\phi R_n \geq R_u$,

$$1.0(50)(0.375)[5(1.24) + \ell_b] \geq 240, \text{ Solution is: } \{6.6 \leq \ell_b\} \text{ in.}$$

Use AISC Equation J10-4 to determine the value of $\ell_b$ required to prevent web crippling. For $\phi R_n \geq R_u$,

$$\phi(0.80)t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u$$

$$0.75(0.80)(0.375)^2\left[1 + 3\left(\frac{\ell_b}{13.9}\right)\left(\frac{0.375}{0.645}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.645)}{0.375}} \geq 240$$

Solution is: $\{8.373 \leq \ell_b\}$ in. Use a bearing length of 8½ in.

(b) Service load = $R_a$ = 150 kips

Determine the length of bearing $\ell_b$ required to prevent web yielding. From AISC Equation J10-2, the allowable strength for this limit state is

$$\frac{R_n}{\Omega} = \frac{(5k + \ell_b)F_y t_w}{\Omega}$$

For $\frac{R_n}{\Omega} \geq R_a$

$$\frac{[5(1.24) + \ell_b](50)(0.375)}{1.50} \geq 150, \text{ Solution is: } \{5.8 \leq \ell_b\} \text{ in.}$$

Use AISC Equation J10-4 to determine the value of $\ell_b$ required to prevent web crippling. For

$$\frac{R_n}{\Omega} \geq R_a$$

$$\frac{0.80 t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}}{\Omega} \geq R_a$$

$$\frac{0.80(0.375)^2\left[1 + 3\left(\frac{\ell_b}{13.9}\right)\left(\frac{0.375}{0.645}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.645)}{0.375}}}{2.00} \geq 150$$

Solution is: $\{7.197 \leq \ell_b\}$ in. Use a bearing length of 8 in.

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**5.14-2**

(a) The factored load reaction is

$$R_u = 1.2D + 1.6L = 1.2(20) + 1.6(50) = 104.0 \text{ kips}$$

Determine the length of bearing $\ell_b$ required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is $R_n = F_y t_w(2.5k + \ell_b)$

For $\phi R_n \geq R_u$,

$$1.0(50)(0.490)[2.5(1.34) + \ell_b] \geq 104.0, \text{ Solution is: } \{0.8949 \leq \ell_b\} \text{ in.}$$

Use AISC Equation J10-5 to determine the value of $\ell_b$ required to prevent web crippling. Assume that $\ell_b/d > 0.2$ and try the second form of the equation, J10-5b. For $\phi R_n \geq R_u$,

$$\phi(0.40)t_w^2\left[1+\left(\frac{4\ell_b}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.490)^2\left[1+\left(\frac{4\ell_b}{26.9}-0.2\right)\left(\frac{0.490}{0.745}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.745)}{0.490}} \geq 104.0,$$

Solution is: $\{0.9973 \leq \ell_b\}$ in.

Check the assumption:

$$\frac{\ell_b}{d} = \frac{0.9973}{26.9} = 3.707\times10^{-2} < 0.2 \qquad \text{(N.G.)}$$

For $\ell_b/d < 0.2$, AISC Equation J10-5a applies:

$$\phi(0.40)t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.490)^2\left[1+3\left(\frac{\ell_b}{26.9}\right)\left(\frac{0.490}{0.745}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.745)}{0.490}} \geq 104.0,$$

Solution is: $\{-0.4636 \leq \ell_b\}$

$\frac{\ell_b}{d} < 0.2$ as assumed.

Try $\ell_b = 6$ in. Determine $B$.

Since the support area will be larger than the bearing plate,

$$P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f_c'A_1 \qquad \text{(AISC Eq. J8-2)}$$

As an initial trial, assume conservatively that the full area of the support is in contact, and

$$P_p = 0.85f_c'A_1 \qquad \text{(AISC Eq. J8-1)}$$

Let $\phi_cP_p > R_u$ : $\quad 0.65(0.85)(3)A_1 \geq 104.0$ in.$^2$ $\quad\Rightarrow\quad A_1 \geq 62.75$ in.$^2$

Then $B \geq 62.75/6 = 10.46$ in. Use 11 in.

Try a $6\times11$ plate. Support width is then $8\times13$.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(13)}{6(11)}} = 1.255$$

Let $\phi_cP_p > R_u$ : $\quad \phi_c\left(0.85f_c'A_1\sqrt{A_2/A_1}\right) > R_u$

$0.65(0.85)(3)A_1(1.255) \geq 104.0$, Solution is: $\{50.00 \leq A_1\}$ in.$^2$

For a 6 x 11 plate, $A_1 = 66 \text{ in.}^2 > 50 \text{ in.}^2$ (OK)

Check upper limit of AISC Equation J8-2:

$$1.7f_c'A_1 = 1.7(3)(66) = 337 \text{ kips}$$

$$0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} = 0.85(3)(66)(1.255) = 211 \text{ kips} < 337 \text{ kips} \qquad \text{(OK)}$$

Plate thickness: $n = \dfrac{B-2k}{2} = \dfrac{11-2(1.34)}{2} = 4.16 \text{ in.}$

$$t \geq \sqrt{\frac{2.22R_u n^2}{B\ell_b F_y}} = \sqrt{\frac{2.22(104.0)(4.16)^2}{11(6)(36)}} = 1.30 \text{ in., use } 1\tfrac{5}{16} \text{ in.}$$

Use a PL $1\frac{5}{16} \times 6 \times 0' - 11"$

(b) The service load reaction is

$$R_a = D + L = 20 + 50 = 70 \text{ kips}$$

Determine the length of bearing $\ell_b$ required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is $R_n = F_y t_w(2.5k + \ell_b)$

For $\dfrac{R_n}{\Omega} \geq R_a$

$$\frac{1}{1.50}(50)(0.490)[2.5(1.34) + \ell_b] \geq 70, \text{ Solution is: } \{0.9357 \leq \ell_b\} \text{ in.}$$

Use AISC Equation J10-5 to determine the value of $\ell_b$ required to prevent web crippling. Assume that $\ell_b/d < 0.2$ and try the first form of the equation, J10-5a. For $\dfrac{R_n}{\Omega} \geq R_a$,

$$\frac{1}{\Omega}(0.40t_w^2)\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_a$$

$$\frac{1}{2.00}[0.40(0.490)^2]\left[1 + 3\left(\frac{\ell_b}{26.9}\right)\left(\frac{0.490}{0.745}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.745)}{0.490}} \geq 70,$$

Solution is: $\{-0.3064 \leq \ell_b\}$

$\ell_b < 0.2$ as assumed.

Try $\ell_b = 6$ in. Determine $B$.

Since the support area will be larger than the bearing plate,

$$P_p = 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f_c' A_1 \qquad \text{(AISC Eq. J8-2)}$$

As an initial trial, assume conservatively that the full area of the support is in contact, and

$$P_p = 0.85 f_c' A_1 \qquad \text{(AISC Eq. J8-1)}$$

Let $\frac{P_p}{\Omega} > R_a$ : $\qquad \frac{0.85(3)A_1}{2.31} \geq 70 \qquad \Rightarrow \qquad A_1 \geq 63.41 \text{ in.}^2$

Then $B \geq 63.41/6 = 10.57$ in.

Since flange width is 10.0 in., try a $6 \times 11$ plate. Support width is then $8 \times 13$.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(13)}{6(11)}} = 1.255$$

Let $\frac{P_p}{\Omega} > R_a$ : $\qquad \frac{1}{\Omega}\left(0.85 f_c' A_1 \sqrt{A_2/A_1}\right) > R_a$

$$\frac{0.85(3)A_1(1.255)}{2.31} \geq 70, \text{ Solution is: } \{50.53 \leq A_1\}$$

$$B = 50.53/6 = 8.422 \text{ in.}$$

Since flange width is 10.0 in., try a $6 \times 11$ plate. Support width is then $8 \times 13$.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(13)}{6(11)}} = 1.255$$

$$\frac{0.85(3)A_1(1.255)}{2.31} \geq 70, \text{ Solution is: } \{50.53 \leq A_1\} \text{ in.}^2$$

For a $6 \times 11$ plate, $A_1 = 66 \text{ in.}^2 > 50.53 \text{ in.}^2$ (OK)

Check upper limit of AISC Equation J8-2:

$$1.7 f_c' A_1 = 1.7(3)(72) = 367 \text{ kips}$$

$$0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} = 0.85(3)(66)(1.255) = 211.2 \text{ kips} < 367 \text{ kips} \qquad \text{(OK)}$$

Plate thickness: $\qquad n = \frac{B - 2k}{2} = \frac{11 - 2(1.34)}{2} = 4.16$ in.

$$t \geq \sqrt{\frac{3.34 R_a n^2}{B \ell_b F_y}} = \sqrt{\frac{3.34(70)(4.16)^2}{11(6)(36)}} = 1.305 \text{ in., use } 1\tfrac{5}{16} \text{ in.}$$

<u>Use a PL $1\frac{5}{16} \times 6 \times 0' - 11''$</u>

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**5.14-3**

(a) The factored load is $P_u = 1.2D + 1.6L = 1.2(75) + 1.6(185) = 386.0$ kips

Compute the required bearing area. From AISC Equation J8-2,

$$P_p = 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f_c' A_1$$

For $\phi_c P_p \geq P_u$,

$$0.65\left[0.85(3.5)A_1\sqrt{\frac{16(16)}{A_1}}\right] \geq 386, \text{ Solution is: } \{155.6 \leq A_1\} \text{ in.}^2$$

Check upper limit:

$$\phi_c 1.7 f_c' A_1 = 0.65(1.7)(3.5)(155.6) = 601.8 \text{ kips} > 386 \text{ kips} \qquad \text{(OK)}$$

The plate must be as large as the column: $b_f d = 12.3(13.1) = 161.1 \text{ in.}^2 > 155.6 \text{ in.}^2$

Try $B = N = 14$ in., with $A_1$ provided $= 14(14) = 196 \text{ in.}^2 > 161.1 \text{ in.}^2$

$$m = \frac{N - 0.95d}{2} = \frac{14 - 0.95(13.1)}{2} = 0.7775 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{14 - 0.8(12.3)}{2} = 2.08 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{13.1(12.3)} = 3.173 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(0.7775, 2.08, 3.173) = 3.173 \text{ in.}$$

$$\text{Required } t = \ell\sqrt{\frac{2P_u}{0.90 F_y BN}} = 3.173\sqrt{\frac{2(386)}{0.90(36)(14)(14)}} = 1.106 \text{ in.; use } 1\tfrac{1}{8} \text{ in.}$$

<u>Use a PL $1\frac{1}{8} \times 14 \times 14$</u>

(b) The applied load is $P_a = D + L = 75 + 185 = 260$ kips.

Compute the required bearing area. Using AISC Equation J8-2,

$$\frac{P_p}{\Omega_c} \geq P_a$$

$$\frac{1}{2.31}\left[0.85(3.5)A_1\sqrt{\frac{16(16)}{A_1}}\right] \geq 260, \text{ Solution is: } \{159.2 \leq A_1\} \text{ in.}^2$$

The upper limit is

$$\frac{1}{\Omega_c}(1.7f_c{'}A_1) = \frac{1}{2.31}[(1.7)(3.5)(159.2)] = 410.1 \text{ kips} > 260 \text{ kips} \qquad \text{(OK)}$$

The plate must be as large as the column:

$$b_f d = 12.3(13.1) = 161.1 \text{ in.}^2 > 159.2 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try $B = N = 14$ in., with $A_1$ provided $= 14(14) = 196$ in.$^2 > 161.1$ in.$^2$

$$m = \frac{N - 0.95d}{2} = \frac{14 - 0.95(13.1)}{2} = 0.7775 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{14 - 0.8(12.3)}{2} = 2.08 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{13.1(12.3)} = 3.173 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(0.7775, 2.08, 3.173) = 3.173 \text{ in.}$$

From Equation 5.20,

$$t \geq \ell\sqrt{\frac{2P_a}{F_y BN/1.67}} = 3.173\sqrt{\frac{2(260)}{36(14)(14)/1.67}} = 1.113 \text{ in.}$$

<u>Use a PL $1\frac{1}{8} \times 14 \times 14$</u>

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**<u>5.14-4</u>**

(a) The factored load is $P_u = 1.2D + 1.6L = 1.2(35) + 1.6(35) = 98.0$ kips

Compute the required bearing area. From AISC Equation J8-2,

$$P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \le 1.7f_c'A_1$$

For $\phi_c P_p \ge P_u$,

$$0.65\left[0.85(3)A_1\sqrt{\frac{12(12)}{A_1}}\right] \ge 98, \text{ Solution is: } \{24.28 \le A_1\} \text{ in.}^2$$

Check upper limit:

$$\phi_c 1.7f_c'A_1 = 0.65(1.7)(3)(24.28) = 80.49 \text{ kips} < 98 \text{ kips} \quad \text{(N.G.)}$$

Let $\phi_c 1.7f_c'A_1 = P_u$, $\quad 0.65(1.7)(3)A_1 = 98$, Solution is: $\{A_1 = 29.56\}$ in.$^2$

The plate must be as large as the column:

$$b_f d = 7.96(9.73) = 77.5 \text{ in.}^2 > 29.56 \text{ in.}^2 \quad \text{(N.G.)}$$

Try $B = N = 12$ in., with $A_1$ provided $= 12(12) = 144$ in.$^2 > 77.5$ in.$^2$

$$m = \frac{N - 0.95d}{2} = \frac{12 - 0.95(9.73)}{2} = 1.378 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{12 - 0.8(7.96)}{2} = 2.816 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{9.73(7.96)} = 2.2 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.378, 2.816, 2.2) = 2.816 \text{ in.}$$

$$\text{Required } t = \ell\sqrt{\frac{2P_u}{0.90F_yBN}} = 2.816\sqrt{\frac{2(98)}{0.90(36)(12)(12)}} = 0.577 \text{ in.; use } \tfrac{5}{8} \text{ in.}$$

Use a PL $\frac{5}{8} \times 12 \times 12$

(b) The applied load is $P_a = D + L = 35 + 35 = 70$ kips.

Compute the required bearing area. Using AISC Equation J8-2,

$$\frac{P_p}{\Omega_c} \ge P_a$$

$$\frac{1}{2.31}\left[0.85(3)A_1\sqrt{\frac{12(12)}{A_1}}\right] \ge 70, \text{ Solution is: } \{27.92 \le A_1\} \text{ in.}^2$$

The upper limit is

$$\frac{1}{\Omega_c}(1.7f_c{'}A_1) = \frac{1}{2.31}[(1.7)(3)(27.92)] = 61.64 \text{ kips} < 70 \text{ kips} \qquad \text{(N.G.)}$$

The plate must be as large as the column:

$$b_f d = 7.96(9.73) = 77.5 \text{ in.}^2$$

Try $B = N = 12$ in., with $A_1$ provided = 12(12) = 144 in.$^2$ > 77.5 in.$^2$

$$m = \frac{N - 0.95d}{2} = \frac{12 - 0.95(9.73)}{2} = 1.378 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{12 - 0.8(7.96)}{2} = 2.816 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{9.73(7.96)} = 2.2 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.378, 2.816, 2.2) = 2.816 \text{ in.}$$

From Equation 5.20,

$$t \geq \ell\sqrt{\frac{2P_a}{F_y BN/1.67}} = 2.816\sqrt{\frac{2(70)}{36(12)(12)/1.67}} = 0.598 \text{ in.; use } \tfrac{5}{8} \text{ in.}$$

Use a PL $\frac{5}{8} \times 12 \times 12$

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**5.15-1**

Compute the nominal flexural strength for $x$-axis bending. The following data for a W21 × 55 are obtained from the $Z_x$ table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 6.11 \text{ ft}, \ L_r = 17.4 \text{ ft}$$

The unbraced length $L_b = 16$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_{px}$$

$$M_{px} = F_y Z_x = 50(126) = 6300 \text{ in.-kips}$$

If the beam weight is neglected, $C_b$ can be obtained from Figure 5.15 as $C_b = 1.32$

$$M_{nx} = 1.32\left[6300 - (6300 - 0.7 \times 50 \times 110)\left(\frac{16 - 6.11}{17.4 - 6.11}\right)\right]$$

$$= 5483 \text{ in.-kips} = 456.9 \text{ ft-kips} \qquad (< M_{px})$$

For the $y$ axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(18.4) = 920.0 \text{ in.-kips} = 76.67 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{18.4}{11.8} = 1.560 < 1.6 \quad \therefore\ M_{ny} = M_{py} = 76.67 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_b M_{nx} = 0.90(456.9) = 411.2 \text{ ft-kips},$$

$$\phi_b M_{ny} = 0.90(76.67) = 69.00 \text{ ft-kips}$$

$$P_u = 1.2(15) + 1.6(15) = 42.0 \text{ kips}, \ w_u = 1.2(0.055) = 0.066 \text{ kips/ft}$$

$$M_{ux} = \frac{42.0(16)}{4} + \frac{0.066(16)^2}{8} = 170.1 \text{ ft-kips}$$

For y-axis bending,

$$1.2(1.5) + 1.6(1.5) = 4.2 \text{ kips}$$

$$M_{uy} = \frac{2(4.2)(16)}{4} = 33.6 \text{ ft-kips}$$

Check interaction equation 5.23:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{170.1}{411.3} + \frac{33.6}{69.00} = 0.901 < 1.00 \ \text{(OK)}$$

The Specification is satisfied.

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{456.9}{1.67} = 273.6 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{76.67}{1.67} = 45.91 \text{ ft-kips}$$

$$M_{ax} = \frac{30(16)}{4} + \frac{0.055(16)^2}{8} = 121.8 \text{ ft-kips}$$

For y-axis bending,

$$M_{ay} = \frac{2(3)(16)}{4} = 24.0 \text{ ft-kips}$$

Check interaction equation 5.24:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{121.8}{273.6} + \frac{24}{45.91} = 0.968 < 1.0 \quad \text{(OK)}$$

The Specification is satisfied.

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**5.15-2**

(a) LRFD solution

Strong-axis bending:

$$P_u = 1.6\left(\frac{4}{5} \times 30\right) = 38.4 \text{ kips}, \qquad M_{ux} = \frac{38.4(30)}{4} = 288.0 \text{ ft-kips}$$

From the beam design charts, for $L_b = 30$ ft and $C_b = 1.0$, $\phi_b M_{nx} = 593$ ft-kips

For $C_b = 1.32$ (from Figure 5.15),

$$\phi_b M_{nx} = 1.32 \times 593 = 782.8 \text{ ft-kips} > \phi_b M_{px} = 720 \text{ ft-kips}$$

$\therefore$ use $\phi_b M_{nx} = \phi_b M_{px} = 720$ ft-kips

Weak-axis bending:

$$P_u = 1.6\left(\frac{3}{5} \times 30\right) = 28.8 \text{ kips}, \qquad M_{uy} = \frac{28.8(30)}{4} = 216.0 \text{ ft-kips}$$

From the $Z_y$ table, $\phi_b M_{ny} = 348$ ft-kips

Check interaction equation 5.23:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{288.0}{720} + \frac{216.0}{348} = 1.021 > 1.0 \qquad \text{(N.G.)}$$

Not Satisfactory. Result of interaction equation = 1.02

(b) ASD solution

Strong-axis bending:

$$P_a = \frac{4}{5} \times 30 = 24.0 \text{ kips}, \qquad M_{ax} = \frac{24.0(30)}{4} = 180.0 \text{ ft-kips}$$

From the beam design charts, for $L_b = 30$ ft and $C_b = 1.0$, $\frac{M_{nx}}{\Omega_b} = 395$ ft-kips

For $C_b = 1.32$ (from Figure 5.15),

$$\frac{M_{nx}}{\Omega_b} = 1.32 \times 395 = 521.4 \text{ ft-kips} > \frac{M_{px}}{\Omega_b} = 479 \text{ ft-kips}$$

$$\therefore \text{ use } \frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 479 \text{ ft-kips}$$

For y-axis bending,

$$P_a = \frac{3}{5} \times 30 = 18.0 \text{ kips}, \qquad M_{ay} = \frac{18.0(30)}{4} = 135 \text{ ft-kips}$$

From the $Z_y$ table, $\frac{M_{nx}}{\Omega_b} = 231$ ft-kips

Check interaction equation 5.24:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{180.0}{479} + \frac{135}{231} = 0.960 < 1.0 \qquad \text{(OK)}$$

<u>Satisfactory. Result of interaction equation = 0.960</u>

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**<u>5.15-3</u>**

Compute the nominal flexural strength for $x$-axis bending. The following data for a W18 × 76 are obtained from the $Z_x$ table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 9.22 \text{ ft}, \; L_r = 27.1 \text{ ft}$$

The unbraced length $L_b = 12$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_{px}$$

$$M_{px} = F_y Z_x = 50(163) = 8150 \text{ in.-kips}$$

From Figure 5.15, $C_b = 1.14$

$$M_{nx} = 1.14\left[8150 - (8150 - 0.7 \times 50 \times 146)\left(\frac{12 - 9.22}{27.1 - 9.22}\right)\right]$$

$$= 8752 \text{ in.-kips} > M_{px}$$

$$\therefore \text{ use } M_{nx} = M_{px} = 8150 \text{ in.-kips} = 679.2 \text{ ft-kips}$$

For the $y$ axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(42.2) = 2110 \text{ in.-kips} = 175.8 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{42.2}{27.6} = 1.52 < 1.6 \qquad \therefore M_{ny} = M_{py} = 175.8 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_b M_{nx} = 0.90(679.2) = 611.3 \text{ ft-kips}, \qquad \phi_b M_{ny} = 0.90(175.8) = 158.2 \text{ ft-kips}$$

$$w_{ux} = 1.2(0.076) + 1.6\left(\frac{2}{\sqrt{5}}\right)(3.5) = 5.100 \text{ kips/ft}$$

$$M_{ux} = \frac{5.100(12)^2}{8} = 91.8 \text{ ft-kips}$$

$$w_{uy} = 1.6\left[\frac{1}{\sqrt{5}}(3.5)\right] = 2.504 \text{ kips/ft}$$

$$M_{uy} = \frac{2.504(12)^2}{8} = 45.07 \text{ ft-kips}$$

Check interaction equation 5.23 (use 1/2 of weak-axis bending strength):

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{91.8}{611.2} + \frac{45.07}{158.2/2} = 0.720 < 1.0 \qquad \text{(OK)}$$

<u>W18 × 76 is satisfactory.</u>

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{679.2}{1.67} = 406.7 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{158.2}{1.67} = 94.73 \text{ ft-kips}$$

$$w_{ax} = 3.5\left(2/\sqrt{5}\right) + 0.076 = 3.206 \text{ kips/ft}, \qquad M_{ax} = \frac{3.206(12)^2}{8} = 57.71 \text{ ft-kips}$$

$$w_{ay} = 3.5\left(1/\sqrt{5}\right) = 1.565 \text{ kips/ft}, \qquad M_{ay} = \frac{1.565(12)^2}{8} = 28.17 \text{ ft-kips}$$

Check interaction equation 5.24 (use 1/2 of weak-axis bending strength):

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{57.71}{406.7} + \frac{28.17}{105.3/2} = 0.677 < 1.0 \qquad \text{(OK)}$$

W18 × 76 is satisfactory

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## 5.15-4

(a) Strong-axis bending strength:

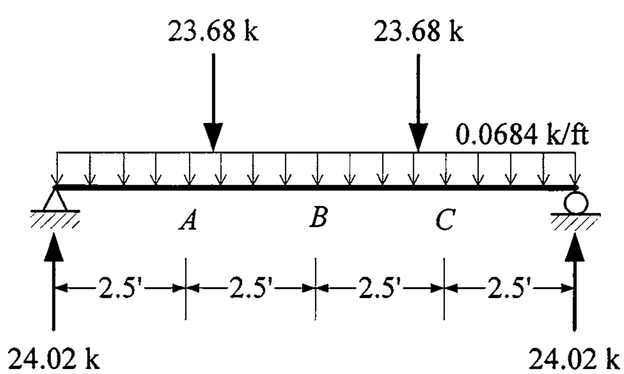
From the beam design charts, for $L_b = 10$ ft and $C_b = 1.0$, $\phi_b M_{nx} = 342$ ft-kips

Compute $C_b$.

$$w_{ux} = 1.2w_D = 1.2(0.057) = 0.0684 \text{ kips/ft}$$

$$P_{ux} = 1.2P_{Dx} + 1.6P_{Lx} = 1.2(0.3 \times 20)\left(\frac{4}{5}\right) + 1.6(0.7 \times 20)\left(\frac{4}{5}\right) = 23.68 \text{ kips}$$

$$\text{Reaction} = \frac{w_{ux}L}{2} + P_{ux} = \frac{0.0684(10)}{2} + 23.68 = 24.02 \text{ kips}$$




Dividing the unbraced length into 4 equal segments of 10/4 = 2.5 ft each and labeling the 3 interior points $A, B$, and $C$, we obtain

$$M_A = M_C = 24.02(2.5) - 0.0684(2.5)^2/2 = 59.84 \text{ ft-kips}$$

$M_B = M_{max} = 24.02(5) - 0.0684(5)^2/2 - 23.68(2) = 71.89$ ft-kips

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(71.89)}{2.5(71.89) + 3(59.84) + 4(71.89) + 3(59.84)} = 1.087$$

For $C_b = 1.087$,

$\phi_b M_{nx} = 1.087 \times 342 = 371.8$ ft-kips $< \phi_b M_{px} = 394$ ft-kips

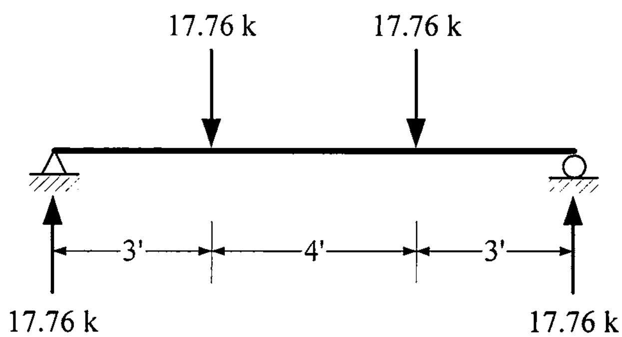
$\therefore$ use $\phi_b M_{nx} = 371.8$ ft-kips

Weak-axis bending strength: From the $Z_y$ table, $\phi_b M_{ny} = 70.9$ ft-kips

Weak axis bending moment:

Reaction $= P_{uy} = 1.2P_{Dy} + 1.6P_{Ly}$

$$= 1.2(0.3 \times 20)\left(\frac{3}{5}\right) + 1.6(0.7 \times 20)\left(\frac{3}{5}\right) = 17.76 \text{ kips}$$




$M_{uy} = 17.76(3) = 53.28$ ft-kips

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{71.89}{371.8} + \frac{53.28}{70.9/2} = 1.70 < 1.0 \quad \text{(N.G)}$$

<u>Not adequate. Result of interaction equation = 1.70</u>

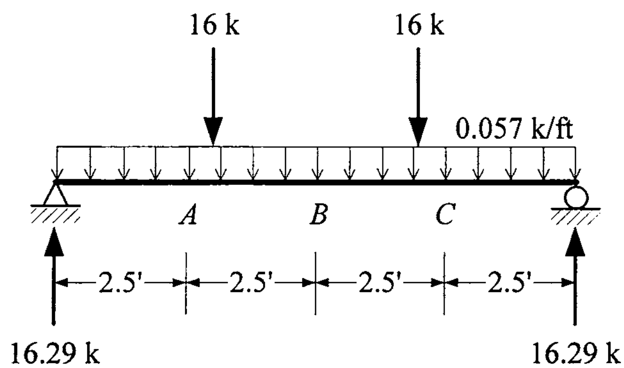
(b) Strong-axis bending strength:

From the beam design charts, for $L_b = 10$ ft and $C_b = 1.0$, $\frac{M_{nx}}{\Omega_b} = 228$ ft-kips

Compute $C_b$.

$$w_{ax} = w_D = 0.057 \text{ kips/ft}, \qquad P_{ax} = P_{Dx} + P_{Lx} = 20\left(\frac{4}{5}\right) = 16.0 \text{ kips}$$

$$\text{Reaction} = \frac{w_{ax}L}{2} + P_{ax} = \frac{0.057(10)}{2} + 16.0 = 16.29 \text{ kips}$$




Dividing the unbraced length into 4 equal segments of $10/4 = 2.5$ ft each and labeling the 3 interior points $A, B$, and $C$, we obtain

$$M_A = M_C = 16.29(2.5) - 0.057(2.5)^2/2 = 40.55 \text{ ft-kips}$$

$$M_B = M_{\max} = 16.29(5) - 0.057(5)^2/2 - 16(2) = 48.74 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(48.74)}{2.5(48.74) + 3(40.55) + 4(48.74) + 3(40.55)} = 1.088$$

For $C_b = 1.088$,

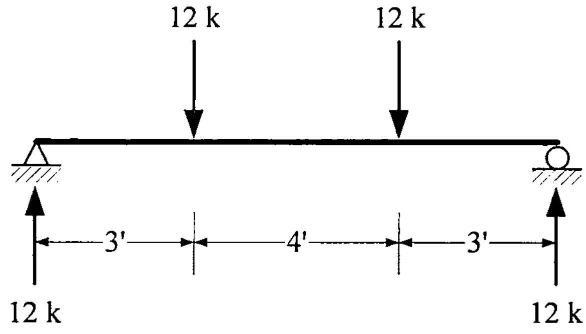
$$\frac{M_{nx}}{\Omega_b} = 1.088 \times 228 = 248.1 \text{ ft-kips} < \frac{M_{px}}{\Omega_b} = 262 \text{ ft-kips}$$

$$\therefore \text{ use } \frac{M_{nx}}{\Omega_b} = 248.1 \text{ ft-kips}$$

Weak-axis bending strength: From the $Z_y$ table, $\frac{M_{ny}}{\Omega_b} = 47.2$ ft-kips

Weak axis bending moment:

$$\text{Reaction} = P_{ay} = P_{Dy} + P_{Ly} = 20\left(\frac{3}{5}\right) = 12.0 \text{ kips}$$




$M_{ay} = 12(3) = 36$ ft-kips

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{48.74}{248.1} + \frac{36}{47.2/2} = 1.72 < 1.0 \qquad \text{(N.G.)}$$

<u>Not adequate. Result of interaction equation = 1.72</u>

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**5.15-5**

Compute the nominal flexural strength for $x$-axis bending. The following data for a W16 × 40 are obtained from the $Z_x$ table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 5.55 \text{ ft}, \; L_r = 15.9 \text{ ft}$$

The unbraced length $L_b = 10$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_{px}$$

$$M_{px} = F_yZ_x = 50(73.0) = 3650 \text{ in.-kips}$$

From Figure 5.15, $C_b = 1.14$

$$M_{nx} = 1.14\left[3650 - (3650 - 0.7 \times 50 \times 64.7)\left(\frac{10 - 5.55}{15.9 - 5.55}\right)\right]$$

$$= 3482 \text{ in.-kips} = 290.2 \text{ ft-kips} \qquad (< M_{px})$$

For the $y$ axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(12.7) = 635.0 \text{ in.-kips} = 52.92 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{12.7}{8.25} = 1.539 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 52.92 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_b M_{nx} = 0.90(290.2) = 261.2 \text{ ft-kips},$$

$$\phi_b M_{ny} = 0.90(52.92) = 47.63 \text{ft-kips}$$

$$w_{ux} = 1.6(2)(4/5) = 2.56 \text{ kips/ft}, \qquad M_{ux} = \frac{2.56(10)^2}{8} = 32.0 \text{ ft-kips}$$

$$w_{uy} = 1.6(2)(3/5) = 1.92 \text{ kips/ft}, \qquad M_{uy} = \frac{1.92(10)^2}{8} = 24.0 \text{ ft-kips}$$

Check interaction equation 5.23.

For the loading condition of Figure P5.15-5(a), use the full weak-axis bending strength:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{32.0}{261.0} + \frac{24.0}{47.63} = 0.626 < 1.0 \qquad \text{(OK)}$$

Beam is satisfactory for the loading of Fig. P5.15-5(a).

For the loading condition of Figure P5.15-5(b), use 1/2 of the weak-axis bending strength:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{32.0}{261.0} + \frac{24.0}{47.63/2} = 1.13 > 1.0 \qquad \text{(N.G.)}$$

Beam is unsatisfactory for the loading of Fig. P5.15-5(b).

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{290.2}{1.67} = 173.8 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{52.92}{1.67} = 31.69 \text{ ft-kips}$$

$$w_{ax} = 2(4/5) = 1.6 \text{ kips/ft}, \qquad M_{ax} = \frac{1.6(10)^2}{8} = 20.0 \text{ ft-kips}$$

$$w_{ay} = 2(3/5) = 1.2 \text{ kips/ft}, \qquad M_{ay} = \frac{1.2(10)^2}{8} = 15.0 \text{ ft-kips}$$

Check interaction equation 5.24:

For the loading condition of Figure P5.15-5(a), use the full weak-axis bending strength:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{20.0}{173.8} + \frac{15.0}{31.69} = 0.588 < 1.0 \qquad \text{(OK)}$$

Beam is satisfactory for the loading of Fig. P5.15-5(a)

For the loading condition of Figure P5.15-5(b), use 1/2 of the weak-axis bending strength:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{20.0}{173.8} + \frac{15.0}{31.69/2} = 1.06 > 1.0 \qquad \text{(N.G.)}$$

Beam is unsatisfactory for the loading of Fig. P5.15-5(b)

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**5.15-6**

Before proceeding to the LRFD and ASD solutions, we will compute the nominal flexural strength about the $x$ and $y$ axes. First, determine the strong-axis bending strength. Neither the beam design charts nor the $Z$ tables include shapes smaller than W8, so the flexural strength of the W6 × 12 must be computed. From the dimensions and properties tables, the shape is compact (no footnote). The following properties of a W6 ×12 will be needed:

$A = 3.55 \text{ in.}^2$, $S_x = 7.31 \text{ in.}^3$, $Z_x = 8.30 \text{ in.}^3$ $I_y = 2.99 \text{ in.}^4$, $r_y = 0.918$ in., $S_y = 1.50 \text{ in.}^3$, $r_{ts} = 1.08$ in., $h_o = 5.75$ in., $J = 0.0903 \text{ in.}^4$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(0.918)\sqrt{\frac{29000}{50}} = 38.91 \text{ in.} = 3.243 \text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(1.08)\frac{29000}{0.7(50)} \sqrt{\frac{0.0903(1.0)}{7.31(5.75)} + \sqrt{\frac{0.0903(1.0)}{7.31(5.75)}^2 + 6.76\left(\frac{0.7(50)}{29000}\right)^2}}$$

$$= 134.6 \text{ in.} = 11.22 \text{ ft}$$

For $L_b = 10$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.14$ (see Fig. 5.15 in textbook).

$M_p = F_yZ_x = 50(8.30) = 415.0$ in.-kips = 34. 58 ft-kips

$$M_n = 1.14\left[415 - (415 - 0.7 \times 50 \times 7.31)\left(\frac{10 - 3.243}{11.22 - 3.243}\right)\right]$$

$= 319.4$ in.-kips = 26. 62 ft-kips $< M_p$

For the $y$ axis, since the shape is compact, there is no flange local buckling and

$M_{ny} = M_{py} = F_yZ_y = 50(2.32) = 116.0$ in.-kips = 9. 667 ft-kips

Check the upper limit:

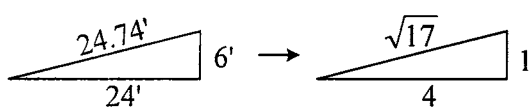
$$\frac{Z_y}{S_y} = \frac{2.32}{1.50} = 1.55 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 9.667 \text{ ft-kips}$$

(a) LRFD solution

$\phi_bM_{nx} = 0.90(26.62) = 23.96$ ft-kips, $\quad \phi_bM_{ny} = 0.90(9.667) = 8.700$ ft-kips

Roof load: Combination 3 controls.

$w_u = 1.2D + 1.6S = 1.2(40/2) + 1.6(40/2) = 56.0$ psf




Tributary width $= \dfrac{\sqrt{17}}{4}(6) = 6.185$ ft

Purlin load $= 56.0(6.185) = 346.4$ lb/ft

Component normal to roof $= w_{ux} = \dfrac{4}{\sqrt{17}}(346.4) = 336.1$ lb/ft

Component parallel to roof $= w_{uy} = \dfrac{1}{\sqrt{17}}(346.4) = 84.01$ lb/ft

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3361)(10)^2 = 4.201 \text{ ft-kips}$$

$$M_{uy} = \frac{1}{8}w_{uy}L^2 = \frac{1}{8}(0.08401)(10)^2 = 1.05 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{4.201}{23.96} + \frac{1.05}{8.700/2} = 0.417 < 1.0 \qquad \text{(OK)}$$

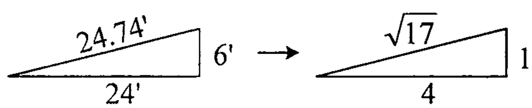
Adequate. Result of interaction equation = 0.415

(b) ASD solution

$$\frac{M_{nx}}{\Omega_b} = \frac{26.62}{1.67} = 15.94 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{9.667}{1.67} = 5.789 \text{ ft-kips}$$

Roof load: Combination 3 controls.

$$w_a = D + S = 40 \text{ psf}$$




$$\text{Tributary width} = \frac{\sqrt{17}}{4}(6) = 6.185 \text{ ft}$$

$$\text{Purlin load} = 40(6.185) = 247.4 \text{ lb/ft}$$

$$\text{Component normal to roof} = w_{ax} = \frac{4}{\sqrt{17}}(247.4) = 240.0 \text{ lb/ft}$$

$$\text{Component parallel to roof} = w_{ay} = \frac{1}{\sqrt{17}}(247.4) = 60.0 \text{ lb/ft}$$

$$M_{ax} = \frac{1}{8}w_{ax}L^2 = \frac{1}{8}(0.240)(10)^2 = 3.0 \text{ ft-kips}$$

$$M_{ay} = \frac{1}{8}w_{ay}L^2 = \frac{1}{8}(0.060)(10)^2 = 0.75 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

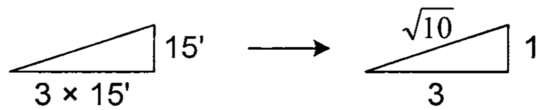
$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{3.0}{15.94} + \frac{0.75}{5.789/2} = 0.447 \qquad \text{(OK)}$$

Adequate. Result of interaction equation = 0.447

**5.15-7**

Dead load: assume purlin weighs 20 lb/ft.

Other dead load:




Tributary width = $\frac{\sqrt{10}}{3}\left(\frac{15}{2}\right) = 7.906$ ft

$w_D = 7.906(16) + 20 = 146.5$ lb/ft

Snow: $w_S = \left(\frac{15}{2}\right)(20) = 150.0$ lb/ft

Load combination 3 controls.

$$w_u = 1.2w_D + 1.6w_S = 1.2(0.1465) + 1.6(0.1500) = 0.4158 \text{ kips/ft}$$

Component normal to roof = $w_{ux} = \frac{3}{\sqrt{10}}(0.4158) = 0.3945$ kips/ft

Component parallel to roof = $w_{uy} = \frac{1}{\sqrt{10}}(0.4158) = 0.1315$ kips/ft

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3945)(18)^2 = 15.98 \text{ ft-kips}$$

To compute $M_{uy}$, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a two-span continuous beam, the maximum moment is

$$M_{uy} = 0.125w_{uy}\left(\frac{L}{2}\right)^2 = 0.125(0.1315)\left(\frac{18}{2}\right)^2 = 1.331 \text{ ft-kips}$$

Try a W6 × 12:

Determine the strong-axis bending strength. The beam design charts in the *Manual* do not contain curves for shapes smaller than W8, so the flexural strength of the W6 × 12 must be computed. From the dimensions and properties tables, the shape is compact (no footnote). The following properties of a W6 ×12 will be needed:

$A = 3.55$ in.$^2$, $S_x = 7.31$ in.$^3$, $Z_x = 8.30$ in.$^3$ $I_y = 2.99$ in.$^4$, $r_y = 0.918$ in., $S_y = 1.50$ in.$^3$,

$r_{ts} = 1.08$ in., $h_o = 5.75$ in., $J = 0.0903$ in.$^4$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(0.918)\sqrt{\frac{29000}{50}} = 38.91 \text{ in.} = 3.243 \text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(1.08)\frac{29000}{0.7(50)} \sqrt{\frac{0.0903(1.0)}{7.31(5.75)} + \sqrt{\frac{0.0903(1.0)}{7.31(5.75)}^2 + 6.76\left(\frac{0.7(50)}{29000}\right)^2}}$$

$$= 134.6 \text{ in.} = 11.22 \text{ ft}$$

For $L_b = 18/2 = 9$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.30$ (see Table 3-1 in the *Manual*).

$$M_p = F_y Z_x = 50(8.30) = 415.0 \text{ in.-kips} = 34.58 \text{ ft-kips}$$

$$M_n = 1.30\left[415 - (415 - 0.7 \times 50 \times 7.31)\left(\frac{9 - 3.243}{11.22 - 3.243}\right)\right]$$

$$= 390.2 \text{ in.-kips} = 32.52 \text{ ft-kips} < M_p$$

$$\phi_b M_{nx} = 0.90(32.52) = 29.27 \text{ ft-kips}$$

For the $y$ axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(2.32) = 116.0 \text{ in.-kips} = 9.667 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{2.32}{1.50} = 1.55 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 9.667 \text{ ft-kips}$$

$$\phi_b M_{ny} = 0.90(9.667) = 8.700 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{15.98}{29.27} + \frac{1.331}{8.700/2} = 0.852 < 1.0 \qquad \text{(OK)}$$

Weight assumption and shear strength are OK: <u>Use a W6 × 12</u>

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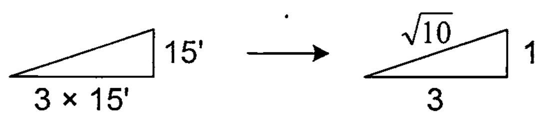
**<u>5.15-8</u>**

Dead load: assume purlin weighs 10 lb/ft.

Other dead load:




$$\text{Tributary width} = \frac{\sqrt{10}}{3}\left(\frac{15}{2}\right) = 7.906 \text{ ft}$$

$$w_D = 7.906(16) + 10 = 136.5 \text{ lb/ft}$$

Snow: $w_S = \left(\frac{15}{2}\right)(20) = 150.0$ lb/ft

Load combination 3 controls.

$$w_u = 1.2w_D + 1.6w_S = 1.2(0.1365) + 1.6(0.1500) = 0.4038 \text{ kips/ft}$$

$$\text{Component normal to roof} = w_{ux} = \frac{3}{\sqrt{10}}(0.4038) = 0.3831 \text{ kips/ft}$$

$$\text{Component parallel to roof} = w_{uy} = \frac{1}{\sqrt{10}}(0.4038) = 0.1277 \text{ kips/ft}$$

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3831)(18)^2 = 15.52 \text{ ft-kips}$$

To compute $M_{uy}$, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a three-span continuous beam, the maximum moment is

$$M_{uy} = 0.10w_{uy}\left(\frac{L}{3}\right)^2 = 0.10(0.1277)\left(\frac{18}{3}\right)^2 = 0.4597 \text{ ft-kips}$$

Try a W6 × 9:

Determine the strong-axis bending strength. The beam design charts in the *Manual* do not contain curves for shapes smaller than W8, so the flexural strength of the W6 × 9 must be computed. From the dimensions and properties tables, the shape is noncompact (see footnote).

Compute the strength based on the limit state of flange local buckling.

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$M_p = F_yZ_x = 50(6.23) = 311.5 \text{ in.-kips}$$

$$\lambda = \frac{b_f}{2t_f} = 9.16$$

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152, \qquad \lambda_r = 1.0\sqrt{\frac{29,000}{50}} = 24.08$$

$$M_n = 311.5 - (311.5 - 0.7 \times 50 \times 5.56)\left(\frac{9.16 - 9.152}{24.08 - 9.152}\right)$$

$$= 311.4 \text{ in.-kips} = 25.95 \text{ ft-kips}$$

Lateral-torsional buckling:

The following properties of a W6 × 9 will be needed:

$A = 2.68$ in.$^2$, $S_x = 5.56$ in.$^3$, $Z_x = 6.23$ in.$^3$ $I_y = 2.20$ in.$^4$, $r_y = 0.905$ in., $S_y = 1.11$ in.$^3$, $Z_y = 1.72$ in.$^3$, $r_{ts} = 1.06$ in., $h_o = 5.69$ in., $J = 0.0405$ in.$^4$

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(0.905)\sqrt{\frac{29000}{50}} = 38.36 \text{ in.} = 3.197 \text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o} + \sqrt{\left(\frac{Jc}{S_xh_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$= 1.95(1.06)\frac{29000}{0.7(50)}\sqrt{\frac{0.0405(1.0)}{5.56(5.69)} + \sqrt{\frac{0.0405(1.0)}{5.56(5.69)}^2 + 6.76\left(\frac{0.7(50)}{29000}\right)^2}}$$

$$= 117.0 \text{ in.} = 9.75 \text{ ft}$$

For $L_b = 18/3 = 6$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.01$ (see Table 3-1 in the *Manual*).

$$M_p = F_y Z_x = 50(6.23) = 311.5 \text{ in.-kips} = 25.96 \text{ ft-kips}$$

$$M_n = 1.01\left[311.5 - (311.5 - 0.7 \times 50 \times 5.56)\left(\frac{6 - 3.197}{9.75 - 3.197}\right)\right]$$

$$= 264.1 \text{ in.-kips} = 22.01 \text{ ft-kips} < M_p$$

Lateral-torsional buckling controls.

$$\phi_b M_{nx} = 0.90(22.01) = 19.81 \text{ ft-kips}$$

Weak-axis bending strength: Since the shape is noncompact, use AISC Equation F6-2:

$$M_{ny} = M_{py} - (M_{py} - 0.7F_y S_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$M_{py} = F_y Z_y = 50(1.72) = 86.0 \text{ in.-kips}$$

$$M_{ny} = 86.0 - (86.0 - 0.7 \times 50 \times 1.11)\left(\frac{9.16 - 9.152}{24.08 - 9.152}\right)$$

$$= 85.97 \text{ in.-kips} = 7.164 \text{ ft-kips}$$

$$\phi_b M_{ny} = 0.90(7.164) = 6.448 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{15.52}{19.81} + \frac{0.4597}{6.448/2} = 0.926 < 1.0 \qquad \text{(OK)}$$

Weight and shear strength are OK: <u>Use a W6 × 9</u>

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# CHAPTER 6 - BEAM-COLUMNS

## 6.2-1

(a) LRFD solution:

From the column load tables, the compressive design strength of a W12 × 106 with $F_y = 50$ ksi and $K_yL = 1.0 \times 14 = 14$ feet is

$$\phi_c P_n = 1130 \text{ kips}$$

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$,

$$\phi_b M_n = 597 \text{ ft-kips} \qquad \text{(Since the bending moment is uniform, } C_b = 1.0.\text{)}$$

The factored axial compressive load is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.5 \times 250) + 1.6(0.5 \times 250) = 350.0 \text{ kips}$$

The factored bending moment is

$$M_u = 1.2M_D + 1.6M_L = 1.2(0.5 \times 240) + 1.6(0.5 \times 240) = 336.0 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{350}{1130} = 0.3097 > 0.2 \quad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{350}{1130} + \frac{8}{9}\left(\frac{336}{597} + 0\right)$$

$$= 0.810 < 1.00 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

(b) ASD solution:

From the column load tables, the allowable compressive strength of a W12 × 106 with $F_y = 50$ ksi and $K_yL = 1.0 \times 14 = 14$ feet is

$$\frac{P_n}{\Omega_c} = 755 \text{ kips}$$

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 398 \text{ ft-kips} \qquad \text{(Since the bending moment is uniform, } C_b = 1.0.\text{)}$$

The total axial compressive load is $P_a = 250$ kips

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{250}{755} = 0.3311 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{250}{755} + \frac{8}{9}\left(\frac{240}{398} + 0\right)$$

$$= 0.867 < 1.0 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

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**<u>6.2-2</u>**

Compute the compressive strength (this shape is not in the column load tables).

For a W18 × 97, $A_g = 28.5 \text{ in.}^2$, $r_y = 2.65$ in., and the shape is not slender (no footnote).

$$\frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.65} = 90.57 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(90.57)^2} = 34.89 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 90.57 < 113.4$, use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/34.89)}(50) = 27.45 \text{ ksi}$$

$$P_n = F_{cr}A_g = 27.45(28.5) = 782.3 \text{ kips}$$

(a) $\phi_c P_n = 0.90(782.3) = 704.1$ kips

From the beam design charts in Part 5 of the *Manual*, for $L_b = 20$ ft,

$$\phi_b M_n = 639 \text{ ft-kips for } C_b = 1.$$

For this case , $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$,

$$\phi_b M_n = 1.14 \times 639 = 728.5 \text{ ft-kips} < \phi_b M_p = 791 \text{ ft-kips}$$

Factored axial load $= P_u = 1.2D + 1.6L = 1.2(10) + 1.6(20) = 44.0$ kips

$$\frac{P_u}{\phi_c P_n} = \frac{44.0}{704.1} = 6.249 \times 10^{-2} < 0.2 \therefore \text{ use Eq. 6.4 (AISC Eq. H1-1b):}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{44.0}{2(704.1)} + \left(\frac{M_{ux}}{728.5} + 0\right) = 1.0$$

$$M_{ux} = 705.7 \text{ ft-kips}$$

Let $\frac{1}{8}w_u L^2 = M_{ux}$ : $\qquad \frac{1}{8}w_u(20)^2 = 705.7 \qquad \Rightarrow \qquad w_u = 14.11$ kips/ft

$$w_u = 1.2w_D + 1.6w_L$$

$14.11 = 1.2(0.097) + 1.6w_L,$ $\underline{w_L = 7.52 \text{ kips/ft}}$

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{782.3}{1.67} = 468.4 \text{ kips}$

From the beam design charts in Part 5 of the *Manual*, for $L_b = 20$ ft,

$$\frac{M_n}{\Omega_b} = 426 \text{ ft-kips for } C_b = 1.$$

For this case , $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$,

$$\frac{M_n}{\Omega_b} = 1.14 \times 426 = 485.6 \text{ ft-kips} < \frac{M_p}{\Omega_b} = 526 \text{ ft-kips}$$

Axial load $= P_a = D + L = 10 + 20 = 30$ kips

$$\frac{P_a}{P_n/\Omega_c} = \frac{30}{468.4} = 6.405 \times 10^{-2} < 0.2$$

$\therefore$ use Equation 6.6 (AISC Equation H1-1b)

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{30}{2(468.4)} + \left(\frac{M_{ax}}{485.6} + 0\right) = 1.0$$

$$M_{ax} = 470.0 \text{ ft-kips}$$

Let $\frac{1}{8}w_a L^2 = M_{ax}$ : $\quad \frac{1}{8}w_a(20)^2 = 470.0 \quad \Rightarrow \quad w_a = 9.4$ kips/ft

$w_D + w_L = 0.097 + w_L = 9.4$ $\underline{w_L = 9.30 \text{ kips/ft}}$

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**6.6-1**

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1L)^2} = \frac{\pi^2(0.8)EI_x}{(K_xL)^2} = \frac{\pi^2(0.8)(29000)(933)}{(1.0 \times 14 \times 12)^2} = 7569 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{240}{240}\right) = 1.0$$

(a) LRFD solution:

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.25 \times 250) + 1.6(0.75 \times 250) = 375.0 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.00P_u/P_{e1})} = \frac{1.0}{1 - (375/7569)} = 1.052$$

$\underline{B_1 = 1.05}$

(b) ASD solution: $P_a = 250$ kips

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-(1.60)(250)/7569}$$

$$= 1.056 \qquad \underline{B_1 = 1.06}$$

---

**6.6-2**

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(1750)}{(1.0\times 20\times 12)^2} = 8696 \text{ kips}$$

$$C_m = 1.0$$

(a) LRFD solution:

$$P_u = 1.2P_D + 1.6P_L = 1.2(10) + 1.6(20) = 44.0 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(44.0/8696)} = 1.01$$

$$\underline{B_1 = 1.01}$$

(b) ASD solution: $P_a = 10 + 20 = 30.0$ kips

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-(1.60)(30.0)/8696}$$

$$= 1.01$$

$$\underline{B_1 = 1.01}$$

---

**6.6-3**

$$K_xL = 1.0(14) = 14 \text{ ft}, \qquad K_yL = 1.0(14) = 14 \text{ ft.}$$

(a) LRFD solution:

From the column load tables, for $KL = 14$ ft, $\phi_c P_n = 1130$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$, $\phi_b M_n = 642$ ft-kips and $\phi_b M_p = 646$ ft-kips. For $C_b = 1.6$,

$$\phi_b M_n = 1.6(642) = 1027 \text{ ft-kips} > \phi_b M_p \;\therefore\; \text{use } \phi_b M_n = \phi_b M_p = 646 \text{ ft-kips}$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.33\times 340) + 1.6(0.67\times 340) = 499.1 \text{ kips}$$

$$M_{nt} = 1.2M_D + 1.6M_L = 1.2(0.33\times 250) + 1.6(0.67\times 250) = 367.0 \text{ ft-kips}$$

For the axis of bending, $C_m = 1.0$ and

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2(29000)(1110)}{(14 \times 12)^2} = 1.126 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.00P_u/P_{e1})} = \frac{1.0}{1 - (499.1/11260)} = 1.046$$

$$M_u = B_1M_{nt} + B_2M_{\ell t} = 1.046(367) + 0 = 383.9 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{499.1}{1130} = 0.4417 > 0.2 \;\therefore\; \text{use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.4417 + \frac{8}{9}\left(\frac{383.9}{646} + 0\right)$$

$$= 0.970 < 1.0 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

(b) ASD solution:

From the column load tables, for $KL = 14$ ft, $\dfrac{P_n}{\Omega_c} = 751$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$, $\dfrac{M_n}{\Omega_b} = 428$ ft-kips and $\dfrac{M_p}{\Omega_b} = 430$ ft-kips. For $C_b = 1.6$,

$$\frac{M_n}{\Omega_b} = 1.6(428) = 684.8 \text{ ft-kips} > \frac{M_p}{\Omega_b} \;\therefore\; \text{use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 430 \text{ ft-kips}$$

$$P_a = 340 \text{ kips}, \; M_{nt} = 250 \text{ ft-kips}$$

For the axis of bending, $C_m = 1.0$ and

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2(29000)(1110)}{(14 \times 12)^2} = 1.126 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{1.0}{1 - 1.60(340/11260)} = 1.051$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.051(240) + 0 = 252.2 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{340}{751} = 0.4527 > 0.2 \;\therefore\; \text{use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4527 + \frac{8}{9}\left(\frac{252.2}{430} + 0\right) = 0.974$$

$$= 0.974 < 1.0 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

**6.6-4**

(a) LRFD solution:

The factored-load axial force is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.30 \times 120) + 1.6(0.70 \times 120) = 177.6 \text{ kips}$$

The factored-load end moments are

$$M_{top} = 1.2M_D + 1.6M_L = 1.2(0.30 \times 135) + 1.6(0.70 \times 135)$$

$$= 199.8 \text{ ft-kips}$$

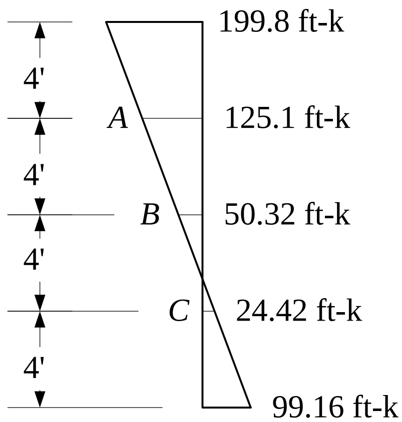
$$M_{bot} = 1.2M_D + 1.6M_L = 1.2(0.30 \times 67) + 1.6(0.70 \times 67) = 99.16 \text{ ft-kips}$$

From the column load tables, for $KL = 16$ ft, $\phi_c P_n = 499$ kips

From the beam design charts in Part 3 of the *Manual*, for $L_b = 16$ ft and $C_b = 1.0$,

$$\phi_b M_n = 283.5 \text{ ft-kips}, \ \phi_b M_p = 324 \text{ ft-kips.}$$

Compute $C_b$:




$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(199.8)}{2.5(199.8) + 3(125.1) + 4(50.32) + 3(24.42)} = 2.173$$

For $C_b = 2.173$, $\phi_b M_n = 2.173(283.5) = 616$ ft-kips

Since 616 ft-kips $> \phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 324$ ft-kips

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{99.16}{199.8}\right) = 0.4015$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(475)}{(0.9 \times 16 \times 12)^2} = 4553 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00 P_u/P_{e1})} = \frac{0.4015}{1-(177.6/4553)}$$

$$= 0.4178 < 1.0 \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(199.8) + 0 = 199.8 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{177.6}{499} = 0.3559 > 0.2$$

$\therefore$ use Equation 6.3 (AISC Equation H1-1a)

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3559 + \frac{8}{9}\left(\frac{199.8}{324} + 0\right)$$

$$= 0.904 < 1.0 \quad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

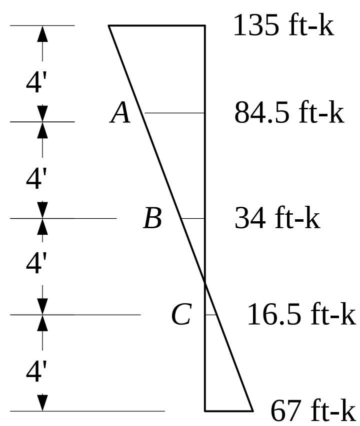
(b) ASD solution:

From the column load tables, for $KL = 16$ ft, $\frac{P_n}{\Omega_c} = 332$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 16$ ft and $C_b = 1.0$,

$\frac{M_n}{\Omega_b} = 189$ ft-kips and $\frac{M_p}{\Omega_b} = 216$ ft-kips.

Compute $C_b$:




$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135)}{2.5(135) + 3(84.5) + 4(34) + 3(16.5)} = 2.173$$

For $C_b = 2.173$,

$$\frac{M_n}{\Omega_b} = 2.173(189) = 411 \text{ ft-kips} > \frac{M_p}{\Omega_b} \quad \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 216 \text{ ft-kips}$$

$$P_a = 120 \text{ kips}, \; M_{nt} = 135 \text{ ft-kips}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{67}{135}\right) = 0.4015$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(475)}{(0.9 \times 16 \times 12)^2} = 4553 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.4015}{1-1.60(120/4553)}$$

$$= 0.4192 < 1.0 \;\therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.0(135) + 0 = 135 \text{ ft-kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{120}{332} = 0.3614 > 0.2$$

$\therefore$ use Equation 6.5 (AISC Equation H1-1a)

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.3614 + \frac{8}{9}\left(\frac{135}{216} + 0\right)$$

$$= 0.917 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

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**6.6-5**

(a) LRFD solution:

$$P_u = 1.2(20) + 1.6(20) = 56.0 \text{ kips},$$

$$M_{nt} = 1.2(32.5) + 1.6(32.5) = 91.0 \text{ ft-kips}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{91.5}{91.5}\right) = 1.0$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(171)}{(10 \times 12)^2} = 3399 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(56.0/3399)} = 1.017$$

$$M_u = B_1M_{nt} = 1.017(91) = 92.55 \text{ ft-kips}$$

Compute the moment strength. For this loading, $C_b = 1.0$. From the beam design charts with $L_b = 10$ ft and $C_b = 1.0$, $\phi_b M_n = 134$ ft-kips.

From the column load tables with $KL = 10$ ft, $\phi_c P_n = 330$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{56.0}{330} = 0.1697 < 0.2 \therefore \text{ use Equation 6.4 (AISC Eq. H1-1b):}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{56.0}{2(330)} + \left(\frac{92.55}{134} + 0\right)$$

$$= 0.776 < 1.0 \qquad \text{(OK)}$$

Member is satisfactory.

(b) ASD solution: $P_a = 40$ kips, $M_{nt} = 65$ ft-kips

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{65}{65}\right) = 1.0$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(171)}{(10 \times 12)^2} = 3399 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.60 P_a / P_{e1})} = \frac{1.0}{1 - 1.60(40/3399)} = 1.019$$

$$M_a = B_1 M_{nt} = 1.019(65) = 66.24 \text{ ft-kips}$$

Compute the moment strength. For this loading, $C_b = 1.0$. From the beam design charts with $L_b = 10$ ft and $C_b = 1.0$, $\frac{M_n}{\Omega_b} = 89.5$ ft-kips.

From the column load tables with $KL = 10$ ft, $\frac{P_n}{\Omega_c} = 220$ kips.

$$\frac{P_a}{P_n/\Omega_c} = \frac{40}{220} = 0.1818 < 0.2$$

$\therefore$ use Equation 6.6 (AISC Equation H1-1b)

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{40}{2(220)} + \left(\frac{66.24}{89.5} + 0\right)$$

$$= 0.831 < 1.0 \qquad \text{(OK)}$$

Member is satisfactory.

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**6.6-6**

The factored-load axial force is $P_u = 285$ kips

The factored-load end moments are

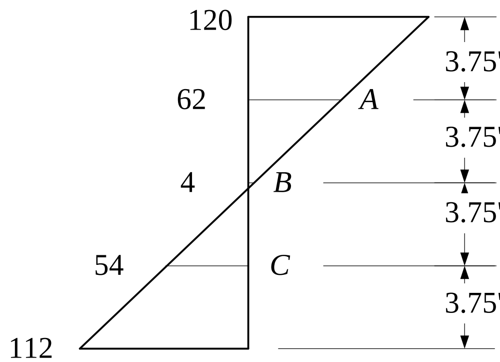
$$M_{utop} = 120 \text{ ft-kips}, \qquad M_{ubot} = 112 \text{ ft-kips}$$

From the column load tables, for $KL = 15$ ft, $\phi_c P_n = 525$ kips

From the beam design charts in Part 3 of the *Manual*, for $L_b = 15$ ft and $C_b = 1.0$,

$$\phi_b M_n = 289.5 \text{ ft-kips}, \quad \phi_b M_p = 324 \text{ ft-kips}.$$

Compute $C_b$:




$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(120)}{2.5(120) + 3(62) + 4(4) + 3(54)} = 2.259$$

For $C_b = 2.259$, $\phi_b M_n = 2.259(289.5) = 654.0$ ft-kips

Since 654.0 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 324$ ft-kips

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{285}{525} = 0.5429 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5429 + \frac{8}{9}\left(\frac{120.0}{324} + 0\right)$$

$$= 0.872 < 1.0 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

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**<u>6.6-7</u>**

(a) LRFD solution:

$$P_u = 1.2(70) + 1.6(170) = 356.0 \text{ kips}$$

$$w_u = 1.2(1.5) + 1.6(3.5) = 7.4 \text{ kips/ft}$$

$$Q_u = 1.2(7) + 1.6(18) = 37.2 \text{ kips}$$

$$M_{nt} = \frac{1}{8}w_u L^2 + \frac{Q_u L}{4} = \frac{1}{8}(7.4)(16)^2 + \frac{37.2(16)}{4} = 385.6 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 8$ ft,

$$\phi_b M_n = \phi_b M_p = 551 \text{ ft-kips. } (L_b < L_p)$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(716)}{(16 \times 12)^2} = 5559 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(356/5559)} = 1.068$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.068(385.6) + 0 = 411.8 \text{ ft-kips}$$

Compressive strength: $K_y L = 8$ ft, $K_x L = 16$ ft,

$$\frac{K_x L}{r_x/r_y} = \frac{16}{1.74} = 9.195 \text{ ft} > 8 \text{ ft.}$$

From the column load tables, for $KL = 9.195$ ft, $\phi_c P_n = 1304$ kips

$$\frac{P_u}{\phi_c P_n} = \frac{356}{1304} = 0.2730 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.2730 + \frac{8}{9}\left(\frac{411.8}{551} + 0\right)$$

$$= 0.937 < 1.0 \quad \text{(OK)}$$

<u>Member is satisfactory.</u>

(b) ASD solution:

$P_a = 70 + 170 = 240$ kips, $w_a = 1.5 + 3.5 = 5.0$ kips/ft, $Q_a = 7 + 18 = 25$ kips

$$M_{nt} = \frac{1}{8}w_a L^2 + \frac{Q_a L}{4} = \frac{1}{8}(5.0)(16)^2 + \frac{25(16)}{4} = 260.0 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 8$ ft,

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 367 \text{ ft-kips. } (L_b < L_p)$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(716)}{(16 \times 12)^2} = 5559 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(240/5559)} = 1.074$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.074(260.0) + 0 = 279.2 \text{ ft-kips}$$

Compressive strength: $K_yL = 8$ ft, $K_xL = 16$ ft,

$$\frac{K_xL}{r_x/r_y} = \frac{16}{1.74} = 9.195 \text{ ft} > 8 \text{ ft.}$$

From the column load tables, for $KL = 9.195$ ft, $\frac{P_n}{\Omega_c} = 870.3$ kips

$$\frac{P_a}{P_n/\Omega_c} = \frac{240}{870.3} = 0.2758 > 0.2$$

∴ use Equation 6.5 (AISC Equation H1-1a)

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.2758 + \frac{8}{9}\left(\frac{279.2}{367} + 0\right)$$

$$= 0.952 < 1.0 \qquad \text{(OK)}$$

Member is satisfactory.

---

**6.6-8**

This shape is not in the column load tables, so the axial compressive strength must be computed. Also, the footnote in the dimensions and properties table indicates that a W33 × 118 is slender for compression. First, compute the flexural buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(11 \times 12)}{2.32} = 56.90 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since 56.90 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(56.90)^2} = 88.40 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/88.40)}(50) = 39.46 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 39.46(34.7) = 1369 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 54.5$$

From AISC Table B4.1, case 10 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29000}{50}} = 35.88$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{F_y}}$ , the web is slender.

For the flange,

$$\frac{b_f}{2t_f} = 7.76 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29000}{50}} = 13.49 \therefore \text{ flange is not slender}$$

Because the web is a stiffened element, $Q_s = 1.0$, and $Q_a$ must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where $b/t$ is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 39.46$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29000}{39.46}} = 40.39$$

Since $\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$ , local buckling must be considered. From AISC Equation E7-17, the reduced width of the web is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

$$= 1.92(0.550)\sqrt{\frac{29000}{39.46}}\left[1 - \frac{0.34}{54.5}\sqrt{\frac{29000}{39.46}}\right] = 23.79 \text{ in.}$$

Unreduced width is

$$b = d - 2k_{des} = 32.9 - 2(1.44) = 30.02 \text{ in.}$$

Reduced area is

$$A_e = A_g - t_w(b - b_e) = 34.7 - 0.550(30.02 - 23.79) = 31.27 \text{ in.}^2$$

$$Q_a = \frac{A_e}{A_g} = \frac{31.27}{34.7} = 0.9012$$

$$Q = Q_sQ_a = 1.0(0.9012) = 0.9012$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.9012(50)}} = 119.5$$

$$\frac{KL}{r} = 56.90 < 119.5 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9012\left(0.658^{\frac{0.9012(50)}{88.40}}\right)50 = 36.40 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 36.40(34.7) = 1263 \text{ kips} \quad \therefore \text{ web local buckling controls.}$$

(a) LRFD solution:

$$\phi_c P_n = 0.90(1263) = 1137 \text{ kips}$$

The factored-load axial force is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.6 \times 625) + 1.6(0.4 \times 625) = 850.0 \text{ kips}$$

The factored-load end moments are

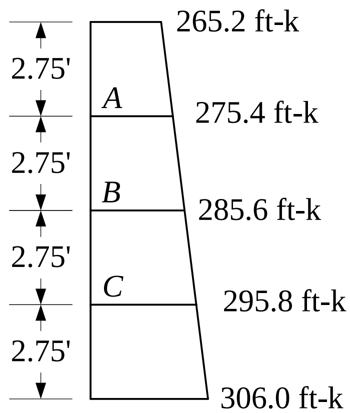
$$M_{top} = 1.2M_D + 1.6M_L = 1.2(0.6 \times 195) + 1.6(0.4 \times 195) = 265.2 \text{ ft-kips}$$

$$M_{bot} = 1.2M_D + 1.6M_L = 1.2(0.6 \times 225) + 1.6(0.4 \times 225) = 306.0 \text{ ft-kips}$$

From the beam design charts in Part 3 of the *Manual*, for $L_b = 11$ ft and $C_b = 1.0$,

$$\phi_b M_n = 1443 \text{ ft-kips}, \quad \phi_b M_p = 1560 \text{ ft-kips}.$$

Compute $C_b$:




$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(306.0)}{2.5(306.0) + 3(275.4) + 4(285.6) + 3(295.8)} = 1.056$$

For $C_b = 1.056$, $\phi_b M_n = 1.056(1443) = 1524 \text{ ft-kips} < \phi_b M_p = 1560$ ft-kips.

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{265.2}{306.0}\right) = 0.9467$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(5900)}{(11 \times 12)^2} = 9.692 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.00 P_u / P_{e1})} = \frac{0.9467}{1 - (850/96920)}$$

$$= 0.955 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(306) + 0 = 306 \text{ ft-kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{850}{1137} = 0.7476 > 0.2$$

$\therefore$ use Equation 6.3 (AISC Equation H1-1a)

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.7476 + \frac{8}{9}\left(\frac{306}{1524} + 0\right) = 0.9261$$

$$= 0.926 < 1.0 \qquad \text{(OK)}$$

<u>This member satisfies the AISC Specification</u>

(b) ASD solution:

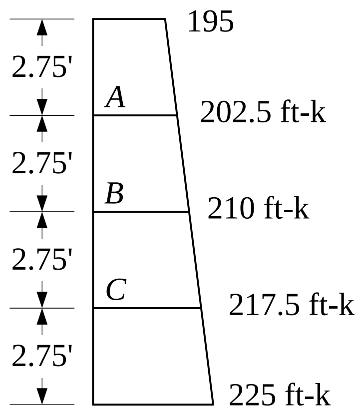
$$P_a = 625 \text{ kips}, \; M_{top} = 195 \text{ ft-kips}, \; M_{bot} = 225 \text{ ft-kips}$$

$$\frac{P_n}{\Omega_c} = \frac{1263}{1.67} = 756.3 \text{ kips}$$

From the design charts in Part 3 of the *Manual*, for $L_b = 11$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 962 \text{ ft-kips and } \frac{M_p}{\Omega_b} = 1040 \text{ ft-kips.}$$

Compute $C_b$:




$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(225)}{2.5(225) + 3(202.5) + 4(210) + 3(217.5)} = 1.056$$

For $C_b = 1.056$,

$$\frac{M_n}{\Omega_b} = 1.056(962) = 1016 \text{ ft-kips} < \frac{M_p}{\Omega_b}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{195}{225}\right) = 0.9467$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(5900)}{(11 \times 12)^2} = 9.692 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.60 P_a / P_{e1})} = \frac{0.9467}{1 - 1.60(625/96920)}$$

$$= 0.9566 < 1.0 \therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(225) + 0 = 225 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{625}{756.3} = 0.8264 > 0.2$$

$\therefore$ use Equation 6.5 (AISC Equation H1-1a)

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.8264 + \frac{8}{9}\left(\frac{225}{1016} + 0\right)$$

$$= 1.02 > 1.0 \quad \text{(N.G.)}$$

This member does not satisfy the AISC Specification.

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**6.6-9**

(a) LRFD Solution:

The service load moments at each end are

$$M_D = 0.33(150) = 49.50 \text{ ft-kips} \quad \text{and} \quad M_L = 0.67(150) = 100.5 \text{ ft-kips}$$

The factored-load moment at each end is

$$1.2 M_D + 1.6 M_L = 1.2(49.50) + 1.6(100.5) = 220.2 \text{ ft-kips}$$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(662)}{(1.0 \times 15 \times 12)^2} = 5848 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{220.2}{220.2}\right) = 0.2$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.0P_u/P_{e1})} = \frac{0.2}{1-(P_u/5848)}$$

Assume $B_1 = 1.0$ and check it later.

$$\therefore\ M_{ux} = B_1 M_{nt} = 1.0(220.2) = 220.2 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 15$ ft,

$$\phi_b M_n = 424 \text{ ft-kips for } C_b = 1.0, \text{ and } \phi_b M_p = 446 \text{ ft-kips}$$

From Figure 5.15e in the textbook, $C_b = 2.27$.

For $C_b = 2.27$, $\phi_b M_n = 2.27(424) = 962.5$ ft-kips

Since 962.5 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 446$ ft-kips

Determine the axial compressive design strength. From the column load tables with $KL = 15$ ft,

$$\phi_c P_n = 809 \text{ kips}$$

Assume that $\dfrac{P_u}{\phi_c P_n} > 0.2$ and use Equation 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$

$$\text{Let } \frac{P_u}{809} + \frac{8}{9}\left(\frac{220.2}{446} + 0\right) = 1.0, \text{ Solution is: } \{P_u = 454.0\}$$

Check assumptions.

$$\frac{P_u}{\phi_c P_n} = \frac{454.0}{809} > 0.2 \qquad \text{(OK)}$$

$$B_1 = \frac{C_m}{1-(1.0P_u/P_{e1})} = \frac{0.2}{1-(454.0/5848)} = 0.217 < 1.0$$

$\therefore$ use $B_1 = 1.0$ (as assumed; OK)

Let $1.2D + 1.6L = P_u$

$$1.2(0.33P) + 1.6(0.67P) = 454.0, \text{ Solution is: } \{P = 309.3\}$$

<u>$P = 309$ kips</u>

(b) ASD solution:

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(662)}{(1.0 \times 15 \times 12)^2} = 5848 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{150}{150}\right) = 0.2$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.2}{1-(1.60P_a/5848)}$$

Assume $B_1 = 1.0$ and check it later.

$$\therefore\ M_r = M_{ax} = B_1 M_{nt} = 1.0(150) = 150 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 15$ ft,

$$\frac{M_n}{\Omega_b} = 282 \text{ ft-kips for } C_b = 1.0, \text{ and } \frac{M_p}{\Omega_b} = 297 \text{ ft-kips}$$

From Figure 5.15e in the textbook, $C_b = 2.27$.

For $C_b = 2.27$, $\dfrac{M_n}{\Omega_b} = 2.27(282) = 640.1$ ft-kips

Since 640.1 ft-kips > $\dfrac{M_p}{\Omega_b}$, use $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 297$ ft-kips

From the column load tables with $KL = 15$ ft, $P_n/\Omega_c = 538$ kips

Assume that $\dfrac{P_a}{P_n/\Omega_c} > 0.2$ and use Equation 6.5 (AISC Eq. H1-1a):

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) \le 1.0$$

Let $\dfrac{P_a}{538} + \dfrac{8}{9}\left(\dfrac{150}{297} + 0\right) = 1.0$, Solution is: $\{P_a = 296\}$

Check assumptions.

$$\frac{P_a}{P_n/\Omega_c} = \frac{296}{538} > 0.2 \qquad \text{(OK)}$$

$$B_1 = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.2}{1-1.60(296)/5848} = 0.218 < 1.0$$

$\therefore$ use $B_1 = 1.0$ (as assumed; OK) $\underline{P = 296 \text{ kips}}$

---

**6.6-10**

(a) LRFD solution:

$$P_u = 1.2(0.25 \times 66) + 1.6(0.75 \times 66) = 99.0 \text{ kips}$$

$$Q_u = 1.2(0.25 \times 6) + 1.6(0.75 \times 6) = 9.0 \text{ kips}$$

$$M_{nt} = \frac{Q_u L}{4} = \frac{9(12)}{4} = 27.0 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\phi_b M_n = 71 \text{ ft-kips}, \ \phi_b M_p = 86.6 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\phi_b M_n = 1.32(71) = 93.72 \text{ ft-kips} > \phi_b M_p \therefore \text{ use } \phi_b M_n = \phi_b M_p = 86.6 \text{ ft-kips}$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(82.7)}{(12 \times 12)^2} = 1142 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(99.0/1142)} = 1.095$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.095(27.0) + 0 = 29.57 \text{ ft-kips}$$

Compressive strength: A W8 × 24 is not in the column load tables, so its axial compressive strength must be computed. The shape is not slender for compression (no footnote).

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(12 \times 12)}{1.61} = 89.44 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since 89.44 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(89.44)^2} = 35.78 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/35.78)}(50) = 27.86 \text{ ksi}$$

The nominal strength is $P_n = F_{cr} A_g = 27.86(7.08) = 197.2$ kips

$$\phi_c P_n = 0.90(197.2) = 177.5 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{99.0}{177.5} = 0.5577 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5577 + \frac{8}{9}\left(\frac{29.57}{86.6} + 0\right)$$

$$= 0.861 < 1.0 \quad \text{(OK)}$$

<u>Member is satisfactory.</u>

(b) ASD solution:

$$P_a = 66 \text{ kips}, \qquad M_{nt} = \frac{Q_a L}{4} = \frac{6(12)}{4} = 18 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 12$ ft,

$$\frac{M_n}{\Omega_b} = 47.5 \text{ ft-kips}, \quad \frac{M_p}{\Omega_b} = 57.6 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\frac{M_n}{\Omega_b} = 1.32(47.5) = 62.7 \text{ ft-kips} > \frac{M_p}{\Omega_b} \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 57.6 \text{ ft-kips}$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(82.7)}{(12 \times 12)^2} = 1142 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60 P_a/P_{e1})} = \frac{1.0}{1-1.60(53/1142)} = 1.08$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.08(18) + 0 = 19.44 \text{ ft-kips}$$

Compressive strength: A W8 × 24 is not in the column load tables, so its axial compressive strength must be computed. From part (a),

$$P_n = 197.2 \text{ kips} \quad \Rightarrow \quad \frac{P_n}{\Omega_c} = \frac{197.2}{1.67} = 118.1 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{66}{118.1} = 0.4488 > 0.2$$

$\therefore$ use Equation 6.5 (AISC Equation H1-1a)

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4488 + \frac{8}{9}\left(\frac{19.44}{57.6} + 0\right)$$

$$= 0.749 < 1.0 \quad \text{(OK)}$$

<u>Member is satisfactory.</u>

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**<u>6.6-11</u>**

$$P_u = 602 \text{ kips}$$

$$M_{ntx} = M_{ux} = 420 \text{ ft-kips}$$

$$M_{nty} = M_{uy} = 150 \text{ ft-kips}$$

From the $Z_x$ Table, $L_p = 14.1 \text{ ft} > L_b = 13$ ft, therefore the moment strength is

$$\phi_b M_{nx} = \phi_b M_{px} = 975 \text{ ft-kips}$$

From the $Z_y$ table, $\phi_b M_{ny} = \phi_b M_{py} = 499$ ft-kips.

Determine the compressive strength. For $KL = 1.0(13) = 13$ feet, the axial compressive design strength from the column load tables is $\phi_c P_n = 1720$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{602}{1720} = 0.35 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.35 + \frac{8}{9}\left(\frac{420}{975} + \frac{150}{499}\right)$$

$$= 1.00 \qquad \text{(OK)}$$

<u>The W14 × 145 is adequate.</u>

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**<u>6.6-12</u>**

(a) LRFD solution:

$$P_u = 1.2(20/2) + 1.6(20/2) = 28.0 \text{ kips}$$

$$Q_u = 1.2(40/2) + 1.6(40/2) = 56.0 \text{ kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.060) = 0.072 \text{ kips/ft}$$

$$M_{ntx} = \frac{(4/5)Q_u L}{4} + \frac{w_{ux} L^2}{8} = \frac{(4/5)(56.0)(12)}{4} + \frac{0.072(12)^2}{8}$$

$$= 135.7 \text{ ft-kips}$$

$$M_{nty} = \frac{(3/5)Q_u L}{4} = \frac{(3/5)(56.0)(12)}{4} = 100.8 \text{ ft-kips}$$

Strong-axis bending strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\phi_b M_{nx} = 309 \text{ ft-kips}, \ \ \phi_b M_{px} = 320 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\phi_b M_{nx} = 1.32(309) = 407.9 \text{ ft-kips} > \phi_b M_{px}$$

$$\therefore \text{ use } \phi_b M_{nx} = \phi_b M_{px} = 320 \text{ ft-kips}$$

Weak-axis bending strength: From the $Z_y$ tables, $\phi_b M_{ny} = \phi_b M_{py} = 150$ ft-kips

Compute the amplified moments. Use $C_{mx} = C_{my} = 1.0$.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(394)}{(12 \times 12)^2} = 5438 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{C_{mx}}{1 - (1.00 P_u / P_{e1x})} = \frac{1.0}{1 - (28.0/5438)} = 1.005$$

$$M_{ux} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.005(135.7) + 0 = 136.4 \text{ ft-kips}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_1 L)^2} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(134)}{(12 \times 12)^2} = 1850 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - (\alpha P_r / P_{e1y})} = \frac{C_{my}}{1 - (1.00 P_u / P_{e1y})} = \frac{1.0}{1 - (28.0/1854)} = 1.015$$

$$M_{uy} = B_1 M_{nty} + B_2 M_{\ell ty} = 1.015(100.8) + 0 = 102.3 \text{ ft-kips}$$

Compressive strength: From the column load tables, for a W10 × 68 with $KL = 12$ ft,

$$\phi_c P_n = 714 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{28.0}{714} = 3.922 \times 10^{-2} < 0.2 \therefore \text{ use Eq. 6.4 (AISC Eq. H1-1b)}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.03922}{2} + \left(\frac{136.4}{320} + \frac{102.3}{150}\right)$$

$$= 1.13 > 1.0 \text{ (N.G.)}$$

Member is unsatisfactory.

(b) ASD Solution

$$P_a = 20 \text{ kips}, \qquad Q_a = 40 \text{ kips}$$

$$w_a = w_D = 0.060 \text{ kips/ft}$$

$$M_{ntx} = \frac{(4/5)Q_a L}{4} + \frac{w_{ax}L^2}{8} = \frac{(4/5)(40)(12)}{4} + \frac{0.060(12)^2}{8} = 97.08 \text{ ft-kips}$$

$$M_{nty} = \frac{(3/5)Q_a L}{4} = \frac{(3/5)(40)(12)}{4} = 72.0 \text{ ft-kips}$$

Strong-axis bending strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\frac{M_{nx}}{\Omega_b} = 206 \text{ ft-kips}, \quad \frac{M_{px}}{\Omega_b} = 320 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\frac{M_{nx}}{\Omega_b} = 1.32(206) = 271.9 \text{ ft-kips} < \frac{M_{px}}{\Omega_b} \therefore \text{ use } \frac{M_{nx}}{\Omega_b} = 271.9 \text{ ft-kips}$$

Weak-axis bending strength: From the $Z_y$ tables, $\dfrac{M_{ny}}{\Omega_b} = \dfrac{M_{py}}{\Omega_b} = 100$ ft-kips

Compute the amplified moments. Use $C_{mx} = C_{my} = 1.0$.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(394)}{(12 \times 12)^2} = 5438 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1 - (1.60 P_a/P_{e1x})} = \frac{1.0}{1 - 1.60(20)/5438} = 1.006$$

$$M_{ax} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.006(97.08) + 0 = 97.66 \text{ ft-kips}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_1 L)^2} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(134)}{(12 \times 12)^2} = 1850 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - (\alpha P_r/P_{e1y})} = \frac{C_{my}}{1 - (1.60 P_a/P_{e1y})} = \frac{1.0}{1 - 1.60(20)/1850} = 1.018$$

$$M_{ay} = B_1 M_{nty} + B_2 M_{\ell ty} = 1.018(72.0) + 0 = 73.30 \text{ ft-kips}$$

Find the compressive strength. For $KL = 12$ feet, the axial compressive strength from the column load tables is $\dfrac{P_n}{\Omega_c} = 475$ kips.

Check the interaction formula:

$$\frac{P_a}{P_n/\Omega_c} = \frac{20}{475} = 4.211 \times 10^{-2} < 0.2$$

$\therefore$ use Equation 6.6 (AISC Equation H1-1b)

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{0.04211}{2} + \left(\frac{97.66}{271.9} + \frac{73.30}{100}\right)$$

$$= 1.11 > 1.0 \qquad \text{(N.G.)}$$

Member is unsatisfactory.

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**6.6-13**

Since a W21 × 93 is not in the column load tables, the axial compressive design strength must be computed.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{0.65(20 \times 12)}{1.84} = 84.78$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29000)}{(84.78)^2} = 39.82 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since $KL/r = 84.78 < 113.4,$ use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/39.82)}(50) = 29.56 \text{ ksi}$$

$$P_n = F_{cr}A_g = 29.56(27.3) = 807.0 \text{ kips}$$

(a) LRFD solution:

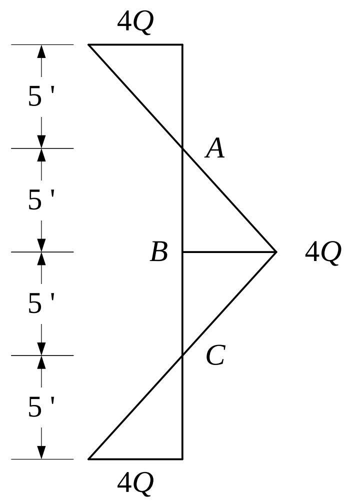
$$P_u = 1.2(70) + 1.6(200) = 404.0 \text{ kips}$$

$$\text{Maximum moment} = M_{nt} = \frac{Q_u L}{8} = \frac{1.6Q(20)}{8} = 4Q$$

Determine the moment strength. From the beam design charts, for $L_b = 20$ ft and $C_b = 1,$

$$\phi_b M_n = 534 \text{ ft-kips}$$

Compute $C_b$:




$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(4Q)}{2.5(4Q) + 3(0) + 4(4Q) + 3(0)} = 1.923$$

For $C_b = 1.923$,

$$\phi_b M_n = 1.923(534) = 1027 \text{ ft-kips} > \phi_b M_p = 829 \text{ ft-kips}$$

$\therefore$ use $\phi_b M_n = \phi_b M_p = 829$ ft-kips

Compute the amplified moment. Use $C_m = 1.0$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(2070)}{(0.65 \times 20 \times 12)^2} = 2.435 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{C_{mx}}{1 - (1.00 P_u / P_{e1x})} = \frac{1.0}{1 - (404/24350)} = 1.017$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.017(4Q) + 0 = 4.068Q \text{ ft-kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{404}{0.90(807)} = 0.5562 > 0.2 \quad \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5562 + \frac{8}{9}\left(\frac{4.068Q}{829} + 0\right)$$

Let

$$0.5562 + \frac{8}{9}\left(\frac{4.068Q}{829} + 0\right) = 1, \text{ Solution is: } \{Q = 101.7\}$$

<u>Maximum $Q = 102$ kips</u>

(b)

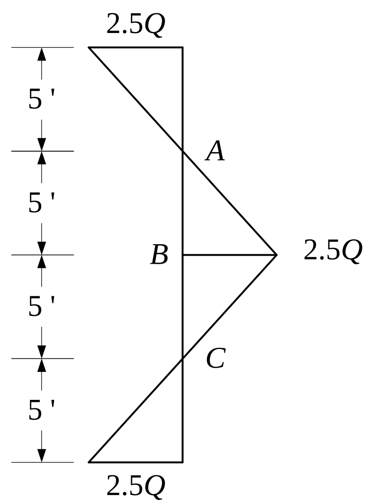
$$P_a = D + L = 70 + 200 = 270 \text{ kips}$$

Maximum moment = $M_{nt} = \dfrac{Q_a L}{8} = \dfrac{Q(20)}{8} = 2.5Q$

Determine the moment strength. From the beam design charts, for $L_b = 20$ ft and $C_b = 1$,

$$\frac{M_n}{\Omega_b} = 356 \text{ ft-kips}$$

Compute $C_b$:




$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(2.5Q)}{2.5(2.5Q) + 3(0) + 4(2.5Q) + 3(0)} = 1.923$$

For $C_b = 1.923$,

$$\frac{M_n}{\Omega_b} = 1.923(356) = 684.6 \text{ ft-kips} > \frac{M_p}{\Omega_b} = 551 \text{ ft-kips}$$

$\therefore$ use $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 551$ ft-kips

Compute the amplified moment. Use $C_m = 1.0$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(2070)}{(0.65 \times 20 \times 12)^2} = 2.435 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1 - (1.60P_a/P_{e1x})} = \frac{1.0}{1 - 1.60(270/24350)}$$

$$= 1.018$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.018(2.5Q) + 0 = 2.545Q \text{ ft-kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{270}{807/1.67} = 0.5587 > 0.2 \;\therefore\; \text{use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.5587 + \frac{8}{9}\left(\frac{2.545Q}{551} + 0\right)$$

Let

$$0.5587 + \frac{8}{9}\left(\frac{2.545Q}{551} + 0\right) = 1, \text{ Solution is: } \{Q = 107.5\}$$

<u>Maximum $Q = 108$ kips</u>

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**6.7-1**

Determine the axial compressive design strength. Use $K_x$ for the unbraced condition.

$$\frac{K_xL}{r_x/r_y} = \frac{1.7(14)}{2.44} = 9.754 \text{ ft} < K_yL = 14 \text{ ft}$$

From the column load tables with $KL = 14$ ft, $\phi_cP_n = 701$ kips

$$\frac{P_u}{\phi_cP_n} = \frac{400}{701} = 0.5706 > 0.2 \;\therefore\; \text{use Eq. 6.3 (AISC Eq. H1-1a).}$$

Check the braced condition first. For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{24}{45}\right) = 0.3867$$

$$P_{e1} = \frac{\pi^2EI}{(K_1L)^2} = \frac{\pi^2EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(795)}{(1.0 \times 14 \times 12)^2} = 8062 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1 - (1.00P_u/P_{e1x})} = \frac{0.3867}{1 - (400/8062)}$$

$$= 0.407 < 1.0 \qquad \therefore \text{ use } B_1 = 1.0$$

Sway condition: use

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \dfrac{1.00(6000)}{40000}} = 1.176$$

The total amplified moment at the top is

$$M_{u\,top} = B_1M_{nt} + B_2M_{\ell t} = 1.0(45) + 1.176(40) = 92.04 \text{ ft-kips}$$

The total amplified moment at the bottom is

$$M_{u\,bot} = 1.0(24) + 1.176(95) = 135.7 \text{ ft-kips}$$

Use $M_u = 135.7$ ft-kips. Compute the moment strength. From the beam design charts with $L_b = 14$ ft,

$$\phi_bM_n = 431 \text{ ft-kips for } C_b = 1.0 \text{ and } \phi_bM_p = 472 \text{ ft-kips}$$

Using the total amplified moment, compute $C_b$:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135.7)}{2.5(135.7) + 3(35.11) + 4(21.83) + 3(78.77)} = 2.208$$

For $C_b = 2.208$, $\phi_b M_n = 2.208(431) = 951.7$ ft-kips

Since 951.7 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 472$ ft-kips

Eq. 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5706 + \frac{8}{9}\left(\frac{135.7}{472} + 0\right)$$

$$= 0.826 < 1.0 \quad \text{(OK)}$$

<u>Member is satisfactory.</u>

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**<u>6.7-2</u>**

(a) LRFD solution

Determine the axial compressive design strength. Use $K_x$ for the unbraced condition.

$$\frac{K_x L}{r_x/r_y} = \frac{1.2(16)}{2.44} = 7.869 \text{ ft} < K_y L = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $\phi_c P_n = 697$ kips

Check load combination 2. For the braced condition,

$$P_{nt} = 1.2P_D + 1.6P_L = 1.2(120) + 1.6(240) = 528.0 \text{ kips}$$

$$M_{nt}(\text{top}) = 1.2(15) + 1.6(40) = 82.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 1.2(18) + 1.6(48) = 98.4 \text{ ft-kips}$$

For the axis of bending, $C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{82.0}{98.4}\right) = 0.2667$

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(881)}{(0.85 \times 16 \times 12)^2} = 9467 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.00P_u/P_{e1})} = \frac{0.2667}{1 - (528/9467)}$$

$$= 0.2825 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(98.4) + 0 = 98.4 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 16$ ft,

$\phi_b M_n = 462$ ft-kips for $C_b = 1.0$ and $\phi_b M_p = 521$ ft-kips

Compute $C_b$:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(98.4)}{2.5(98.4) + 3(36.9) + 4(8.20) + 3(53.3)} = 2.239$$

For $C_b = 2.239, \phi_b M_n = 2.239(462) = 1034$ ft-kips

Since 1034 ft-kips $> \phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 521$ ft-kips

$$\frac{P_u}{\phi_c P_n} = \frac{528}{697} = 0.7575 > 0.2 \ \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a).}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.7575 + \frac{8}{9}\left(\frac{98.4}{521} + 0\right)$$

$$= 0.925 < 1.0 \qquad \text{(OK)}$$

Check load combination 4: $1.2D + 1.0W + 0.5L$

For the braced condition,

$$P_{nt} = 1.2P_D + 0.5P_L = 1.2(120) + 0.5(240) = 264.0 \text{ kips}$$

$$M_{nt}(\text{top}) = 1.2(15) + 0.5(40) = 38.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 1.2(18) + 0.5(48) = 45.6 \text{ ft-kips}$$

For the sway condition, combination 4 is $1.0W$.

$$M_{\ell t}(\text{top}) = M_{\ell t}(\text{bot}) = 1.0W = 130 \text{ ft-kips}$$

$$P_{\ell t} = 1.0W = 1.0(30) = 30 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{38.0}{45.6}\right) = 0.2667$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - 1.0[(P_{nt} + P_{\ell t})/P_{e1}]}$$

$$= \frac{0.2667}{1 - 1.0[(264.0 + 30)]/9467} = 0.275 < 1.0 \ \therefore \text{ use } B_1 = 1.0$$

For the sway condition, for the axis of bending,

$$P_{e2} = \frac{\pi^2 EI}{(K_2 L)^2} = \frac{\pi^2 (29000)(881)}{(1.2 \times 16 \times 12)^2} = 4750 \text{ kips}$$

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \approx \frac{1}{1 - \dfrac{\alpha P_{nt}}{P_{e2}}} = \frac{1}{1 - \dfrac{1.0(264)}{4750}} = 1.059$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(45.6) + 1.059(130) = 183.3 \text{ ft-kips}$$

$$P_u = P_{nt} + B_2 P_{\ell t} = 264.0 + 1.059(30) = 295.8$$

$$\frac{P_u}{\phi_c P_n} = \frac{295.8}{697} = 0.4244 > 0.2 \therefore \text{ use AISC Eq. H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.4244 + \frac{8}{9}\left(\frac{183.3}{521} + 0\right)$$

$$= 0.737 < 1.0 \quad \text{(OK)}$$

(Load combination 2 controls.) Member is adequate.

(b) ASD solution

Determine the axial compressive design strength. Use $K_x$ for the unbraced condition.

$$\frac{K_x L}{r_x/r_y} = \frac{1.2(16)}{2.44} = 7.869 \text{ ft} < K_y L = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $P_n/\Omega_c = 464$ kips

Check load combination 2. For the braced condition,

$$P_{nt} = P_D + P_L = 120 + 240 = 360 \text{ kips}$$

$$M_{nt}(\text{top}) = 15 + 40 = 55 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 18 + 48 = 66 \text{ ft-kips}$$

For the axis of bending, $C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{55}{66}\right) = 0.2667$

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(881)}{(0.85 \times 16 \times 12)^2} = 9467 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{0.2667}{1 - 1.60(360)/9467}$$

$$= 0.2840 < 1.0 \therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(66) + 0 = 66 \text{ ft-kips}$$

$$P_a = P_{nt} + B_2 P_{\ell t} = 360 + 0 = 360$$

Compute the moment strength. From the beam design charts with $L_b = 16$ ft,

$$\frac{M_n}{\Omega_b} = 308 \text{ ft-kips for } C_b = 1.0 \text{ and } \frac{M_p}{\Omega_b} = 347 \text{ ft-kips}$$

For $C_b = 2.239$ (see part a), $\frac{M_n}{\Omega_b} = 2.239(308) = 689.6$ ft-kips

Since 689.6 ft-kips $> \frac{M_p}{\Omega_b}$, use $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 347$ ft-kips

$$\frac{P_a}{P_n/\Omega_c} = \frac{360}{464} = 0.7759 > 0.2 \therefore \text{ use Equation 6.5 (AISC Eq. H1-1a):}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.7759 + \frac{8}{9}\left(\frac{66}{347} + 0\right)$$

$$= 0.945 < 1.0 \quad \text{(OK)}$$

Check load combination 6a: $D + 0.75L + 0.75(0.6W)$

For the braced condition,

$$P_{nt} = P_D + 0.75P_L = 120 + 0.75(240) = 300.0 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{300}{464} = 0.6466 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$M_{nt}(\text{top}) = 15 + 0.75(40) = 45.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 18 + 0.75(48) = 54.0 \text{ ft-kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{45}{54}\right) = 0.2667$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{0.2667}{1 - 1.60(300)/9467}$$

$$= 0.281 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

For the sway condition,

$$M_{\ell t}(\text{top}) = M_{\ell t}(\text{bot}) = 0.75(0.6)W = 0.75(0.6)(130) = 58.5 \text{ ft-kips}$$

$$P_{\ell t} = 0.75(0.6)(30) = 13.5 \text{ kips}$$

For the axis of bending,

$$P_{e2} = \frac{\pi^2 EI}{(K_2L)^2} = \frac{\pi^2(29000)(881)}{(1.2 \times 16 \times 12)^2} = 4750 \text{ kips}$$

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \approx \frac{1}{1 - \dfrac{\alpha P_{\ell t}}{P_{e2}}} = \frac{1}{1 - \dfrac{1.6(13.5)}{4750}} = 1.005$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.0(45.6) + 1.005(58.5) = 104.4 \text{ ft-kips}$$

$$P_a = P_{nt} + B_2P_{\ell t} = 300.0 + 1.005(13.5) = 313.6 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{313.6}{464} = 0.6759 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

Equation 6.5 (AISC Eq. H1-1a):

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.6759 + \frac{8}{9}\left(\frac{104.4}{347} + 0\right)$$

$$= 0.943 < 1.0 \quad \text{(OK)}$$

<u>Member is adequate.</u>

**6.8-1**

(a) LRFD solution

The factored axial load is

$$P_u = 1.2(0.30 \times 236) + 1.6(0.70 \times 236) = 349.3 \text{ kips}$$

The factored moment is

$$M_{ntx} = 1.2(0.30 \times 168) + 1.6(0.70 \times 168) = 248.6 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(248.6) = 248.6 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 15 \text{ ft.}$$

From Table 6-1, Try a W12 × 79, with $p = 1.24 \times 10^{-3}$ and $b_x = 2.11 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_u = (1.24 \times 10^{-3})(349.3) = 0.4331 > 0.2 \quad \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check (remember that $B_1$ has not yet been computed and $C_b$ has not been accounted for)

$$\begin{aligned} pP_r + b_xM_{rx} + b_yM_{ry} &= pP_u + b_xM_{ux} + b_yM_{uy} \\ &= (1.24 \times 10^{-3})(349.3) + (2.11 \times 10^{-3})(248.6) + 0 \\ &= 0.958 < 1.0 \qquad \text{(OK)} \end{aligned}$$

Calculate $B_1$:

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(662)}{(15 \times 12)^2} = 5848 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{349.3}{5848}} = 0.638 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.)

From Figure 5.15g, $C_b = 1.67$. Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{2.11 \times 10^{-3}}{1.67} = 1.26 \times 10^{-3}$$

For $L_b = 0, b_x = 1.99 \times 10^{-3}$

Use $b_x = 1.99 \times 10^{-3}$ (the larger value)

Check Equation 6.9: $p = 1.24 \times 10^{-3},\ b_x = 1.99 \times 10^{-3}$

$$pP_u + b_x M_{ux} + b_y M_{uy} = (1.24 \times 10^{-3})(349.3) + (1.99 \times 10^{-3})(248.6)$$

$$= 0.928 < 1.0 \quad \text{(OK)}$$

Use a W12 × 79

(b) ASD Solution

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x} M_{ntx} = 1.0(168) = 168 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 15 \text{ ft.}$$

From Table 6-1, Try a W12 × 79, with $p = 1.86 \times 10^{-3}$ and $b_x = 3.16 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_a = (1.86 \times 10^{-3})(236) = 0.4390 > 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check (remember that $B_1$ has not yet been computed and $C_b$ has not been accounted for),

$$pP_r + b_x M_{rx} + b_y M_{ry} = pP_a + b_x M_{ax} + b_y M_{ay}$$

$$= (1.86 \times 10^{-3})(236) + (3.16 \times 10^{-3})(168) + 0$$

$$= 0.970 < 1.0 \quad \text{(OK)}$$

Calculate $B_1$:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29000)(662)}{(15 \times 12)^2} = 5848 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60 P_a}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{1.60(236)}{5848}} = 0.6414 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as}$$

assumed

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.)

From Figure 5.15g, $C_b = 1.67$. Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{3.16 \times 10^{-3}}{1.67} = 1.89 \times 10^{-3}$$

For $L_b = 0, b_x = 2.99 \times 10^{-3}$

Use $b_x = 2.99 \times 10^{-3}$ (the larger value)

Check Equation 6.9: $p = 1.86 \times 10^{-3},\ b_x = 2.99 \times 10^{-3}$

$$pP_a + b_xM_{ax} + b_yM_{ay} = (1.86 \times 10^{-3})(236) + (2.99 \times 10^{-3})(168)$$

$$= 0.941 < 1.0 \qquad \text{(OK)}$$

Use a W12 × 79

---

**6.8-2**

The factored axial load is

$P_u = 400$ kips

The factored moments are

$M_{ux}$ (top) = 182 ft-kips

$M_{ux}$ (bot) = 140 ft-kips

The effective length for compression and the unbraced length for bending are the same:

$KL = L_b = 12$ ft.

From Table 6-1, Try a W10 × 77, with $p = 1.23 \times 10^{-3}$ and $b_x = 2.50 \times 10^{-3}$.

Determine which interaction equation to use:

$pP_r = pP_u = (1.23 \times 10^{-3})(399) = 0.4908 > 0.2 \therefore$ Equation 6.9 controls.

As a preliminary check (remember that $C_b$ has not been accounted for)

$$pP_r + b_xM_{rx} + b_yM_{ry} = pP_u + b_xM_{ux} + b_yM_{uy}$$

$$= (1.23 \times 10^{-3})(399) + (2.50 \times 10^{-3})(182) + 0$$

$$= 0.946 < 1.0 \qquad \text{(OK)}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.)

Compute $C_b$ and modify $b_x$ to account for $C_b$.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(182)}{2.5(182) + 3(150.5) + 4(161) + 3(171.5)} = 1.102$$

$$\frac{b_x}{C_b} = \frac{2.50 \times 10^{-3}}{1.102} = 2.27 \times 10^{-3}$$

For $L_b = 0,\ b_x = 2.43 \times 10^{-3}$

Use $b_x = 2.43 \times 10^{-3}$ (the larger value)

Check Equation 6.9: $p = 1.23 \times 10^{-3},\ b_x = 2.43 \times 10^{-3}$

$$pP_u + b_x M_{ux} + b_y M_{uy} = (1.23 \times 10^{-3})(399) + (2.43 \times 10^{-3})(182)$$

$$= 0.933 < 1.0 \qquad \text{(OK)}$$

<u>Use a W10 × 77</u>

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**<u>6.8-3</u>**

(a) LRFD Solution

$$P_u = 1.2(0.5 \times 45) + 1.6(0.5 \times 45) = 63.0 \text{ kips}$$

$$w_u = 1.2(0.5 \times 5) + 1.6(0.5 \times 5) = 7.0 \text{ kips/ft}$$

$$M_{ntx} = \frac{1}{8}(7.0)(12)^2 = 126.0 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x} M_{ntx} = 1.0(126) = 126 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 39, with $p = 2.84 \times 10^{-3}$ and $b_x = 5.67 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_u = (2.84 \times 10^{-3})(63) = 0.1789 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check (remember that $B_1$ has not yet been computed and $C_b$ has not been accounted for)

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) = 0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(2.84 \times 10^{-3})(63) + \frac{9}{8}((5.67 \times 10^{-3})(126) + 0)$$

$$= 0.893 < 1.0 \qquad \text{(OK)}$$

Calculate $B_1$: Use $C_{mx} = 1.0$ (transversely-loaded member)

$$P_{e1x} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29000)(209)}{(12 \times 12)^2} = 2885 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{1.0}{1 - \dfrac{63}{2885}} = 1.022$$

$$M_{ux} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.022(126) + 0 = 128.8 \text{ ft-kips}$$

From Figure 5.15 in the textbook, $C_b = 1.14$. Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{5.67 \times 10^{-3}}{1.14} = 4.97 \times 10^{-3}$$

For $L_b = 0,\ b_x = 5.06 \times 10^{-3}$

Use $b_x = 5.06 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $p = 2.84 \times 10^{-3},\ b_x = 5.06 \times 10^{-3}$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(2.84 \times 10^{-3})(63) + \frac{9}{8}((5.06 \times 10^{-3})(126) + 0)$$

$$= 0.807 < 1.0 \qquad \text{(OK)}$$

Try the next lighter shape, a W10 × 33 with $p = 3.42 \times 10^{-3}$ and $b_x = 7.00 \times 10^{-3}$.

$pP_r = pP_u = (3.42 \times 10^{-3})(63) = 0.2155 > 0.2 \therefore$ Equation 6.9 controls.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(171)}{(12 \times 12)^2} = 2360 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{1.0}{1 - \dfrac{63}{2360}} = 1.027$$

$$M_{ux} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.027(126) + 0 = 129.4 \text{ ft-kips}$$

Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{7.00 \times 10^{-3}}{1.14} = 6.14 \times 10^{-3}$$

For $L_b = 0,\ b_x = 6.11 \times 10^{-3}$

Use $b_x = 6.14 \times 10^{-3}$ (the larger value)

Check Equation 6.9: $p = 3.42 \times 10^{-3},\ b_x = 6.14 \times 10^{-3}$

$$pP_u + b_x M_{ux} + b_y M_{uy} = (3.42 \times 10^{-3})(63) + (6.14 \times 10^{-3})(126)$$

$$= 0.989 < 1.0 \qquad \text{(OK)}$$

Use a W10 × 33

(b) ASD Solution

$$P_a = 45 \text{ kips}, \qquad M_{ntx} = \frac{1}{8}(5)(12)^2 = 90.0 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x} M_{ntx} = 1.0(90) = 90 \text{ ft-kips}$$

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 33, with $p = 5.15 \times 10^{-3}$ and $b_x = 10.5 \times 10^{-3}$.

$$pP_r = pP_a = (5.15 \times 10^{-3})(45) = 0.2318 > 0.2 \;\therefore\; \text{Equation 6.9 controls.}$$

preliminary check:

$$pP_a + b_xM_{ax} + b_yM_{ay} = (5.15 \times 10^{-3})(45) + (10.5 \times 10^{-3})(90)$$

$$= 1.18 > 1.0 \qquad \text{(N.G.)}$$

Try a W10 × 39. $\quad p = 4.28 \times 10^{-3}, b_x = 8.53 \times 10^{-3}$

$$pP_r = pP_a = (4.28 \times 10^{-3})(45) = 0.1926 < 0.2 \;\therefore\; \text{Equation 6.10 controls.}$$

As a preliminary check (remember that $B_1$ has not yet been computed and $C_b$ has not been accounted for)

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(4.28 \times 10^{-3})(45) + \frac{9}{8}((8.53 \times 10^{-3})(90) + 0)$$

$$= 0.960 < 1.0 \qquad \text{(OK)}$$

Calculate $B_1$: Use $C_{mx} = 1.0$ (transversely-loaded member)

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(209)}{(12 \times 12)^2} = 2885 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{1.0}{1 - \dfrac{1.60(45)}{2885}} = 1.026$$

$$M_{ax} = B_{1x}M_{ntx} = 1.026(90) = 92.34 \text{ ft-kips}$$

From Figure 5.15 in the textbook, $C_b = 1.14$. Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{8.53 \times 10^{-3}}{1.14} \times 10^{-3} = 7.48 \times 10^{-6}$$

For $L_b = 0$, $b_x = 7.61 \times 10^{-3}$

Use $b_x = 7.61 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $\quad p = 4.28 \times 10^{-3},\ b_x = 7.61 \times 10^{-3}$

$$0.5pP_a + \frac{9}{8}(b_xM_{ax} + b_yM_{ay}) = 0.5(4.28 \times 10^{-3})(45) + \frac{9}{8}((7.61 \times 10^{-3})(90) + 0)$$

$$= 0.867 < 1.0 \qquad \text{(OK)}$$

<u>Use a W10 × 39</u>

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**6.8-4**

$P_u = 140$ kips

The factored moments at the top are

$M_{ntx} = M_{nty} = 150$ ft-kips

The factored moments at the bottom are

$M_{ntx} = M_{nty} = 75$ ft-kips

Check the W10s. Try a W10 ×112. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 0.983 \times 10^{-3},\ b_x = 1.69 \times 10^{-3},\ \ b_y = 3.43 \times 10^{-3}$$

$$pP_u = (0.983 \times 10^{-3})(140) = 0.1376 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) = 0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(0.983 \times 10^{-3})(140) + \frac{9}{8}((1.69 \times 10^{-3})(150) + (3.43 \times 10^{-3})(150))$$

$$= 0.933 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 96. From Table 6-1,

$$p = 1.05 \times 10^{-3},\ b_x = 1.70 \times 10^{-3},\ \ b_y = 3.51 \times 10^{-3}$$

$$pP_u = (1.05 \times 10^{-3})(139.5) = 0.1465 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.05 \times 10^{-3})(140) + \frac{9}{8}((1.70 \times 10^{-3})(150) + (3.51 \times 10^{-3})(150))$$

$$= 0.953 < 1.0 \qquad \text{(OK)}$$

Check the W14s. Try a W14 × 90. From Table 6-1,

$$p = 1.02 \times 10^{-3},\ b_x = 1.57 \times 10^{-3},\ \ b_y = 3.26 \times 10^{-3}$$

$$pP_u = (1.02 \times 10^{-3})(140) = 0.1428 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.02 \times 10^{-3})(140) + \frac{9}{8}((1.57 \times 10^{-3})(150) + (3.26 \times 10^{-3})(150))$$

$$= 0.887 < 1.0 \qquad \text{(OK)}$$

Since this shape is adequate for $C_b = 1.0$, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.

<u>Use a W14 × 90</u>

**6.8-5**

(a) LRFD Solution

$$P_u = 1.2(135) + 1.6(415) = 826.0 \text{ kips}$$

The factored moments are

$$M_{ntx}\text{ (top)} = 1.2(90) + 1.6(270) = 540.0 \text{ ft-kips}$$

$$M_{ntx}\text{ (bot)} = 1.2(30) + 1.6(90) = 180.0 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(540) = 540 \text{ ft-kips}$$

Try a W14 × 159. From Table 6-1, with $KL = L_b = 20$ ft,

$$p = 0.619 \times 10^{-3},\ b_x = 0.863 \times 10^{-3}$$

$$pP_u = (0.619 \times 10^{-3})(826) = 0.5113 > 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$pP_r + b_xM_{rx} + b_yM_{ry} = pP_u + b_xM_{ux} + b_yM_{uy}$$

$$= (0.619 \times 10^{-3})(826) + (0.863 \times 10^{-3})(540) + 0$$

$$= 0.977 < 1.0 \qquad \text{(OK)}$$

Calculate $B_1$ for the axis of bending:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{180}{540}\right) = 0.4667$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(1900)}{(0.8 \times 20 \times 12)^2} = 1.475 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.4667}{1 - \dfrac{826}{14750}}$$

$$= 0.4944 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.)

Compute $C_b$ and modify $b_x$ to account for $C_b$.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(540)}{2.5(540) + 3(0) + 4(180) + 3(360)} = 2.143$$

$$\frac{b_x}{C_b} = \frac{0.863 \times 10^{-3}}{2.143} = 0.403 \times 10^{-3}$$

For $L_b = 0,\ b_x = 0.826 \times 10^{-3}$

Use $b_x = 0.826 \times 10^{-3}$ (the larger value)

$$pP_u + b_x M_{ux} + b_y M_{uy} = (0.619 \times 10^{-3})(826) + (0.826 \times 10^{-3})(540) + 0$$

$$= 0.957 < 1.0 \quad \text{(OK)}$$ Use a W14 × 159

(a) ASD Solution

The axial service load is $P_a = P_D + P_L = 135 + 415 = 550$ kips

The service-load moments are

$M_{ntx}$ (top) $= M_D + M_L = 90 + 270 = 360$ ft-kips

$M_{ntx}$ (bot) $= M_D + M_L = 30 + 90 = 120$ ft-kips

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x} M_{ntx} = 1.0(360) = 360 \text{ ft-kips}$$

Try a W14 × 159. From Table 6-1, with $KL = L_b = 20$ ft,

$$p = 0.931 \times 10^{-3},\ b_x = 1.30 \times 10^{-3}$$

$$pP_u = (0.931 \times 10^{-3})(550) = 0.5121 > 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$pP_r + b_x M_{rx} + b_y M_{ry} = pP_a + b_x M_{ax} + b_y M_{ay}$$

$$= (0.931 \times 10^{-3})(550) + (1.30 \times 10^{-3})(360) + 0$$

$$= 0.980 < 1.0$$

Calculate $B_1$:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{120}{360}\right) = 0.4667$$

$$P_{e1x} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29000)(1900)}{(0.8 \times 20 \times 12)^2} = 1.475 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60 P_a}{P_{e1x}}} = \frac{0.4667}{1 - \dfrac{1.60(550)}{14750}}$$

$$= 0.4963 < 1.0 \qquad \therefore\ B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of $C_b$ and an adjustment of $b_x$, are not necessary.)

Compute $C_b$ and modify $b_x$ to account for $C_b$.

$$C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(360)}{2.5(360) + 3(0) + 4(120) + 3(240)} = 2.143$$

$$\frac{b_x}{C_b} = \frac{1.30 \times 10^{-3}}{2.143} = 6.07 \times 10^{-4}$$

For $L_b = 0,\ b_x = 1.24 \times 10^{-3}$

Use $b_x = 1.24 \times 10^{-3}$ (the larger value)

Check Equation 6.9: $p = 0.931 \times 10^{-3},\ b_x = 1.24 \times 10^{-3}$

$$pP_a + b_xM_{ax} + b_yM_{ay} = (0.931 \times 10^{-3})(550) + (1.24 \times 10^{-3})(360)$$

$$= 0.958 \qquad \text{(OK)}$$

Use a W14 × 159

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**6.8-6**

(a) LRFD Solution

$$P_u = 1.2(92/2) + 1.6(92/2) = 128.8 \text{ kips}$$

The factored moments at the top are

$$M_{ntx} = 1.2(160/2) + 1.6(160/2) = 224.0 \text{ ft-kips}$$

$$M_{nty} = 1.2(24/2) + 1.6(24/2) = 33.6 \text{ ft-kips}$$

The factored moments at the bottom are

$$M_{ntx} = 1.2(214/2) + 1.6(214/2) = 299.6 \text{ ft-kips}$$

$$M_{nty} = 1.2(31/2) + 1.6(31/2) = 43.4 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ux} = B_{1x}M_{ntx} = 1.0(299.6) = 299.6 \text{ ft-kips}$$

$$M_{uy} = B_{1y}M_{nty} = 1.0(43.4) = 43.4 \text{ ft-kips}$$

Check the W10s. Try a W10 ×100. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.11 \times 10^{-3},\ b_x = 1.93 \times 10^{-3},\ b_y = 3.89 \times 10^{-3}$$

$$pP_u = (1.11 \times 10^{-3})(128.8) = 0.1430 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.11 \times 10^{-3})(128.8) + \frac{9}{8}((1.93 \times 10^{-3})(299.6) + (3.89 \times 10^{-3})(43.4))$$

$$= 0.912 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 79. From Table 6-1,

$$p = 1.28 \times 10^{-3},\ b_x = 2.13 \times 10^{-3},\ b_y = 4.37 \times 10^{-3}$$

$$pP_u = (1.28 \times 10^{-3})(128.8) = 0.1649 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.28 \times 10^{-3})(128.8) + \frac{9}{8}((2.13 \times 10^{-3})(299.6) + (4.37 \times 10^{-3})(43.4))$$

$$= 1.01 > 1.0 \qquad \text{(but close; retain this possibility)}$$

Check the W14s. Try a W14 × 82. From Table 6-1,

$$p = 1.44 \times 10^{-3},\ b_x = 1.92 \times 10^{-3},\ b_y = 5.29 \times 10^{-3}$$

$$pP_u = (1.44 \times 10^{-3})(128.8) = 0.1855 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.44 \times 10^{-3})(128.8) + \frac{9}{8}((1.92 \times 10^{-3})(299.6) + (5.29 \times 10^{-3})(43.4))$$

$$= 0.998 < 1.0 \qquad \text{(OK)}$$

Try a W12 × 79 (the lightest),

$$p = 1.28 \times 10^{-3},\ b_x = 2.13 \times 10^{-3},\ b_y = 4.37 \times 10^{-3}$$

Calculate $B_1$ for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{224.0}{299.6}\right) = 0.3009$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{33.6}{43.4}\right) = 0.2903$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(662)}{(0.8 \times 16 \times 12)^2} = 8031 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.3009}{1 - \dfrac{128.8}{8031}}$$

$$= 0.3058 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(216)}{(16 \times 12)^2} = 1677 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{P_u}{P_{e1y}}} = \frac{0.2903}{1 - \dfrac{128.8}{1677}}$$

$$= 0.3145 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(299.6)}{2.5(299.6) + 3(93.1) + 4(37.8) + 3(168.7)} = 2.222$$

$$\frac{b_x}{C_b} = \frac{2.13 \times 10^{-3}}{2.222} = 9.586 \times 10^{-4}$$

For $L_b = 0,\ b_x = 1.99 \times 10^{-3}$

Use $b_x = 1.99 \times 10^{-3}$ (the larger value)

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.28 \times 10^{-3})(128.8) + \frac{9}{8}((1.99 \times 10^{-3})(299.6) + (4.37 \times 10^{-3})(43.4))$$

$= 0.967 < 1.0$ (OK) <u>Use a W12 × 79</u>

(b) ASD Solution

The axial service load is $P_a = 92$ kips

The service-load moments at the top are

$M_{ntx} = 160$ ft-kips, $M_{nty} = 24$ ft-kips

The service-load moments at the bottom are

$M_{ntx} = 214$ ft-kips, $M_{nty} = 31$ ft-kips

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ax} = B_{1x} M_{ntx} = 1.0(214) = 214 \text{ ft-kips}$$

$$M_{ay} = B_{1y} M_{nty} = 1.0(31) = 31 \text{ ft-kips}$$

Check the W10s. Try a W10 ×100. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.67 \times 10^{-3},\ b_x = 2.90 \times 10^{-3},\ b_y = 5.84 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.67 \times 10^{-3})(92)$$

$$= 0.1536 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(1.67 \times 10^{-3})(92) + \frac{9}{8}((2.90 \times 10^{-3})(214) + (5.84 \times 10^{-3})(31))$$

$$= 0.979 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 79. From Table 6-1,

$$p = 1.92 \times 10^{-3}, \ b_x = 3.21 \times 10^{-3}, \ b_y = 6.56 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.92 \times 10^{-3})(92)$$

$$= 0.1766 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(1.92 \times 10^{-3})(92) + \frac{9}{8}((3.21 \times 10^{-3})(214) + (6.56 \times 10^{-3})(31))$$

$$= 1.09 > 1.0 \qquad \text{(N.G.)}$$

Check the W14s. Try a W14 × 82. From Table 6-1,

$$p = 2.16 \times 10^{-3}, \ b_x = 2.89 \times 10^{-3}, \ b_y = 7.95 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (2.16 \times 10^{-3})(92)$$

$$= 0.1987 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(2.16 \times 10^{-3})(92) + \frac{9}{8}((2.89 \times 10^{-3})(214) + (7.95 \times 10^{-3})(31))$$

$$= 1.07 > 1.0 \qquad \text{(but close; retain this possibility.)}$$

Try a W14 × 82. Calculate $B_1$ for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{160}{214}\right) = 0.3009$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{24}{31}\right) = 0.2903$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(881)}{(0.8 \times 16 \times 12)^2} = 1.069 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{0.3009}{1 - \dfrac{1.60(92)}{10690}}$$

$$= 0.3051 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(148)}{(16 \times 12)^2} = 1149 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{1.60P_a}{P_{e1y}}} = \frac{0.2903}{1 - \dfrac{1.60(92)}{1149}} = 0.3330 < 1.0$$

$$\therefore B_{1y} = 1.0 \text{ as assumed}$$

Compute $C_b$ and modify $b_x$ to account for $C_b$.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(31)}{2.5(31) + 3(10.25) + 4(3.5) + 3(17.25)} = 2.227$$

$$\frac{b_x}{C_b} = \frac{2.89 \times 10^{-3}}{2.227} = 1.30 \times 10^{-3}$$

For $L_b = 0,\ b_x = 2.56 \times 10^{-3}$

Use $b_x = 2.56 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $p = 2.16 \times 10^{-3},\ b_x = 2.56 \times 10^{-3},\ b_y = 7.95 \times 10^{-3}$

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(2.16 \times 10^{-3})(92) + \frac{9}{8}((2.56 \times 10^{-3})(214) + (7.95 \times 10^{-3})(31))$$

$$= 0.993 < 1.0 \qquad \text{(OK)}$$

Use a W14 × 82

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**6.8-7**

(a) LRFD Solution

$$P_u = 1.2(0.25 \times 80) + 1.6(0.75 \times 80) = 120.0 \text{ kips}$$

The factored moments at the top are

$$M_{ntx} = 1.2(0.25 \times 133) + 1.6(0.75 \times 133) = 199.5 \text{ ft-kips}$$

$$M_{nty} = 1.2(0.25 \times 43) + 1.6(0.75 \times 43) = 64.5 \text{ ft-kips}$$

The factored moments at the bottom are

$$M_{ntx} = 1.2(0.25 \times 27) + 1.6(0.75 \times 27) = 40.5 \text{ ft-kips}$$

$$M_{nty} = 1.2(0.25 \times 9) + 1.6(0.75 \times 9) = 13.5 \text{ ft-kips}$$

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ux} = B_{1x}M_{ntx} = 1.0(199.5) = 199.5 \text{ ft-kips}$$

$$M_{uy} = B_{1y}M_{nty} = 1.0(64.5) = 64.5 \text{ ft-kips}$$

Check the W10s. Try a W10 ×77. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.46 \times 10^{-3},\ b_x = 2.62 \times 10^{-3},\ \ b_y = 5.16 \times 10^{-3}$$

$$pP_u = (1.46 \times 10^{-3})(120) = 0.1752 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.46 \times 10^{-3})(120) + \frac{9}{8}((2.62 \times 10^{-3})(199.5) + (5.16 \times 10^{-3})(64.5))$$

$$= 1.05 > 1.0 \qquad \text{(but close; retain this as a possibility.)}$$

Check the W12s. Try a W12 × 72. From Table 6-1,

$$p = 1.41 \times 10^{-3},\ b_x = 2.37 \times 10^{-3},\ \ b_y = 4.82 \times 10^{-3}$$

$$pP_u = (1.41 \times 10^{-3})(120) = 0.1692 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.41 \times 10^{-3})(120) + \frac{9}{8}((2.37 \times 10^{-3})(199.5) + (4.82 \times 10^{-3})(64.5))$$

$$= 0.966 < 1.0 \qquad \text{(OK)}$$

Check the W14s. Try a W14 × 68. From Table 6-1,

$$p = 1.73 \times 10^{-3},\ b_x = 2.38 \times 10^{-3},\ \ b_y = 6.42 \times 10^{-3}$$

$$pP_u = (1.73 \times 10^{-3})(120) = 0.2076 > 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$pP_u + b_xM_{ux} + b_yM_{uy}$$

$$= (1.73 \times 10^{-3})(120) + (2.38 \times 10^{-3})(199.5) + (6.42 \times 10^{-3})(64.5)$$

$= 1.10 > 1.0$ (N.G.)

Try a W12 × 72, $p = 1.41 \times 10^{-3}$, $b_x = 2.37 \times 10^{-3}$, $b_y = 4.82 \times 10^{-3}$

Calculate $B_1$ for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{40.5}{199.5}\right) = 0.6812$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{13.5}{64.5}\right) = 0.6837$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(597)}{(16 \times 12)^2} = 4635 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6812}{1 - \dfrac{120}{4635}} = 0.6993 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(195)}{(16 \times 12)^2} = 1514 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{P_u}{P_{e1y}}} = \frac{0.6837}{1 - \dfrac{120}{1514}} = 0.7426 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

Since the amplification factors are as assumed, and this shape is adequate for $C_b = 1.0$, computation of the actual value of $C_b$ and an adjustment of $b_x$ are not necessary. The preliminary evaluation is sufficient.

<u>Use a W12 × 72</u>

(a) ASD Solution (abbreviated version; rejected trials not shown.)

The axial service load is $P_a = 80$ kips

The service-load moments at the top are

$M_{ntx} = 133$ ft-kips, $M_{nty} = 43$ ft-kips

The service-load moments at the bottom are

$M_{ntx} = 27$ ft-kips, $M_{nty} = 9$ ft-kips

The amplification factor $B_1$ can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ax} = B_{1x} M_{ntx} = 1.0(133) = 133 \text{ ft-kips}$$

$$M_{ay} = B_{1y} M_{nty} = 1.0(43) = 43 \text{ ft-kips}$$

Try a W12 × 72. From Table 6-1,

$p = 2.12 \times 10^{-3}$, $b_x = 3.56 \times 10^{-3}$, $b_y = 7.24 \times 10^{-3}$

$pP_u = (2.12 \times 10^{-3})(80) = 0.1696 < 0.2$ $\therefore$ Equation 6.10 controls.

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(2.12 \times 10^{-3})(80) + \frac{9}{8}((3.56 \times 10^{-3})(133) + (7.24 \times 10^{-3})(43))$$

$$= 0.968 < 1.0 \qquad \text{(OK)}$$

Calculate $B_1$ for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{27}{133}\right) = 0.6812$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{9}{43}\right) = 0.6837$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(597)}{(16 \times 12)^2} = 4635 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{1.60P_a}{P_{e1x}}} = \frac{0.6812}{1 - \frac{1.60(80)}{4635}}$$

$$= 0.7005 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29000)(195)}{(16 \times 12)^2} = 1514 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \frac{1.60P_a}{P_{e1y}}} = \frac{0.6837}{1 - \frac{1.60(80)}{1514}} = 0.7468 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as}$$

assumed

Since the amplification factors are as assumed, and this shape is adequate for $C_b = 1.0$, computation of the actual value of $C_b$ and an adjustment of $b_x$ are not necessary. The preliminary evaluation is sufficient.

Use a W12 × 72

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**6.8-8**

Assume that $B_1 = B_2 = 1.0$ for purposes of making a trial selection.

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(270) + 1.0(30) = 300.0 \text{ ft-kips}$$

Try a W12 × 65. From Table 6-1, For $KL = L_b = 16$ ft,

$$p = 1.56 \times 10^{-3}, \; b_x = 2.66 \times 10^{-3}$$

Determine the critical effective length for compression. Use the sway condition.

$$\frac{K_x L}{r_x / r_y} = \frac{2.0(16)}{1.75} = 18.29 \text{ ft} > K_y L = 16 \text{ ft.}$$

$\therefore$ use $KL = 18.29$ ft for $p$ (use $L_b = 16$ ft for $b_x$). From Table 6-1, $p = 1.54 \times 10^{-3}$ (by interpolation).

$$pP_u = (1.54 \times 10^{-3})(75) = 0.1155 < 0.2 \quad \therefore \text{ Equation 6.10 controls.}$$

Consider the braced condition first.

For the axis of bending,

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_{1x}L)^2} = \frac{\pi^2(29000)(533)}{(16 \times 12)^2} = 4138 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{75}{4138}} = 0.6111 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

Sway condition:

For the axis of bending,

$$P_{e2x} = \frac{\pi^2 EI_x}{(K_{2x}L)^2} = \frac{\pi^2(29000)(533)}{(2.0 \times 16 \times 12)^2} = 1035 \text{ kips}$$

Assume that $P_{story} = P_u$ and $P_{e\,story} = P_{e2}$ :

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \dfrac{1.00(75)}{1035}} = 1.078$$

The total amplified moment is

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(270) + 1.078(30) = 302.3 \text{ ft-kips}$$

Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{2.66 \times 10^{-3}}{1.67} = 1.59 \times 10^{-3}$$

For $L_b = 0$, $b_x = 2.50 \times 10^{-3}$

Use $b_x = 2.50 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $\quad p = 1.54 \times 10^{-3}$, $b_x = 2.50 \times 10^{-3}$

$$0.5pP_u + \left(\frac{9}{8}\right)(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.54 \times 10^{-3})(75) + \frac{9}{8}((2.50 \times 10^{-3})(302.3) + 0)$$

$$= 0.908 < 1 \qquad \text{(OK)}$$

<u>Use a W12 × 65</u>

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**6.8-9**

(a) LRFD Solution

Load combinations involving dead load ($D$), roof live load ($L_r$), and wind load ($W$) are as follows:

Load Combination 2: $1.2D + 0.5L_r$

$$P_{nt} = 1.2(19) + 0.5(33) = 39.3 \text{ kips}$$

$$P_{\ell t} = 0$$

$$M_{nt} = 1.2(79) + 0.5(130) = 159.8 \text{ ft-kips}$$

$$M_{\ell t} = 0$$

(Since the frame and loading are symmetrical, there are no sidesway moments for this load combination.)

Load Combination 3: $1.2D + 1.6L_r + 0.5W$

$$P_{nt} = 1.2(19) + 1.6(33) + 0.5(-11) = 70.1 \text{ kips}$$

$$P_{\ell t} = 0.5(1.4) = 0.7 \text{ kips}$$

$$M_{nt} = 1.2(79) + 1.6(130) + 0.5(-46) = 279.8 \text{ ft-kips}$$

$$M_{\ell t} = 0.5(32) = 16.0 \text{ ft-kips}$$

Load Combination 4: $1.2D + 1.0W + 0.5L_r$

$$P_{nt} = 1.2(19) + 1.0(-11) + 0.5(33) = 29.7 \text{ kips}$$

$$P_{\ell t} = 1.0(1.4) = 1.4 \text{ kips}$$

$$M_{nt} = 1.2(79) + 1.0(-46) + 0.5(130) = 113.8 \text{ ft-kips}$$

$$M_{\ell t} = 1.0(32) = 32.0 \text{ ft-kips}$$

Load combination 3 will govern. Use $P_{nt} = 70.1$ kips, $P_{\ell t} = 0.7$ kips, $M_{nt} = 279.8$ ft-kips, and $M_{\ell t} = 16.0$ ft-kips. For purposes of selecting a trial shape, assume $B_1 = 1.0$.

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} = R_M \frac{H}{\Delta_H/L} = 0.85\left(\frac{3.6}{1/400}\right) = 1224$$

(The unfactored horizontal load $H = 3.6$ kips is used because the drift index is based on the maximum drift caused by *service* loads.)

$$B_2 = \frac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1-\dfrac{\alpha P_{nt}}{P_{e\,story}}} = \frac{1}{1-\dfrac{1.00(70.1)}{1224}} = 1.061$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(279.8) + 1.061(16.0) = 296.8 \text{ ft-kips}$$

$$P_u = P_{nt} + B_2 P_{\ell t} = 70.1 + 1.061(0.7) = 70.84 \text{ kips}$$

Try a W14 × 61. From Table 6-1, using $KL = L_b = 18$ ft,

$$p = 2.19 \times 10^{-3},\ b_x = 2.84 \times 10^{-3}$$

Determine the critical effective length for compression. Use the sway condition. Estimate the effective length factor as $K_x = 2.0$.

$$\frac{K_x L}{r_x/r_y} = \frac{2.0(18)}{2.44} = 14.75 \text{ ft} < K_y L = 18 \text{ ft.} \quad \therefore\ KL = 18 \text{ ft as assumed}$$

$$pP_u = (2.19 \times 10^{-3})(70.84) = 0.1551 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

Consider the braced condition first. Use $K_x = 1.0$.

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29000)(640)}{(1.0 \times 18 \times 12)^2} = 3926 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-1.0[(P_{nt}+P_{\ell t})/P_{e1}]}$$

$$= \frac{0.6}{1-1.0[(70.1+0.7)]/3926} = 0.611 < 1.0$$

$$\therefore\ B_1 = 1.0 \text{ as assumed}$$

Since $B_1 = 1.0$ is the value originally assumed, and $B_2$ will not change, the previously computed value of $M_u = 296.8$ ft-kips is unchanged.

Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{2.84 \times 10^{-3}}{1.67} = 1.701 \times 10^{-3}$$

For $L_b = 0,\ b_x = 2.32 \times 10^{-3}$

Use $b_x = 2.32 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $\quad p = 2.19 \times 10^{-3},\ b_x = 2.32 \times 10^{-3}$

$$0.5pP_u + \left(\frac{9}{8}\right)(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(2.19 \times 10^{-3})(70.84) + \frac{9}{8}((2.32 \times 10^{-3})(296.8) + 0)$$

$$= 0.852 < 1 \qquad \text{(OK)}$$

Use a W14 × 61

(b) ASD Solution

Potential load combinations involving dead load ($D$), roof live load ($L_r$), and wind load ($W$) are as follows:

Load Combination 3: $D + L_r$

$$P_{nt} = 19 + 33 = 52 \text{ kips}$$

$$P_{\ell t} = 0$$

$$M_{nt} = 79 + 130 = 209 \text{ ft-kips}$$

$$M_{\ell t} = 0$$

(Since the frame and loading are symmetrical, there are no sidesway moments for this load combination.)

Load Combination 5: $D \pm 0.6W$

$$P_{nt} = 19 + (-11 + 1.4) = 9.4 \text{ kips}$$

$$P_{\ell t} = 1.4 \text{ kips}$$

$$M_{nt} = 79 + (-46) = 33 \text{ ft-kips}$$

$$M_{\ell t} = 32 \text{ ft-kips}$$

Load Combination 6a: $D + 0.75(0.6)W + 0.75L_r$

$$P_{nt} = 19 + 0.75(0.6)(-11 + 1.4) + 0.75(33) = 39.43 \text{ kips}$$

$$P_{\ell t} = 0.75(0.6)(1.4) = 0.63 \text{ kips}$$

$$M_{nt} = 79 + 0.75(0.6)(-46) + 0.75(130) = 155.8 \text{ ft-kips}$$

$$M_{\ell t} = 0.75(0.6)32 = 14.4 \text{ ft-kips}$$

Assume load combination 3 controls. After a shape is selected, check combination 6a. For purposes of selecting a trial shape, assume $B_1 = 1.0$.

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(209) + 0 = 209 \text{ ft-kips}$$

$$P_a = P_{nt} + B_2 P_{\ell t} = 52 + 0 = 52 \text{ kips}$$

Try a W14 × 61. From Table 6-1, using $KL = L_b = 18$ ft,

$$p = 3.29 \times 10^{-3},\ b_x = 4.27 \times 10^{-3}$$

Determine the critical effective length for compression. Use the sway condition. Estimate the effective length factor as $K_x = 2.0$

$$\frac{K_x L}{r_x/r_y} = \frac{2.0(18)}{2.44} = 14.75 \text{ ft} < K_y L = 18 \text{ ft.} \quad \therefore KL = 18 \text{ ft as assumed}$$

$$pP_a = (3.29 \times 10^{-3})(52) = 0.1711 < 0.2 \ \therefore \text{ Equation 6.10 controls.}$$

For the braced condition, use $K_x = 1.0$.

$$P_{e1} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29000)(640)}{(1.0 \times 18 \times 12)^2} = 3926 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{0.6}{1 - 1.60(52/3926)}$$

$$= 0.6130 < 1.0 \qquad B_1 = 1.0 \text{ as assumed}$$

Modify $b_x$ to account for $C_b$.

$$\frac{b_x}{C_b} = \frac{4.27 \times 10^{-3}}{1.67} = 2.56 \times 10^{-3}$$

For $L_b = 0,\ b_x = 3.49 \times 10^{-3}$

Use $b_x = 3.49 \times 10^{-3}$ (the larger value)

Check Equation 6.10: $p = 3.29 \times 10^{-3},\ b_x = 3.49 \times 10^{-3}$

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(3.29 \times 10^{-3})(52) + \frac{9}{8}((3.49 \times 10^{-3})(209) + 0)$$

$$= 0.906 < 1.0 \qquad \text{(OK)}$$

Check load combination 6a: $P_{nt} = 39.43$ kips, $M_{nt} = 155.8$ ft-kips, $P_{\ell t} = 0.63$ kips, and $M_{\ell t} = 14.4$ ft-kips

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{0.6}{1 - 1.60(39.43)/3926}$$

$$= 0.6098 < 1.0 \qquad B_1 = 1.0 \text{ as assumed}$$

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} = R_M \frac{H}{\Delta_H/L} = 0.85\left(\frac{3.6}{1/400}\right) = 1224$$

(The unfactored horizontal load $H = 3.6$ kips is used because the drift index is based on the maximum drift caused by *service* loads.)

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \dfrac{1.60(39.43)}{1224}} = 1.054$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(155.8) + 1.054(14.4) = 171.0 \text{ ft-kips}$$

$$P_a = P_{nt} + B_2 P_{\ell t} = 39.43 + 1.054(0.63) = 40.09 \text{ kips}$$

$$pP_a = (3.29 \times 10^{-3})(40.09) = 0.1319 < 0.2 \;\therefore\; \text{Equation 6.10 controls.}$$

$$p = 3.29 \times 10^{-3},\; b_x = 3.49 \times 10^{-3}$$

$$0.5pP_a + \left(\frac{9}{8}\right)(b_x M_{ax} + b_y M_{ay})$$

$$= 0.5(3.29 \times 10^{-3})(40.09) + \frac{9}{8}((3.49 \times 10^{-3})(171.0) + 0)$$

$$= 0.737 < 1.0 \quad \text{(OK)}$$

Use a W14 × 61

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**6.8-10**

(a) LRFD Solution

Determine the total vertical load to be stabilized by the bracing.

Load combination 2: $1.2D + 0.5L_r$

$$\Sigma P_u = [1.2(0.860)(45) + 0.5(1.45)(45)] \times 3 = 237.2 \text{ kips}$$

Load Combination 3: $1.2D + 1.6L_r + 0.5W$

$$\Sigma P_u = [1.2(0.860)(45) + 1.6(1.45)(45) + 0.5(-0.510)(45)] \times 3$$

$$= 418.1 \text{ kips}$$

Combination 3 controls; $P_r = \Sigma P_u = 418.1$ kips

From AISC Equation A-6-1, the lateral shear to be resisted is

$$P_{rb} = 0.004P_r = 0.004(418.1) = 1.672 \text{ kips}$$

Design both braces as tension-only members, and let the entire force be resisted by one brace. Since $P_{rb}$ is the horizontal component of the brace force (see Figure 6.24a in the textbook), the brace force is

$$F = \frac{P_{rb}}{\cos\theta}, \text{ where } \theta = \tan^{-1}(18/45) = 21.80°$$

$$F = \frac{1.672}{\cos(21.80°)} = 1.801 \text{ kips}$$

Based on the limit state of tension yielding, the required area is

$$A = \frac{F}{0.9F_y} = \frac{1.801}{0.9(36)} = 5.559 \times 10^{-2} \text{ in.}^2$$

From AISC Equation A-6-2, the required lateral stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) = \frac{1}{0.75}\left[\frac{2(418.1)}{(18 \times 12)}\right] = 5.162 \text{ kips/in.}$$

The length of the brace is

$$L = (18 \times 12)/\sin(21.80°) = 581.6 \text{ in.}$$

Let

$$\frac{AE}{L}\cos^2\theta = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) = 5.162$$

$$A = \frac{5.162L}{E\cos^2\theta} = \frac{5.162(581.6)}{29000\cos^2(21.8°)} = 0.120 \text{ in.}^2$$

The stiffness requirement controls.

Use a tension brace with a cross-sectional area of at least 0.120 in.$^2$

(b) ASD Solution

Determine the total vertical load to be stabilized by the bracing.

Load combination 3 controls: $D + L_r$

$$P_r = \Sigma P_a = [0.860(45) + 1.45(45)] \times 3 = 311.9 \text{ kips}$$

From AISC Equation A-6-1, the lateral shear to be resisted is

$$P_{rb} = 0.004P_r = 0.004(311.9) = 1.248 \text{ kips}$$

Design both braces as tension-only members, and let the entire force be resisted by one brace. Since $P_{rb}$ is the horizontal component of the brace force (see Figure 6.24a in the textbook), the brace force is

$$F = \frac{P_{rb}}{\cos\theta}, \text{ where } \theta = \tan^{-1}(18/45) = 21.80°$$

$$F = \frac{1.248}{\cos(21.80°)} = 1.344 \text{ kips}$$

Based on the limit state of tension yielding, the required area is

$$A = \frac{F}{F_y/\Omega} = \frac{1.344}{36/2.00} = 7.467 \times 10^{-2} \text{ in.}^2$$

From AISC Equation A-6-2, the required lateral stiffness is

$$\beta_{br} = \Omega\left(\frac{2P_r}{L_b}\right) = 2.00\left[\frac{2(311.9)}{(18\times 12)}\right] = 5.776 \text{ kips/in.}$$

The length of the brace is

$$L = (18\times 12)/\sin(21.80°) = 581.6 \text{ in.}$$

Let

$$\frac{AE}{L}\cos^2\theta = \Omega\left(\frac{2P_r}{L_b}\right) = 5.776$$

$$A = \frac{5.776L}{E\cos^2\theta} = \frac{5.776(581.6)}{29000\cos^2(21.8°)} = 0.134 \text{ in.}^2$$

The stiffness requirement controls.

<u>Use a tension brace with a cross-sectional area of at least 0.134 in.$^2$</u>

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**6.9-1**

The loads transmitted by the purlins are as follows:

Snow: $20(3)(25) = 1500$ lb = 1.500 kips

Roof: metal deck: 2 psf
roofing: 4 psf
Insulation: 3 psf
Total: 9 psf

$9(3)(25) = 675.0$ lb

Purlins: $8.5(25) = 212.5$ lb

Total dead load = $675 + 212.5 = 887.5$ lb = 0.8875 kips

(a) LRFD Solution

Load combination 3 will control:

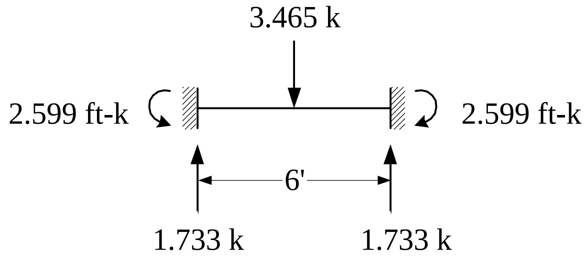
$$P_u = 1.2D + 1.6S = 1.2(0.8875) + 1.6(1.500) = 3.465 \text{ kips}$$

The fixed-end moment for each top chord member is

$$M = M_{nt} = \frac{P_u L}{8} = \frac{3.465(6)}{8} = 2.599 \text{ ft-kips}$$

The reaction at each end of the fixed-end top chord member is

$$\frac{P_u}{2} = \frac{3.465}{2} = 1.733 \text{ kips}$$




Total interior panel point load = $3.465 + 2(1.733) = 6.931$ kips

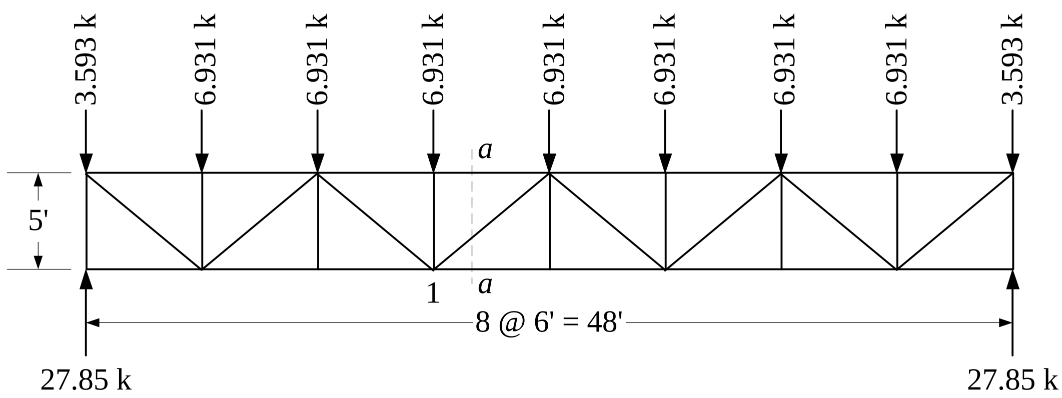
Exterior panel point load:

Snow: $1500/2 = 750.0$ lb
Roof: $675/2 = 337.5$ lb
Purlins: 212.5 lb

$$P_u = 1.2(0.3375 + 0.2125) + 1.6(0.750) = 1.860 \text{ kips}$$

Total exterior panel point load = $1.733 + 1.860 = 3.593$ kips

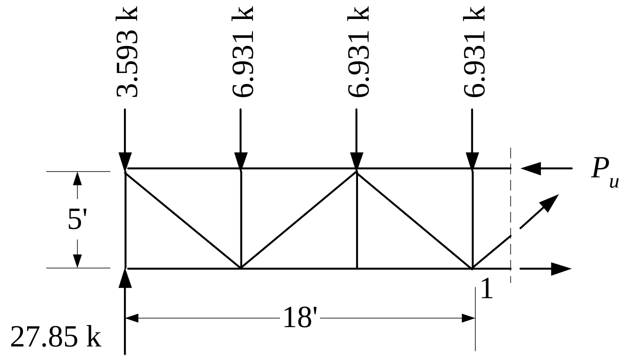
Truss reaction at each end = $\dfrac{2(3.593) + 7(6.931)}{2} = 27.85$ kips




Consider a free body of the portion of the truss left of section a-a:

$$\sum M_1 = (27.85 - 3.593)(18) - 6.931(12 + 6) - P_u(5) = 0,$$

$$P_u = 62.37 \text{ kips}$$




Design for an axial compressive load of 62.37 kips and a bending moment of 2.599 ft-kips.

Try a WT5 × 15. From the column load tables with $K_xL = 6$ ft and $K_yL = 3$ft,

$$\phi_c P_n = 166 \text{ kips}$$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(9.28)}{(6 \times 12)^2} = 512.4 \text{ kips}$$

$$C_m = 1.0 \text{ (transversely-loaded member)}$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{1.0}{1 - \dfrac{62.37}{512.4}} = 1.139$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.139(2.599) + 0 = 2.96 \text{ ft-kips}$$

The shape is compact for flexure. Check the limit state of yielding. Because the maximum moment is a fixed-end moment, the stem will be in compression, assuming that the flange is at the top. For stems in compression,

$$M_n = M_p = F_y Z_x \le M_y \qquad \text{(AISC Equation F9-3)}$$

Since $S_x < Z_x$, the yield moment $M_y$ will control.

$$M_n = M_y = F_y S_x = 50(2.24) = 112.0 \text{ in.-kips}$$

Check lateral-torsional buckling. From AISC Equation F9-5,

$$B = \pm 2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} = -2.3\left(\frac{5.24}{6 \times 12}\right)\sqrt{\frac{8.35}{0.310}} = -0.8687$$

(The minus sign is used when the stem is in compression anywhere along the unbraced length.) From AISC Equation F9-4,

$$M_n = M_{cr} = \frac{\pi\sqrt{EI_yGJ}}{L_b}\left[B + \sqrt{1+B^2}\right]$$

$$= \frac{\pi\sqrt{29000(8.35)(11200)0.310)}}{6\times 12}\left[-0.8687 + \sqrt{1+(-0.8687)^2}\right]$$

$$= 576.8 \text{ in.-kips}$$

The limit state of yielding controls.

$$\phi_b M_n = 0.90(112.0) = 100.8 \text{ in.-kips} = 8.4 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_u}{\phi_c P_n} = \frac{62.37}{166} = 0.3757 > 0.2 \ \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3757 + \frac{8}{9}\left(\frac{2.96}{8.4} + 0\right)$$

$$= 0.689 < 1.0 \quad \text{(OK)}$$

Use a WT5 × 15.

(b) ASD Solution

Load combination 3 will control:

$$P_a = D + S = 0.8875 + 1.500 = 2.388 \text{ kips}$$

The fixed-end moment for each top chord member is

$$M = M_{nt} = \frac{P_a L}{8} = \frac{2.388(6)}{8} = 1.791 \text{ ft-kips}$$

The reaction at each end of the fixed-end top chord member is

$$\frac{P_a}{2} = \frac{2.388}{2} = 1.194 \text{ kips}$$

Total interior panel point load = $2.388 + 2(1.194) = 4.776$ kips

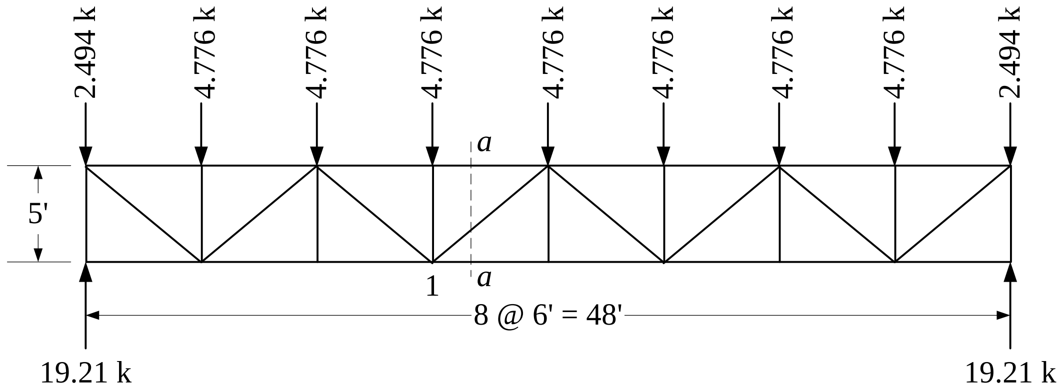
Exterior panel point load:

Snow: 1500/2 = 750.0 lb
Roof: 675/2 = 337.5 lb
Purlins: 212.5 lb

$$P_a = 0.750 + 0.3375 + 0.2125 = 1.3 \text{ kips}$$

Total exterior panel point load = $1.194 + 1.3 = 2.494$ kips

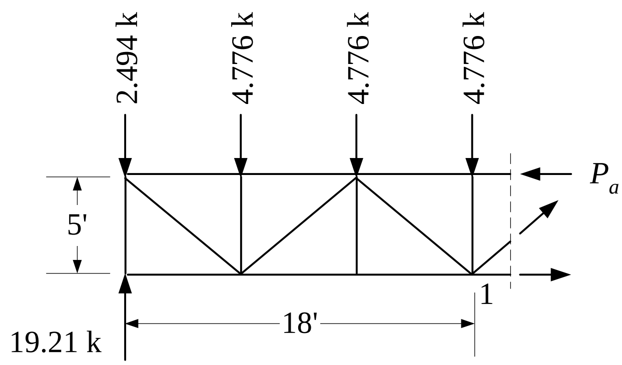
Truss reaction at each end = $\dfrac{2(2.494)+7(4.776)}{2} = 19.21$ kips




Consider a free body of the portion of the truss left of section a-a:

$$\sum M_1 = (19.21 - 2.494)(18) - 4.776(12+6) - P_a(5) = 0$$

$$P_a = 42.98 \text{ kips}$$




Design for an axial compressive load of 42.98 kips and a bending moment of 1.791 ft-kips.

Try a WT5 × 15. From the column load tables with $K_xL = 6$ ft and $K_yL = 3$ft,

$$\frac{P_n}{\Omega_c} = 110 \text{ kips}$$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(9.28)}{(6\times 12)^2} = 512.4 \text{ kips}$$

$C_m = 1.0$ (transversely-loaded member)

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(42.98)/512.4}$$

$$= 1.155$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.155(1.791) + 0 = 2.069 \text{ ft-kips}$$

From part (a), $M_n = 112.0$ in.-kips

$$\frac{M_n}{\Omega_b} = \frac{112.0}{1.67} = 67.07 \text{ in.-kips} = 5.589 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = \frac{42.98}{110} = 0.3907 > 0.2 \;\therefore\; \text{use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.3907 + \frac{8}{9}\left(\frac{2.069}{5.589} + 0\right)$$

$$= 0.720 < 1.0 \qquad \text{(OK)}$$

Use a WT5 × 15.

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**6.9-2**

For the deck and built-up roof, the tributary width of top chord is $46.6/8 = 5.825$ ft.

$(2+5)(5.825)(25) = 1019$ lb

Purlins: $12(25) = 300.0$ lb

For the snow, the tributary width of top chord is $45/8 = 5.625$ ft.

$18(5.625)(25) = 2531$ lb

Assume a truss weight of 10% of the other gravity loads:

$0.10(1019 + 300 + 2531) = 385.0$ lb

Total dead load = $1019 + 300 + 385 = 1704$ lb

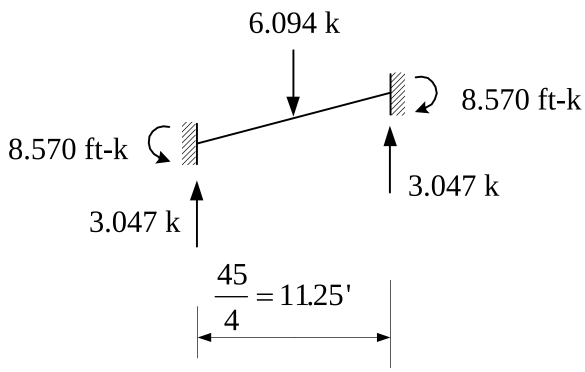
(a) LRFD Solution

Load combination 3 will control:

$$P_u = 1.2D + 1.6S = 1.2(1.704) + 1.6(2.531) = 6.094 \text{ kips}$$

The fixed-end moment for each top chord member is (see figure)

$$M = M_{nt} = \frac{P_u L}{8} = \frac{6.094(11.25)}{8} = 8.570 \text{ ft-kips}$$




Total interior panel point load = $6.094 + 2(3.047) = 12.19$ kips

Exterior panel point load:

Snow: $2531/2 = 1266$ lb

Deck and roof: $1019/2 = 509.5$ lb

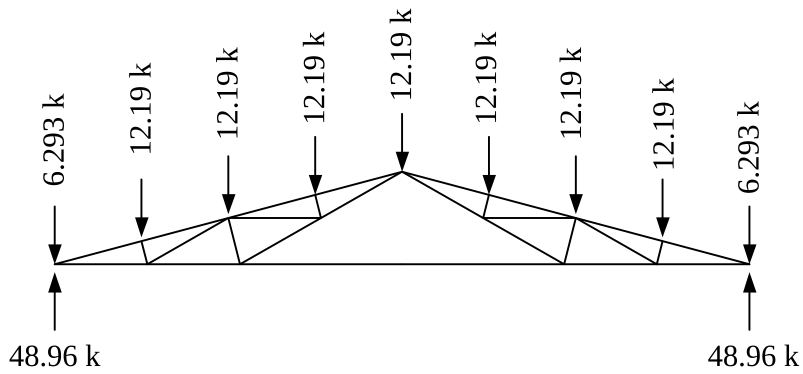
Purlins: 300 lb

Truss weight: $0.10(1266 + 509.5 + 300) = 207.6$ lb

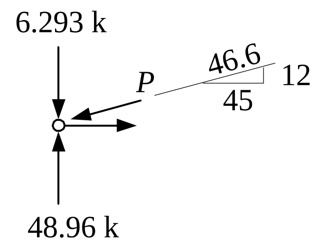
$$P_u = 1.2(0.5095 + 0.300 + 0.2076) + 1.6(1.266) = 3.246 \text{ kips}$$

Total exterior panel point load = $3.047 + 3.246 = 6.293$ kips

Truss reaction at each end = $\frac{7(12.19) + 2(6.293)}{2} = 48.96$ kips




The maximum top chord load is in the member adjacent to the support. Consider a free body of the pin at the left support:




$$\sum F_y = 48.96 - 6.293 - \frac{12}{46.6}P = 0, \qquad P = 165.7 \text{ kips compression}$$

Design for an axial compressive load of 165.7 kips and a bending moment of 8.570 ft-kips.

Try a WT7 × 34

From the column load tables with $K_xL = 46.6/4 = 11.65$ ft and $K_yL = 11.65/2 = 5.825$ ft,

$$\phi_c P_n = 290.5 \text{ kips}$$

Since bending is about the $x$ axis and the member is braced against sidesway,

$$M_{nt} = 8.570 \text{ ft-kips and } M_{\ell t} = 0$$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(32.6)}{(11.65 \times 12)^2} = 477.4 \text{ kips}$$

$$C_m = 1.0 \text{ (transversely-loaded member)}$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{1.0}{1 - \dfrac{165.7}{477.4}} = 1.532$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.532(8.570) + 0 = 13.13 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

Check the limit state of yielding. Because the maximum moment is a fixed-end moment, the stem will be in compression, assuming that the flange is at the top. For stems in compression,

$$M_n = M_p = F_y Z_x \le M_y \qquad \text{(AISC Equation F9-3)}$$

Since $S_x < Z_x$, the yield moment $M_y$ will control.

$$M_n = M_y = F_y S_x = 50(5.69) = 284.5 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(284.5) = 256.1 \text{ in.-kips} = 21.34 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_u}{\phi_c P_n} = \frac{165.7}{290.5} = 0.5704 > 0.2 \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5704 + \frac{8}{9}\left(\frac{13.13}{21.34} + 0\right)$$

$$= 1.12 > 1.0 \qquad \text{(N.G.)}$$

Try a WT7 × 37

From the column load tables with $K_x L = 11.65$ ft and $K_y L = 5.825$ ft,

$$\phi_c P_n = 318.2 \text{ kips}$$

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29000)(36.0)}{(11.65 \times 12)^2} = 527. \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{1.0}{1 - \dfrac{165.7}{527.2}} = 1.458$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.458(8.570) + 0 = 12.50 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_y Z_x \leq M_y$$

Since $S_x < Z_x$, the yield moment $M_y$ will control.

$$M_n = M_y = F_y S_x = 50(6.25) = 312.5 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(312.5) = 281.3 \text{ in.-kips} = 23.44 \text{ ft-kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{165.7}{318.2} = 0.5207 > 0.2 \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5207 + \frac{8}{9}\left(\frac{12.50}{23.44} + 0\right)$$

$$= 0.995 < 1.0 \quad \text{(OK)}$$

Use a WT7 × 37

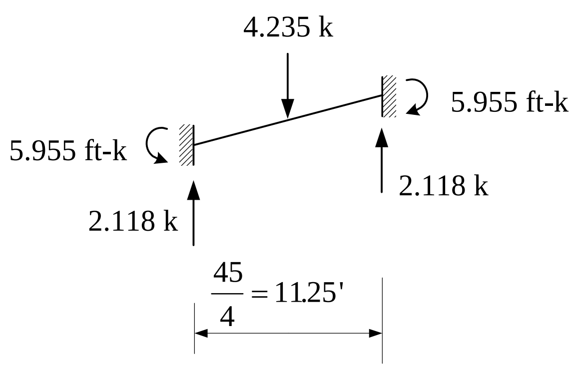
(b) ASD Solution

Load combination 3 will control:

$$P_a = D + S = 1.704 + 2.531 = 4.235 \text{ kips}$$

The fixed-end moment for each top chord member is (see figure)

$$M = M_{nt} = \frac{P_a L}{8} = \frac{4.235(11.25)}{8} = 5.955 \text{ ft-kips}$$




Total interior panel point load $= 4.235 + 2(2.118) = 8.471$ kips

Exterior panel point load:

Snow: $2531/2 = 1266$ lb

Deck and roof: $1019/2 = 509.5$ lb

Purlins: 300 lb

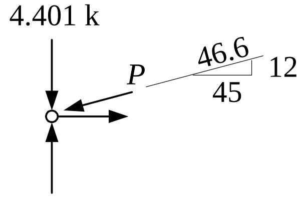
Truss weight: $0.10(1266 + 509.5 + 300) = 207.6$ lb

$P_a = 1.266 + 0.5095 + 0.300 + 0.2076 = 2.283$ kips

Total exterior panel point load $= 2.118 + 2.283 = 4.401$ kips

Truss reaction at each end $= \dfrac{7(8.471) + 2(4.401)}{2} = 34.05$ kips

The maximum top chord load is in the member adjacent to the support. Consider a free body of the pin at the left support:




$$\sum F_y = 34.05 - 4.401 - \frac{12}{46.6}P = 0, \qquad P = 115.1 \text{ kips compression}$$

Design for an axial compressive load of 115.1 kips and a bending moment of 5.955 ft-kips.

Try a WT7 × 37

From the column load tables with $K_xL = 11.65$ ft and $K_yL = 5.825$ ft,

$$\frac{P_n}{\Omega_c} = 211.4 \text{ kips}$$

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29000)(36.0)}{(11.65 \times 12)^2} = 527.2 \text{ kips}$$

$C_m = 1.0$ (transversely-loaded member)

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{1.0}{1 - 1.60(115.1)/527.2}$$

$$= 1.537$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.537(5.955) + 0 = 9.153 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_y Z_x \le M_y$$

Since $S_x < Z_x$, the yield moment $M_y$ will control.

$$M_n = M_y = F_y S_x = 50(6.25) = 312.5 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{312.5}{1.67} = 187.1 \text{ in.-kips} = 15.59 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = \frac{115.1}{211.4} = 0.5445 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.5445 + \frac{8}{9}\left(\frac{9.153}{15.59} + 0\right)$$

$$= 1.07 > 1.0 \qquad \text{(N.G.)}$$

Try a WT7 × 41

From the column load tables with $K_x L = 11.65$ ft and $K_y L = 5.825$ ft, $\frac{P_n}{\Omega_c} = 237.0$ kips

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2(29000)(41.2)}{(11.65 \times 12)^2} = 603.4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{1.0}{1 - 1.60(115.1)/603.4} = 1.439$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.439(5.955) + 0 = 8.569 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_y Z_x \le M_y$$

Since $S_x < Z_x$, the yield moment $M_y$ will control.

$$M_n = M_y = F_y S_x = 50(7.14) = 357.0 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{357.0}{1.67} = 213.8 \text{ in.-kips} = 17.82 \text{ ft-kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{115.1}{237.0} = 0.4857 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4857 + \frac{8}{9}\left(\frac{8.569}{17.82} + 0\right)$$

$$= 0.913 < 1.0 \qquad \text{(OK)}$$

<u>Use a WT7 × 41</u>

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## CHAPTER 7 - SIMPLE CONNECTIONS

**7.3-1**

(a) Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 2.75 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in.= actual $\ell_e$ (OK)

(b) Check bearing on gusset plate (it has the smaller thickness; everything else is the same).

For A242 steel, $F_u = 70$ ksi

$$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the holes nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(1.031)(3/8)(70) = 32.48 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(3/8)(70) = 55.12 \text{ kips} > 32.48 \text{ kips}$$

$$\therefore \text{ use } R_n = 32.48 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 2.75 - \frac{15}{16} = 1.813 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(1.813)(3/8)(70) = 57.11 \text{ kips}$$

$$2.4dtF_u = 55.12 \text{ kips} < 57.11 \text{ kips} \;\therefore \text{ use } R_n = 55.12 \text{ kips}$$

For the connection, the total bearing strength is

$32.48 + 2(55.12) = 142.7$ kips $\qquad$ $\underline{R_n = 143 \text{ kips}}$

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**7.3-2**

(a) Minimum spacing = $2\frac{2}{3}d = 2.667(3/4) = 2.0$ in. < 2.5 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.25 in. < 1.5 in. (OK)

(b) Check bearing on gusset plate (it has the smaller thickness; everything else is the same).

$$h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For the holes nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u = 1.2(1.094)(3/8)(58) = 28.55 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(3/4)(3/8)(58) = 39.15 \text{ kips} > 28.55 \text{ kips}$$

$$\therefore \text{ use } R_n = 28.55 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 3 - \frac{13}{16} = 2.188 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u = 1.2(2.188)(3/8)(58) = 57.11 \text{ kips}$$

$$2.4dtF_u = 39.15 \text{ kips} < 57.11 \text{ kips} \ \therefore \text{ use } R_n = 39.15 \text{ kips}$$

For the connection, the total bearing strength is

$$2(28.55) + 4(39.15) = 214 \text{ kips} \qquad \underline{R_n = 214 \text{ kips}}$$

---

**7.4-1**

(a) Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 3 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2 in. (OK)

(b) Design strengths. Bolt shear: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

The nominal shear capacity of one bolt is

$$R_n = F_{nv}A_b = 27(0.6013) = 16.24 \text{ kips}$$

Check bearing on gusset plate. $h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 3 - \frac{15/16}{2} = 2.531$ in.

$$R_n = 1.2\ell_c tF_u = 1.2(2.531)(3/8)(58) = 66.06 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68 \text{ kips} < 66.06 \text{ kips}$$

$$\therefore \text{ use } R_n = 45.68 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u = 1.2(2.063)(3/8)(58) = 53.84 \text{ kips}$$

$$2.4dtF_u = 45.68 \text{ kips} < 53.84 \text{ kips} \;\therefore\; \text{use } R_n = 45.68 \text{ kips}$$

(The bearing strength of the channel does not need to be checked. Its thickness is greater than that of the gusset plate, and its ultimate tensile stress is larger.)

For the connection, the strength for each bolt is the minimum of the shear and bearing values. In this connection, shear controls for each bolt, and the total design strength is

$$\phi R_n = 0.75(16.24) \times 6 = \underline{73.1 \text{ kips}}$$

(c) Allowable strengths: From part (b),

nominal shear strength = 16.24 kips/bolt

nominal bearing strength for the edge bolts = 45.68 kips

nominal bearing strength for the interior bolts = 45.68 kips

For the connection, the strength for each bolt is the minimum of the shear and bearing values. In this connection, shear controls for each bolt, and the total allowable strength is

$$\frac{R_n}{\Omega} = \frac{16.24}{2.00} \times 6 = \underline{48.7 \text{ kips}}$$

---

**7.4-2**

(a) Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 2.75 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. = actual $\ell_e$ (OK)

(b) Bolt shear: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Shear capacity of one bolt is

$$R_n = F_{nv}A_b = 54(0.6013) \times 2 = 64.94 \text{ kips (double shear)}$$

For bearing, the steel is the same for the tension member and the splice plates, the edge distances are the same for the tension member and the splice plates, and the combined thickness of the splice plates is the same as the thickness of the tension member. As a consequence, the bearing strength will be the same for both the tension member and the splice plates.

Check bearing on the tension member. $h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031$ in.

$$R_n = 1.2\ell_c tF_u = 1.2(1.031)(1/2)(58) = 35.88 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(1/2)(58) = 60.9 \text{ kips} > 35.88 \text{ kips}$$

$$\therefore \text{ use } R_n = 35.88 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 2.75 - \frac{15}{16} = 1.813 \text{ in.}$$

$R_n = 1.2\ell_c tF_u = 1.2(1.813)(1/2)(58) = 63.09$ kips

$2.4dtF_u = 60.9$ kips < 63.09 kips $\therefore$ use $R_n = 60.9$ kips

For the connection, the strength for each bolt is the minimum of the shear and bearing values. Bearing controls for each bolt:

$R_n = 2(35.88) + 2(60.9) = 194$ kips $\qquad$ $\underline{R_n = 194 \text{ kips}}$

---

**7.4-3**

$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418$ in.$^2$

Assume that the bolt threads are in the plane of shear. The nominal shear capacity of one bolt in double shear is

$R_n = F_{nv}A_b = 54(0.4418) \times 2 = 47.71$ kips

The nominal bearing strength of one bolt (gusset plate thickness controls) is

$2.4dtF_u = 2.4(3/4)(3/8)(58) = 39.15$ kips < 47.71 kips for shear

$\therefore$ Bearing strength controls for each bolt.

(a) LRFD solution: $\phi R_n = 0.75(39.15) = 29.36$ kips/bolt

$P_u = 1.2D + 1.6L = 1.2(14) + 1.6(25) = 56.8$ kips

Number of bolts required is $\frac{56.8}{29.36} = 1.93$ $\qquad$ Use 2 bolts

(b) ASD solution: $\frac{R_n}{\Omega} = \frac{39.15}{2.00} = 19.58$ kips/bolt

$P_a = D + L = 14 + 25 = 39$ kips

Number of bolts required is $\frac{39}{19.58} = 1.99$ $\qquad$ Use 2 bolts

---

**7.4-4**

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

Nominal shear capacity of one bolt in double shear is

$$R_n = F_{nv}A_b = 54(0.6013) \times 2 = 64.94 \text{ kips}$$

Since no bolt spacing or edge distances are given, assume that the upper limit of $2.4dtF_u$ controls for bearing strength. Since 3/8 in. < 1/4 + 1/4 = 1/2 in., the bearing strength of one bolt is

$$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68 \text{ kips} < 57.72 \text{ kips for shear}$$

$\therefore$ Bearing strength controls.

(a) LRFD solution: $\phi R_n = 0.75(45.68) = 34.26 \text{ kips/bolt}$

$$P_u = 1.2D + 1.6L = 1.2(0.25 \times 60) + 1.6(0.75 \times 60) = 90.0 \text{ kips}$$

Number of bolts required is $\dfrac{90}{34.26} = 2.63,$ use 4 bolts for symmetry on each side.

<u>Use 8 bolts</u>

(b) ASD solution: $\dfrac{R_n}{\Omega} = \dfrac{45.68}{2.00} = 19.58 \text{ kips/bolt}, \qquad P_a = D + L = 60 \text{ kips}$

Number of bolts required is $\dfrac{60}{19.58} = 3.06,$ use 4 bolts for symmetry on each side.

<u>Use 8 bolts</u>

---

**7.4-5**

Bolt shear: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Nominal shear capacity of one bolt is

$$R_n = F_{nv}A_b = 54(0.4418) = 23.86 \text{ kips}$$

Check bearing on tension member (it has the smaller edge distance; everything else is the same):

$$h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For the holes nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(1.094)(5/16)(58) = 23.79 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(3/4)(5/16)(58) = 32.63 \text{ kips} > 23.79 \text{ kips}$$

$$\therefore \text{ use } R_n = 23.79 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 3.5 - \frac{13}{16} = 2.688 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(2.688)(5/16)(58) = 58.46 \text{ kips}$$

$$2.4dtF_u = 32.63 \text{ kips} < 58.46 \text{ kips} \therefore \text{ use } R_n = 32.63 \text{ kips}$$

For the connection, the strength for each bolt is the minimum of the shear and bearing values:

Edge bolts: bearing controls: $R_n = 23.79$ kips/bolt

Interior bolts: shear controls: $R_n = 23.86$ kips/bolt

For the connection, the nominal strength is

$$2(23.79) + 4(23.86) = 143.0 \text{ kips}$$

(a) LRFD solution: $\phi R_n = 0.75(143.0) = 107.2$ kips

Let $1.2D + 1.6(2D) = 107.2$, Solution is: $\{D = 24.36\}$

$D = 24.36$ kips, $L = 48.72$ kips, <u>Total = 24.36 + 48.72 = 73.1 kips</u>

(b) ASD solution: $\dfrac{R_n}{\Omega} = \dfrac{143.0}{2.00} = 71.5 \text{ kips} = P_a$ <u>Total = 71.5 kips</u>

---

**7.4-6**

Bolt shear (assume that the threads are in shear): $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013$ in.$^2$

Nominal shear capacity of one bolt is

$$R_n = F_{nv} A_b \times 2 \text{ (for double shear)} = 54(0.6013) \times 2 = 64.94 \text{ kips/bolt}$$

Check bearing on the gusset plate (it is thinner than the combined thickness of the angles):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(1.531)(3/8)(58) = 39.96 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68 \text{ kips} > 39.96 \text{ kips}$$

$$\therefore \text{ use } R_n = 39.96 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(2.063)(3/8)(58) = 53.84 \text{ kips}$$

$$2.4dtF_u = 45.68 \text{ kips} < 53.84 \text{ kips} \therefore \text{ use } R_n = 45.68 \text{ kips}$$

For the connection, the strength for each bolt is the minimum of the shear and bearing values. Bearing controls for each bolt:

$$R_n = 39.96 + 4(45.68) = 222.7 \text{ kips}$$

(a) LRFD Solution

$$\phi R_n = 0.75(222.7) = 167 \text{ kips}$$

$$P_u = 1.2D + 1.6L = 1.2(40) + 1.6(100) = 208 \text{ kips} > 167 \text{ kips} \qquad \text{(N.G.)}$$

<u>connection does not have enough capacity.</u>

(b) ASD Solution

$$\frac{R_n}{\Omega} = \frac{222.7}{2.00} = 111 \text{ kips}$$

$$P_a = D + L = 40 + 100 = 140 \text{ kips} > 111 \text{ kips} \qquad \text{(N.G.)}$$

<u>connection does not have enough capacity.</u>

---

**<u>7.6-1</u>**

Bolt shear (assume that the threads are in shear): $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013$ in.$^2$

Nominal shear capacity of one bolt is

$$R_n = F_{nv}A_b \times 2 \text{ (for double shear)} = 54(0.6013) \times 2 = 64.94 \text{ kips/bolt}$$

Check bearing on gusset plate (it is thinner than the combined thickness of the angles, the edge distance is the same as for the angles, and the ultimate tensile stress $F_u$ is smaller):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u = 1.2(1.531)(5/8)(58) = 66.60 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(5/8)(58) = 76.13 \text{ kips} > 66.60 \text{ kips}$$

$$\therefore \text{ use } R_n = 66.60 \text{ kips}$$

For the other bolts,

$$\ell_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u = 1.2(2.063)(5/8)(58) = 89.74 \text{ kips}$$

$$2.4dtF_u = 76.13 \text{ kips} < 89.74 \text{ kips} \therefore \text{ use } R_n = 76.13 \text{ kips}$$

For the connection, the strength for each bolt is the minimum of the shear and bearing values:

Edge bolts: shear controls: $R_n = 64.94$ kips/bolt

Interior bolts: shear controls: $R_n = 64.94$ kips/bolt

For the connection, the strength is

$$8(64.94) = 519.5 \text{ kips}$$

Tension on the gross section: $A_g = 2(7.13) = 14.26 \text{ in.}^2$

$$P_n = F_y A_g = 50(14.26) = 713.0 \text{ kips}$$

Net section:

$$A_n = A_g - \Sigma td_h = 2\left[7.13 - 2\left(\frac{5}{8}\right)\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 11.76 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.72}{9} = 0.8089$$

$$A_e = A_g U = 11.76(0.8089) = 9.513 \text{ in.}^2$$

$$P_n = F_u A_e = 65(9.513) = 618.3 \text{ kips}$$

(a) LRFD solution. Compute the design strength:

For the bolts, $\phi R_n = 0.75(519.5) = 389.6$ kips

For tension on the gross section, $\phi_t P_n = 0.90(713.0) = 641.7$ kips

For tension on the net section, $\phi_t P_n = 0.75(618.3) = 463.7$ kips

The bolt strength controls. $P_u = 389.6$ kips. Since $D > 8L$, load combination 1 controls.

$$1.4D = 1.4(8.5L) = 389.6 \qquad \Rightarrow \qquad L = 32.74 \text{ kips}$$

$$P = D + L = 8.5(32.74) + 32.74 = 311.0 \text{ kips} \qquad \underline{P = 311 \text{ kips}}$$

(b) ASD solution. Compute the allowable strength:

For the bolts, $\dfrac{R_n}{\Omega} = \dfrac{519.5}{2.00} = 259.7$ kips

For tension on the gross section, $\dfrac{R_n}{\Omega_t} = \dfrac{713.0}{1.67} = 426.9$ kips

For tension on the net section, $\dfrac{R_n}{\Omega_t} = \dfrac{618.3}{2.00} = 309.1$ kips

The bolt strength controls. $P_a = 259.7$ kips. $\underline{P = 260 \text{ kips}}$

---

**<u>7.6-2</u>**

Determine the nominal shear and bearing strengths per bolt. The shear strength is

$$R_n = F_{nv}A_b = 68(0.6013) \times 2 = 81.78 \text{ kips/bolt}$$

Bearing: $h = d + \dfrac{1}{16} = \dfrac{7}{8} + \dfrac{1}{16} = \dfrac{15}{16}$ in.

Edge bolts: $\ell_c = \ell_e - \dfrac{h}{2} = 1.5 - \dfrac{15/16}{2} = 1.031$ in.

$$R_n = 1.2\ell_c tF_u = 1.2(1.031)(0.5)(58) = 35.88 \text{ kips/bolt}$$

Upper limit $= 2.4dtF_u = 2.4(7/8)(0.5)(58) = 60.9$ kips/bolt $>$ 35.88 kips/bolt

$\therefore$ use $R_n = 35.88$ kips/bolt

Other bolts: $\ell_c = s - h = 3 - \dfrac{15}{16} = 2.063$ in.

$$R_n = 1.2\ell_c tF_u = 1.2(2.063)(0.5)(58) = 71.79 \text{ kips/bolt}$$

$$2.4dtF_u = 60.9 \text{ kips/bolt} < 71.79 \text{ kips/bolt} \therefore \text{ use } R_n = 60.9 \text{ kips/bolt}$$

(a) LRFD Solution

$$P_u = 1.2P_D + 1.6P_L = 1.2(35) + 1.6(86) = 179.6 \text{ kips}$$

Select a trial number of bolts based on shear, then check bearing. The shear strength is

$\phi R_n = 0.75(81.78) = 61.34$

No. bolts required = $\dfrac{P_u}{\phi R_n} = \dfrac{179.6}{61.34} = 2.93$; Try 4 on each side for symmetry.

For the connection, the strength for each bolt is the minimum of the shear and bearing values. Bearing controls for each bolt in this connection:

$\phi R_n = 0.75[2(35.88) + 2(60.9)] = 145.2$ kips < 179.6 kips (N.G.)

Try 2 rows of 3 bolts on each side (for symmetry):

Total $\phi R_n = 0.75[3(35.88) + 3(60.9)] = 217.8$ kips >179.6 kips (OK)

Use 6 bolts each side, 12 total

(b) ASD Solution

$P_a = P_D + P_L = 35 + 86 = 121.0$ kips

Select a trial number of bolts based on shear, then check bearing. The shear strength is

$\dfrac{R_n}{\Omega} = \dfrac{81.78}{2.00} = 40.89$ kips/bolt

No. bolts required = $\dfrac{121}{40.89} = 2.96$; Try 4 on each side for symmetry.

For the connection, the strength for each bolt is the minimum of the shear and bearing values. Bearing controls for each bolt in this connection:

$\dfrac{R_n}{\Omega} = \dfrac{1}{2.00}[2(35.88) + 2(60.9)] = 96.8$ kips < 121 kips (N.G.)

Try 2 rows of 3 bolts on each side (for symmetry):

Total $R_n/\Omega = [3(35.88) + 3(60.9)]/2.00 = 145$ kips >121 kips (OK)

Use 6 bolts each side, 12 total

---

**7.6-3**

LRFD Solution

$$P_u = 1.2D + 1.6L = 1.2(45) + 1.6(90) = 198.0 \text{ kips}$$

The bearing strength of the gusset plate is smaller than the bearing strength of the member. For one bolt,

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(3/8)(65) = 38.39 \text{ kips}$$

(a) For shear strength, assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\phi R_n = 0.75F_{nv}A_b = 0.75(27)(0.6013) = 12.18 \text{ kips/bolt} < 38.39 \text{ kips/bolt}$$

$$\text{Number required} = 198/12.18 = 16.3$$

Use an even number for symmetry. Use 18 bolts

(b) $\phi R_n = 0.75F_{nv}A_b = 0.75(54)(0.6013) = 24.35 \text{ kips/bolt} < 38.39 \text{ kips/bolt}$

Number required $= 198/24.35 = 8.13$ Use 10 bolts

(c) $\phi R_n = 0.75F_{nv}A_b = 0.75(68)(0.6013) = 30.67 \text{ kips/bolt} < 38.39 \text{ kips/bolt}$

Number required $= 198/30.67 = 6.46$ Use 8 bolts

ASD Solution

$$P_a = D + L = 45 + 90 = 135 \text{ kips}$$

The bearing strength of the gusset plate is smaller than the bearing strength of the member. For one bolt,

$$\frac{R_n}{\Omega} = \frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(3/8)(65)}{2.00} = 25.59 \text{ kips}$$

(a) For shear strength, assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{27(0.6013)}{2.00} = 8.118 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$$

Number required = $135/8.118 = 16.6$

Use an even number for symmetry. Use 18 bolts

(b) $\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{54(0.6013)}{2.00} = 16.23 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$

Number required = $135/16.23 = 8.32$ Use 10 bolts

(c) $\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{68(0.6013)}{2.00} = 20.44 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$

Number required = $135/20.44 = 6.60$ Use 8 bolts

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**7.6-4**

(a)

| Bolt Diameter (in.) | Single-shear design strength (kips) | Slip-critical design strength, one slip plane (kips) | Single-shear allowable strength (kips) | Slip-critical allowable strength, one slip plane (kips) |
|---|---|---|---|---|
| 1/2 | 7.952 | 4.068 | 5.301 | 2.712 |
| 5/8 | 12.425 | 6.441 | 8.283 | 4.294 |
| 3/4 | 17.892 | 9.492 | 11.928 | 6.328 |
| 7/8 | 24.353 | 13.221 | 16.236 | 8.814 |
| 1 | 31.809 | 17.289 | 21.206 | 11.526 |
| 1 1/8 | 40.258 | 18.984 | 26.839 | 12.656 |
| 1 1/4 | 49.701 | 24.069 | 33.134 | 16.046 |
| 1 3/8 | 60.138 | 28.815 | 40.092 | 19.210 |
| 1 1/2 | 71.569 | 34.917 | 47.713 | 23.278 |

(b) Shear never controls in a slip-critical connection; the slip-critical strength is always smaller.

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**7.6-5**

Determine the nominal strengths for all limit states. For bolt shear,

$$A_b = \pi d^2/4 = \pi(1.125)^2/4 = 0.994 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 54(0.994) = 53.68 \text{ kips/bolt}$$

Slip-critical strength: From AISC Table J3-1, the minimum bolt tension is $T_b = 56$ kips. From AISC Equation J3-4,

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(56)(1.0) = 18.98 \text{ kips/bolt}$$

Bearing: $h = d + \frac{1}{16} = 1\frac{1}{8} + \frac{1}{16} = 1.188 \text{ in.}$

Edge bolts: $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{1.188}{2} = 1.406 \text{ in.}$

$$R_n = 1.2\ell_c tF_u) = 1.2(1.406)(1/2)(58) = 48.93 \text{ kips/bolt}$$

Upper limit $= 2.4dtF_u) = 2.4(1.125)(1/2)(58)$

$$= 78.3 \text{ kips/bolt} > 48.93 \text{ kips/bolt} \therefore \text{ use } R_n = 48.93 \text{ kips/bolt}$$

Other bolts: $\ell_c = s - h = 3 - 1.188 = 1.812 \text{ in.}$

$$R_n = 1.2\ell_c tF_u) = 1.2(1.812)(1/2)(58) = 63.06 \text{ kips/bolt}$$

$$2.4dtF_u) = 78.3 \text{ kips/bolt} > 63.06 \text{ kips/bolt} \therefore \text{ use } R_n = 63.06 \text{ kips/bolt}$$

Tension on the gross section: $A_g = 0.5(6.5) = 3.25 \text{ in.}^2$

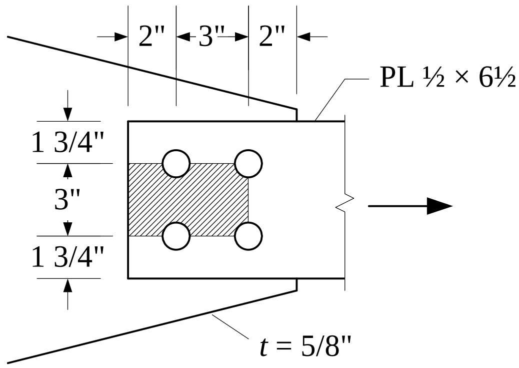
$$P_n = F_y A_g = 36(3.25) = 117.0 \text{ kips}$$

Tension on the net section:

$$A_e = A_n = A_g - \Sigma t d_h = 3.25 - 2\left(\frac{1}{2}\right)\left(1\frac{1}{8} + \frac{1}{8}\right) = 2.0 \text{ in.}^2$$

$$P_n = F_u A_e = 58(2.0) = 116.0 \text{ kips}$$

Check block shear (tension member controls).




$$A_{gv} = 2 \times \frac{1}{2}(3 + 2) = 5.0 \text{ in.}^2$$

$$A_{nv} = 2 \times \frac{1}{2}\left[3 + 2 - 1.5\left(1\frac{1}{8} + \frac{1}{8}\right)\right] = 3.125 \text{ in.}^2$$

$$A_{nt} = \frac{1}{2}\left(3 - 1\frac{1}{4}\right) = 0.875 \text{ in.}^2$$

For this type of block shear, $U_{bs} = 1.0$. From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.125) + 1.0(58)(0.875) = 159.5 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.0) + 1.0(58)(0.875) = 158.8 \text{ kips}$$

The nominal block shear strength is therefore 158.8 kips.

(a) LRFD solution. Compute design strengths:

For bolt shear, $\phi R_n = 0.75(53.68) = 40.26$ kips/bolt

For bolt slip, $\phi R_n = 1.00(18.98) = 18.98$ kips/bolt

For bearing, $\phi R_n = 0.75(48.93) = 36.70$ kips/bolt

The total bolt strength is

$$\phi R_n = 4(18.98) = 75.92 \text{ kips}$$

For tension on the gross section, $\phi R_n = 0.90(117.0) = 105.3$ kips

For tension on the net section, $\phi R_n = 0.75(116.0) = 87.0$ kips

For block shear, $\phi R_n = 0.75(158.8) = 119.1$ kips

Bolt strength controls: $\phi R_n = 75.92 = P_u$

$$1.2D + 1.6L = 1.2D + 1.6(3D) = 75.92, \text{ Solution is: } \{D = 12.65\}$$

$$P = D + L = 12.65 + 3(12.65) = 50.6 \text{ kips}$$ $\underline{P = 50.6 \text{ kips}}$

(b) ASD solution. Compute allowable strengths:

For bolt shear, $\dfrac{R_n}{\Omega} = \dfrac{53.68}{2.00} = 26.84$ kips/bolt

For bolt slip, $\dfrac{R_n}{\Omega} = \dfrac{18.98}{1.50} = 12.65$ kips/bolt

For bearing, $\dfrac{R_n}{\Omega} = \dfrac{48.93}{2.00} = 24.46$ kips/bolt

The total bolt strength is

$$\frac{R_n}{\Omega} = 4(12.65) = 50.6 \text{ kips}$$

For tension on the gross section, $\dfrac{R_n}{\Omega_t} = \dfrac{117.0}{1.67} = 70.06$ kips

For tension on the net section, $\dfrac{R_n}{\Omega_t} = \dfrac{116.0}{2.00} = 58.0$ kips

For block shear, $\dfrac{R_n}{\Omega} = \dfrac{158.8}{2.00} = 79.4$ kips

Bolt strength controls: $P_a = 50.6$ kips $\underline{P = 50.6 \text{ kips}}$

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**7.6-6**

Before proceeding to the LRFD and ASD solutions, compute the nominal bolt shear, bearing, and block shear strengths

Check slenderness: $\dfrac{L}{r_{\min}} = \dfrac{L}{r_z} = \dfrac{9(12)}{0.756} = 143 < 300$ (OK)

For shear,

$$A_b = \pi d^2/4 = \pi(1.125)^2/4 = 0.994 \text{ in.}^2$$

$R_n = F_{nv}A_b = 54(0.994) = 53.68$ kips/bolt

Slip-critical strength: From AISC Table J3-1, the minimum bolt tension is $T_b = 56$ kips. From AISC Equation J3-4,

$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(56)(1.0) = 18.98$ kips/bolt

Bearing: $h = d + \frac{1}{16} = 1\frac{1}{8} + \frac{1}{16} = 1.188$ in.

Edge bolt: $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{1.188}{2} = 1.406$ in.

$R_n = 1.2\ell_c tF_u) = 1.2(1.406)(3/8)(58) = 36.70$ kips/bolt

Upper limit = $2.4dtF_u) = 2.4(1.125)(3/8)(58) = 58.73$

$= 58.73$ kips/bolt > 36.70 kips/bolt $\therefore$ use $R_n = 36.70$ kips/bolt

Other bolts: $\ell_c = s - h = 3.5 - 1.188 = 2.312$ in.

$R_n = 1.2\ell_c tF_u) = 1.2(2.312)(3/8)(58) = 60.34$ kips/bolt

$2.4dtF_u) = 58.73$ kips/bolt < 60.34 kips/bolt $\therefore$ use $R_n = 58.73$ kips/bolt

For the connection, the strength for each bolt is the minimum of the shear, slip-critical, and bearing values. The slip-critical strength controls for each bolt in this connection:

Total $R_n = 6(18.98) = 113.9$ kips

Tension on the gross section: $P_n = F_y A_g = 36(4.50) = 162.0$ kips

Tension on the net section: Use a hole diameter of $1\frac{1}{8} + \frac{1}{8} = 1.25$

$A_n = A_g - \Sigma td_h = 4.50 - \left(\frac{1}{2}\right)(1.25) = 3.875 \text{ in.}^2$

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{17.5} = 0.9526$

$A_e = A_n U = 3.875(0.9526) = 3.691 \text{ in.}^2$

$P_n = F_u A_e = 58(3.691) = 214.1$ kips

Check block shear on the angle.

$$A_{gv} = \frac{1}{2}[5(3.5) + 2] = 9.75 \text{ in.}^2$$

$$A_{nv} = \frac{1}{2}[5(3.5) + 2 - 0.5(1.25)] = 9.438 \text{ in.}^2$$

$$A_{nt} = \frac{1}{2}[2.5 - 0.5(1.25)] = 0.9375 \text{ in.}^2$$

For this type of block shear, $U_{bs} = 1.0$. From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(9.438) + 1.0(58)(0.9375) = 382.8 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(9.75) + 1.0(58)(0.9375) = 265.0 \text{ kips}$$

The nominal block shear strength is therefore 265.0 kips.

(a) LRFD solution. Compute design strengths:

The bolt strength is $\phi R_n = 1.00(132.9) = 133 \text{ kips}$

For tension on the gross section, $\phi R_n = 0.90(162.0) = 146 \text{ kips}$

For tension on the net section, $\phi R_n = 0.75(214.1) = 161 \text{ kips}$

For block shear, $\phi R_n = 0.75(265.0) = 199 \text{ kips}$

The bolt strength controls. $\phi R_n = 133 \text{ kips}$

For load combination 2, $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0 \text{ kips}$

For load combination 4, $P_u = 1.2D + 1.6W + 0.5L = 1.2(20) + 1.6(20) + 0.5(60)$

$$= 86.0 \text{ kips}$$

$\therefore$ combination 2 controls; $P_u = 120 \text{ kips} < 133 \text{ kips}$ (OK)

member and connection are satisfactory.

(b) ASD Solution. Compute allowable strengths:

The bolt strength is $R_n/\Omega = 132.9/1.50 = 88.6$ kips

For tension on the gross section, $R_n/\Omega = 162.0/1.67 = 97.0$ kips

For tension on the net section, $R_n/\Omega = 214.1/2.00 = 107$ kips

For block shear, $R_n/\Omega = 265.0/2.00 = 133$ kips

Slip-critical strength controls. $R_n/\Omega = 88.6$ kips

For load combination 2, $P_a = D + L = 20 + 60 = 80$ kips

For load combination 6, $P_a = D + 0.75W + 0.75L = 20 + 0.75(20) + 0.75(60) = 80$ kips

$\therefore P_u = 80 \text{ kips} < 88.6 \text{ kips}$ (OK)

<u>member and connection are satisfactory</u>

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**<u>7.7-1</u>**

(a) LRFD Solution

$$P_u = 1.2D + 1.6L = 1.2(40) + 1.6(80) = 176.0 \text{ kips}$$

From *Manual* Table 7-1, the bolt shear strength is $\phi r_n = 59.9$ kips

From *Manual* Table 7-3, the bolt slip-critical strength is $\phi r_n = 28.8$ kips

Number of bolts required = $\dfrac{176}{28.8} = 6.11$ try 8 bolts for symmetry.

Minimum spacing = $2\frac{2}{3}d = 2.667\left(1\dfrac{3}{8}\right) = 3.667$ in., try 4 in.

Minimum edge distance from AISC Table J3.4 = $1\dfrac{1}{4} \times d = 1\dfrac{1}{4} \times 1\dfrac{3}{8}$

$$= 1.72 \text{ in., try 2 in.}$$

Check bearing. To determine which component to check, compare the product of the thickness and the ultimate stress (the edge distances and spacings are the same for both components). For the gusset plate,

$$tF_u = 0.5(58) = 29.0 \text{ kips/in.}$$

For the tension member, $F_u = 70$ ksi for A242 steel, and

$$t_w F_u = 0.448(70) = 31.36 \text{ kips/in.} > 29.0 \text{ kip/in}$$

$\therefore$ check bearing on the gusset plate, with $t = 1/2$ in. and $F_u = 58$ ksi.

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in.,

$$\phi r_n = t(66.9) = 0.5(66.9) = 33.5 \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of $2\frac{2}{3}d$ (slightly less than the actual spacing),

$$\phi r_n = t(66.9) = 0.5(116) = 58.0 \text{ kips}$$

Slip critical strength controls. Since the number of bolts was determined from the slip-critical strength, 6 bolts are adequate.

Use 8 bolts in two rows, with $\ell_e = 2$ in. and $s = 4$ in.

(b) ASD Solution

$$P_a = D + L = 40 + 80 = 120 \text{ kips}$$

From *Manual* Table 7-1, the bolt shear strength is $\dfrac{r_n}{\Omega} = 40.0$ kips

From *Manual* Table 7-3, the bolt slip-critical strength is $\dfrac{r_n}{\Omega} = 19.2$ kips

Number of bolts required = $\dfrac{120}{19.2} = 6.25$; try 8 bolts for symmetry.

Minimum spacing = $2\frac{2}{3}d = 2.667\left(1\frac{3}{8}\right) = 3.667$ in., try 4 in.

Minimum edge distance from AISC Table J3.4 = $1\frac{1}{4} \times d = 1\frac{1}{4} \times 1\frac{3}{8} = 1.72$ in., try 2 in.

Check bearing. To determine which component to check, compare the product of the thickness and the ultimate stress (the edge distances and spacings are the same for both components). For the gusset plate,

$$tF_u = 0.5(58) = 29.0 \text{ kips/in.}$$

For the tension member, $F_u = 70$ ksi for A242 steel, and

$$t_w F_u = 0.448(70) = 31.36 \text{ kips/in.} > 29.0 \text{ kip/in}$$

$\therefore$ check bearing on the gusset plate, with $t = 1/2$ in. and $F_u = 58$ ksi.

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in.,

$$\frac{r_n}{\Omega} = t(44.6) = 0.5(44.6) = 22.3 \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of $2\frac{2}{3}d$ (slightly less than the actual spacing),

$$\frac{r_n}{\Omega} = t(77.6) = 0.5(77.6) = 38.8 \text{ kips}$$

Slip critical strength controls. Since the number of bolts was determined from the slip-critical strength, 8 bolts are adequate.

Use 8 bolts in two rows, with $\ell_e = 2$ in. and $s = 4$ in.

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**7.7-2**

Load Combination 2: $1.2D + 1.6L = 1.2(50) + 1.6(100) = 220.0$ kips

Load Combination 4: $1.2D + 1.0W + 0.5L = 1.2(50) + 1.0(45) + 0.5(100) = 155.0$ kips

Combination 2 controls; use $P_u = 220$ kips. Slip-critical strength will control over shear. Use the following spreadsheet table to help select the number of bolts.

| Diameter (in.) | $T_b$ (kips) | $\phi R_n$ / bolt (kips) | No. bolts required |
|---|---|---|---|
| 1/2 | 12 | 4.068 | 54.1 |
| 5/8 | 19 | 6.441 | 34.2 |
| 3/4 | 28 | 9.492 | 23.2 |
| 7/8 | 39 | 13.221 | 16.6 |
| 1 | 51 | 17.289 | 12.7 |
| 1 1/8 | 56 | 18.984 | 11.6 |
| 1 1/4 | 71 | 24.069 | 9.1 |
| 1 3/8 | 85 | 28.815 | 7.6 |
| 1 1/2 | 103 | 34.917 | 6.3 |

Try ten $1\frac{1}{4}$-inch diameter bolts in two lines. $\phi R_n = 10(24.07) = 240.7$ kips > 220 kips (OK)

Select a tension member.

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{220}{0.9(36)} = 6.79 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{220}{0.75(58)} = 5.06 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try L8 × 6 × $\frac{5}{8}$

$$A_g = 8.41 \text{ in.}^2 > 6.79 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = r_z = 1.29 \text{ in.} > 0.80 \text{ in.} \quad \text{(OK)}$$

$$A_n = 8.41 - 2\left(1\frac{1}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 6.691 \text{ in.}^2$$

Use the alternative value of $U$ from AISC Table D3.1, Case 8 (4 or more bolts per line):

$$A_e = A_n U = 6.691(0.80) = 5.35 \text{ in.}^2 > 5.06 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667\left(1\frac{1}{4}\right) = 3.334 \text{ in. Use } 3\tfrac{1}{2} \text{ in.}$$

Minimum edge distance from AISC Table J3.4 = $1\frac{5}{8}$ in. Use 2 in.

Check bearing:

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in.,

$$\phi r_n = t(66.9) = 0.375(70.1) = 26.3 \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of $2\frac{2}{3}d$ (slightly less than the actual spacing),

$$\phi r_n = t(105) = 0.375(105) = 39.4 \text{ kips}$$

Bolt strength is controlled by the slip-critical limit state, and no further check is necessary.

Check block shear on the gusset plate. For hole diameters, use $1\frac{1}{4} + \frac{1}{8} = 1.375$ in.

$$A_{gv} = \frac{3}{8}[4(3.5) + 2] \times 2 = 12.0 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[16 - 4.5(1.375)] \times 2 = 7.359 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.375)] = 0.6094 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(7.359) + 1.0(58)(0.6094) = 291.4 \text{ kips}$$

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(12.0) + 1.0(58)(0.6094) = 294.5 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 291.4 kips

The *design* block shear strength is $\phi R_n = 0.75(291.4) = 219 \text{ kips} < P_u = 220 \text{ kips}$ (N.G.)

Increase the block shear strength by increasing the edge distance and spacing.

Try $\ell_e = 2.5$ in. and $s = 4$ in.

$$A_{gv} = \frac{3}{8}[4(4) + 2.5] \times 2 = 13.88 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[18.5 - 4.5(1.375)] \times 2 = 9.234 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.375)] = 0.6094 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(9.234) + 1.0(58)(0.6094) = 356.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(13.88) + 1.0(58)(0.6094) = 335.2 \text{ kips}$$

$$\phi R_n = 0.75(335.2) = 251 \text{ kips} > 220 \text{ kips} \qquad \text{(OK)}$$

Use an L8 × 6 × $\frac{5}{8}$ and ten $1\frac{1}{4}$-inch diameter Group A bolts in two lines.

Use $\ell_e = 2.5$ in., $s = 4$ in., and the workable gage distances from AISC Table 1-7A.

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(b) ASD Solution

Load Combination 2: $D + L = 50 + 100 = 150$ kips

Load Combination 6: $D + 0.75L + 0.75(0.6)W = 50 + 0.75(100) + 0.75(0.6)(45) =$ 145.3 kips

Combination 2 controls; use $P_a = 150$ kips. Slip-critical strength will control over shear. Use the following spreadsheet table to help select the number of bolts.

| Diameter (in.) | $T_b$ (kips) | $R_n/\Omega$ per bolt (kips) | No. bolts required |
|---|---|---|---|
| 1/2 | 12 | 2.712 | 55.3 |
| 5/8 | 19 | 4.294 | 34.9 |
| 3/4 | 28 | 6.328 | 23.7 |
| 7/8 | 39 | 8.814 | 17.0 |
| 1 | 51 | 11.526 | 13.0 |
| 1 1/8 | 56 | 12.656 | 11.9 |
| 1 1/4 | 71 | 16.046 | 9.3 |
| 1 3/8 | 85 | 19.210 | 7.8 |
| 1 1/2 | 103 | 23.278 | 6.4 |

Try ten $1\frac{1}{4}$-inch diameter bolts in two lines. $\dfrac{R_n}{\Omega} = 10(16.05) = 161 \text{ kips} > 150 \text{ kips}$ (OK)

Select a tension member.

$$\text{Required } A_g = \frac{P_a}{0.6F_y} = \frac{150}{0.6(36)} = 6.94 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{150}{0.5(58)} = 5.17 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try L8 × 6 × $\frac{5}{8}$

$$A_g = 8.41 \text{ in.}^2 > 6.94 \text{ in.}^2 \quad \text{(OK)} \qquad r_{\min} = r_z = 1.29 \text{ in.} > 0.80 \text{ in.} \quad \text{(OK)}$$

$$A_n = 8.41 - 2\left(1\frac{1}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 6.691 \text{ in.}^2$$

Use the alternative value of $U$ from AISC Table D3.1, Case 8 (4 or more bolts per line):

$$A_e = A_n U = 6.691(0.80) = 5.35 \text{ in.}^2 > 5.17 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667\left(1\frac{1}{4}\right) = 3.334 \text{ in. Use 4 in.}$$

Minimum edge distance from AISC Table J3.4 = $1\frac{5}{8}$ in. Use $2\frac{1}{2}$ in. Check bearing:

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in. (this is less than the actual edge distance and is conservative),

$$\frac{r_n}{\Omega} = t(46.8) = 0.375(136) = \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of $2\frac{2}{3}d$ (less than the actual spacing),

$$\frac{r_n}{\Omega} = t(70.3) = 0.375(70.3) = 26.36 \text{ kips}$$

Bolt strength is controlled by the slip-critical limit state, and no further check is necessary.

Check block shear on the gusset plate. For hole diameters, use $1\frac{1}{4} + \frac{1}{8} = 1.375$ in.

$$A_{gv} = \frac{3}{8}[4(4) + 2.5] \times 2 = 13.88 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[18.5 - 4.5(1.375)] \times 2 = 9.234 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.375)] = 0.6094 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(9.234) + 1.0(58)(0.6094) = 356.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(13.88) + 1.0(58)(0.6094) = 335.2 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{335.2}{2.00} = 168 \text{ kips} > P_a = 150 \text{ kips} \qquad \text{(OK)}$$

Use an L8 × 6 × $\frac{5}{8}$ and ten $1\frac{1}{4}$-inch diameter Group A bolts in two lines.

Use $\ell_e = 2.5$ in., $s = 4$ in., and the workable gage distances from AISC Table 1-7A.

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**7.7-3**

(a) LRFD Solution: $P_u = 1.2D + 1.6L = 1.2(45) + 1.6(105) = 222.0$ kips

Try $1\frac{1}{8}$-in. diameter bolts. Slip-critical strength will control over shear:

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(56)(2) = 37.97 \text{ kips/bolt}$$

$$\phi R_n = 0.75(37.97) = 28.48 \text{ kips/bolt}$$

Number of bolts required $= \dfrac{222}{28.48} = 7.80$ try 8 bolts in two rows.

Select a tension member. Required $A_g = \dfrac{P_u}{0.9F_y} = \dfrac{222}{0.9(36)} = 6.85 \text{ in.}^2$

Required $A_e = \dfrac{P_u}{0.75F_u} = \dfrac{222}{0.75(58)} = 5.10 \text{ in.}^2$

Required $r_{\min} = \dfrac{L}{300} = \dfrac{15 \times 12}{300} = 0.6 \text{ in.}$

Try $2L7 \times 4 \times \frac{7}{16}$ LLBB

$A_g = 9.26 \text{ in.}^2 > 6.85 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 1.55 \text{ in.} > 0.6 \text{ in.}$ (OK)

$A_n = 9.26 - 4\left(1\frac{1}{8} + \frac{1}{8}\right)\left(\frac{7}{16}\right) = 7.073 \text{ in.}^2$

Use the alternative value of $U$ from AISC Table D3.1, Case 8:

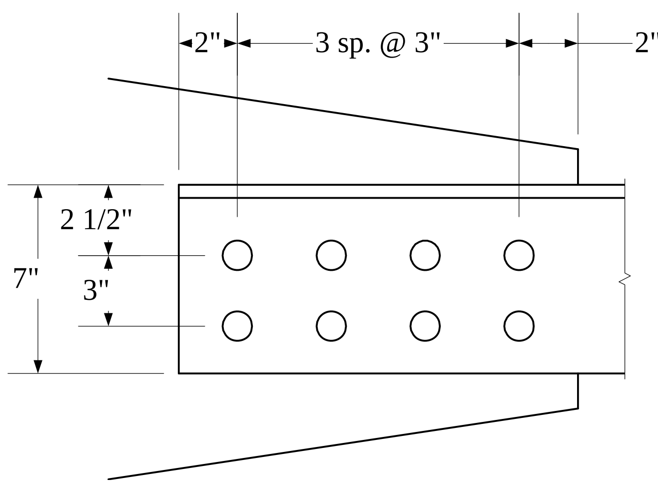
$A_e = A_n U = 7.073(0.80) = 5.66 \text{ in.}^2 > 5.10 \text{ in.}^2$ (OK)

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

Minimum spacing $= 2\frac{2}{3}d = 2.667(1.125) = 3.0 \text{ in.}$ Use 3 in.

Minimum edge distance from AISC Table J3.4 $= 1\frac{1}{2}$ in. Use 2 in.

Try the following layout.



Check bearing (gusset plate controls).

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in.,

$$\phi r_n = t(73.4) = 0.375(73.4) = 27.53 \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of 3 in.,

$$\phi r_n = t(94.6) = 0.375(94.6) = 35.48 \text{ kips}$$

For the connection, the strength for each bolt is the minimum of the slip-critical and bearing values:

Edge bolts: bearing controls: $\phi r_n = 27.53$ kips/bolt

Interior bolts: slip controls: $\phi r_n = 28.48$ kips/bolt

Total strength $= 2(27.53) + 6(28.48) = 226 \text{ kips} > P_u = 220$ kips (OK)

Check block shear on the gusset plate. For hole diameters, use $1\frac{1}{8} + \frac{1}{8} = 1.25$ in.

$$A_{gv} = \frac{3}{8}[3(3) + 2] \times 2 = 8.25 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[11 - 3.5(1.25)] \times 2 = 4.969 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.25)] = 0.6563 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(4.969) + 1.0(58)(0.6563) = 211.0 \text{ kips}$$

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(8.25) + 1.0(58)(0.6563) = 216.3 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 211.0 kips

The *design* block shear strength is $\phi R_n = 0.75(211.0)$

$$= 158.3 \text{ kips} < P_u = 220 \text{ kips} \quad \text{(N.G.)}$$

Increase the bolt spacing. Try $s = 5$ in.

$$A_{gv} = \frac{3}{8}[3(5) + 2] \times 2 = 12.75 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[17 - 3.5(1.25)] \times 2 = 9.469 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.25)] = 0.6563 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

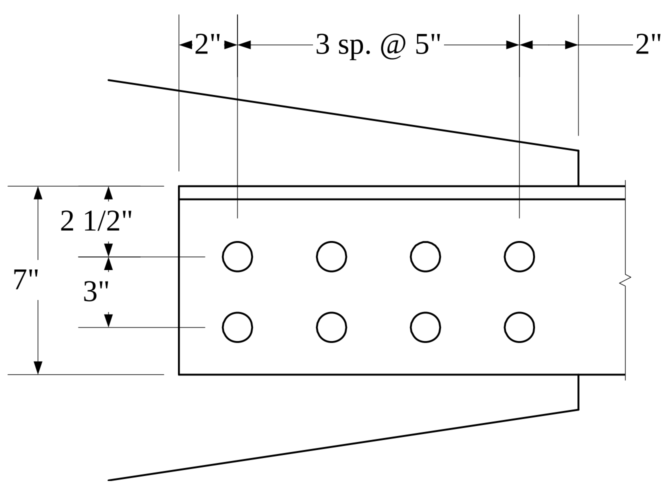
$$= 0.6(58)(9.469) + 1.0(58)(0.6563) = 367.6 \text{ kips}$$

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(12.75) + 1.0(58)(0.6563) = 313.5 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 313.5 kips

The *design* block shear strength is $\phi R_n = 0.75(313.5) = 235 \text{ kips} > P_u = 220 \text{ kips}$ (OK)

Use 2L7 × 4 × $\frac{7}{16}$ LLBB and eight $1\frac{1}{8}$-inch diameter Group A bolts in two lines as shown.




(b) ASD Solution: $P_a = D + L = 45 + 105 = 150 \text{ kips}$

Try $1\frac{1}{8}$-in. diameter bolts. Slip-critical strength will control over shear:

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(56)(2) = 37.97 \text{ kips/bolt}$$

$$\frac{R_n}{\Omega} = \frac{37.97}{2.00} = 18.99 \text{ kips/bolt}$$

Number of bolts required = $\frac{150}{18.99} = 7.90$ try 8 bolts in two rows.

Select a tension member. Required $A_g = \dfrac{P_a}{0.6F_y} = \dfrac{150}{0.6(36)} = 6.94 \text{ in.}^2$

Required $A_e = \dfrac{P_a}{0.5F_u} = \dfrac{150}{0.5(58)} = 5.17 \text{ in.}^2$

Required $r_{\min} = \dfrac{L}{300} = \dfrac{15 \times 12}{300} = 0.6 \text{ in.}$

Try 2L7 × 4 × $\frac{7}{16}$ LLBB

$A_g = 9.26 \text{ in.}^2 > 6.94 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 1.55 \text{ in.} > 0.6 \text{ in.}$ (OK)

$A_n = 9.26 - 4\left(1\frac{1}{8} + \frac{1}{8}\right)\left(\frac{7}{16}\right) = 7.073 \text{ in.}^2$

Use the alternative value of $U$ from AISC Table D3.1, Case 8:

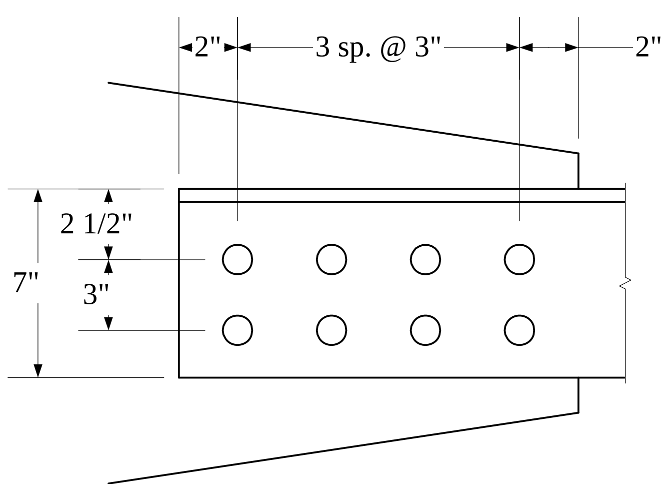
$A_e = A_nU = 7.301(0.80) = 5.84 \text{ in.}^2 > 5.17 \text{ in.}^2$ (OK)

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

Minimum spacing = $2\frac{2}{3}d = 2.667(1.125) = 3.0$ in. Use 3 in.

Minimum edge distance from AISC Table J3.4 = $1\frac{3}{4}$ in. Use 2 in.

Try the following layout.



Check bearing (gusset plate controls).

Edge bolts: From *Manual* Table 7-5 and an edge distance of 2 in.,

$$\frac{r_n}{\Omega} = t(48.9) = 0.375(48.9) = 18.34 \text{ kips}$$

Inner bolts: From *Manual* Table 7-4 and a spacing of 3 in.,

$$\frac{r_n}{\Omega} = t(63.1) = 0.375(63.1) = 23.66 \text{ kips}$$

For the connection, the strength for each bolt is the minimum of the slip-critical and bearing values:

Edge bolts: bearing controls: $\frac{r_n}{\Omega} = 18.34$ kips/bolt

Interior bolts: slip controls: $\frac{r_n}{\Omega} = 18.99$ kips/bolt

Total allowable strength $= 2(18.34) + 6(18.99) = 151 \text{ kips} > P_a = 150$ kips (OK)

Check block shear on the gusset plate. For hole diameters, use $1\frac{1}{8} + \frac{1}{8} = 1.25$ in.

$$A_{gv} = \frac{3}{8}[3(3) + 2] \times 2 = 8.25 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[11 - 3.5(1.25)] \times 2 = 4.969 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.25)] = 0.6563 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(4.969) + 1.0(58)(0.6563) = 211.0 \text{ kips}$$

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(8.25) + 1.0(58)(0.6563) = 216.3 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 211.0 kips

The *allowable* block shear strength is

$$\frac{R_n}{\Omega} = \frac{211.0}{2.00} = 106 \text{ kips} < P_a = 150 \text{ kips (N.G.)}$$

Increase the bolt spacing. Try $s = 5$ in.

$$A_{gv} = \frac{3}{8}[3(5) + 2] \times 2 = 12.75 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[17 - 3.5(1.25)] \times 2 = 9.469 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3 - 1(1.25)] = 0.6563 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

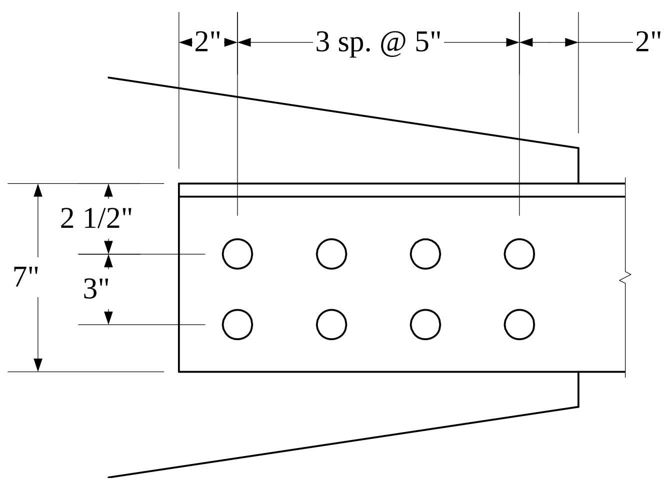
$$= 0.6(58)(9.469) + 1.0(58)(0.6563) = 367.6 \text{ kips}$$

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(12.75) + 1.0(58)(0.6563) = 313.5 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 313.5 kips

The *allowable* block shear strength is

$$\frac{R_n}{\Omega} = \frac{313.5}{2.00} = 157 \text{ kips} > P_a = 150 \text{ kips (OK)}$$




Use 2L7 × 4 × $\frac{7}{16}$ LLBB and eight $1\frac{1}{8}$-inch diameter Group A bolts in two lines as shown.

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**7.8-1**

$$b = \frac{5.5 - t_w}{2} = \frac{5.5 - 0.585}{2} = 2.458 \text{ in}$$

$$a = \frac{b_f - 5.5}{2} = \frac{10.4 - 5.5}{2} = 2.45 \text{ in.}$$

$$1.25b = 1.25(2.458) = 3.073 \text{ in.} > 2.45 \text{ in.} \therefore \text{ use } a = 2.45 \text{ in.}$$

$$b' = b - \frac{d}{2} = 2.458 - \frac{3/4}{2} = 2.083 \text{ in.}$$

$$a' = a + \frac{d}{2} = 2.45 + \frac{3/4}{2} = 2.825 \text{ in.}$$

$$d' = d + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.,}$$

$$p = 10/3 = 3.333 \text{ in.}$$

$$2b = 2(2.458) = 4.916 > 3.333 \therefore \text{use } p = 3.333 \text{ in.}$$

$$\delta = 1 - d'/p = 1 - (7/8)/3.333 = 0.7375 \text{ in.}$$

$$A_b = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

(a) LRFD solution.

$$B = \phi R_n = \phi F_{nt} A_b = 0.75(90)(0.4418) = 29.82 \text{ kips}$$

$$\text{Total factored load} = 1.2D + 1.6L = 1.2(25) + 1.6(75) = 150.0 \text{ kips}$$

$$T = \frac{150}{6} = 25.0 \text{ kips/bolt}$$

$$\frac{B}{T} - 1 = \frac{29.82}{25} - 1 = 0.1928, \qquad \frac{a'}{b'} = \frac{2.825}{2.083} = 1.356$$

$$\alpha = \frac{\left(\frac{B}{T} - 1\right)\frac{a'}{b'}}{\delta\left[1 - \left(\frac{B}{T} - 1\right)\frac{a'}{b'}\right]} = \frac{0.1928(1.356)}{0.7375[1 - 0.1928(1.356)]} = 0.4799 < 1.0$$

$$\text{Required } t_f = \sqrt{\frac{4Tb'}{\phi_b p F_u(1 + \delta\alpha)}} = \sqrt{\frac{4(25)(2.083)}{0.90(3.333)(65)[1 + 0.7375(0.4799)]}}$$

$$= 0.888 \text{ in.}$$

Actual thickness = 0.985 in. > 0.888 in. (OK)

Compute the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{4Tb'}{\phi_b p t_f^2 F_u} - 1\right) = \frac{1}{0.7375}\left[\frac{4(25)(2.083)}{0.90(3.333)(0.985)^2(65)} - 1\right]$$

$$= 0.1371$$

$$\delta\alpha = 0.7375(0.1371) = 0.1011$$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 25\left[1 + \frac{0.1}{(1+0.1011)}\frac{2.083}{2.825}\right] = 26.67$$

$$= 26.67 \text{ kips} < \phi R_n = 29.82 \text{ kips} \qquad \text{(OK)}$$

<u>Connection is adequate. Required $t_f = 0.888$ in. < 0.985 in.</u>

(b) ASD solution

The allowable tensile strength of one bolt is

$$B = \frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90.0(0.4418)}{2.00} = 19.88 \text{ kips}$$

The total applied load is

$$D + L = 25 + 75 = 100 \text{ kips}$$

The external load per bolt, excluding prying force, is $T = 100/6 = 16.67$ kips.

$$\frac{B}{T} - 1 = \frac{19.88}{16.67} - 1 = 0.1926, \qquad \frac{a'}{b'} = \frac{2.825}{2.083} = 1.356$$

$$\alpha = \frac{[(B/T-1)](a'/b')}{\delta\{1-[(B/T)-1](a'/b')\}} = \frac{0.1926(1.356)}{0.7375[1-0.1926(1.356)]} = 0.4794$$

From Equation 7.19,

$$\text{Required } t_f = \sqrt{\frac{\Omega_b 4Tb'}{pF_u(1+\delta\alpha)}} = \sqrt{\frac{1.67(4)(16.67)(2.083)}{3.333(65)[1+0.7375(0.4794)]}}$$

$$= 0.889 \text{ in.} < 0.985 \text{ in.} \qquad \text{(OK)}$$

Determine the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{\Omega_b 4Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.7375}\left(\frac{1.67(4)(16.67)(2.083)}{3.333(0.985)^2(65)} - 1\right) = 0.1404$$

$$B_c = T\left[1+\frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 16.67\left[1+\frac{0.7375(0.1404)}{(1+0.7375\times 0.1404)}\frac{2.083}{2.825}\right]$$

$$= 17.82 \text{ kips} < 19.88 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.889$ in. < 0.985 in.

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**7.8-2**

For $b$, use the distance from the bolt centerline to the mid-thickness of the angle leg.

$$b = \frac{5.375 - 3/8 - 5/8}{2} = 2.188 \text{ in}$$

$$a = \frac{4+4+3/8-5.375}{2} = 1.5 \text{ in.}$$

$$1.25b = 1.25(2.188) = 2.735 \text{ in.} > 1.5 \text{ in.} \therefore \text{ use } a = 1.5 \text{ in.}$$

$$b' = b - \frac{d}{2} = 2.188 - \frac{1/2}{2} = 1.938 \text{ in.}$$

$$a' = a + \frac{d}{2} = 1.5 + \frac{1/2}{2} = 1.75 \text{ in.}$$

$$d' = d + \frac{1}{8} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8} \text{ in.,} \qquad p = \frac{7}{2} = 3.5 \text{ in.}$$

$$2b = 2(2.188) = 4.376 > 3.5 \therefore \text{ use } p = 3.5 \text{ in.}$$

$$\delta = 1 - d'/p = 1 - (5/8)/3.5 = 0.8214 \text{ in.}$$

$$A_b = \frac{\pi(1/2)^2}{4} = 0.1963 \text{ in.}^2$$

(a) LRFD solution.

$$B = \phi R_n = \phi F_{nt} A_b = 0.75(90)(0.1963) = 13.25 \text{ kips}$$

$$\text{Total factored load} = 1.2D + 1.6L = 1.2(6) + 1.6(15) = 31.2 \text{ kips}$$

$$T = \frac{31.2}{4} = 7.8 \text{ kips/bolt}$$

$$\frac{B}{T} - 1 = \frac{13.25}{7.8} - 1 = 0.6987, \qquad \frac{a'}{b'} = \frac{1.75}{1.938} = 0.9030$$

$$\alpha = \frac{\left(\frac{B}{T} - 1\right)\frac{a'}{b'}}{\delta\left[1 - \left(\frac{B}{T} - 1\right)\frac{a'}{b'}\right]} = \frac{0.6987(0.9030)}{0.8214[1 - 0.6987(0.9030)]} = 2.081 > 1.0$$

$\therefore$ use $\alpha = 1.0$

$$\text{Required } t_f = \sqrt{\frac{4Tb'}{\phi_b p F_u (1 + \delta\alpha)}} = \sqrt{\frac{4(7.8)(1.938)}{0.90(3.5)(58)[1 + 0.8214(1.0)]}}$$

$$= 0.426 \text{ in.}$$

Actual thickness = 5/8 in. > 0.426 in. (OK)

Compute the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{4Tb'}{\phi_b p t_f^2 F_u} - 1\right) = \frac{1}{0.8214}\left[\frac{4(7.8)(1.938)}{0.90(3.5)(5/8)^2(58)} - 1\right] = -0.1860$$

Since $\alpha$ must be between 0 and 1 inclusive, use $\alpha = 0$

$$\delta\alpha = 0.8214(0) = 0$$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1 + \delta\alpha)}\frac{b'}{a'}\right] = 7.8\left[1 + \frac{0}{(1 + 0)}\left(\frac{1.938}{1.75}\right)\right] = 7.8$$

$$= 7.8 \text{ kips} < \phi R_n = 13.25 \text{ kips} \quad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.426$ in. < 5/8 in.

(b) ASD solution

The allowable tensile strength of one bolt is

$$B = \frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90.0(0.1963)}{2.00} = 8.834 \text{ kips}$$

The total applied load is

$$D + L = 6 + 15 = 21 \text{ kips}$$

The external load per bolt, excluding prying force, is $T = 21/4 = 5.25$ kips.

$$\frac{B}{T} - 1 = \frac{8.834}{5.25} - 1 = 0.6827, \qquad \frac{a'}{b'} = \frac{1.75}{1.938} = 0.9030$$

$$\alpha = \frac{[(B/T-1)](a'/b')}{\delta\{1-[(B/T)-1](a'/b')\}} = \frac{0.6827(0.9030)}{0.8214[1-0.6827(0.9030)]} = 1.96 > 1.0$$

$\therefore$ use $\alpha = 1.0$

From Equation 7.19,

$$\text{Required } t_f = \sqrt{\frac{\Omega_b 4Tb'}{pF_u(1+\delta\alpha)}} = \sqrt{\frac{1.67(4)(5.25)(1.938)}{3.5(58)(1+0.8214(1.0))}}$$

$$= 0.429 \text{ in.} < 5/8 \text{ in.} \qquad \text{(OK)}$$

Determine the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{\Omega_b 4Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.8214}\left(\frac{1.67(4)(5.25)(1.938)}{3.5(5/8)^2(58)} - 1\right) = -0.1740$$

Since $\alpha$ must be between 0 and 1 inclusive, use $\alpha = 0$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 5.25\left[1 + \frac{0}{(1+0)}\frac{1.938}{1.75}\right]$$

$$= 5.25 \text{ kips} < 8.834 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.429$ in. < 5/8 in.

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**7.9-1**

Nominal shear strength (assume that the threads are in shear):

$$A_b = \frac{\pi d^2}{4} = \frac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 54(0.6013) = 32.47 \text{ kips/bolt}$$

Bearing strength (the WT flange controls): $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

For the hole nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{0.9375}{2} = 1.531$ in.

$$R_n = 1.2\ell_c tF_u = 1.2(1.531)(0.640)(65) = 76.43 \text{ kips}$$

$$2.4dtF_u = 2.4(7/8)(0.640)(65) = 87.36 \text{ kips}$$

$\therefore$ use $R_n = 76.43$ kips

For the other bolts, $\ell_c = s - h = 3 - 0.9375 = 2.063$ in.

$$R_n = 1.2\ell_c t F_u = 1.2(2.063)(0.640)(65) = 103.0 \text{ kips}$$

$$2.4dtF_u = 87.36 \text{ kips} < 103.0 \text{ kips} \;\therefore\; \text{use } R_n = 87.36 \text{ kips}$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(84) + 1.6(66) = 206.4 \text{ kips}$$

$$T_u = \text{Tensile force} = 206.4\cos 30° = 178.7 \text{ kips, or } \frac{178.7}{8} = 22.34 \text{ kips/bolt}$$

$V_u$ = Shear/bearing force = $206.4\sin 30° = 103.2$ kips, or $\dfrac{103.2}{8} = 12.9$ kips/bolt

The design shear strength is $\phi R_n = 0.75(32.47) = 24.35 \text{ kips} > 12.9 \text{ kips}$ (OK)

For the design bearing strength, conservatively use the smaller of the two strengths computed:

$$\phi R_n = 0.75(76.43) = 57.3 \text{ kips} > 12.9 \text{ kips} \qquad \text{(OK)}$$

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{12.9}{0.6013} = 21.45 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)}(21.45) = 69.33 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(69.33)(0.6013) = 31.3 \text{ kips/bolt} > 22.3 \text{ kips/bolt} \quad \text{(OK)}$$

<u>The connection is adequate.</u>

(b) ASD solution

$$P_a = D + L = 84 + 66 = 150 \text{ kips}$$

$$T_a = \text{Tensile force} = 150\cos 30° = 129.9 \text{ kips, or } \frac{129.9}{8} = 16.24 \text{ kips/bolt}$$

$$V_a = \text{Shear/bearing force} = 150\sin 30° = 75.00 \text{ kips, or } \frac{75.00}{8} = 9.375$$

kips/bolt

The allowable shear strength is $\frac{R_n}{\Omega} = \frac{32.47}{2.00} = 16.23$ kips > 9.38 kips (OK)

For the allowable bearing strength, conservatively use the smaller of the two strengths computed:

$$\frac{R_n}{\Omega} = \frac{76.43}{2.00} = 38.2 \text{ kips} > 9.375 \text{ kips} \qquad \text{(OK)}$$

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{9.375}{0.6013} = 15.59 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{54}(15.59) = 65.03 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{65.03(0.6013)}{2.00} = 19.6 \text{ kips/bolt} > 16.2 \text{ kips/bolt} \quad \text{(OK)}$$

The connection is adequate.

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**7.9-2**

Nominal shear strength:

$$A_b = \frac{\pi d^2}{4} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 54(0.4418) = 23.86 \text{ kips/bolt}$$

Bearing strength ($t_f = 0.605$ in. for both W and WT):

$$R_n = 2.4dtF_u = 2.4(3/4)(0.605)(65) = 70.79 \text{ kips}$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(30) + 1.6(70) = 148.0 \text{ kips}$$

$$T_u = \text{Tensile force} = 148\left(\frac{2}{\sqrt{5}}\right) = 132.4 \text{ kips, or } \frac{132.4}{6} = 22.1 \text{ kips/bolt}$$

$V_u$ = Shear/bearing force = $148\left(\frac{1}{\sqrt{5}}\right) = 66.19$ kips, or $\frac{66.19}{6} = 11.03$ kips/bolt

The design shear strength is $\phi R_n = 0.75(23.86) = 17.90$ kips > 11.0 kips (OK)

The design bearing strength is $\phi R_n = 0.75(70.79) = 53.1$ kips > 11.0 kips (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}, \qquad f_{rv} = \frac{11.03}{0.4418} = 24.97 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)}(24.97) = 61.51 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(61.51)(0.4418) = 20.4 \text{ kips/bolt} < 22.1 \text{ kips/bolt} \quad \text{(N.G.)}$$

The connection is not adequate.

(b) ASD solution

$$P_a = D + L = 30 + 70 = 100 \text{ kips}$$

$$T_a = \text{Tensile force} = 100\left(\frac{2}{\sqrt{5}}\right) = 89.44 \text{ kips, or } \frac{89.44}{6} = 14.9 \text{ kips/bolt}$$

$$V_a = \text{Shear/bearing force} = 100\left(\frac{1}{\sqrt{5}}\right) = 44.72 \text{ kips, or}$$

$$\frac{44.72}{6} = 7.453 \text{ kips/bolt}$$

The allowable shear strength is $\frac{R_n}{\Omega} = \frac{23.86}{2.00} = 11.93$ kips > 7.453 kips (OK)

The allowable bearing strength is $\frac{R_n}{\Omega} = \frac{70.79}{2.00} = 35.4$ kips > 7.453 kips (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt}, \qquad f_{rv} = \frac{7.453}{0.4418} = 16.87 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{54}(16.87) = 60.77 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{60.77(0.4418)}{2.00} = 13.4 \text{ kips/bolt } < 14.9 \text{ kips/bolt (N.G.)}$$

<u>The connection is not adequate.</u>

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**<u>7.9-3</u>**

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0 \text{ kips}$$

$$T_u = \frac{3}{5}(120) = 72.0 \text{ kips}, \qquad V_u = \frac{4}{5}(120) = 96.0 \text{ kips}$$

Assume that the tension strength will control:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(54)} f_{rv} = 117.0 - 2.222 f_{rv} \le 90$$

$$\phi F'_{nt} = 0.75(117 - 2.222 f_{rv}) \le 0.75(90) = 87.75 - 1.667 f_{rv} \le 67.5$$

Let $\phi F'_{nt} = \frac{72}{\Sigma A_b}$ and $f_{rv} = \frac{96}{\Sigma A_b}$

$$\frac{72}{\Sigma A_b} = 87.75 - 1.667\left(\frac{96}{\Sigma A_b}\right)$$

$72 = 87.75\Sigma A_b - 160.0$, Solution is: $\left\{\Sigma A_b = 2.644 \text{ in.}^2\right\}$

$$\text{Required } A_b = \frac{\Sigma A_b}{12} = \frac{2.644}{12} = 0.2203 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.2203)}{\pi}} = 0.5296 \text{ in.}$$

Try $\frac{7}{8}$ -in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

$$\text{Required } N_b = \frac{\Sigma A_b}{A_b} = \frac{2.644}{0.6013} = 4.397, \text{ Try 6 bolts for symmetry.}$$

First, check the upper limit on $F'_{nt}$:

$$f_{rv} = \frac{96}{\Sigma A_b} = \frac{96}{6(0.6013)} = 26.61 \text{ ksi}$$

$$F'_{nt} = 117 - 2.222 f_{rv} = 117 - 2.222(26.61) = 57.87 \text{ ksi} < 90 \text{ ksi (OK)}$$

Check shear: $V_u/\text{bolt} = 96/6 = 16.0$ kips/bolt

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.6013) = 24.35 \text{ kips/bolt} > 16.0 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength (the WT flange thickness controls):

$$\phi(2.4 dt F_u) = 0.75(2.4)(7/8)(0.345)(65)$$

$$= 35.3 \text{ kips/bolt} > 16.0 \text{ kips/bolt} \quad \text{(OK)}$$

Use 6 bolts.

(b) ASD solution

$$T_a = \frac{3}{5}(80) = 48 \text{ kips}, \qquad V_a = \frac{4}{5}(80) = 64 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3 F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{54} f_{rv} \le 90 = 117 - 3.333 f_{rv} \le 90$$

$$\frac{F'_{nt}}{\Omega} = \frac{(117 - 3.333 f_{rv})}{2.00} \le \frac{90}{2.00} = 58.5 - 1.667 f_{rv} \le 45$$

Let $\dfrac{F'_{nt}}{\Omega} = \dfrac{48}{\Sigma A_b}$ and $f_{rv} = \dfrac{64}{\Sigma A_b}$

where $\Sigma A_b$ is the total bolt area. Substituting and solving for $\Sigma A_b$, we get

$$\frac{48}{\Sigma A_b} = 58.5 - 1.667\left(\frac{64}{\Sigma A_b}\right)$$

$$48 = 58.5 \Sigma A_b - 106.7, \text{ Solution is: } \left\{\Sigma A_b = 2.644 \text{ in.}^2\right\}$$

Try $\frac{7}{8}$ -in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

The number of bolts required is $N_b = \dfrac{\Sigma A_b}{A_b} = \dfrac{2.644}{0.6013} = 4.397$

Try six bolts for symmetry. First, check the upper limit on $F'_{nt}$ :

$$f_{rv} = \frac{64}{\Sigma A_b} = \frac{64}{6(0.6013)} = 17.74 \text{ ksi}$$

$$F'_{nt} = 117 - 3.333 f_{rv} = 117 - 3.333(17.74) = 57.87 \text{ ksi} < 90 \text{ ksi} \qquad \text{(OK)}$$

Check shear.

$$V_a/\text{bolt} = 64/6 = 10.7 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{2.00} = \frac{54(0.6013)}{2.00} = 16.24 \text{ kips/bolt} > 10.7 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength (the WT flange thickness controls):

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(0.345)(65)}{2.00} = 23.6 \text{ kips/bolt} > 10.7 \text{ kips/bolt} \quad \text{(OK)}$$

<u>Use 6 bolts.</u>

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**<u>7.9-4</u>**

(a) LRFD solution

Tension member-to-gusset plate connection:

$$P_u = 1.2D + 1.6L = 1.2(0.25 \times 120) + 1.6(0.75 \times 120) = 180 \text{ kips}$$

Bolt shear strength:

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(68)(0.6013) \times 2 \text{ shear planes} = 61.33 \text{ kips/bolt}$$

Check bearing on the gusset plate assuming upper limit controls.

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(7/8)(58) = 79.9 \text{ kips/bolt}$$

Slip-critical strength: From AISC Table J3.1, $T_b = 49$ kips

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(49)(2) = 33.22 \text{ kips/bolt}$$

$$\phi R_n = 1.0(33.22) = 33.22 \text{ kips/bolt} \quad \text{(controls)}$$

Number required = 180/33.22 = 5.42 bolts. Use 6.

Gusset plate-to-column angles connection:

Bearing on the angles will control, because $2 \times \dfrac{3}{8}$ in. < $\dfrac{7}{8}$ in.

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(2 \times \frac{3}{8})(58) = 68.5 \text{ kips/bolt}$$

Shear strength and slip-critical strength are same as for tension member-to-gusset plate connection, $\therefore$ use 6 bolts.

Connection angles-to-column flange connection: assume that the slip-critical strength will control. Select the number of bolts based on slip-critical strength, then check tension.

$$T_u = V_u = \frac{1}{\sqrt{2}}(180) = 127.3 \text{ kips}$$

$$\text{Reduction factor} = k_{sc} = 1 - \frac{T_u}{D_u T_b n_b} = 1 - \frac{127.3}{1.13(49)n_b}$$

Let $127.3 = n_b\left[38.76\left(1 - \dfrac{127.3}{1.13(49)n_b}\right)\right]$, Solution is: $\{n_b = 5.583\}$

Use 6 rows of 2 bolts for symmetry and to match the arrangement of gusset plate-to-connection angle bolts. Check tension.

$$\text{Shear/tension load per bolt} = 127.3/12 = 10.61 \text{ kips}$$

$$\phi R_n = \phi F_{nt} A_b = 0.75(90)(0.6013) = 40.6 \text{ kips/bolt} > 10.61 \text{ kips/bolt (OK)}$$

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667(7/8) = 2.33 \text{ in.}$$

Minimum edge distance from AISC Table J3.4 = $1\frac{1}{8}$ in.

Try $\ell_e = 1\frac{1}{2}$ in. and $s = 2\frac{1}{2}$ in. for all connection elements and check bearing.

For the angles-to-column connection, the angle thickness will control (3/8 in. < 0.695 in.):

$$h = \frac{7}{8} + \frac{1}{16} = 0.9375 \text{ in.}$$

For the hole nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{0.9375}{2} = 1.031$ in.

$$\phi R_n = \phi(1.2\ell_c tF_u) = 0.75(1.2)(1.031)(3/8)(58)$$

$$= 20.2 \text{ kips} > 10.61 \text{ kips (OK)}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(3/8)(58) = 34.3 \text{ kips/bolt}$$

For the other bolts, $\ell_c = s - h = 2.5 - 0.9375 = 1.563$ in.

$$\phi R_n = \phi(1.2\ell_c tF_u) = 0.75(1.2)(1.563)(3/8)(58)$$

$$= 30.6 \text{ kips} > 10.61 \text{ kips (OK)}$$

For the tension member connection, the gusset plate thickness will control:

For the hole nearest the edge,

$$\phi R_n = \phi(1.2\ell_c tF_u) = 0.75(1.2)(1.031)(7/8)(58)$$

$$= 47.1 \text{ kips} > 180/6 = 30 \text{ kips (OK)}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(7/8)(58) = 79.9 \text{ kips/bolt}$$

For the other bolts,

$$\phi R_n = \phi(1.2\ell_c tF_u) = 0.75(1.2)(1.563)(7/8)(58) = 71.4 \text{ kips} > 30 \text{ kips (OK)}$$

For the gusset plate-to-column connection angles,

$$\phi R_n = \frac{2(3/8)}{7/8} \times 47.1 = 40.4 \text{ kips/bolt} > 30 \text{ kips/bolt (OK)}$$

Summary:
Use 6 bolts for tension member-to-gusset plate connection;
use 6 bolts for gusset-plate-to connection angles connection;
use 12 bolts for connection angles to column flange connection;
use edge distances of $1\frac{1}{2}$ in. and bolt spacings of $2\frac{1}{2}$ in. throughout.

(b) ASD solution

Tension member-to-gusset plate connection:

$$P_a = D + L = 120 \text{ kips}$$

Bolt shear strength:

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nv}A_b = \frac{1}{2.00}(68)(0.6013) \times 2 \text{ shear planes} = 40.89 \text{ kips/bolt}$$

Check bearing on the gusset plate assuming upper limit controls.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(7/8)(58) = 53.3 \text{ kips/bolt}$$

Slip-critical strength: From AISC Table J3.1, $T_b = 49$ kips

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(49)(2) = 33.22 \text{ kips/bolt}$$

$$\frac{R_n}{\Omega} = \frac{33.22}{1.50} = 22.15 \text{ kips/bolt (controls)}$$

Number required $= 120/22.15 = 5.42$ bolts. Use 6.

Gusset plate-to-column angles connection:

Bearing on the angles will control, because $2 \times \frac{3}{8}$ in. < $\frac{7}{8}$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(2 \times \frac{3}{8})(58) = 45.7 \text{ kips/bolt}$$

Shear strength and slip-critical strength are same as for tension member-to-gusset plate connection, $\therefore$ use 6 bolts.

Connection angles-to-column flange connection: assume that the slip-critical strength will control. Select the number of bolts based on slip-critical strength, then check tension.

$$T_a = V_a = \frac{1}{\sqrt{2}}(120) = 84.85 \text{ kips}$$

$$\text{Reduction factor} = k_{sc} = 1 - \frac{1.5T_a}{D_u T_b n_b} = 1 - \frac{1.5(84.85)}{1.13(49)n_b}$$

Let $84.85 = n_b\left[25.84\left(1 - \frac{1.5(84.85)}{1.13(49)n_b}\right)\right]$, Solution is: $\{n_b = 5.58\}$

Use 6 rows of 2 bolts for symmetry and to match the arrangement of gusset plate-to-connection angle bolts. Check tension.

Shear/tension load per bolt $= 84.85/12 = 7.07$ kips

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nt}A_b = \frac{1}{2.00}(90)(0.6013) = 27.1 \text{ kips/bolt} > 7.07 \text{ kips/bolt (OK)}$$

Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in.

Minimum edge distance from AISC Table J3.4 $= 1\frac{1}{8}$ in.

Try $\ell_e = 1\frac{1}{2}$ in. and $s = 2\frac{1}{2}$ in. for all connection elements and check bearing.

For the angles-to-column connection, the angle thickness will control (3/8 in. $< 0.695$ in.):

$$h = \frac{7}{8} + \frac{1}{16} = 0.9375 \text{ in.}$$

For the hole nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{0.9375}{2} = 1.031$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c tF_u) = \frac{1}{2.00}(1.2)(1.031)(3/8)(58)$$

$$= 13.5 \text{ kips} > 7.07 \text{ kips (OK)}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(3/8)(58) = 22.8 \text{ kips/bolt}$$

For the other bolts, $\ell_c = s - h = 2.5 - 0.9375 = 1.563$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c tF_u) = \frac{1}{2.00}(1.2)(1.563)(3/8)(58)$$

$$= 20.4 \text{ kips} > 7.07 \text{ kips (OK)}$$

For the tension member connection, the gusset plate thickness will control:

For the hole nearest the edge,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c tF_u) = \frac{1}{2.00}(1.2)(1.031)(7/8)(58)$$

$$= 31.4 \text{ kips} > 120/6 = 20 \text{ kips (OK)}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(7/8)(58) = 53.3 \text{ kips/bolt}$$

For the other bolts,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c tF_u) = \frac{1}{2.00}(1.2)(1.563)(7/8)(58)$$

$$= 47.6 \text{ kips} > 20 \text{ kips (OK)}$$

For the gusset plate-to-column connection angles,

$$\frac{R_n}{\Omega} = \frac{2(3/8)}{7/8} \times 31.4 = 26.9 \text{ kips/bolt} > 20 \text{ kips/bolt (OK)}$$

Summary:

Use 6 bolts for tension member-to-gusset plate connection;
use 6 bolts for gusset-plate-to connection angles connection;
use 12 bolts for connection angles to column flange connection;
use edge distances of $1\frac{1}{2}$ in. and bolt spacings of $2\frac{1}{2}$ in. throughout.

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**7.9-5**

Let $\theta$ = angle that load makes with the horizontal = $\arctan(12.5/13) = 43.88°$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(55) + 1.6(145) = 298.0 \text{ kips}$$

$$T_u = 298\cos(43.88°) = 214.8 \text{ kips}, \qquad V_u = 298\sin(43.88°) = 206.6 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)} f_{rv} \leq 90$$

$$= 117 - 2.222 f_{rv} \leq 90$$

$$\phi F'_{nt} = 0.75(117 - 2.222 f_{rv}) \leq 0.75(90)$$

$$= 87.75 - 1.667 f_{rv} \leq 67.5$$

Let $\phi F'_{nt} = \dfrac{214.8}{\Sigma A_b}$ and $f_{rv} = \dfrac{206.6}{\Sigma A_b}$

$$\frac{214.8}{\Sigma A_b} = 87.75 - 1.667\left(\frac{206.6}{\Sigma A_b}\right)$$

$214.8 = 87.75\Sigma A_b - 344.3$, Solution is: $\left\{\Sigma A_b = 6.372 \text{ in.}^2\right\}$

Required $A_b = \dfrac{\Sigma A_b}{12} = \dfrac{6.372}{12} = 0.531 \text{ in.}^2$

Required diameter $= d_b = \sqrt{\dfrac{4A_b}{\pi}} = \sqrt{\dfrac{4(0.531)}{\pi}} = 0.8222$ in.

Try $\frac{7}{8}$ -in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Check the upper limit on $F_t$:

$$f_{rv} = \frac{206.6}{\Sigma A_b} = \frac{206.6}{12(0.6013)} = 28.63 \text{ ksi}$$

$F'_{nt} = 117 - 2.222 f_{nv} = 117 - 2.222(28.63) = 53.38 \text{ ksi} < 90 \text{ ksi}$ (OK)

Check shear: $V_u/\text{bolt} = 206.6/12 = 17.2$ kips/bolt

$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.6013) = 24.35 \text{ kips/bolt} > 17.2 \text{ kips/bolt}$ (OK)

Check bearing. $h = \dfrac{7}{8} + \dfrac{1}{16} = 0.9375$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \dfrac{h}{2} = 2.5 - \dfrac{0.9375}{2} = 2.031$ in.

$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.031)(1.11)(65) = 132 \text{ kips} > 17.2 \text{ kips}$ (OK)

$\phi(2.4 d t F_u) = 0.75(2.4)(7/8)(1.11)(65) = 114 \text{ kips/bolt} > 17.2 \text{ kips}$ (OK)

For the other holes, $\ell_c = s - h = 4 - 0.9375 = 3.063$ in. Since this is larger than $\ell_c$ for the edge bolts, no further check is necessary.

Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 4 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2.5 in. (OK)

<u>Required $d = 0.822$ in.; use $\frac{7}{8}$-in.-diameter bolts</u>

(b) ASD solution

$$P_a = D + L = 55 + 145 = 200 \text{ kips}$$

$$T_a = 200\cos(43.88°) = 144.2 \text{ kips}, \qquad V_a = 200\sin(43.88°) = 138.6 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{54}f_{rv} \le 90$$

$$= 117 - 3.333f_{rv} \le 90$$

$$\frac{F'_{nt}}{\Omega} = \frac{117 - 3.333f_{rv}}{2.00} \le \frac{90}{2.00}$$

$$= 58.5 - 1.667f_{rv} \le 45$$

Let $\frac{F'_{nt}}{\Omega} = \frac{144.2}{\Sigma A_b}$ and $f_{rv} = \frac{138.6}{\Sigma A_b}$

$$\frac{144.2}{\Sigma A_b} = 58.5 - 1.667\left(\frac{138.6}{\Sigma A_b}\right)$$

$144.2 = 58.5\Sigma A_b - 231$, Solution is: $\left\{\Sigma A_b = 6.414 \text{ in.}^2\right\}$

Required $A_b = \frac{\Sigma A_b}{12} = \frac{6.414}{12} = 0.5345 \text{ in.}^2$

Required diameter = $d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5345)}{\pi}} = 0.8249$ in.

Try $\frac{7}{8}$ -in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Check the upper limit on $F_t$:

$$f_{rv} = \frac{138.6}{\Sigma A_b} = \frac{138.6}{12(0.6013)} = 19.21 \text{ ksi}$$

$F'_{nt} = 117 - 3.75f_{rv} = 117 - 3.75(19.21) = 44.96$ ksi < 90 ksi (OK)

Check shear: $V_a$/bolt = $138.6/12 = 11.6$ kips/bolt

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{54(0.6013)}{2.00} = 16.24 \text{ kips/bolt} > 11.6 \text{ kips/bolt} \qquad \text{(OK)}$$

Check bearing. $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 2.5 - \frac{0.9375}{2} = 2.031$ in.

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c tF_u}{\Omega} = \frac{1.2(2.031)(1.11)(65)}{2.00} = 87.9 \text{ kips} > 11.6 \text{ kips} \quad \text{(OK)}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(1.11)(65)}{2.00} = 75.8 \text{ kips/bolt} > 11.6 \text{ kips} \quad \text{(OK)}$$

For the other holes, $\ell_c = s - h = 4 - 0.9375 = 3.063$ in. Since this is larger than $\ell_c$ for the edge bolts, no further check is necessary.

Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 4 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2.5 in. (OK)

<u>Required $d = 0.825$ in.; use $\frac{7}{8}$-in.-diameter bolts</u>

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## <u>7.11-1</u>

(a) Tension member gross section strength

$$P_n = F_yA_g = 50(2.50) = 125.0 \text{ kips}$$

$$\phi_t P_n = 0.90(125) = 112.5 \text{ kips}$$

Net section strength: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.428}{13} = 0.9671$

$$A_e = A_gU = 2.50(0.9671) = 2.418 \text{ in.}^2$$

$$P_n = F_uA_e = 65(2.418) = 157.2 \text{ kips}$$

$$\phi_t P_n = 0.75(157.2) = 117.9 \text{ kips}$$

Weld strength is

$$R_n = 0.707w(0.6F_{EXX}) = 0.707(1/8)(0.6 \times 70) = 3.712 \text{ kips/in.}$$

$$\phi R_n = 0.75(3.712) = 2.784 \text{ kips/in.}$$

Base metal shear strength of the plate:

$$\text{Yielding: } \phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(3/8)$$

$$= 8.1 \text{ kips/in.} > 2.784 \text{ kips/in.}$$

$$\text{Rupture: } \phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(3/8)$$

$$= 9.788 \text{ kips/in.} > 2.784 \text{ kips/in.}$$

Base metal shear strength of the tension member:

$$\text{Yielding: } \phi R_n = \phi(0.6F_y t) = 1.00(0.6)(50)(0.179)$$

$$= 5.37 \text{ kips/in.} > 2.784 \text{ kips/in.}$$

$$\text{Rupture: } \phi R_n = \phi(0.6F_u t) = 0.75(0.6)(65)(0.179)$$

$$= 5.236 \text{ kips/in.} > 2.784 \text{ kips/in.}$$

The weld strength is smaller than the base metal strength.

$$\text{Total weld strength} = 2.784(13 + 13) = 72.38 \text{ kips}$$

Weld strength controls overall: $P_u = 72.38$ kips. Let

$$1.2D + 1.6(3D) = 72.38, \text{ Solution is: } \{D = 12.06\}$$

$$P = D + L = 12.06 + 3(12.06) = 48.24 \text{ kips} \qquad \underline{P = 48.2 \text{ kips}}$$

(b) ASD solution

Tension member gross section strength

$$P_n = F_y A_g = 50(2.50) = 125.0 \text{ kips}$$

$$\frac{P_n}{\Omega_t} = \frac{125}{1.67} = 74.85 \text{ kips}$$

Net section strength: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.428}{13} = 0.9671$

$A_e = A_g U = 2.50(0.9671) = 2.418 \text{ in.}^2$

$P_n = F_u A_e = 65(2.418) = 157.2 \text{ kips}$

$\frac{P_n}{\Omega_t} = \frac{157.2}{2.00} = 78.6 \text{ kips}$

Weld strength:

$R_n = 0.707w(0.6F_{EXX}) = 0.707(1/8)(0.6 \times 70) = 3.712 \text{ kips/in.}$

$\frac{R_n}{\Omega} = \frac{3.712}{2.00} = 1.856 \text{ kips/in.}$

Base metal shear strength of the plate:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(36)(3/8)}{1.50} = 5.4 \text{ kips/in} > 1.856 \text{ kips/in.}$

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(58)(3/8)}{2.00} = 6.525 \text{ kips/in.} > 1.856 \text{ kips/in.}$

Base metal shear strength of the tension member:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(50)(0.179)}{1.50} = 3.58 \text{ kips/in} > 1.856 \text{ kips/in.}$

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(65)(0.179)}{2.00} = 3.491 \text{ kips/in.} > 1.856 \text{ kips/in.}$

The weld strength is smaller than the base metal strength.

Total weld strength $= 1.856(13 + 13) = 48.26 \text{ kips}$

Weld strength controls overall: $P_a = 48.26$ kips. $\underline{P = 48.3 \text{ kips}}$

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**7.11-2**

(a) LRFD solution. Gross section: $A_g = 5.12 \text{ in.}^2$

$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(5.12) = 165.9 \text{ kips}$

Net section: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{5} = 0.8342$

$$A_e = A_g U = 5.12(0.8342) = 4.271 \text{ in.}^2$$

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(4.271) = 185.8 \text{ kips}$$

Weld:

$$\phi R_n = \phi[0.707w(0.6F_{EXX})] = 0.75(0.707)(3/16)(0.6 \times 70) = 4.176 \text{ kips/in.}$$

(Alternate: $\phi R_n = 1.392 \times 3$ sixteenths $= 4.176$ kips/in.)

For the strength of the connection, investigate the two options given in AISC J2.4(c).

1. Use the basic weld strength for both the longitudinal and transverse welds.

$$\phi R_n = 4.176(5 + 5 + 5) = 62.64 \text{ kips (for one angle)}$$

2. Use 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld.

$$\phi R_n = 0.85(4.176)(5 + 5) + 1.5(4.176)(5) = 66.82 \text{ kips (for one angle)}$$

The larger value may be used. For two angles, the total weld strength is

$$66.82 \times 2 = 133.6 \text{ kips}$$

Check block shear on the gusset plate in lieu of base metal shear strength.

$$A_{gv} = A_{nv} = \frac{3}{8}(5) \times 2 = 3.750 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}(5) = 1.875 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6 F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(3.750) + 1.0(58)(1.875) = 239.3 \text{ kips}$$

with an upper limit of

$$0.6 F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(3.750) + 1.0(58)(1.875) = 189.8 \text{ kips}$$

Block shear design strength = $\phi R_n = 0.75(189.8) = 142.4$ kips

Weld shear controls, and $P_u = 133.6$ kips. Let

$$1.2D + 1.6(2D) = 133.6\text{, Solution is: } \{D = 30.36\}$$

$$P = D + L = 30.36 + 2(30.36) = 91.1 \text{ kips} \qquad \underline{P = 91.1 \text{ kips}}$$

(b) ASD solution

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \frac{36(5.12)}{1.67} = 110.4 \text{ kips}$$

Net section: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{5} = 0.8342$

$$A_e = A_g U = 5.12(0.8342) = 4.271 \text{ in.}^2$$

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{1}{2.00}(58)(4.271) = 123.9 \text{ kips}$$

Weld:

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}[0.707w(0.6F_{EXX})] = \frac{1}{2.00}(0.707)(3/16)(0.6 \times 70)$$

$$= 2.784 \text{ kips/in.}$$

(Alternate: $\frac{R_n}{\Omega} = 0.9279 \times 3$ sixteenths = 2.784 kips/in.)

For the strength of the connection, investigate the two options given in AISC J2.4(c).

1. Use the basic weld strength for both the longitudinal and transverse welds.

$$\frac{R_n}{\Omega} = 2.784(5 + 5 + 5) = 41.76 \text{ kips (for one angle)}$$

2. Use 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld.

$$\frac{R_n}{\Omega} = 0.85(2.784)(5 + 5) + 1.5(2.784)(5) = 44.54 \text{ kips (for one angle)}$$

The larger value may be used. For two angles, the total allowable weld strength = $44.54 \times 2 = 89.1$ kips

Check block shear on the gusset plate in lieu of base metal shear strength. From the

LRFD solution, the nominal block shear strength is

$$R_n = 189.8 \text{ kips}$$

The allowable block shear strength is

$$\frac{R_n}{\Omega} = \frac{189.8}{2.00} = 94.9 \text{ kips}$$

Weld strength controls: $\underline{P_a = 89.1 \text{ kips}}$

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**7.11-3**

From Table 2-4 in Part 2 of the *Manual*, for A242 steel, $F_y = 50$ ksi and $F_u = 70$ ksi

(a) LRFD solution

Tension member gross section: $A_g = (5/8)(7) = 4.375 \text{ in.}^2$

$$\phi_t P_n = 0.90 F_y A_g = 0.90(50)(4.375) = 196.9 \text{ kips}$$

Net section:

$$\phi_t P_n = 0.75 F_u A_e = 0.75(70)(4.375) = 229.7 \text{ kips}$$

The weld strength is

$$\phi R_n = 1.392 \times 5 \text{ sixteenths} = 6.96 \text{ kips/in.}$$

There is no base metal shear in this connection.

For a 7-in. length, $\phi R_n = 6.96 \times 7 = 48.72$ kips

Total weld strength = $2(48.72) = 97.44$ kips (controls). Let

$$1.2D + 1.6(2.5D) = 97.44, \text{ Solution is: } \{D = 18.74\}$$

$$P = D + L = 18.74 + 2.5(18.74) = 65.59 \text{ kips}$$ $\underline{P = 65.6 \text{ kips}}$

(b) ASD solution

Tension member gross section strength: $A_g = (5/8)(7) = 4.375 \text{ in.}^2$

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \frac{50(4.375)}{1.67} = 131.0 \text{ kips}$$

Net section strength: $\dfrac{P_n}{\Omega_t} = \dfrac{F_u A_e}{\Omega_t} = \dfrac{70(4.375)}{2.00} = 153.1 \text{ kips}$

Weld strength: $\dfrac{R_n}{\Omega} = 0.9279 \times 5$ sixteenths = 4.640 kips/in.

Total weld strength = $4.640(7 + 7) = 64.96$ kips (controls) <u>$P = 65.0$ kips</u>

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**<u>7.11-4</u>**

(a) LRFD solution. Gross section: For the outer member,

$$A_g = (5/16)(3) \times 2 = 1.875 \text{ in.}^2$$

For the inner member,

$$A_g = (1/2)(6) = 3.0 \text{ in.}^2$$

Outer member controls: $\phi_t P_n = 0.90 F_y A_g = 0.90(36)(1.875) = 60.75 \text{ kips}$

Net section: $A_e = A_g$

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(1.875) = 81.56 \text{ kips}$$

Weld: $\phi R_n = 1.392 \times 3$ sixteenths = 4.176 kips/in

1. Basic weld strength for both the longitudinal and transverse welds:

$$\phi R_n = 4.176(3 + 3 + 3) \times 2 = 75.17 \text{ kips}$$

2. 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld:

$$\phi R_n = [0.85(4.176)(3 + 3) + 1.5(4.176)(3)] \times 2 = 80.18 \text{ kips}$$

Use the larger value of 80.18 kips.

Check block shear on the inner member in lieu of base metal shear strength.

$$A_{gv} = A_{nv} = \frac{1}{2}(3) \times 2 = 3.0 \text{ in.}^2$$

$$A_{nt} = \frac{1}{2}(3) = 1.5 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.0) + 1.0(58)(1.5) = 191.4 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.0) + 1.0(58)(1.5) = 151.8 \text{ kips}$$

Block shear design strength = $\phi R_n = 0.75(151.8) = 113.9$ kips

Gross section tensile strength controls, and $P_u = 60.75$ kips. Let

$$1.2D + 1.6(3D) = 60.75, \text{ Solution is: } \{D = 10.13\}$$

$$P = D + 3L = 10.13 + 3(10.13) = 40.5 \text{ kips} \qquad \underline{P = 40.5 \text{ kips}}$$

(b) ASD solution

Gross section: $\dfrac{P_n}{\Omega_t} = \dfrac{F_yA_g}{\Omega_t} = \dfrac{36(1.875)}{1.67} = 40.4$ kips

Net section:

$$\frac{P_n}{\Omega_t} = \frac{F_uA_e}{\Omega_t} = \frac{58(1.875)}{2.00} = 54.4 \text{ kips}$$

Weld: $\dfrac{R_n}{\Omega} = 0.9279 \times 3$ sixteenths = 2.784 kips/in.

1. Basic weld strength for both the longitudinal and transverse welds:

$$\frac{R_n}{\Omega} = 2.784(3 + 3 + 3) \times 2 = 50.11 \text{ kips}$$

2. 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld:

$$\frac{R_n}{\Omega} = [0.85(2.784)(3 + 3) + 1.5(2.784)(3)] \times 2 = 53.45 \text{ kips}$$

Use the larger value of 53.45 kips.

Check block shear on the gusset plate in lieu of base metal shear strength. From the

LRFD solution, the nominal block shear strength is

$$R_n = 151.8 \text{ kips}$$

The allowable block shear strength is

$$\frac{R_n}{\Omega} = \frac{151.8}{2.00} = 75.9 \text{ kips}$$

Gross section tensile strength controls: $\underline{P_a = 40.4 \text{ kips}}$

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**7.11-5**

From AISC Table J2.4, the minimum weld size is 3/16 inch (based on the angle thickness). Maximum size = 5/16 − 1/16 = 1/4 in.

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(50) = 104.0 \text{ kips}$

Try $w$ = ¼ in., $\phi R_n = 1.392 \times 4$ sixteenths = 5.568 kips/in.

The base metal shear yield strength (gusset plate controls) is

$$0.6F_y t = 0.6(36)\left(\frac{3}{8}\right) = 8.1 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(58)\left(\frac{3}{8}\right) = 9.788$ kips/in.

Base metal shear yield strength for the angles is

$$0.6F_y t = 0.6(50)\left(\frac{5}{16} \times 2\right) = 18.75 \text{ kips/in.}$$

and the shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{3}{8} \times 2\right) = 21.94$ kips/in.

The weld strength of 5.568 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{104}{5.568} = 18.68 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{18.68 - 5}{2} = 6.84 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(5.568) = 4.733 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(5.568) = 8.352 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$104 - 5(8.352) = 62.24 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{62.24}{2(4.733)} = 6.575 \text{ in.}$$

The first option requires shorter longitudinal welds. Try a 5-inch transverse weld and two 7-inch longitudinal welds. Check the block shear strength of the gusset plate.

$$A_{gv} = A_{nv} = 2 \times \frac{3}{8}(7) = 5.25 \text{ in.}^2 \qquad A_{nt} = \frac{3}{8}(5) = 1.875 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(5.25) + 1.0(58)(1.875) = 291.5 \text{ kips}$$

with an upper limit of

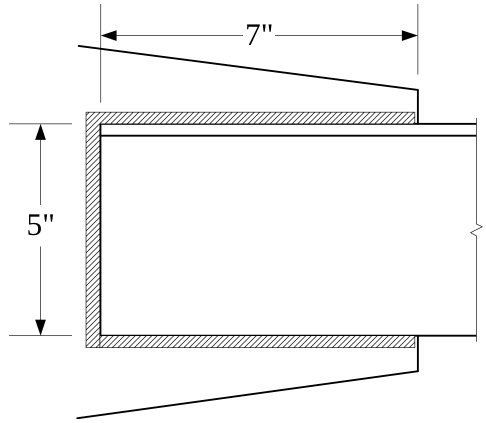
$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.25) + 1.0(58)(1.875)$$

$$= 222.2 \text{ kips (controls)}$$

The design strength is

$$\phi R_n = 0.75(222.2) = 167 \text{ kips} > 104 \text{ kips} \qquad \text{(OK)}$$

Use ¼-in. fillet welds as shown.




(b) ASD solution: $P_a = D + L = 20 + 50 = 70$ kips

Try $w = ¼$ in., $R_n/\Omega = 0.9279 \times 4$ sixteenths = 3.712 kips/in.

Base metal shear strength (gusset plate controls): The allowable shear yield strength is

$$0.4F_y t = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

and the allowable shear rupture strength is

$$0.3F_u t = 0.3(58)\left(\frac{3}{8}\right) = 6.525 \text{ kips/in.}$$

The weld strength of 3.712 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{70}{3.712} = 18.86 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{18.86 - 5}{2} = 6.93 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(3.712) = 3.155 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(3.712) = 5.568 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$70 - 5(5.568) = 42.16 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{42.16}{2(3.155)} = 6.681 \text{ in.}$$

The first option requires shorter longitudinal welds, but try a 5-inch transverse weld and two 7-inch longitudinal welds. Check the block shear strength of the gusset plate. From the LRFD solution, $R_n = 222.2$ kips, and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{222.2}{2.00} = 111 \text{ kips} > 70 \text{ kips} \qquad \text{(OK)}$$

Use ¼-in. fillet welds as shown in the figure above (in LRFD solution).

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**7.11-6**

From AISC Table J2.4, the minimum weld size is 1/8 inch (based on the angle thickness). Maximum size = 1/4 − 1/16 = 3/16 in.

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(14) + 1.6(25) = 56.8$ kips

For one angle, $P_u = 56.8/2 = 28.4$ kips

Try $w = 1/8$ in., $\phi R_n = 1.392 \times 2$ sixteenths = 2.784 kips/in.

Compare the base metal shear strength of the gusset plate with that of the two angles. The plate thickness of 3/8 in. is smaller than $2 \times 1/4 = 1/2$ inch. Therefore, the weld strength cannot exceed the base metal shear strength for a thickness of $3/8 \div 2 = 3/16$ in.

The base metal shear yield strength is

$$0.6F_yt = 0.6(36)\left(\frac{3}{16}\right) = 4.05 \text{ kips/in.}$$

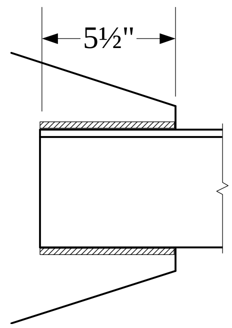
Shear rupture strength is $0.45F_ut = 0.45(58)\left(\frac{3}{16}\right) = 4.894$ kips/in.

The weld strength of 2.784 kips/in. governs. Try two longitudinal welds:

$$\text{total required length of weld} = \frac{28.4}{2.784} = 10.2 \text{ in.}$$

$$\text{length of each longitudinal weld} = \frac{10.2}{2} = 5.1 \text{ in.}$$

<u>Use 1/8-in. fillet welds as shown.</u>




(b) ASD solution: $P_a = D + L = 14 + 25 = 39$ kips

For one angle, $P_a = 39/2 = 19.5$ kips

Try $w = 1/8$ in., $R_n/\Omega = 0.9279 \times 2$ sixteenths = 1.856 kips/in.

Compare the base metal shear strength of the gusset plate with that of the two angles. The plate thickness of 3/8 in. is smaller than $2 \times 1/4 = 1/2$ inch. Therefore, the weld strength cannot exceed the base metal shear strength for a thickness of $3/8 \div 2 = 3/16$ in.

The base metal shear yield strength is

$$0.4F_y t = 0.4(36)\left(\frac{3}{16}\right) = 2.7 \text{ kips/in.}$$

and the allowable shear rupture strength is

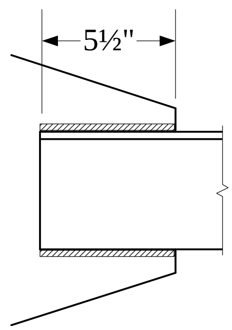
$$0.3F_u t = 0.3(58)\left(\frac{3}{16}\right) = 3.263 \text{ kips/in.}$$

The weld strength of 1.856 kips/in. governs. Try two longitudinal welds.

$$\text{total required length of weld} = \frac{19.5}{1.856} = 10.51 \text{ in.}$$

length of longitudinal welds = $\dfrac{10.51}{2} = 5.255$ in.

Use 1/8-in. fillet welds as shown

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**7.11-7**

From AISC Table J2.4, the minimum weld size is 3/16 inch (based on the gusset plate thickness). Maximum size = $0.400 - 1/16 = 0.338$ in., or 5/16 in. (to the nearest 1/16 in.)

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(40) + 1.6(100) = 208.0$ kips

Try $w = 3/16$ in., $\phi R_n = 1.392 \times 3$ sixteenths = 4.176 kips/in.

The base metal shear yield strength (gusset plate controls) is

$$0.6F_y t = 0.6(36)\left(\frac{3}{8}\right) = 8.1 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(58)\left(\dfrac{3}{8}\right) = 9.788$ kips/in.

The weld strength of 4.176 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

total required length of weld = $\dfrac{208.0}{4.176} = 49.81$ in.

length of longitudinal welds = $\dfrac{49.81 - 9.00}{2} = 20.41$ in.

For the second option, the strength of the longitudinal welds is

$$0.85(4.176) = 3.550 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(4.176) = 6.264 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$208.0 - 9(6.264) = 151.6 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{151.6}{2(3.550)} = 21.35 \text{ in.}$$

To minimize the length of the connection, use the maximum weld size permitted. Use $w = 5/16$ in.

$$\phi R_n = 1.392 \times 5 \text{ sixteenths} = 6.96 \text{ kips/in}$$

First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{208}{6.96} = 29.89 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{29.89 - 9.00}{2} = 10.45 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(6.96) = 5.916 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(6.96) = 10.44 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$208 - 9(10.44) = 114.0 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{114.0}{2(5.916)} = 9.635 \text{ in.}$$

The second option requires shorter longitudinal welds. Try a 9-inch transverse weld and two10-inch longitudinal welds. Check the block shear strength of the gusset plate.

$$A_{gv} = A_{nv} = 2 \times \frac{3}{8}(10) = 7.5 \text{ in.}^2 \qquad A_{nt} = \frac{3}{8}(9) = 3.375 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(7.5) + 1.0(58)(3.375) = 456.8 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(7.5) + 1.0(58)(3.375) = 357.8 \text{ kips (controls)}$$

The design strength is

$$\phi R_n = 0.75(357.8) = 268 \text{ kips} > 208 \text{ kips} \qquad \text{(OK)}$$

Use 5/16-in. fillet welds, with a 9-in. transverse weld and two 10-in. longitudinal welds.

(b) ASD solution: $P_a = 40 + 100 = 140$ kips

To minimize the length of the connection, use the maximum weld size permitted. Use $w = 5/16$ in.

$$R_n/\Omega = 0.9279 \times 5 \text{ sixteenths} = 4.640 \text{ kips/in.}$$

Base metal shear strength (gusset plate controls): The allowable shear yield strength is

$$0.4F_yt = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

and the allowable shear rupture strength is

$$0.3F_ut = 0.3(58)\left(\frac{3}{8}\right) = 6.525 \text{ kips/in.}$$

The weld strength of 4.640 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{140}{4.640} = 30.17 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{30.17 - 9}{2} = 10.59 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(4.640) = 3.944 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(4.640) = 6.96 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$140 - 9(6.96) = 77.36 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{77.36}{2(3.944)} = 9.807 \text{ in.}$$

The second option requires shorter longitudinal welds. Try a 9-inch transverse weld and two 10-inch longitudinal welds. Check the block shear strength of the gusset plate. From the LRFD solution, $R_n = 357.8$ kips, and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{357.8}{2.00} = 179 \text{ kips} > 140 \text{ kips} \qquad \text{(OK)}$$

Use 5/16-in. fillet welds, with a 9-in. transverse weld and two 10-in. longitudinal welds.

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**7.11-8**

Tension member gross section: $\phi_t P_n = 0.90 F_y A_g = 0.90(36)(1.93) = 62.53$ kips

For the net section, assume $U = 0.80$: $A_e = A_g U = 1.93(0.80) = 1.544 \text{ in.}^2$

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(1.544) = 67.16 \text{ kips.} \quad \therefore \text{ use } P_u = 62.53 \text{ kips}$$

For $t = 1/4$ in., min. $w = 1/8$ in., and max $w = 1/4 - 1/16 = 3/16$ in.

Try two longitudinal E70 fillet welds, and to minimize the length of the connection, try $w = 3/16$ inch.

$$\phi R_n = 1.392 \times 3 \text{ sixteenths} = 4.176 \text{ kips/in.}$$

The base metal shear yield strength (angle controls) is

$$0.6F_y t = 0.6(36)\left(\frac{1}{4}\right) = 5.4 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(58)\left(\frac{1}{4}\right) = 6.525$ kips/in.

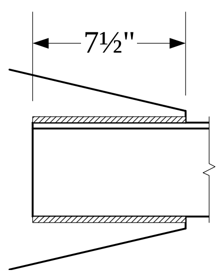
The weld strength of 4.176 kips/in. governs.

$$\text{Required length} = \frac{62.53}{4.176} = 14.97 \text{ in.}$$

Check assumed value of $U$ :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.08}{8} = 0.865 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$

Use $\frac{3}{16}$-in. E70 fillet welds as shown.

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**7.11-9**

$$P_a = 12 + 36 = 48 \text{ kips or } 48/2 = 24 \text{ kips/angle}$$

$$\text{Req'd } A_g = \frac{P_a}{0.6F_y} = \frac{24}{0.6(36)} = 1.11 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_a}{0.5F_u} = \frac{24}{0.5(58)} = 0.828 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{12(12)}{300} = 0.48 \text{ in.}$$

Try 2L2½ × 2½ × ¼, $A_g = 1.19$ in.$^2$ (for one angle) > 1.11 in.$^2$ (OK)

From the properties table for the double-angle section, $r_{\min} = 0.764$ in. > 0.48 OK)

Net section: Assume $U = 0.80$ :

$$A_e = A_g U = 1.19(0.80) = 0.952 \text{ in.}^2 > 0.828 \text{ in.}^2 \text{ (OK)}$$

Weld size: min. $w = 1/8$ in. and max $w = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$ in.

Try $w = 1/8$ in., $R_n/\Omega = 0.9279 \times 2$ sixteenths = 1.856 kips/in.

The base metal shear yield strength (angle controls) is

$$0.4F_y t = 0.4(36)\left(\frac{1}{4}\right) = 3.6 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(58)\left(\frac{1}{4}\right) = 4.35$ kips/in.

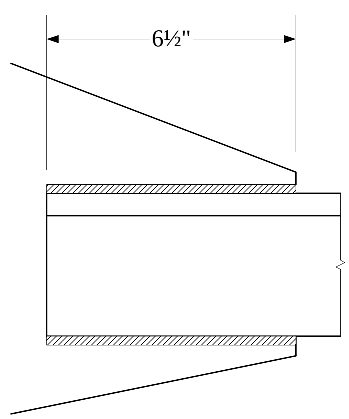
The weld strength of 1.856 kips/in. governs.

$$\text{Required length} = \frac{24}{1.856} = 12.93 \text{ in., try two 6½-in. longitudinal welds.}$$

Check assumed value of $U$ :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.711}{6.5} = 0.891 > \text{assumed value of 0.80} \qquad \text{(OK)}$$

<u>Use 2L2½ × 2½ × ¼, welded with $\frac{1}{8}$-in. E70 fillet welds as shown.</u>



**7.11-10**

(a)  LRFD solution

From load combination 2, $P_u = 1.2D + 1.6L = 1.2(45) + 1.6(60) = 150.0$ kips

From load combination 4,

$$P_u = 1.2D + 1.0W + 0.5L = 1.2(45) + 1.0(62) + 0.5(60) = 146.0 \text{ kips}$$

Combination 2 controls, and $P_u = 150$ kips

$$\text{Req'd } A_g = \frac{P_u}{0.9F_y} = \frac{150}{0.9(50)} = 3.33 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_u}{0.75F_u} = \frac{150}{0.75(65)} = 3.08 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{18(12)}{300} = 0.72 \text{ in.}$$

Try C10 × 15.3,       $A_g = 4.48 \text{ in.}^2 > 3.33 \text{ in.}^2$  (OK)

$$r_{\min} = 0.711 \text{ in.} \approx 0.72 \text{ in. (OK)}$$

Net section:  Assume $U = 0.80$ :

$$A_e = A_g U = 4.48(0.80) = 3.58 \text{ in.}^2 > 3.33 \text{ in.}^2 \text{ (OK)}$$

Weld size:  $t_{PL} = 3/8$ in.,       $t_w = 0.240$ in.,       $w_{\min} = \frac{1}{8}$ in.

Try a $\frac{1}{8}$-in. fillet weld; use E70 electrodes.

$$\phi R_n = 1.392 \times 2 \text{ sixteenths} = 2.784 \text{ kips/in.}$$

The base metal shear yield strength (use the gusset plate) is

$$0.6F_y t = 0.6(36)\left(\frac{3}{8}\right) = 8.1 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(58)\left(\frac{3}{8}\right) = 9.788$ kips/in.

The weld strength of 2.784 kips/in. governs.  Both longitudinal and transverse welds will be used.  To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c).  First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{150}{2.784} = 53.88 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{53.88 - 10.0}{2} = 21.94 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(2.784) = 2.366 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(2.784) = 4.176 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

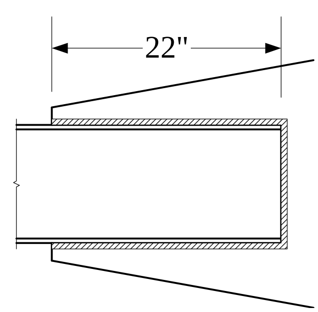
$$150 - 10.0(4.176) = 108.2 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{108.2}{2(2.366)} = 22.87 \text{ in}$$

The first option results in a slightly shorter connection. Use a transverse weld and two 22-inch longitudinal welds. Check assumed value of $U$:

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.634}{22} = 0.971 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$




Check block shear on gusset plate.

The shear areas are $A_{gv} = A_{nv} = \frac{3}{8}(22)(2) = 16.5 \text{ in.}^2$

The tension area is $A_{nt} = \frac{3}{8}(10) = 3.75 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(16.5) + 1.0(58)(3.75) = 791.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(16.5) + 1.0(58)(3.75) = 573.9 \text{ kips}$$

Use $R_n = 573.9$ kips, $\phi R_n = 0.75(573.9) = 430 \text{ kips} > 150 \text{ kips}$ (OK)

Use a C10 × 15.3, welded with $\frac{1}{8}$-in. E70 fillet welds as shown.

(b) ASD solution

Load combination 6a controls:

$$P_a = D + 0.75L + 0.75(0.6)W = 45 + 0.75(60) + 0.75(0.6)(62) = 117.9 \text{ kips}$$

$$\text{Req'd } A_g = \frac{P_a}{0.6F_y} = \frac{117.9}{0.6(50)} = 3.93 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_a}{0.5F_u} = \frac{117.9}{0.5(65)} = 3.63 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{18(12)}{300} = 0.72 \text{ in.}$$

Try C10 × 15.3, $A_g = 4.48 \text{ in.}^2 > 3.93 \text{ in.}^2$ (OK)

$r_{\min} = 0.711 \text{ in.} \approx 0.72 \text{ in.}$ (OK)

Net section: Assume $U = 0.90$ :

$$A_e = A_gU = 4.48(0.90) = 4.03 \text{ in.}^2 > 3.63 \text{ in.}^2 \text{ (OK)}$$

Weld size: $t_{PL} = 3/8$ in., $t_w = 0.240$ in.

$w_{\min} = \frac{1}{8}$ in., $w_{\max} = 0.24 - 1/16 = 0.178$ in. or $\frac{1}{8}$ in.

Use a $\frac{1}{8}$-in. fillet weld; use E70 electrodes.

$$\frac{R_n}{\Omega} = 0.9279 \times 2 \text{ sixteenths} = 1.856 \text{ kips/in.}$$

The base metal shear yield strength (gusset plate controls) is

$$0.4F_y t = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(58)\left(\frac{3}{8}\right) = 6.525$ kips/in.

The weld strength of 1.856 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{117.9}{1.856} = 63.52 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{63.52 - 10.0}{2} = 26.76 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(1.856) = 1.578 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(1.856) = 2.784 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$117.9 - 10.0(2.784) = 90.06 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{90.06}{2(1.856)} = 24.26 \text{ in}$$

The second option results in a shorter connection. Use a transverse weld and two 24.5-inch longitudinal welds. Check assumed value of $U$ :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.634}{24.5} = 0.974 > \text{assumed value of } 0.90 \qquad \text{(OK)}$$

Check block shear on gusset plate.

The shear areas are $A_{gv} = A_{nv} = \frac{3}{8}(24.5)(2) = 18.38 \text{ in.}^2$

The tension area is $A_{nt} = \frac{3}{8}(10) = 3.75 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(18.38) + 1.0(58)(3.75) = 857.1 \text{ kips}$$

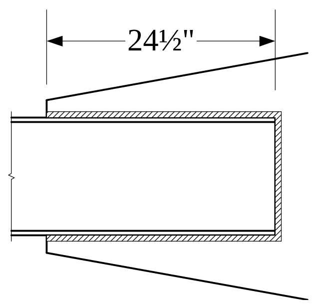
with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(18.38) + 1.0(58)(3.75) = 614.5 \text{ kips}$$

Use $R_n = 614.5$ kips

$$\frac{R_n}{\Omega} = \frac{614.5}{2.00} = 307 \text{ kips} > 117.9 \text{ kips} \qquad \text{(OK)}$$

Use a C10 × 15.3, welded with $\frac{1}{8}$-in. E70 fillet welds as shown.



# CHAPTER 8 - ECCENTRIC CONNECTIONS

## 8.2-1

Direct shear components:

$$P_x = \frac{3}{5}(20) = 12 \text{ kips}, \qquad P_y = \frac{4}{5}(20) = 16 \text{ kips}$$

$$p_{cx} = \frac{12}{4} = 3 \text{ kips} \rightarrow \qquad p_{cy} = \frac{16}{4} = 4 \text{ kips} \downarrow$$

Eccentricity: $e_x = 10$ in., $e_y = 9 + 1.5 - 4.5 = 6$ in.

$$M = 12(6) + 16(10) = 232 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2 + y^2) = 2[(4.5)^2 + (1.5)^2] = 45.0 \text{ in.}^2$$

Top bolt is critical. $x = 0$, $y = 9/2 = 4.5$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{232(4.5)}{45} = 23.2 \text{ kips} \rightarrow$$

$$\sum p_x = 3 + 23.2 = 26.2 \text{ kips} \rightarrow \qquad \sum p_y = 4 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(26.2)^2 + (4)^2} = 26.5 \text{ kips}$$

$\underline{p = 26.5 \text{ kips}}$

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## 8.2-2

Eccentricity: $e_x = 3 + 3 + 4 = 10$ in., $e_y = \frac{5.5}{2} = 2.75$ in.

$$\sum(x^2 + y^2) = 2[(6)^2 + (3)^2 + (3)^2 + (6)^2] + 10(2.75)^2 = 255.6 \text{ in.}^2$$

(a) Direct shear components:

$$P_x = P\cos 30° = 0.866P, \qquad P_y = P\sin 30° = 0.5P$$

$$p_{cx} = \frac{0.866P}{10} = 0.0866P \text{ kips} \rightarrow \qquad p_{cy} = \frac{0.5P}{10} = 0.05P \text{ kips} \downarrow$$

$$M = 0.5P(10) + 0.866P(2.75) = 7.382P \text{ in.-kips} \curvearrowright$$

Top right bolt is critical. $x = 6$ in., $y = 5.5/2 = 2.75$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{7.382P(2.75)}{255.6} = 7.942 \times 10^{-2}P \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{7.382P(6)}{255.6} = 0.1733P \text{ kips} \rightarrow$$

$$\sum p_x = 0.0866P + 7.942 \times 10^{-2}P = 0.166P \text{ kips} \rightarrow$$

$$\sum p_y = 0.05P + 0.1733P = 0.2233P \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(0.166P)^2 + (0.2233P)^2} = 0.2782P \text{ kips}$$

Slip-critical strength will control over shear. Assuming Class A surfaces,

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(28)(1) = 9.492 \text{ kips/bolt}$$

(a) $\phi R_n = 1.0(9.492) = 9.492$ kips/bolt

Let $P = P_u$, and $0.2782P_u = 9.492$ $\qquad$ $\underline{P_u = 34.1 \text{ kips}}$

(b) $\frac{R_n}{\Omega} = \frac{9.492}{1.5} = 6.328$ kips/bolt

Let $P = P_a$, and $0.2782P_a = 6.328$ $\quad \Rightarrow \quad$ $\underline{P_u = 34.1 \text{ kips}}$

---

**8.2-3**

Direct shear component: $\quad p_{cy} = \frac{65}{5} = 13.0 \text{ kips} \downarrow$

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(3)}{5} = 1.8 \text{ in.}, \qquad \bar{y} = \frac{3+2(7)}{5} = 3.4 \text{ in.}$$

Eccentricity: $\quad e_x = 3 + 2 + 6 - 1.8 = 9.2$ in.

$$M = 65(9.2) = 598.0 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2+y^2) = (1.8)^2(2) + (3-1.8)^2(3) + (3.4)^2(2) + (3.4-3)^2 + (7-3.4)^2(2)$$

$$= 60.0 \text{ in.}^2$$

Top right bolt is critical. $\quad x = 3 - 1.8 = 1.2$ in., $\qquad y = 3 + 4 - 3.4 = 3.6$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{598(3.6)}{60} = 35.88 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{598(1.2)}{60} = 11.96 \text{ kips} \downarrow$$

$$\sum p_x = 35.88 \text{ kips} \rightarrow \qquad \sum p_y = 13 + 11.96 = 24.96 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(35.88)^2 + (24.96)^2} = 43.71 \text{ kips}$$

$\underline{p = 43.7 \text{ kips}}$

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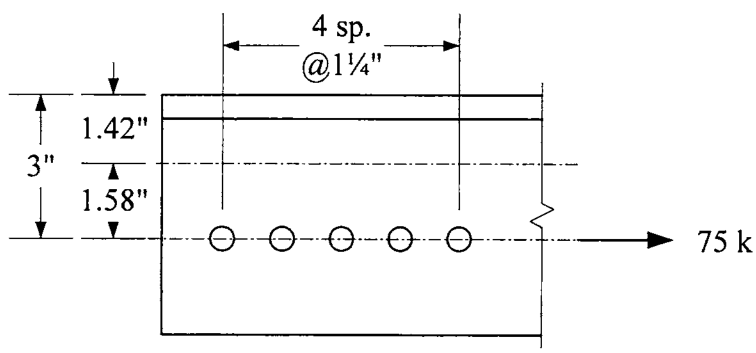
**8.2-4**

Direct shear component: $p_{cx} = \frac{75}{5} = 15.0 \text{ kips} \rightarrow$




$$M = 75(1.58) = 118.5 \text{ in.-kips}$$

$$\sum(x^2+y^2) = (1.25)^2(2) + (2.5)^2(2) = 15.63 \text{ in.}^2$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{118.5(2.5)}{15.63} = 18.95 \text{ kips} \uparrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(15.0)^2 + (18.95)^2} = 24.17 \text{ kips}$$

Without considering eccentricity, $p = p_{cx} = 15.0$ kips

Difference = 24.17 − 15 = 9.17 kips (61%) $\qquad$ Difference = 9.17 kips

---

**8.2-5**

Direct shear components:

$$p_{cx} = \frac{15}{5} = 3 \text{ kips} \leftarrow \qquad p_{cy} = \frac{30}{5} = 6 \text{ kips} \downarrow$$

Determine location of centroid with respect to lower right bolt:

$$\bar{x} = \frac{2(3)}{5} = 1.2 \text{ in.}, \qquad \bar{y} = 3 \text{ in.}$$

Eccentricity: $e_x = 1.2 + 4 = 5.2$ in., $e_y = 3 + 2 = 5$ in.

$$M = 30(5.2) - 15(5) = 81.0 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2 + y^2) = 3(1.2)^2 + 2(1.8)^2 + 4(3)^2 = 46.8 \text{ in.}^2$$

Lower right bolt is critical. $x = 1.2$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{81(3)}{46.8} = 5.192 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{81(1.2)}{46.8} = 2.077 \text{ kips} \downarrow$$

$$\sum p_x = 3 + 5.192 = 8.192 \text{ kips} \leftarrow \qquad \sum p_y = 6 + 2.077 = 8.077 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(8.192)^2 + (8.077)^2} = 11.5 \text{ kips}$$

$\underline{p = 11.5 \text{ kips}}$

---

**8.2-6**

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(1) + 6(1)}{6} = 1.5 \text{ in.}, \qquad \bar{y} = \frac{3(1) + 6(1) + 9(3)}{6} = 6.0 \text{ in.}$$

Eccentricity: $e_x = 12.5 - 1.5 = 11.0$ in., $e_y = 6 + 1.5 = 7.5$ in.

$$\sum(x^2 + y^2) = (1.5)^2(4) + (1.5)^2 + (4.5)^2 + (3)^2(3) + (3)^2 + (6)^2 = 103.5 \text{ in.}^2$$

(a) $P_u = 1.2D + 1.6L = 1.2(22) + 1.6(44) = 96.8$ kips

Direct shear components:

$$P_{ux} = \frac{3}{5}(96.8) = 58.08 \text{ kips} \rightarrow \qquad P_{uy} = \frac{4}{5}(96.8) = 77.44 \text{ kips} \downarrow$$

$$p_{cx} = \frac{58.08}{6} = 9.68 \text{ kips} \rightarrow \qquad p_{cy} = \frac{77.44}{6} = 12.91 \text{ kips} \downarrow$$

$$M = 77.44(11.0) - 58.08(7.5) = 416.2 \text{ in.-kips} \curvearrowright$$

Top right bolt is critical. $x = 4.5$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{416.2(3)}{103.5} = 12.06 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{416.2(4.5)}{103.5} = 18.10 \text{ kips} \downarrow$$

$$\sum p_x = 9.68 + 12.06 = 21.74 \text{ kips} \rightarrow \qquad \sum p_y = 12.91 + 18.10 = 31.01 \text{ kips}$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(21.74)^2 + (31.01)^2} = 37.87 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)A_b = 37.87 \qquad \Rightarrow \qquad A_b = 0.9351 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.9351$, Solution is: $\{d = 1.091\}$ in. <u>Use $1\frac{1}{8}$-in. diameter bolts</u>

(b) $P_a = D + L = 22 + 44 = 66$ kips

Direct shear components:

$$P_{ax} = \frac{3}{5}(66) = 39.6 \text{ kips} \rightarrow \qquad P_{ay} = \frac{4}{5}(66) = 52.8 \text{ kips} \downarrow$$

$$p_{cx} = \frac{39.6}{6} = 6.6 \text{ kips} \rightarrow \qquad p_{cy} = \frac{52.8}{6} = 8.8 \text{ kips} \downarrow$$

$$M = 52.8(11.0) - 39.6(7.5) = 283.8 \text{ in.-kips} \curvearrowright$$

Top right bolt is critical. $x = 4.5$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{283.8(3)}{103.5} = 8.226 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{283.8(4.5)}{103.5} = 12.34 \text{ kips} \downarrow$$

$$\sum p_x = 6.6 + 8.226 = 14.83 \text{ kips} \rightarrow \qquad \sum p_y = 8.8 + 12.34 = 21.14 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(14.83)^2 + (21.14)^2} = 25.82 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{54A_b}{2.00} = 25.82 \quad \Rightarrow \quad A_b = 0.9563 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.9563$, Solution is: $\{d = 1.103\}$ in. Use $1\frac{1}{8}$-in. diameter bolts

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**8.2-7**

Determine location of centroid with respect to lower right bolt:

$$\bar{x} = \frac{2(2) + 1(6)}{6} = 1.667 \text{ in.}, \qquad \bar{y} = \frac{2(4) + 1(8)}{6} = 2.667 \text{ in.}$$

Eccentricity: $e_x = 10 + 1.667 = 11.67$ in., $e_y = 10 - 2.667 = 7.333$ in.

$$\sum(x^2 + y^2) = (1.667)^2(3) + (0.3333)^2(2) + (4.333)^2$$

$$+ (2.667)^2(3) + (1.333)^2(2) + (5.333)^2 = 80.67 \text{ in.}^2$$

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(15) + 1.6(35) = 74.0$ kips

$$P_{ux} = \frac{4}{5}(74) = 59.2 \text{ kips} \leftarrow \qquad P_{uy} = \frac{3}{5}(74) = 44.4 \text{ kips} \downarrow$$

Direct shear components: $p_{cx} = \frac{59.2}{6} = 9.867 \text{ kips} \leftarrow$

$$p_{cy} = \frac{44.4}{6} = 7.4 \text{ kips} \downarrow$$

$$M = 44.4(11.67) - 59.2(7.333) = 84.03 \text{ in.-kips} \curvearrowright$$

Check top right bolt. $x = 1.667$ in., $y = 8 - 2.667 = 5.333$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{84.03(5.333)}{80.67} = 5.555 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{84.03(1.667)}{80.67} = 1.736 \text{ kips} \downarrow$$

$$\sum p_x = -9.867 + 5.555 = -4.312 \text{ kips } (\leftarrow)$$

$$\sum p_y = 7.4 + 1.736 = 9.136 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(-4.312)^2 + (9.136)^2} = 10.10 \text{ kips}$$

Check bottom right bolt. $x = 1.667$ in., $y = 2.667$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{84.03(2.667)}{80.67} = 2.778 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{84.03(1.667)}{80.67} = 1.736 \text{ kips} \downarrow$$

$$\sum p_x = 9.867 + 2.778 = 12.65 \text{ kips} \leftarrow \qquad \sum p_y = 7.4 + 1.736 = 9.136 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(12.65)^2 + (9.136)^2} = 15.60 \text{ kips (controls)}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)A_b = 15.60, \text{ Solution is: } \{A_b = 0.3852\} \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.3852$, $d = 0.700$ in.

<u>Required $d = 0.700$ in.; use $\frac{3}{4}$-in. diameter bolts</u>

(b) ASD solution: $P_a = D + L = 15 + 35 = 50$ kips

$$P_{ax} = \frac{4}{5}(50) = 40 \text{ kips} \leftarrow \qquad P_{ay} = \frac{3}{5}(50) = 30 \text{ kips} \downarrow$$

Direct shear components: $p_{cx} = \frac{40}{6} = 6.667$ kips $\leftarrow$ $\qquad p_{cy} = \frac{30}{6} = 5$ kips $\downarrow$

$$M = 30(11.67) - 40(7.333) = 56.78 \text{ in.-kips} \curvearrowright$$

Bottom right bolt is critical. $x = 1.667$ in., $y = 2.667$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{56.78(2.667)}{80.67} = 1.877 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{56.78(1.667)}{80.67} = 1.173 \text{ kips} \downarrow$$

$$\sum p_x = 6.667 + 1.877 = 8.544 \text{ kips} \leftarrow \qquad \sum p_y = 5 + 1.173 = 6.173 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(8.544)^2 + (6.173)^2} = 10.54 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega}$$

$$\frac{54A_b}{2.00} = 10.54, \text{ Solution is: } \{A_b = 0.3904\}$$

From $\frac{\pi d^2}{4} = 0.3904, \ d = 0.705$ in.

Required $d = 0.705$ in.; use $\frac{3}{4}$-in. diameter bolts

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**8.2-8**

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(5.5)}{7} = 2.357 \text{ in.,} \qquad \bar{y} = \frac{2(3.5)+1(7)+2(10.5)}{7} = 5.0 \text{ in.}$$

$$\sum(x^2+y^2) = (2.357)^2(4) + (5.5-2.357)^2(3) + (5)^2(2)$$

$$+ (5-3.5)^2(2) + (7-5)^2(1) + (10.5-5)^2(2) = 170.9 \text{ in.}^2$$

(a) $P_u = 1.2D + 1.6L = 1.2(50) + 1.6(25) = 100.0$ kips

Direct shear components:

$$P_{ux} = 100\cos 60° = 50.0 \text{ kips} \leftarrow \qquad P_{uy} = 100\sin 60° = 86.6 \text{ kips} \downarrow$$

$$p_{cx} = \frac{50}{7} = 7.143 \text{ kips} \leftarrow \qquad p_{cy} = \frac{86.6}{7} = 12.37 \text{ kips} \downarrow$$

Eccentric shear components: Check lower right bolt.

$$M = 86.6(5.5+1.5+6.5-2.357) - 50(10.5+2-5) = 590.0 \text{ in.-kips} \curvearrowright$$

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{590(5)}{170.9} = 17.26 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{590(5.5-2.357)}{170.9} = 10.85 \text{ kips} \downarrow$$

$$\sum p_x = 7.143 + 17.26 = 24.4 \text{ kips} \leftarrow$$

$$\sum p_y = 12.37 + 10.85 = 23.22 \text{ kips} \downarrow$$

$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(24.4)^2 + (23.22)^2} = 33.68$ kips

Slip-critical strength will control over shear. Assuming Class A surfaces,

$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)T_b(1.0) = 0.339\,T_b$ kips/bolt

$\phi R_n = 1.0(0.339T_b) = 0.339\,T_b$

For $0.339T_b = 33.68$, Solution is: $\{T_b = 99.35\}$ kips.

From AISC Table J3.1, $T_b = 103$ kips for a $1\frac{1}{2}$-inch diameter bolt.

Use $1\frac{1}{2}$-inch diameter Group A bolts.

(b) $P_a = D + L = 50 + 25 = 75$ kips

Direct shear components:

$P_{ax} = 75\cos 60° = 37.5$ kips ←  $P_{ay} = 75\sin 60° = 64.95$ kips ↓

$p_{cx} = \dfrac{37.5}{7} = 5.357$ kips ←  $p_{cy} = \dfrac{64.95}{7} = 9.279$ kips ↓

Eccentric shear components: Check lower right bolt.

$M = 64.95(5.5 + 1.5 + 6.5 - 2.357) - 37.5(10.5 + 2 - 5) = 442.5$ in.-kips ↷

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{442.5(5)}{170.9} = 12.95 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{442.5(5.5 - 2.357)}{170.9} = 8.138 \text{ kips} \downarrow$$

$\sum p_x = 5.357 + 12.95 = 18.31$ kips ←

$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(18.31)^2 + (17.42)^2} = 25.27$ kips

Slip-critical strength will control over shear. Assuming Class A surfaces,

$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)T_b(1.0) = 0.339\,T_b$ kips/bolt

$$\frac{R_n}{\Omega} = \frac{0.339T_b}{1.50} = 0.226\,T_b$$

For $0.226T_b = 25.27$, Solution is: $\{T_b = 111.8\}$ kips.

Although the problem statement specifies Group A bolts, the required size is larger than $1\frac{1}{2}$

inches. If Group B bolts are used, a $1\frac{3}{8}$-in. diameter bolt, with $T_b = 121$ kips (AISC Table J3.1), will work.

Use $1\frac{3}{8}$-inch diameter *Group B* bolts.

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**8.2-9**

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(2.5) + 1.6(7.5) = 15.0 \text{ kips}$$

Direct shear components: $p_{cx} = \frac{15}{6} = 2.5 \text{ kips} \rightarrow$

Location of centroid with respect to lower row of bolts:

$$\bar{y} = \frac{3(6) + 3(1)}{6} = 3.5 \text{ in.}$$

$$M = 15(10 + 3.5) = 202.5 \text{ in.-kips} \curvearrowleft$$

$$\sum(x^2 + y^2) = 4(3)^2 + 2(3.5)^2 + (0.5)^2 + 3(2.5)^2 = 79.5 \text{ in.}^2$$

Lower right bolt controls.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{202.5(3.5)}{79.5} = 8.915 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{202.5(3)}{79.5} = 7.642 \text{ kips} \downarrow$$

$$\sum p_x = 2.5 + 8.915 = 11.42 \text{ kips} \rightarrow \qquad \sum p_y = 7.642 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(11.42)^2 + (7.642)^2} = 13.74 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)A_b = 13.74 \qquad \Rightarrow \qquad A_b = 0.3393 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.3393$, $\quad d = 0.657$ in., $\quad$ Try $d = \frac{3}{4}$ inch.

Check bearing. $h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in. For the holes nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{13/16}{2} = 1.594 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.594)(3/8)(58) = 31.2 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(3/8)(58) = 29.36 \text{ kips} < 31.2 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 29.36 \text{ kips/bolt} > 13.74 \text{ kips/bolt} \qquad \text{(OK)}$$

(No need to check the other bolts, since $\ell_c$ will be larger.)

Use $\frac{3}{4}$-in. diameter bolts.

(b) ASD solution: $P_a = D + L = 10$ kips

Direct shear components: $p_{cx} = \frac{10}{6} = 1.667 \text{ kips} \rightarrow$

Lower right bolt controls.

$$M = 10(10 + 3.5) = 135 \text{ in.-kips} \curvearrowleft$$

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{135(3.5)}{79.5} = 5.943 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{135(3)}{79.5} = 5.094 \text{ kips} \downarrow$$

$$\sum p_x = 1.667 + 5.943 = 7.610 \text{ kips} \rightarrow \qquad \sum p_y = 5.094 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(7.610)^2 + (5.094)^2} = 9.158 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\frac{R_n}{\Omega} = \frac{F_{nv} A_b}{\Omega} = \frac{54 A_b}{2.00} = 9.158 \qquad \Rightarrow \qquad A_b = 0.3392 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.3392$, $d = 0.657$ in. Try $d = \frac{3}{4}$ inch.

Check bearing. $h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in. For the holes nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{13/16}{2} = 1.594 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c t F_u}{\Omega} = \frac{1.2(1.594)(3/8)(58)}{2.00} = 20.8 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(3/4)(3/8)(58)}{2.00} = 19.6 \text{ kips} < 20.8 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.6 \text{ kips/bolt} > 9.16 \text{ kips/bolt} \qquad \text{(OK)}$$

(No need to check the other bolts, since $\ell_c$ will be larger.) Use $\frac{3}{4}$-in. diameter bolts.

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**8.2-10**

(a) Direct shear components: $p_{cx} = \frac{43}{6} = 7.167 \text{ kips} \leftarrow$

$M = 43(8) = 344$ in.-kips ↷

$$\sum(x^2 + y^2) = 2(3)^2 + 2(9)^2 + 2(15)^2 = 630 \text{ in.}^2$$

Right-hand bolt controls.

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{344(15)}{630} = 8.19 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(7.167)^2 + (8.19)^2} = 10.9 \text{ kips}$$

$p = 10.9$ kips

(b) Use Table 7-6, Angle = 0°. For $e_x = 8$ in., $n = 6$, and $s = 6$ in.,

$C = 4.47$

Using the table notation, $\text{Bolt force} = r_n = \frac{R_n}{C} = \frac{43}{4.47} = 9.620 \text{ kips}$

$p = 9.62$ kips (11.7% less than force from elastic analysis)

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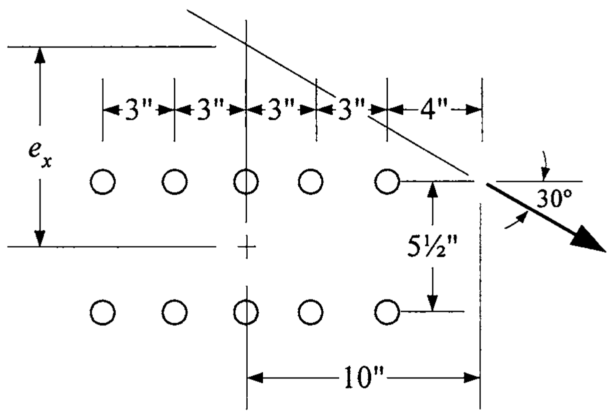
**8.2-11**

Use Table 7-8, Angle = 30°. $n = 5$ and $s = 3$ in.




$$e_x = 10\tan 30° + \frac{5.5}{2} = 8.524 \text{ in.}$$

$$C = 4.756$$

Slip-critical strength:

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(28)(1) = 9.492 \text{ kips/bolt}$$

For the connection, $R_n = CR_n = 4.756(9.492) = 45.14$ kips

(a) LRFD solution

$P_u = \phi R_n = 0.75(45.14) = 33.86$ kips $\underline{P_u = 33.9 \text{ kips}}$

(b) ASD solution

$P_a = \dfrac{R_n}{\Omega} = \dfrac{45.14}{2.00} = 22.57$ kips $\underline{P_a = 22.6 \text{ kips}}$

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**8.2-12**

Use Table 7-13, Angle = 30°. $n = 3$ and $s = 3$ in.

$$e_x = 4 + 4 + 2 - 3\tan 30° = 8.268 \text{ in.}$$

$$C = 5.687 \text{ (by interpolation)}$$

$r_n = F_{nv}A_b = 54(0.4418) = 23.86$ kips/bolt

$R_n = Cr_n = 5.687(23.86) = 135.7$ kips

(a) $\phi R_n = 0.75(135.7) = 101.8$ kips $\qquad$ $\underline{P_u = 102 \text{ kips}}$

(b) $C_{req} = \dfrac{P_u}{\phi r_n} = \dfrac{175}{0.75(23.86)} = 9.779$

For $n = 5$, $C = 10.8$ $\qquad$ $\underline{\text{Use } n = 5 \text{ bolts per vertical row}}$

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**8.3-1**

Nominal bearing strength: $h = 1 + \dfrac{1}{16} = 1.063$ in.

For edge bolts, $\ell_c = \ell_e - \dfrac{h}{2} = 2 - \dfrac{1.063}{2} = 1.469$ in.

For other bolts, $\ell_c = s - h = 14 - 1.063 = 12.94$ in.

Check strength with $\ell_c = 1.469$ in.

$t_f = 1.12$ in. for W10 × 100 or 0.560 in. for WT5 × 24.5 (controls)

$R_n = 1.2\ell_c tF_u = 1.2(1.469)(0.560)(65) = 64.17$ kips

$2.4dtF_u = 02.4(1.0)(0.560)(65) = 87.36$ kips

$\therefore$ use $R_n = 64.17$ kips/bolt

Shear strength: $A_b = \pi d^2/4 = \pi(1.0)^2/4 = 0.7854 \text{ in.}^2$

Assume that threads are in the plane of shear.

$R_n = F_{nv}A_b = 54(0.7854) = 42.41$ kips/bolt

Shear strength, not bearing, controls.

(a) LRFD solution

Factored load $= P_u = 1.2D + 1.6L = 1.2(20) + 1.6(54) = 110.4$ kips

Shear/bearing load per bolt $= 110.4/4 = 27.6$ kips

Shear strength per bolt = $\phi R_n = 0.75(42.41) = 31.81$ kips > 27.6 kips (OK)

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 2r_t(14) = 28r_t. \qquad M_u = P_u e = 110.4(3) = 331.2 \text{ in.-kips}$$

From $28r_t = 331.2,\ r_t = 11.83$ kips

$$f_{rv} = \frac{27.6}{0.7854} = 35.14 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(54)}(35.14) = 38.91 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(38.91)(0.7854) = 22.9 \text{ kips/bolt} > 10.9 \text{ kips/bolt} \quad \text{(OK)}$$

<u>Bolts are adequate</u>

(b) ASD solution

Service load = $P_a = D + L = 20 + 54 = 74$ kips

Shear/bearing load per bolt = $74/4 = 18.5$ kips

Shear strength per bolt = $\dfrac{R_n}{\Omega} = \dfrac{42.41}{2.00} = 21.21$ kips > 17 kips (OK)

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 2r_t(14) = 28r_t. \qquad M_a = P_a e = 74(3) = 222 \text{ in.-kips}$$

From $28r_t = 222,\ r_t = 7.929$ kips

$$f_{rv} = \frac{18.5}{0.7854} = 23.56 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{54}(23.56) = 38.47 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{38.47(0.7854)}{2.00} = 15.11 \text{ kips/bolt} > 10.9 \text{ kips/bolt (OK)}$$

Bolts are adequate

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**8.3-2**

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(30) + 1.6(65) = 140.0 \text{ kips}$$

Shear/bearing load per bolt = $140/8 = 17.5$ kips

Bearing strength: $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For edge bolts, $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

For other bolts, $\ell_c = s - h = 3 - 15/16 = 2.063$ in.

Check strength with $\ell_c = 1.531$ in.

$t_f = 1.11$ in. for W12 × 120 or 0.640 in. for WT6 × 29 (controls)

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.531)(0.640)(65) = 57.32 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.640)(65) = 65.52 \text{ kips} > 57.32 \text{ kips}$$

$\therefore$ use $\phi R_n = 57.32$ kips/bolt > 17.5 kips/bolt (OK)

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.6013) = 24.35 \text{ kips/bolt} > 17.5 \text{ kips/bolt} \quad \text{(OK)}$$

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(7) = 28r_t. \qquad M_u = P_u e = 140(3) = 420 \text{ in.-kips}$$

From $28r_t = 420$, Solution is: $\{r_t = 15\}$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}, \qquad f_{rv} = \frac{17.5}{0.6013} = 29.1 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)}(29.1) = 52.33 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(52.33)(0.6013) = 23.6 \text{ kips/bolt} > 15 \text{ kips/bolt (OK)}$$

Bolts are adequate

(b) ASD Solution

$P_a = 30 + 65 = 95$ kips

Shear/bearing load per bolt = 95/8 = 11.88 kips

Bearing strength: $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For edge bolts, $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

For other bolts, $\ell_c = s - h = 3 - 15/16 = 2.063$ in.

Check strength with $\ell_c = 1.531$ in.

$t_f = 1.11$ in. for W12 × 120 or 0.640 in. for WT6 × 29 (controls)

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c t F_u}{\Omega} = \frac{1.2(1.531)(0.640)(65)}{2.00} = 38.21 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(0.640)(65)}{2.00} = 43.68 \text{ kips} > 38.21 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.21 \text{ kips/bolt} > 11.88 \text{ kips/bolt} \qquad \text{(OK)}$$

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\frac{R_n}{\Omega} = \frac{F_{nv} A_b}{\Omega} = \frac{54(0.6013)}{2.00} = 16.24 \text{ kips/bolt} > 11.88 \text{ kips/bolt} \qquad \text{(OK)}$$

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(7) = 28r_t. \qquad M_u = P_a e = 95(3) = 285 \text{ in.-kips}$$

From $28r_t = 285,\qquad r_t = 10.18$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt},\qquad f_{rv} = \frac{11.88}{0.6013} = 19.76 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{54}(19.76) = 51.13 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{52.33(0.6013)}{2.00} = 15.7 \text{ kips/bolt} > 10.18 \text{ kips/bolt (OK)}$$

Bolts are adequate

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**8.3-3**

Nominal bearing strength: $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

$t_f = 0.605$ in. for W12 × 65 or 0.670 in. for WT15 × 49.5

For edge bolts (applies to WT only), $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{0.9375}{2} = 1.531$ in.

$$R_n = 1.2\ell_c t F_u = 1.2(1.531)(0.670)(65) = 80.01 \text{ kips}$$

$$2.4dtF_u = 2.4(7/8)(0.670)(65) = 91.46 \text{ kips}$$

$\therefore$ use $R_n = 80.01$ kips/bolt for these bolts

For other bolts (use $t_w$ for W), $\ell_c = s - h = 3 - 0.9375 = 2.063$ in.

$$R_n = 1.2\ell_c t F_u = 1.2(2.063)(0.605)(65) = 97.35 \text{ kips}$$

$$2.4dtF_u = 2.4(7/8)(0.605)(65) = 82.58 \text{ kips}$$

$\therefore$ use $R_n = 82.58$ kips/bolt for these bolts

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$R_n = F_{nv}A_b = 54(0.6013) = 32.47 \text{ kips/bolt}$$

Shear strength, not bearing, controls.

(a) LRFD solution

Factored load = $P_u = 1.2D + 1.6L = 1.2(25) + 1.6(75) = 150.0$ kips

Shear/bearing load per bolt = $150/10 = 15$ kips

Shear strength per bolt = $\phi R_n = 0.75(32.47) = 24.35$ kips > 15 kips (OK)

Tension: From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(9) = 36r_t. \qquad M_u = P_u e = 150(6) = 900 \text{ in.-kips}$$

From $36r_t = 900,\ r_t = 25$ kips

$$f_{rv} = \frac{15}{0.6013} = 24.95 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(54)}(24.95) = 61.56 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(61.56)(0.6013) = 27.8 \text{ kips/bolt} > 25 \text{ kips/bolt (OK)}$$

Bolts are adequate

(b) ASD solution

Service load = $P_a = D + L = 25 + 75 = 100$ kips

Shear/bearing load per bolt = $100/10 = 10$ kips

Shear strength per bolt = $\frac{R_n}{\Omega} = \frac{32.47}{2.00} = 16.24$ kips > 10 kips (OK)

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(9) = 36r_t. \qquad M_a = P_a e = 74(3) = 222 \text{ in.-kips}$$

From $36r_t = 222,\ r_t = 6.17$ kips

$$f_{rv} = \frac{10}{0.6013} = 16.63 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{54}(16.63) = 61.57 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{61.57(0.6013)}{2.00} = 18.5 \text{ kips/bolt} > 6.17 \text{ kips/bolt (OK)}$$

Bolts are adequate

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**8.3-4**

(a) LRFD Solution

Factored load $= P_u = 1.2D + 1.6L = 1.2(10) + 1.6(25) = 52.0$ kips

Shear/bearing load per bolt $= 52/6 = 8.667$ kips

Bearing strength: $t_f = 0.560$ in. for W10 × 49 or 0.620 in. for WT5 × 22.5

$$h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For edge bolts (applies to WT only), $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.094)(0.620)(65) = 39.7 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.620)(65) = 54.4 \text{ kips} > 39.7 \text{ kips}$$

$\therefore$ use $\phi R_n = 39.7$ kips/bolt > 8.667 kips/bolt (OK)

For other bolts, $\ell_c = s - h = 3 - 13/16 = 2.188$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.188)(0.560)(65) = 71.7 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.560)(65) = 49.1 \text{ kips} < 71.7 \text{ kips}$$

$\therefore$ use $\phi R_n = 49.1$ kips/bolt > 8.667 kips/bolt (OK)

Shear strength: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) = 17.89 \text{ kips/bolt} > 8.667 \text{ kips (OK)}$$

Tension: Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(6)+2(9)}{6} = 5.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n'r_t d_m = 2r_t(5+1+3/2) = 15.0r_t, \qquad M_u = P_u e = 52(3) = 156.0 \text{ in.-kips}$$

From $15.0r_t = 156.0$, Solution is: $\{r_t = 10.4\}$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{8.667}{0.4418} = 19.62 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)}(19.62) = 73.4 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(73.4)(0.4418) = 24.3 \text{ kips/bolt} > 10.4 \text{ kips/bolt (OK)}$$

Bolts are adequate

(b) ASD Solution

$$P_a = D + L = 10 + 25 = 35 \text{ kips}$$

Shear/bearing load per bolt = $35/6 = 5.833$ kips

Bearing strength: $t_f = 0.560$ in. for W10 × 49 or 0.620 in. for WT5 × 22.5

$$h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For edge bolts (applies to WT only), $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c t F_u) = 0.5(1.2)(1.094)(0.620)(65) = 26.45 \text{ kips}$$

$$\frac{1}{\Omega}(2.4dtF_u) = 0.5(2.4)(3/4)(0.620)(65) = 36.27 \text{ kips} > 26.45 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 26.45 \text{ kips/bolt} > 5.833 \text{ kips/bolt} \qquad \text{(OK)}$$

For other bolts, $\ell_c = s - h = 3 - 13/16 = 2.188$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c t F_u) = 0.5(1.2)(2.188)(0.560)(65) = 47.79 \text{ kips}$$

$$\frac{1}{\Omega}(2.4dtF_u) = 0.5(2.4)(3/4)(0.560)(65) = 32.76 \text{ kips} < 47.79 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 32.76 \text{ kips/bolt} > 5.833 \text{ kips/bolt} \qquad \text{(OK)}$$

Shear strength: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nv}A_b = 0.5(54)(0.4418) = 11.93 \text{ kips/bolt} > 5.833 \text{ kips (OK)}$$

Tension: Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(6) + 2(9)}{6} = 5.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n'r_t d_m = 2r_t(5 + 1 + 3/2) = 15.0r_t, \qquad M_a = P_a e = 35(3) = 105 \text{ in.-kips}$$

From $15.0r_t = 105$, Solution is: $\{r_t = 7.0\}$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}}f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{5.833}{0.4418} = 13.2 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)}(13.2) = 87.67 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F'_{nt}A_b = 0.5(87.67)(0.4418) = 19.4 \text{ kips/bolt} > 7.0 \text{ kips/bolt (OK)}$$

Bolts are adequate

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**8.3-5**

(a) LRFD solution:

$$P_u = 1.2D + 1.6L = 1.2(0.33 \times 105) + 1.6(0.67 \times 105) = 154.1 \text{ kips}$$

Vertical component of load $= \frac{3}{5}(154.1) = 92.46$ kips

Horizontal component of load $= \frac{4}{5}(154.1) = 123.3$ kips

Shear load per bolt $= 92.46/10 = 9.246$ kips

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.6013) = 24.35$ kips/bolt > 9.246 kips (OK)

Tension:

Direct tensile load per bolt = $123.3/10 = 12.33$ kips

For the moment component, the neutral axis of the bolt group is 6 in. from the bottom row of bolts (from symmetry).

$$n' = 4, \qquad d_m = 4.5 + 4.5 = 9.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(9) = 36r_t$$

$$M_u = Pe = 92.46(6) = 554.8 \text{ in.-kips}$$

From $36r_t = 554.8$, Solution is: $\{r_t = 15.41\}$ kips

Total tensile load per bolt = $12.33 + 15.41 = 27.74$ kips

$$f_{rv} = \frac{9.246}{0.6013} = 15.38 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(54)}(15.38) = 82.82 \text{ ksi} < 90 \text{ ksi}, \quad \therefore \text{use } 82.82 \text{ ksi}$$

$\phi R_n = \phi F'_{nt} A_b = 0.75(82.82)(0.6013) = 37.3$ kips/bolt > 27.7 kips/bolt (OK)

Bolts are adequate

(b) ASD solution

$$P_a = D + L = 105 \text{ kips}$$

Vertical component of load = $\frac{3}{5}(105) = 63.0$ kips

Horizontal component of load = $\frac{4}{5}(105) = 84.0$ kips

Shear load per bolt = $63/10 = 6.3$ kips

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{54(0.6013)}{2.00} = 16.24 \text{ kips/bolt} > 6.3 \text{ kips} \qquad \text{(OK)}$$

Direct tensile load per bolt = $84/10 = 8.4$ kips

For the moment component, the neutral axis of the bolt group is 6 in. from the bottom row of bolts (from symmetry).

$$n' = 4, \qquad d_m = 4.5 + 4.5 = 9.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(9) = 36r_t, \qquad M_a = Pe = 63(6) = 378 \text{ in.-kips}$$

From $36r_t = 378$, $\quad r_t = 10.5$ kips

Total tensile load per bolt = $8.4 + 10.5 = 18.9$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{6.3}{0.6013} = 10.48 \text{ ksi}$$

$$= 1.3(90) - \frac{2.00(90)}{54}(10.48) = 82.07 \text{ ksi} < 90 \text{ ksi}$$

The allowable tensile strength is

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{82.07(0.6013)}{2.00} = 24.7 \text{ kips} > 18.9 \text{ kips} \qquad \text{(OK)}$$

Bolts are adequate

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**8.3-6**

(a) $P_u = 1.2D + 1.6L = 1.2(10) + 1.6(40) = 76.0$ kips

Vertical component of load = $\frac{2}{\sqrt{5}}(76) = 67.98$ kips

Horizontal component of load = $\frac{1}{\sqrt{5}}(76) = 33.99$ kips

Direct tensile load per bolt = $33.99/6 = 5.665$ kips. Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(3) + 2(9)}{6} = 4.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t(3 + 6 - 1.5) = 30.0r_t$$

$$M_u = \frac{2}{\sqrt{5}}76(6) + \frac{1}{\sqrt{5}}(76)(4 + 3) = 645.8 \text{ in.-kips}$$

From $30r_t = 645.8$, Solution is: $\{r_t = 21.53\}$ kips

Total tensile load per bolt = $5.665 + 21.53 = 27.20$ kips

Shear/bearing load per bolt = $76/6 = 12.67$ kips

$$A_b = \pi d^2/4 = \pi(1.0)^2/4 = 0.7854 \text{ in.}^2$$

$$\phi R_n = 0.75(54)(0.7854) = 31.81 \text{ kips} > 12.67 \qquad \text{(OK)}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}, \qquad f_{rv} = \frac{12.67}{0.7854} = 16.13 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.90(54)}(16.13) = 87.13 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(87.13)(0.7854)$$

$$= 51.3 \text{ kips/bolt} > 27.20 \text{ kips/bolt} \qquad \text{(OK)}$$

<u>Bolts are adequate</u>

(b) ASD Solution

$$P_a = D + L = 10 + 40 = 50 \text{ kips}$$

Vertical component of load = $\frac{2}{\sqrt{5}}(50) = 44.72$ kips

Horizontal component of load = $\frac{1}{\sqrt{5}}(50) = 22.36$ kips

Direct tensile load per bolt = $22.36/6 = 3.727$ kips. Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(3) + 2(9)}{6} = 4.0 \text{ in.}$$

From Equation 8.6 in the textbook, the resisting moment is

$$M = n'r_t d_m = 4r_t(3 + 6 - 1.5) = 30.0r_t$$

$$M_a = 44.72(6) + 22.36(4 + 3) = 424.8 \text{ in.-kips}$$

From $30r_t = 424.8$, Solution is: $\{r_t = 14.16\}$ kips

Total tensile load per bolt $= 3.727 + 14.16 = 17.89$ kips

Shear/bearing load per bolt $= 44.72/6 = 7.453$ kips

$$A_b = \pi d^2/4 = \pi(1.0)^2/4 = 0.7854 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = 0.5(54)(0.7854) = 21.21 \text{ kips} > 7.453 \qquad \text{(OK)}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}, \qquad f_{rv} = \frac{7.453}{0.7854} = 9.489 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{54}(9.489) = 85.37 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega} F'_{nt} A_b = 0.5(85.37)(0.7854)$$

$$= 33.52 \text{ kips/bolt} > 17.89 \text{ kips/bolt} \qquad \text{(OK)}$$

Bolts are adequate

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**8.3-7**

Nominal bearing strength: $h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in.

For edge bolts, $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094$ in.

For other bolts, $\ell_c = s - h = 3 - 13/16 = 2.188$ in.

Check strength with $\ell_c = 1.094$ in. (the angle controls)

$$R_n = 1.2\ell_c t F_u = 1.2(1.094)(5/16)(58) = 23.79 \text{ kips}$$

$$2.4dtF_u = 2.4(3/4)(5/16)(58) = 32.63 \text{ kips}$$

$\therefore$ use $R_n = 23.79$ kips/bolt

Shear strength: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$R_n = F_{nv}A_b = 54(0.4418) = 23.86 \text{ kips/bolt}$$

Slip-critical strength:

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(28)(1) = 9.492 \text{ kips/bolt}$$

(a) LRFD solution

Shear/bearing load per bolt $= R_u/10 = 0.1R_u$

Bearing: $\phi R_n = 0.75(23.79) = 17.84$ kips

Shear: $\phi R_n = 0.75(23.86) = 17.90$ kips

Slip: $\phi R_n = 1.00(9.492) = 9.492$ kips

Slip is more critical than bearing or shear, and in this type of connection, the tensile force does not reduce the slip-critical strength. Let

$$0.1R_u = 9.492, \qquad R_u = 94.9 \text{ kips}$$

Tension: From Equation 8.6 in the textbook, the resisting moment is

$$M = n'r_t d_m = 4r_t(9) = 36r_t. \qquad M_u = R_u e = 2.25R_u \text{ in.-kips}$$

From $36r_t = 2.25R_u$, $r_t = 0.0625R_u$

Interaction of tension and shear is not a consideration with slip-critical bolts. The design tensile strength per bolt is

$$\phi R_n = \phi F_{nt}A_b = 0.75(90)(0.4418) = 29.82 \text{ kips}$$

Let $0.0625R_u = 29.82, \qquad R_u = 477$ kips

The reaction capacity is therefore based on the slip-critical strength.

$\underline{R_u = 94.9 \text{ kips}}$

(b) ASD solution

Shear/bearing load per bolt $= R_a/10 = 0.1R_a$

Bearing: $\frac{R_n}{\Omega} = \frac{23.79}{2.00} = 11.90$ kips

Shear: $\frac{R_n}{\Omega} = \frac{23.86}{2.00} = 11.93$ kips

Slip: $\frac{R_n}{\Omega} = \frac{9.492}{1.50} = 6.328$ kips

Slip is more critical than bearing or shear, and in this type of connection, the tensile force does not reduce the slip-critical strength. Let

$$0.1R_a = 6.328, \qquad R_a = 63.3 \text{ kips}$$

Tension:

From Equation 8.6 in the textbook, the resisting moment is

$$M = n'r_t d_m = 4r_t(9) = 36r_t. \qquad M_a = R_a e = 2.25R_a$$

From $36r_t = 2.25R_a$, $r_t = 0.0625R_a$

Interaction of tension and shear is not a consideration with slip-critical bolts. The allowable tensile strength per bolt is

$$\frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90(0.4418)}{2.00} = 19.88 \text{ kips}$$

Let $0.0625R_a = 19.88, \qquad R_a = 318$ kips

The reaction capacity is therefore based on the slip-critical strength.

$\underline{R_a = 63.3 \text{ kips}}$

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**8.3-8**

(a) LRFD solution

$$P_{ux} = 1.2D + 1.6L = 1.2(0.30 \times 47) + 1.6(0.70 \times 47) = 69.56 \text{ kips}$$

$$P_{uy} = 1.2D + 1.6L = 1.2(0.30 \times 60) + 1.6(0.70 \times 60) = 88.8 \text{ kips}$$

Assume that tension controls. Select bolt size based on tension, then check the other limit states.

$$\text{Direct tension} = \frac{69.56}{10} = 6.956 \text{ kips/bolt}$$

$$M = n' r_t d_m = 4r_t[(3.5 + 3.5/2)(2)] = 42.0r_t$$

$$M_u = 69.56(3.5 + 3.5 + +2) + 88.8(8) = 1336 \text{ in.-kips}$$

From $42r_t = 1336$, $r_t = 31.81$ kips

Total tensile load per bolt = $6.956 + 31.81 = 38.77$ kips. Let

$$\phi F_{nt} = \frac{38.77}{A_b}$$

$$A_b = \frac{38.77}{\phi F_{nt}} = \frac{38.77}{0.75(90)} = 0.5744 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5744)}{\pi}} = 0.8552 \text{ in.}$$

Try $\frac{7}{8}$-in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Slip-critical strength will control over shear. In this type of connection, the tensile force does not normally reduce the slip-critical strength, but since there is a direct tension component, apply the reduction factor.

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(39)(1) = 13.22 \text{ kips/bolt}$$

$$\phi R_n = 1.0(13.22) = 13.22 \text{ kips/bolt}$$

$$k_{sc} = 1 - \frac{T_u}{D_u T_b n_b} = 1 - \frac{69.56}{1.13(39)(10)} = 0.8422$$

$$k_{sc}(13.22) = 0.8422(13.22) = 11.13 \text{ kips/bolt}$$

$$\text{Required strength per bolt} = \frac{P_{uy}}{n_b} = \frac{88.8}{10} = 8.88 < 11.13 \text{ kips/bolt (OK)}$$

Check bearing (flange of WT controls). $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.531)(0.590)(65)$$

$$= 52.8 \text{ kips} > 8.88 \text{ kips (OK)}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.590)(65) = 60.4 \text{ kips/bolt} > 52.8 \text{ kips}$$

For the other holes, $\ell_c = s - h = 3.5 - 15/16 = 2.56$ in. Since this is larger than $\ell_c$ for the edge bolts, no further check is necessary.

Use $\frac{7}{8}$-inch diameter bolts

(b) ASD Solution

$$P_{ax} = 47 \text{ kips}$$

$$P_{ay} = 60 \text{ kips}$$

Assume that tension controls. Select bolt size based on tension, then check the other limit states.

$$\text{Direct tension} = \frac{47}{10} = 4.7 \text{ kips/bolt}$$

$$M = n' r_t d_m = 4r_t[(3.5 + 3.5/2)(2)] = 42.0 r_t$$

$$M_a = 47(3.5 + 3.5 + +2) + 60(8) = 903.0 \text{ in.-kips}$$

From $42r_t = 903.0$, Solution is: $\{r_t = 21.5\}$ kips

Total tensile load per bolt = $4.7 + 21.5 = 26.2$ kips. Let

$$\frac{F_{nt}}{\Omega} = \frac{26.2}{A_b}$$

$$A_b = \frac{26.2}{F_{nt}/\Omega} = \frac{26.2}{0.5(90)} = 0.5822 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5822)}{\pi}} = 0.861 \text{ in.}$$

Try $\frac{7}{8}$-in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013$ in.$^2$

Slip-critical strength will control over shear. In this type of connection, the tensile force does not normally reduce the slip-critical strength, but since there is a direct tension component, apply the reduction factor.

$$R_n = \mu D_u h_f T_b n_s = 0.30(1.13)(1.0)(39)(1) = 13.22 \text{ kips/bolt}$$

$$\frac{R_n}{\Omega} = \frac{13.22}{1.50} = 8.813 \text{ kips/bolt}$$

$$k_{sc} = 1 - \frac{T_a}{D_u T_b n_b} = 1 - \frac{60}{1.13(39)(10)} = 0.8639$$

$k_{sc}(8.813) = 0.8639(8.813) = 7.614$ kips/bolt

Required strength per bolt $= \dfrac{P_{ay}}{n_b} = \dfrac{60}{10} = 6.0 < 7.614$ kips/bolt (OK)

Check bearing (flange of WT controls). $h = \dfrac{7}{8} + \dfrac{1}{16} = \dfrac{15}{16}$ in.

For the holes nearest the edge, $\ell_c = \ell_e - \dfrac{h}{2} = 2 - \dfrac{15/16}{2} = 1.531$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c t F_u) = \frac{1}{1.50}(1.2)(1.531)(0.590)(65)$$

$$= 46.97 \text{ kips} > 6.0 \text{ kips (OK)}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{1.50}(2.4)(7/8)(0.590)(65) = 53.69 \text{ kips/bolt} > 46.97 \text{ kips}$$

For the other holes, $\ell_c = s - h = 3.5 - 15/16 = 2.56$ in. Since this is larger than $\ell_c$ for the edge bolts, no further check is necessary. Use $\frac{7}{8}$-inch diameter bolts

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## 8.3-9

(a) Factored load: Neglect the beam weight initially, and account for it later.

$$w_u = 1.2w_D + 1.6w_L = 1.6(5) = 8.0 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(8)(30)^2 = 900 \text{ ft-kips}$$

From the $Z_x$ table, Try a W27 × 84:

$$\phi_b M_n = \phi_b M_p = 915 \text{ ft-kips} > 900 \text{ ft-kips (OK)}$$

Check beam weight: $w_u = 1.2w_D + 1.6w_L = 1.2(0.084) + 1.6(5) = 8.101$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(8.101)(30)^2 = 911 \text{ ft-kips} < 915 \text{ ft-kips (OK)}$$

Shear: $V_u = \dfrac{8.101(30)}{2} = 122$ kips

From the $Z_x$ table, $\phi_v V_n = 368 \text{ kips} > 122 \text{ kips (OK)}$

Use a W27 × 84

(b) Use Group A, Type N, bearing bolts.

For the beam-to-angle bolts, design for shear, then check bearing. Try $\frac{3}{4}$-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) \times 2 \text{ shear planes} = 35.79 \text{ kips/bolt}$$

Let $35.79N_b = V_u$ : $\quad 35.79N_b = 122 \quad \Rightarrow \quad N_b = 3.41$

Try 4 bolts in beam-to-angle connection and 8 bolts in angle-to-column connection.

Use a minimum spacing of $3d = 3(3/4) = 2.25$ in.

Minimum edge distance from AISC Table J3.4 = 1in.

Try $s = 3$ in. and $\ell_e = 1\frac{1}{2}$ in. Total length of angles $= 3(3) + 2(1.5) = 12$ in.

Bearing: Check angles first: $h = d + \dfrac{1}{16} = \dfrac{3}{4} + \dfrac{1}{16} = 0.8125$ in.

For edge bolt, $\ell_c = \ell_e - \dfrac{h}{2} = 1.5 - \dfrac{0.8125}{2} = 1.094$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.094)t(58) = 57.11t$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)t(58) = 78.3t > 57.11t$$

$\therefore$ use $\phi R_n = 57.11t$ kips/bolt

For other bolts, $\ell_c = s - h = 3 - 0.8125 = 2.188$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.188)t(58) = 114.2t$$

$\phi(2.4dtF_u) = 78.3t < 114.2t$ $\therefore$ use $\phi R_n = 78.3t$ kips/bolt

Total bearing strength (per angle leg) $= 57.11t + 3(78.3t) = 292.0t$

For two angles, $292.0t \times 2 = 584.0t$

Let $584.0t = V_u = 122 \quad \Rightarrow \quad t = 0.209$ in.

Check beam web. $t_w = 0.515$ in. There are no edge bolts. For the other bolts,

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.188)(0.515)(65) = 65.92 \text{ kips/bolt}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.515)(65) = 45.19 \text{ kips/bolt} < 65.92 \text{ kips/bolt}$$

$\therefore$ use $\phi R_n = 45.19$ kips/bolt

Total bearing strength = $4(45.19) = 180.8$ kips > 122 kips (OK)

Bearing on column flange OK (column flange is thicker than the beam web).

Shear yield strength of angles (AISC J4.2):

$$\phi R_n = \phi(0.60 F_y A_g) \times 2 = 1.0(0.60)(36)(12t) \times 2 = 518.4t$$

Let $518.4t = 122$, Solution is: $\{t = 0.2353\}$ in.

Shear rupture strength of angle (AISC J4.2):

Use hole diameter = $\frac{3}{4} + \frac{1}{8} = 0.875$ in.

$$A_{nv} = 2 \times (12 - 4 \times 0.875)t = 17.0t \text{ in.}^2$$

$$\phi R_n = \phi(0.60 F_u A_{nv}) = 0.75(0.60)(58)(17.0t) = 443.7t \text{ kips}$$

Let $443.7t = 122$, Solution is: $\{t = 0.2750\}$ in.

Block shear strength of angles:

Shear areas: $A_{gv} = t(10.5) \times 2 = 21.0t \text{ in.}^2$

$$A_{nv} = t[10.5 - 3.5(0.875)] \times 2 = 14.88t \text{ in.}^2$$

Tension area: $A_{nt} = t[1.5 - 0.5(0.875)] \times 2 = 2.126t \text{ in.}^2$

$$R_n = 0.6 F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(14.88t) + 1.0(58)(2.126t) = 641.1t \text{ kips}$$

with an upper limit of

$$0.6 F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(21.0t) + 1.0(58)(2.126t) = 576.9t \text{ kips}$$

Let $\phi R_n = 0.75(576.9t) = 122$, Solution is: $\{t = 0.2820\}$ in.

The minimum required angle thickness is 0.2820 in.

Try 2L4 × 4 × $\frac{5}{16}$ × 1 ft - 0 in. $\left(\frac{5}{16} \text{ in.} > 0.2820 \text{ in. required}\right)$

Assume a $\frac{1}{2}$-in. setback and the usual gage distance.

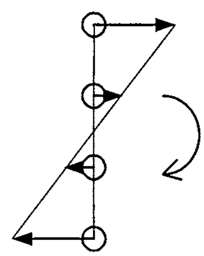
Use $2L4 \times 4 \times \frac{5}{16}$, with

four $\frac{3}{4}$-in. diameter Group A bearing-type bolts in the beam web, and

eight $\frac{3}{4}$-in. diameter Group A bearing-type bolts in the column flange

(c) Check the beam-to-angle connection (shear only), accounting for eccentricity. The direct shear component is

$$p_{cy} = \frac{V_u}{4} = \frac{122}{4} = 30.5 \text{ kips}$$



Eccentric shear component:

$$M = V_u e = 122(2.5) = 305.0 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(4.5)^2 + 2(1.5)^2 = 45 \text{ in.}^2$$

Top bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{305(4.5)}{45} = 30.5 \text{ kips}$$

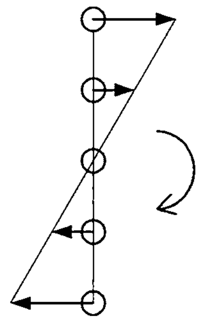
$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(30.5)^2 + (30.5)^2} = 43.1 \text{ kips}$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) \times 2 \text{ shear planes}$$

$$= 35.8 \text{ kips/bolt} < 43.1 \text{ kips/bolt} \quad \text{(N.G.)}$$

Try 5 bolts in the beam-to-angle connection and 10 bolts in the angle-to-column connection.

Direct shear component: $p_{cy} = \frac{V_u}{5} = \frac{122}{5} = 24.4$ kips



Eccentric shear component:

$$M = V_u e = 122(2.5) = 305.0 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(6)^2 + 2(3)^2 = 90 \text{ in.}^2$$

Top bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{305(6)}{90} = 20.33 \text{ kips}$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(20.33)^2 + (24.4)^2} = 31.8 \text{ kips} < 35.8 \text{ kips (OK)}$$

Check the angle-to-column connection, accounting for eccentricity.

Tension: From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t[(3 + 1.5)(2)] = 36.0 r_t$$

$$M_u = V_u e = 122(2.5) = 305.0 \text{ in.-kips}$$

From $36r_t = 305$, $r_t = 8.47$ kips

$$f_{rv} = \frac{122/10}{0.4418} = 27.61 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(54)}(27.61) = 55.64 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(55.64)(0.4418)$$

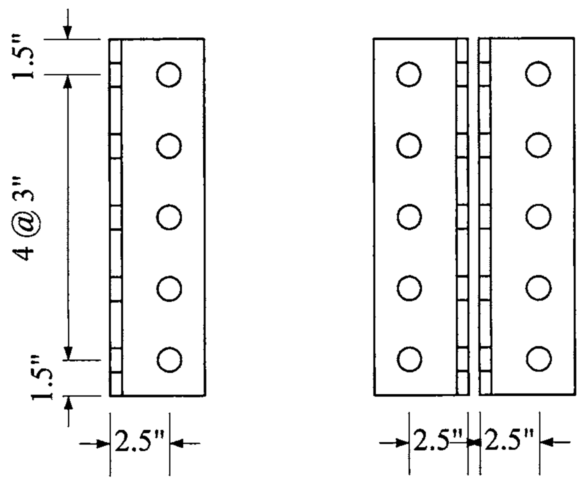
$$= 18.4 \text{ kips/bolt} > 8.47 \text{ kips/bolt} \qquad \text{(OK)}$$

Use 2L4 × 4 × $\frac{5}{16}$, with

five $\frac{3}{4}$ in. diameter Group A bearing-type bolts in the beam web, and

ten $\frac{3}{4}$ in. diameter Group A bearing-type bolts in the column flange

(d)




Beam connection        Column connection

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**8.3-10**

(a) Neglect the beam weight initially, and account for it later.

$w_a = 5$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(5)(30)^2 = 563 \text{ ft-kips}$$

From the $Z_x$ table, Try a W27 × 84:

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 609 \text{ ft-kips} > 563 \text{ ft-kips (OK)}$$

Check beam weight: $w_a = 5 + 0.084 = 5.084$ kips/ft

$$M_a = \frac{1}{8}waL^2 = \frac{1}{8}(5.084)(30)^2 = 572 \text{ ft-kips} < 609 \text{ ft-kips (OK)}$$

Shear: $V_a = \dfrac{5.084(30)}{2} = 76.26$ kips

From the $Z_x$ table, $\dfrac{V_n}{\Omega_v} = 246$ kips > 76.26 kips (OK)

<u>Use a W27 × 84</u>

(b) Use Group A, Type N, bearing bolts.

For the beam-to-angle bolts, design for shear, then check bearing. Try $\frac{3}{4}$-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nv}A_b = 0.5(54)(0.4418) \times 2 \text{ shear planes} = 23.86 \text{ kips/bolt}$$

Let $23.86N_b = V_a$ : $\quad 23.86N_b = 76.26 \quad \Rightarrow \quad N_b = 3.20$

Try 4 bolts in beam-to-angle connection and 8 bolts in angle-to-column connection.

Use a minimum spacing of $3d = 3(3/4) = 2.25$ in.

Minimum edge distance from AISC Table J3.4 = 1 in.

Try $s = 3$ in. and $\ell_e = 1\frac{1}{2}$ in. Total length of angles = $3(3) + 2(1.5) = 12$ in.

Bearing: Check angles first: $h = d + \dfrac{1}{16} = \dfrac{3}{4} + \dfrac{1}{16} = 0.8125$ in.

For edge bolt, $\ell_c = \ell_e - \dfrac{h}{2} = 1.5 - \dfrac{0.8125}{2} = 1.094$ in.

$$= \frac{1}{\Omega}(1.2\ell_c tF_u) = 0.5(1.2)(1.094)t(58) = 38.07t$$

$$\frac{1}{\Omega}(2.4dtF_u) = 0.5(2.4)(3/4)t(58) = 52.2t > 38.07t$$

$$\therefore \text{ use } \phi R_n = 38.07t \text{ kips/bolt}$$

For other bolts, $\ell_c = s - h = 3 - 0.8125 = 2.188$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c tF_u) = 0.5(1.2)(2.188)t(58) = 76.14t$$

$$\frac{1}{\Omega}(2.4dtF_u) = 52.2t < 76.14t \therefore \text{ use } \frac{R_n}{\Omega} = 52.2t \text{ kips/bolt}$$

Total bearing strength (per angle leg) = $38.07t + 3(52.2t) = 194.7t$

For two angles, $194.7t \times 2 = 389.4t$

Let $389.4t = V_a = 76.26 \quad \Rightarrow \quad t = 0.196$ in.

Check beam web. $t_w = 0.515$ in. There are no edge bolts. For the other bolts,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2\ell_c t F_u) = 0.5(1.2)(2.188)(0.515)(65) = 43.95 \text{ kips/bolt}$$

$$\frac{1}{\Omega}(2.4dtF_u) = 0.5(2.4)(3/4)(0.515)(65) = 30.13 \text{ kips/bolt} < 43.95 \text{ kips/bolt}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 30.13 \text{ kips/bolt}$$

Total bearing strength = $4(30.13) = 121$ kips > 76.26 kips (OK)

Bearing on column flange OK (column flange is thicker than the beam web).

Shear yield strength of angles (AISC J4.2):

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.60F_yA_g) \times 2 = \frac{1}{1.5}(0.60)(36)(12t) \times 2 = 345.6t$$

Let $345.6t = 76.26$, Solution is: $\{t = 0.2207\}$ in.

Shear rupture strength of angle (AISC J4.2):

Use hole diameter = $\frac{3}{4} + \frac{1}{8} = 0.875$ in.

$$A_{nv} = 2 \times (12 - 4 \times 0.875)t = 17.0t \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.60F_uA_{nv}) = 0.5(0.60)(58)(17.0t) = 295.8t \text{ kips}$$

Let $295.8t = 76.26$, Solution is: $\{t = 0.2578\}$ in.

Block shear strength of angles:

Shear areas: $A_{gv} = t(10.5) \times 2 = 21.0t$ in.$^2$

$$A_{nv} = t[10.5 - 3.5(0.875)] \times 2 = 14.88t \text{ in.}^2$$

Tension area: $A_{nt} = t[1.5 - 0.5(0.875)] \times 2 = 2.126t$ in.$^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(14.88t) + 1.0(58)(2.126t) = 641.1t \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(21.0t) + 1.0(58)(2.126t) = 576.9t \text{ kips}$$

Let $\frac{R_n}{\Omega} = 0.5(576.9t) = 76.26$, Solution is: $\{t = 0.264\}$ in.

The minimum required angle thickness is 0.264 in.

Try 2L4 × 4 × $\frac{5}{16}$ × 1 ft - 0 in. $\left(\frac{5}{16} \text{ in.} > 0.264 \text{ in. required}\right)$

Assume a $\frac{1}{2}$-in. setback and the usual gage distance.

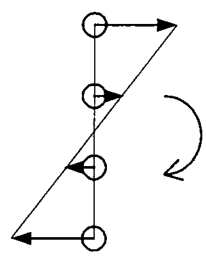
Use 2L4 × 4 × $\frac{5}{16}$, with

four $\frac{3}{4}$-in. diameter Group A bearing-type bolts in the beam web, and

eight $\frac{3}{4}$-in. diameter Group A bearing-type bolts in the column flange

(c) Check the beam-to-angle connection (shear only), accounting for eccentricity. The direct shear component is

$$p_{cy} = \frac{V_a}{4} = \frac{76.26}{4} = 19.07 \text{ kips}$$



Eccentric shear component:

$$M = V_a e = 76.26(2.5) = 190.7 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(4.5)^2 + 2(1.5)^2 = 45 \text{ in.}^2$$

Top bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{190.7(4.5)}{45} = 19.07 \text{ kips}$$

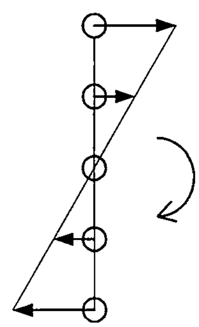
$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(19.07)^2 + (19.07)^2} = 26.97 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega} F_{nv} A_b = 0.5(54)(0.4418) \times 2 \text{ shear planes}$$

$$= 23.9 \text{ kips/bolt} < 26.97 \text{ kips/bolt} \quad \text{(N.G.)}$$

Try 5 bolts in the beam-to-angle connection and 10 bolts in the angle-to-column connection.

Direct shear component: $p_{cy} = \frac{V_a}{5} = \frac{76.26}{5} = 15.25$ kips



Eccentric shear component:

$$M = V_a e = 76.26(2.5) = 190.7 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(6)^2 + 2(3)^2 = 90 \text{ in.}^2$$

Top bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{190.7(6)}{90} = 12.71 \text{ kips}$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(12.71)^2 + (15.25)^2}$$

$$= 19.85 \text{ kips} < 23.9 \text{ kips (OK)}$$

Check the angle-to-column connection, accounting for eccentricity.

Tension: From Equation 8.6 in the textbook, the resisting moment is

$$M = n' r_t d_m = 4r_t[(3 + 1.5)(2)] = 36.0 r_t$$

$$M_a = V_a e = 76.26(2.5) = 190.7 \text{ in.-kips}$$

From $36r_t = 190.7$, $r_t = 5.30$ kips

$$f_{rv} = \frac{76.26/10}{0.4418} = 17.26 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{54}(17.26) = 59.47 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F'_{nt}A_b = 0.5(59.47)(0.4418)$$

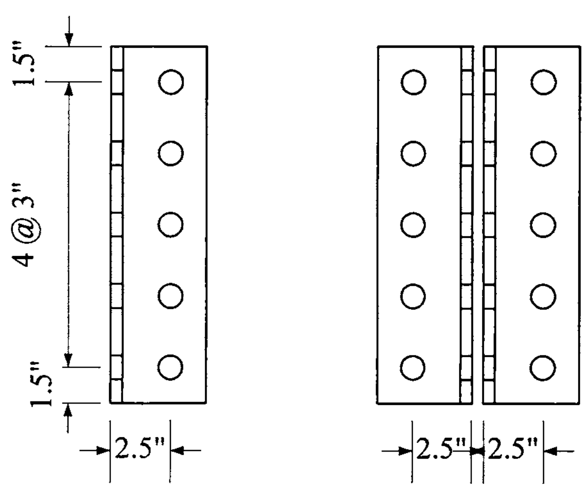
$$= 13.14 \text{ kips/bolt} > 5.30 \text{ kips/bolt} \quad \text{(OK)}$$

Use 2L4 × 4 × $\frac{5}{16}$, with

five $\frac{3}{4}$ in. diameter Group A bearing-type bolts in the beam web, and

ten $\frac{3}{4}$ in. diameter Group A bearing-type bolts in the column flange

(d)




Beam connection    Column connection

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**8.4-1**

Direct shearing stress: $f_{1y} = \frac{8}{7+8} = 0.5333 \text{ kips/in.}\downarrow$

Shearing stress caused by moment: Locate centroid with respect to upper left corner.

$$\bar{x} = \frac{7(3.5)}{15} = 1.633 \text{ in.}, \qquad \bar{y} = \frac{8(4)}{15} = 2.133 \text{ in.}$$

$$M = Pe = 8(15 - 1.633) = 106.9 \text{ in.-kips} \curvearrowright$$

$$I_x = \frac{(8)^3}{12} + 8(4 - 2.133)^2 + 7(2.133)^2 = 102.4 \text{ in.}^4$$

$$I_y = 8(1.633)^2 + \frac{(7)^3}{12} + 7(3.5 - 1.633)^2 = 74.32 \text{ in.}^4$$

$$J = I_x + I_y = 102.4 + 74.32 = 176.7 \text{ in.}^4$$

Lower left corner:

$$f_{2x} = \frac{My}{J} = \frac{106.9(2.133)}{176.7} = 1.290 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{106.9(7 - 1.633)}{176.7} = 3.247 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(1.290)^2 + (0.5333 + 3.247)^2} = 3.99 \text{ kips/in.}$$

$$\underline{f_v = 3.99 \text{ kips/in.}}$$

---

**8.4-2**

Direct shearing stress: $f_{1y} = \dfrac{16}{5+3+5} = 1.231 \text{ kips/in.} \downarrow$

Shearing stress caused by moment: Locate centroid with respect to lower left corner.

$$\bar{x} = \frac{5(2.5) \times 2}{13} = 1.923 \text{ in.}, \qquad \bar{y} = \frac{5(12) + 3(10.5)}{13} = 7.038 \text{ in.}$$

$$M = Pe = 16(12 - 1.923) = 161.2 \text{ in.-kips} \curvearrowright$$

$$I_x = \frac{(3)^3}{12} + 3(10.5 - 7.038)^2 + 5(7.038)^2 + 5(12 - 7.038)^2 = 409.0 \text{ in.}^4$$

$$I_y = 3(1.923)^2 + 2\left[\frac{(5)^3}{12} + 5(2.5 - 1.923)^2\right] = 35.26 \text{ in.}^4$$

$$J = I_x + I_y = 409.0 + 35.26 = 444.3 \text{ in.}^4$$

Lower right corner:

$$f_{2x} = \frac{My}{J} = \frac{161.2(7.038)}{444.3} = 2.554 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{161.2(5 - 1.923)}{444.3} = 1.116 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(2.554)^2 + (1.231 + 1.116)^2} = 3.47 \text{ kips/in.}$$

$$\underline{f_v = 3.47 \text{ kips/in.}}$$

---

**8.4-3**

$$P_x = 70\cos 75^\circ = 18.12 \text{ kips} \leftarrow \qquad P_y = 70\sin 75^\circ = 67.62 \text{ kips} \downarrow$$

$$f_{1x} = \frac{18.12}{18} = 1.007 \text{ kips/in.} \leftarrow \qquad f_{1y} = \frac{67.62}{18} = 3.757 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment: Locate centroid with respect to lower left corner.

$$\bar{x} = \frac{9(4.5) + 3(1.5) + 3(7.5) + 3(9)}{18} = 5.25 \text{ in.}$$

$$\bar{y} = \frac{3(12) + 3(12) + 3(10.5)}{18} = 5.75 \text{ in.}$$

$$M = 18.12(12 - 5.75) + 67.62(6 + 5.25) = 874.0 \text{ in.-kips} \curvearrowleft$$

$$I_x = 9(5.75)^2 + 3(12 - 5.75)^2 \times 2 + \frac{(3)^3}{12} + 3(12 - 1.5 - 5.75)^2 = 601.9 \text{ in.}^4$$

$$I_y = \frac{(9)^3}{12} + 9(0.75)^2 + \frac{(3)^3}{12} + 3(3.75)^2 + \frac{(3)^3}{12} + 3(2.25)^2 + 3(3.75)^2$$

$$= 169.9 \text{ in.}^4$$

$$J = I_x + I_y = 601.9 + 169.9 = 771.8 \text{ in.}^4$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{874.0(12 - 5.75)}{771.8} = 7.078 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{874.0(5.25)}{771.8} = 5.945 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(1.007 + 7.078)^2 + (3.757 + 5.945)^2}$$

$$= 12.6 \text{ kips/in.}$$

<u>$f_v = 12.6$ kips/in.</u>

---

**<u>8.4-4</u>**

(a) $P_u = 1.2D + 1.6L = 1.2(2) + 1.6(6) = 12.0 \text{ kips}$

Load on one angle $= \dfrac{12}{2} = 6.0$ kips, $\quad f_y = \dfrac{6}{3(5)} = 0.4 \text{ kips/in.}\downarrow$

Shearing stress caused by moment: Locate centroid with respect to right side of weld.

$$\bar{x} = \frac{5(2.5) \times 2}{15} = 1.667 \text{ in.}$$

$$M = 6(17 - 1.667) = 92.00 \text{ in.-kips} \curvearrowleft$$

$$I_x = \frac{(5)^3}{12} + 2(5)(2.5)^2 = 72.92 \text{ in.}^4$$

$$I_y = 5(1.667)^2 + 2\left[\frac{(1.667)^3}{3} + \frac{(3.333)^3}{3}\right] = 41.67 \text{ in.}^4$$

$$J = I_x + I_y = 72.92 + 41.67 = 114.6 \text{ in.}^4$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{92(2.5)}{114.6} = 2.007 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{92(5 - 1.667)}{114.6} = 2.676 \text{ kips/in.}\downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(2.007)^2 + (0.4 + 2.676)^2} = 3.67 \text{ kips/in.}$$

Weld strength is

$$\phi R_n = \phi[0.707w(0.6)F_{EXX}] = 0.75(0.707)(3/16)(0.6)(70) = 4.18 \text{ kips/in.}$$

(Alternate: $\phi R_n = 1.392 \times 3$ sixteenths $= 4.18$ kips/in.)

Since 3.67 kips/in. < 4.18 kips/in., <u>weld is adequate.</u>

(b) $P_a = 8$ kips

Load on one angle $= \frac{8}{2} = 4$ kips, $\quad f_y = \frac{4}{3(5)} = 0.2667$ kips/in.↓

Shearing stress caused by moment: Locate centroid with respect to right side of weld.

$$\bar{x} = \frac{5(2.5) \times 2}{15} = 1.667 \text{ in.}$$

$$M = 4(17 - 1.667) = 61.33 \text{ in.-kips} \circlearrowleft$$

$$I_x = \frac{(5)^3}{12} + 2(5)(2.5)^2 = 72.92 \text{ in.}^4$$

$$I_y = 5(1.667)^2 + 2\left[\frac{(1.667)^3}{3} + \frac{(3.333)^3}{3}\right] = 41.67 \text{ in.}^4$$

$$J = I_x + I_y = 72.92 + 41.67 = 114.6 \text{ in.}^4$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{61.33(2.5)}{114.6} = 1.338 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{61.33(5 - 1.667)}{114.6} = 1.784 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(1.338)^2 + (0.2667 + 1.784)^2} = 2.45 \text{ kips/in.}$$

Weld strength is

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}[0.707w(0.6)F_{EXX}] = 0.5(0.707)(3/16)(0.6)(70) = 2.78 \text{ kips/in.}$$

(Alternate: $\frac{R_n}{\Omega} = 0.9279 \times 3$ sixteenths = 2.78 kips/in.)

Since 2.45 kips/in. < 2.78 kips/in., <u>weld is adequate.</u>

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**8.4-5**

Locate centroid with respect to left side of plate.

$$\bar{x} = \frac{6(4 + 3)(2)}{18} = 4.667 \text{ in.}$$

$$I_x = \frac{(6)^3}{12} + 6(6)^2 \times 2 = 450.0 \text{ in.}^4$$

$$I_y = 6(4.667)^2 + 2\left[\frac{(6)^3}{12} + 6(7-4.667)^2\right] = 232.0 \text{ in.}^4$$

$$J = I_x + I_y = 450 + 232 = 682 \text{ in.}^4$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(9) + 1.6(8) = 23.6 \text{ kips}$$

$$P_{ux} = 23.6\sin 30° = 11.80 \text{ kips} \rightarrow \qquad P_{uy} = 23.6\cos 30° = 20.44 \text{ kips} \downarrow$$

$$f_{1x} = \frac{11.80}{18} = 0.6556 \text{ kips/in.} \rightarrow \qquad f_{1y} = \frac{20.44}{18} = 1.136 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment:

$$M = 11.80(6) + 20.44(10 + 12 - 4.667) = 425.1 \text{ in.-kips} \curvearrowright$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{425.1(6)}{682} = 3.740 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{425.1(10-4.667)}{682} = 3.324 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.6556 + 3.740)^2 + (1.136 + 3.324)^2}$$

$$= 6.262 \text{ kips/in.}$$

Required weld size = $\frac{6.262}{1.392} = 4.50$ sixteenths.

For $t = 1/2$ in., min. $w = 3/16$ in. and max $w = 1/2 - 1/16 = 7/16$ in.

<u>Use a $\frac{5}{16}$-in. fillet weld.</u>

(b) ASD solution

$$P_a = D + L = 9 + 8 = 17 \text{ kips}$$

$$P_{ax} = 17\sin 30° = 8.500 \text{ kips} \rightarrow \qquad P_{ay} = 17\cos 30° = 14.72 \text{ kips} \downarrow$$

$$f_{1x} = \frac{8.500}{18} = 0.4722 \text{ kips/in.} \rightarrow \qquad f_{1y} = \frac{14.72}{18} = 0.8178 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment:

$$M = 8.500(6) + 14.72(10 + 12 - 4.667) = 306.1 \text{ in.-kips} \curvearrowright$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{306.1(6)}{682} = 2.693 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{306.1(10 - 4.667)}{682} = 2.394 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.4722 + 2.693)^2 + (0.8178 + 2.394)^2}$$

$$= 4.509 \text{ kips/in.}$$

Required weld size $= \dfrac{4.509}{0.9279} = 4.859$ sixteenths.

For $t = 1/2$ in., min. $w = 3/16$ in. and max $w = 1/2 - 1/16 = 7/16$ in.

Use a $\frac{3}{8}$-in. fillet weld.

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**8.4-6**

(a) $D + L = D + 2D = 3D = 18;\ \ D = 6$ kips, $L = 12$ kips

$$P_u = 1.2D + 1.6L = 1.2(6) + 1.6(12) = 26.4 \text{ kips}$$

$$f_{1y} = \frac{26.4}{5} = 5.28 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment: Locate centroid with respect to lower right.

$$\bar{x} = \frac{3(1.5) + 2(1)}{3 + 2} = 1.3 \text{ in.}, \qquad \bar{y} = \frac{3(6)}{3 + 2} = 3.6 \text{ in.}$$

$$M = 26.4(2 + 1.3) = 87.12 \text{ in.-kips} \curvearrowright$$

$$I_x = 3(6 - 3.6)^2 + 2(3.6)^2 = 43.2 \text{ in.}^4$$

$$I_y = \frac{(3)^3}{12} + 3(1.5 - 1.3)^2 + \frac{(2)^3}{12} + 2(0.3)^2 = 3.217 \text{ in.}^4$$

$$J = I_x + I_y = 43.2 + 3.217 = 46.42 \text{ in.}^4$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{87.12(3.6)}{46.42} = 6.756 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{87.12(1.3)}{46.42} = 2.440 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(6.756)^2 + (5.28 + 2.440)^2} = 10.3 \text{ kips/in.}$$

Weld strength: $\phi R_n = 1.392 \times 8$ sixteenths $= 11.1$ kips/in. > 10.3 kips/in. (OK)

Weld is adequate.

(b) $P_a = 18$ kips

$$f_{1y} = \frac{18}{5} = 3.6 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment: Locate centroid with respect to lower right.

$$\bar{x} = \frac{3(1.5) + 2(1)}{3 + 2} = 1.3 \text{ in.}, \quad \bar{y} = \frac{3(6)}{3 + 2} = 3.6 \text{ in.}$$

$$M = 18(2 + 1.3) = 59.4 \text{ in.-kips} \curvearrowright$$

$$I_x = 3(6 - 3.6)^2 + 2(3.6)^2 = 43.2 \text{ in.}^4$$

$$I_y = \frac{(3)^3}{12} + 3(1.5 - 1.3)^2 + \frac{(2)^3}{12} + 2(0.3)^2 = 3.217 \text{ in.}^4$$

$$J = I_x + I_y = 43.2 + 3.217 = 46.42 \text{ in.}^4$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{59.4(3.6)}{46.42} = 4.607 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{59.4(1.3)}{46.42} = 1.664 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(4.607)^2 + (3.6 + 1.664)^2} = 7.00 \text{ kips/in.}$$

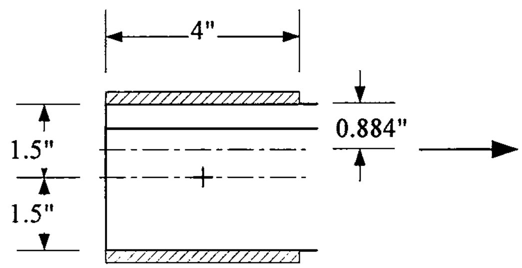
Weld strength:

$\frac{R_n}{\Omega} = 0.9279 \times 8$ sixteenths $= 7.42$ kips/in. > 7.00 kips/in. (OK)

Weld is adequate.

**8.4-7**

If eccentricity is not considered, $f_{1x} = \dfrac{115}{2(4)} = 14.38$ kips/in. →




Load when eccentricity is considered:

$$I_x = 2(4)(1.5)^2 = 18.0 \text{ in.}^4, \qquad I_y = 2\left[\frac{(4)^3}{12}\right] = 10.67 \text{ in.}^4$$

$$J = I_x + I_y = 18.0 + 10.67 = 28.67 \text{ in.}^4$$

$$e = 1.5 - 0.884 = 0.616 \text{ in.}$$

$$M = Pe = 115(0.616) = 70.84 \text{ in.-kips}$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{70.84(1.5)}{28.67} = 3.706 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{70.84(2)}{28.67} = 4.942 \text{ kips/in.} \downarrow$$

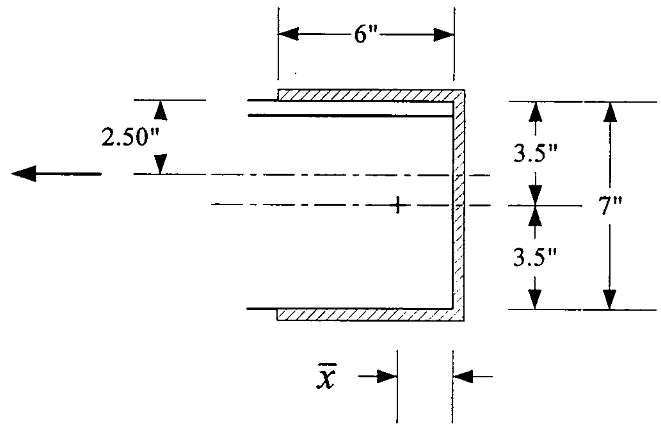
$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(14.38 + 3.706)^2 + (4.942)^2} = 18.75 \text{ kips/in.}$$

Load caused by eccentricity = $18.75 - 14.38 = 4.37$ kips/in.

$\underline{f_v = 4.37 \text{ kips/in.}}$

**8.4-8**

If eccentricity is not considered, $f_{1x} = \dfrac{140}{2(6)+7} = 7.368$ kips/in. ←




Load when eccentricity is considered: Locate centroid with respect to right side.

$$\bar{x} = \frac{6(3)\times 2}{19} = 1.895 \text{ in.,} \qquad \bar{y} = 3.5 \text{ in.}$$

$$I_x = 2(6)(3.5)^2 + \frac{(7)^3}{12} = 175.6 \text{ in.}^4$$

$$I_y = 2\left[\frac{(6)^3}{12} + 6(3-1.895)^2\right] + 7(1.895)^2 = 75.79 \text{ in.}^4$$

$$J = I_x + I_y = 175.6 + 75.79 = 251.4 \text{ in.}^4$$

$$M = 140(3.5 - 2.50) = 140.0 \text{ in.-kips}$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{140(3.5)}{251.4} = 1.949 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{140(6-1.895)}{251.4} = 2.286 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(7.368+1.949)^2 + (2.286)^2} = 9.593 \text{ kips/in.}$$

Load caused by eccentricity = 9.593 − 7.368 = 2.23 kips/in.

$\underline{f_v = 2.23 \text{ kips/in.}}$

**8.4-9**

(a) Compute the tension member available strength. For the gross section:

$$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(4.38) = 141.9 \text{ kips}$$

Net section: Assume that $U = 0.80$. Check once the connection is designed.

$$A_e = A_g U = 4.38(0.80) = 3.504 \text{ in.}^2$$

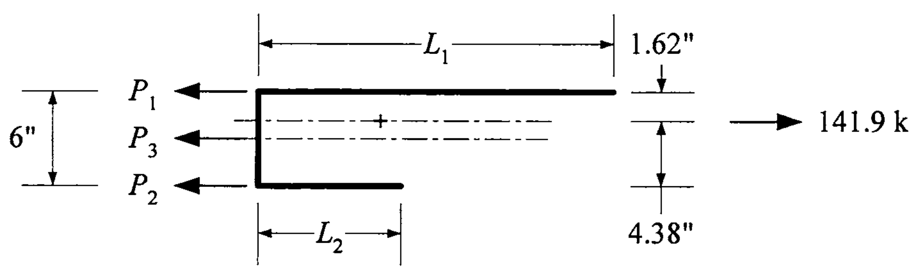
$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(3.504) = 152.4 \text{ kips. Gross section controls.}$$

Determine weld size. Based a thickness of 3/8 inch, min. $w = 3/16$ in.

$$\max w = 3/8 - 1/16 = 5/16 \text{ in.}$$

To minimize the length of the connection, try 5/16 in. fillet welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(5) = 6.96 \text{ kips/in.}$$




$$P_3 = 6(6.96) = 41.76 \text{ kips}$$

$$\sum M_{L2} = 141.9(4.38) - 41.76\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 82.71 \text{ kips}$$

$$\sum F = 141.9 - 41.76 - 82.71 - P_2 = 0 \quad \Rightarrow \quad P_2 = 17.43 \text{ kips}$$

$$L_1 = \frac{P_1}{6.96} = \frac{82.71}{6.96} = 11.9 \text{ in. Use 12 in.}$$

$$L_2 = \frac{P_2}{6.96} = \frac{17.43}{6.96} = 2.50 \text{ in. Use } 2\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is $0.85(6.96) = 5.916$ kips/in.

and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

$P_3 = 6(10.44) = 62.64$ kips

$\sum M_{L2} = 141.9(4.38) - 62.64\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 72.27$ kips

$\sum F = 141.9 - 62.64 - 72.27 - P_2 = 0 \quad \Rightarrow \quad P_2 = 6.99$ kips

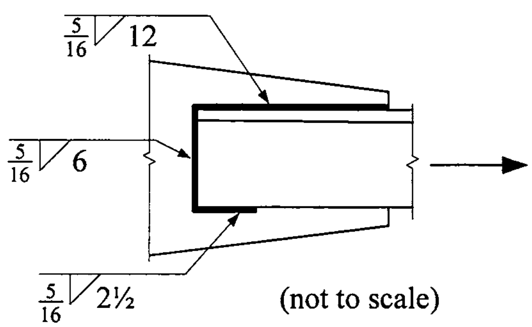
$L_1 = \frac{P_1}{5.916} = \frac{72.27}{5.916} = 12.22$ in. Use 12½ in.

$L_2 = \frac{P_2}{5.916} = \frac{6.99}{5.916} = 1.182$ in. Use 1½ in.

(Min. length = $4w = 4(5/16) = 1.25$ in.)

The total length of weld is nearly the same for both options. Try the first option and check the value of $U$.

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.62}{12} = 0.865 > 0.80$ (OK) Use the weld shown.



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(b) Compute the tension member available strength. For the gross section,

$$\frac{P_n}{\Omega} = \frac{1}{\Omega}F_yA_g = \frac{1}{1.67}(36)(4.38) = 94.42 \text{ kips}$$

Net section: Assume that $U = 0.80$. Check once the connection is designed.

$$A_e = A_gU = 4.38(0.80) = 3.504 \text{ in.}^2$$

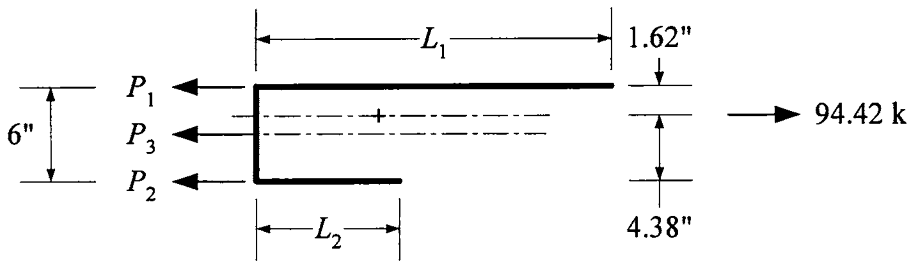
$\frac{P_n}{\Omega} = \frac{1}{\Omega}F_uA_e = 0.5(58)(3.504) = 101.6$ kips. Gross section controls.

Determine weld size. Based a thickness of 3/8 inch, min. $w = 3/16$ in.

max $w = 3/8 - 1/16 = 5/16$ in.

To minimize the length of the connection, try 5/16 in. fillet welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\frac{R_n}{\Omega} = 0.9279D = 0.9279(5) = 4.640 \text{ kips/in.}$$




$$P_3 = 6(4.640) = 27.84 \text{ kips}$$

$$\sum M_{L2} = 94.42(4.38) - 27.84\left(\frac{6}{2}\right) - P_1(6) = 0$$

Solution is: $\{P_1 = 55.01\}$ kips

$$\sum F = 94.42 - 55.01 - 27.84 - P_2 = 0, \text{ Solution is: } \{P_2 = 11.57\} \text{ kips}$$

$$L_1 = \frac{P_1}{6.96} = \frac{55.01}{4.640} = 11.86 \text{ in. Use 12 in.}$$

$$L_2 = \frac{P_2}{6.96} = \frac{11.57}{4.640} = 2.494 \text{ in. Use } 2\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is $0.85(4.640) = 3.944$ kips/in.

and the strength of the transverse weld is $1.5(4.640) = 6.96$ kips/in.

$$P_3 = 6(6.96) = 41.76 \text{ kips}$$

$$\sum M_{L2} = 94.42(4.38) - 41.76\left(\frac{6}{2}\right) - P_1(6) = 0$$

Solution is: $\{P_1 = 48.05\}$ kips

$$\sum F = 94.42 - 41.76 - 48.05 - P_2 = 0, \text{ Solution is: } \{P_2 = 4.61\} \text{ kips}$$

$$L_1 = \frac{P_1}{3.944} = \frac{48.05}{3.944} = 12.18 \text{ in. Use } 12\tfrac{1}{2} \text{ in.}$$

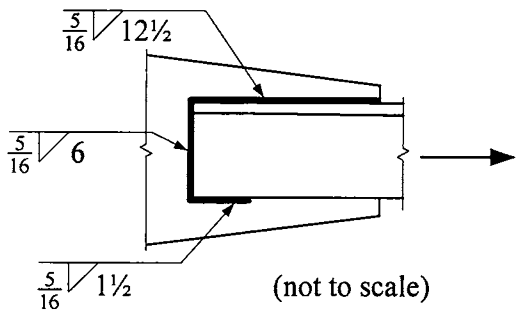
$$L_2 = \frac{P_2}{3.944} = \frac{4.61}{3.944} = 1.169 \text{ in. Use } 1\tfrac{1}{2} \text{ in.}$$

(Min. length = $4w = 4(5/16) = 1.25$ in.)

The total length of weld is nearly the same for both options. Try the first option and check the value of $U$.

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.62}{12} = 0.865 > 0.80 \quad \text{(OK)}$$

Use the weld shown.

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**8.4-10**

Use Table 8-10, Angle = 0° $k\ell = 7$ in., $k = \frac{k\ell}{\ell} = \frac{7}{8} = 0.875$

$x = 0.2043$ (by interpolation), $x\ell = 0.2043(8) = 1.634$ in.

$e_x = 7 + 8 - 1.634 = 13.37$ in.

$a\ell = e_x$, $a = \frac{e_x}{\ell} = \frac{13.37}{8} = 1.671$

$C = 1.006$ by interpolation. If the 8-kip load is a factored load,

$$D_{\text{min}} = \frac{P_u}{\phi C C_1 \ell} = \frac{8}{0.75(1.006)(1.0)(8)} = 1.325$$

$\phi R_n = 1.392D = 1.392(1.325) = 1.84$ kips/in.

If the 9-kip load is a service load,

$$D_{\text{min}} = \frac{\Omega P_a}{C C_1 \ell} = \frac{2.00(8)}{1.006(1.0)(8)} = 1.988$$

$\frac{R_n}{\Omega} = 0.9279D = 0.9279(1.988) = 1.84$ kips/in. $p = 1.84$ kips/in.

---

**8.4-11**

Consider the weld on one angle. Use Table 8-8, Angle = 0°

$$k\ell = 5 \text{ in.}, \qquad k = \frac{k\ell}{\ell} = \frac{5}{5} = 1.0$$

$$x = 0.333, \qquad x\ell = 0.333(5) = 1.665 \text{ in.}$$

$$e_x = 12 + 5 - 1.665 = 15.34 \text{ in.}$$

$$a\ell = e_x, \qquad a = \frac{e_x}{\ell} = \frac{15.34}{5} = 3.068$$

Maximum value of $a$ in the table is 3.00. By extrapolation,

$$C = 1.12 + \left(\frac{1.12 - 1.19}{3.00 - 2.80}\right)(3.068 - 3.00) = 1.096$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(0.25 \times 8) + 1.6(0.75 \times 8) = 12.0 \text{ kips}$$

For one angle, $P_u = \frac{12}{2} = 6.0$ kips

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{6}{0.75(1.096)(1)(5)}$$

$= 1.46$ sixteenths < 3 sixteenths furnished (OK)

Weld is adequate.

(b) ASD solution

$P_a = 8$ kips; For one angle, $P_a = 8/2 = 4$ kips

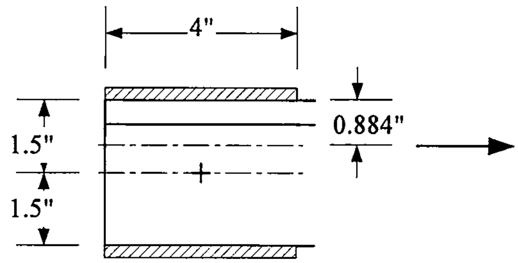
$$D_{\min} = \frac{\Omega P_a}{C C_1 \ell} = \frac{2.00(4)}{1.096(1)(5)}$$

$= 1.46$ sixteenths < 3 sixteenths furnished (OK)

Weld is adequate.

---

**8.4-12**




Use Table 8-4, Angle = 0°

$k\ell = 3$ in., $\quad k = \dfrac{k\ell}{\ell} = \dfrac{3}{4} = 0.75$

$e_x = 1.5 - 0.884 = 0.616$ in.

$a\ell = e_x, \quad a = \dfrac{0.616}{\ell} = \dfrac{0.616}{4} = 0.154$

$C = 3.539$ by interpolation. If the 115-kip load is treated as a factored load,

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{115}{0.75(3.539)(1.0)(4)} = 10.83$$

$\phi R_n = 1.392D = 1.392(10.83) = 15.08$ kips/in.

When eccentricity is not accounted for, $e_x = 0$, $a = 0$, and $C = 3.71$

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{115}{0.75(3.71)(1.0)(4)} = 10.33$$

$\phi R_n = 1.392D = 1.392(10.33) = 14.38$ kips/in.

Difference = $15.08 - 14.38 = 0.70$ kips/in. <u>0.70 kips/inch</u>

---

**8.4-13**

(a) $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(40) = 88.0$ kips

$P_{ux} = 88.0 \sin 15° = 22.78$ kips ← $\qquad P_{uy} = 88.0 \cos 15° = 85.0$ kips ↓

$f_{1x} = \dfrac{22.78}{2(12+4)} = 0.7119$ kips/in. ← $\qquad f_{1y} = \dfrac{85.0}{32} = 2.656$ kips/in. ↓

Shearing stress caused by moment:

$$M = 85.0(2 + 2.5) - 22.78(6) = 245.8 \text{ in.-kips} \curvearrowright$$

$$I_x = 2 \times \frac{(12)^3}{12} + 2(4)(6)^2 = 576.0 \text{ in.}^4$$

$$I_y = 12(2)^2(2) + 2\frac{(4)^3}{12} = 106.7 \text{ in.}^4$$

$$J = I_x + I_y = 576.0 + 106.7 = 682.7 \text{ in.}^4$$

Stress at lower right:

$$f_{2x} = \frac{My}{J} = \frac{245.8(6)}{682.7} = 2.16 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{245.8(2)}{682.7} = 0.7201 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2}$$

$$= \sqrt{(0.7119 + 2.16)^2 + (2.656 + 0.7201)^2} = 4.432 \text{ kips/in.}$$

Required weld size $= \frac{4.432}{1.392} = 3.18$ sixteenths. Use a ¼-in. fillet weld.

(b) Use Table 8-6, Angle = 15°

$$k\ell = 4 \text{ in.,} \qquad k = \frac{k\ell}{\ell} = \frac{4}{12} = 0.3333$$

$$6 \tan 15° = 1.608 \text{ in.,} \qquad e_x = \frac{4}{2} + 2.5 - 1.608 = 2.892 \text{ in.}$$

$$a\ell = e_x, \qquad a = \frac{e_x}{\ell} = \frac{2.892}{12} = 0.241$$

$C = 4.79$ by interpolation

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{88}{0.75(4.79)(1)(12)} = 2.04 \text{ sixteenths}$$

Use a $\frac{3}{16}$-in. fillet weld.

---

**8.4-14**

(a) $P_a = D + L = 20 + 40 = 60$ kips

$P_{ax} = 60\sin 15° = 15.53$ kips ← $\quad P_{ay} = 60\cos 15° = 57.96$ kips ↓

$f_{1x} = \frac{15.53}{2(12+4)} = 0.4853$ kips/in. ← $\quad f_{1y} = \frac{57.96}{32} = 1.811$ kips/in. ↓

Shearing stress caused by moment:

$M = 57.96(2 + 2.5) - 15.53(6) = 167.6$ in.-kips ↻

$I_x = (2)\frac{(12)^3}{12} + 2(4)(6)^2 = 288.0 \text{ in.}^4$

$I_y = 12(2)^2(2) + 2\frac{(4)^3}{12} = 106.7 \text{ in.}^4$

$J = I_x + I_y = 288.0 + 106.7 = 394.7 \text{ in.}^4$

Stress at lower right:

$f_{2x} = \frac{My}{J} = \frac{167.6(6)}{394.7} = 2.548$ kips/in. ←

$f_{2y} = \frac{Mx}{J} = \frac{167.6(2)}{394.7} = 0.8493$ kips/in. ↓

$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.4853 + 2.548)^2 + (1.811 + 0.8493)^2}$

$= 4.035$ kips/in.

Required weld size $= \frac{4.035}{0.9279} = 4.349$ sixteenths. $\quad$ <u>Use a $\frac{5}{16}$-in. fillet weld.</u>

(b) Use Table 8-6, Angle = 15°

$k\ell = 4$ in., $\quad k = \frac{k\ell}{\ell} = \frac{4}{12} = 0.3333$

$6\tan 15° = 1.608$ in., $\quad e_x = \frac{4}{2} + 2.5 - 1.608 = 2.892$ in.

$a\ell = e_x, \quad a = \frac{e_x}{\ell} = \frac{2.892}{12} = 0.241$

$C = 4.798$ by interpolation

$$D_{\min} = \frac{\Omega P_a}{CC_1\ell} = \frac{2.00(60)}{4.798(1)(12)} = 2.08 \text{ sixteenths}$$ Use a $\frac{3}{16}$-in. fillet weld.

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**8.4-15**

(a) $P_u = 1.2D + 1.6L = 1.2(31) + 1.6(31) = 86.8$ kips

For $t$ = 5/16 in., min. $w$ = 3/16 in. and max $w$ = 5/16 – 1/16 = 1/4 in.

Use E70 fillet welds. Try $w$ = 3/16 in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(3) = 4.176 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(5/16) = 6.75$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(5/16) = 8.156$ kips/in.

The weld shear strength controls.

Total length required = $\frac{86.8}{4.176} = 20.79$ in.

Length of longitudinal welds = $\frac{20.79 - 6}{2} = 7.40$ in.; use 7½ in.

For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

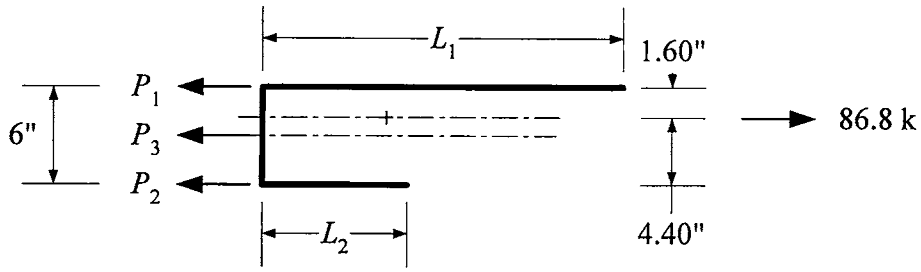
Length of longitudinal welds = $\frac{86.8 - 6.264(6)}{3.550(2)} = 6.93$ in.; use 7 in.

The second options requires shorter longitudinal welds:

Use $\frac{3}{16}$-in. E70 fillet welds, across the end and 7 inches on each side

(b) Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$\phi R_n = 4.176$ kips/in.




$P_3 = 6(4.176) = 25.06$ kips

$\sum M_{L2} = 86.8(4.40) - 25.06\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 51.12$ kips

$\sum F = 86.8 - 25.06 - 51.12 - P_2 = 0 \quad \Rightarrow \quad P_2 = 10.62$ kips

$L_1 = \frac{P_1}{4.176} = \frac{51.12}{4.176} = 12.2$ in. Use 12½ in.

$L_2 = \frac{P_2}{4.176} = \frac{10.62}{4.176} = 2.54$ in. Use 3 in.

For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

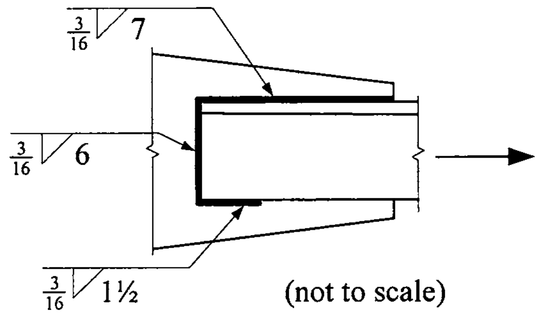
$P_3 = 6(6.264) = 37.58$ kips

$\sum M_{L2} = 86.8(4.40) - 37.58\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 44.86$ kips

$\sum F = 86.8 - 37.58 - 44.86 - P_2 = 0 \quad \Rightarrow \quad P_2 = 4.36$ kips

$L_1 = \frac{P_1}{3.550} = \frac{44.86}{3.550} = 12.6$ in. Use 7 in.

$L_2 = \frac{P_2}{3.550} = \frac{4.36}{3.550} = 1.23$ in. Use 1½ in. Use the weld shown.

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**8.4-16**

(a) $P_a = D + L = 31 + 31 = 62$ kips

For $t = 5/16$ in., min. $w = 3/16$ in. and max $w = 5/16 - 1/16 = 1/4$ in.

Use E70 fillet welds. Try $w = 3/16$ in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$R_n/\Omega = 0.9279D = 0.9279(3) = 2.784 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $R_n/\Omega = 0.4F_y t = 0.4(36)(5/16) = 4.5$ kips/in.

Rupture: $R_n/\Omega = 0.3F_u t = 0.3(58)(5/16) = 5.438$ kips/in.

The weld shear strength controls.

$$\text{Total length required} = \frac{62}{2.784} = 22.27 \text{ in.}$$

$$\text{Length of longitudinal welds} = \frac{22.27 - 6}{2} = 8.14 \text{ in.; use } 8\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(2.784) = 2.366 \text{ kips/in.}$$

and the strength of the transverse weld is $1.5(2.784) = 4.176$ kips/in.

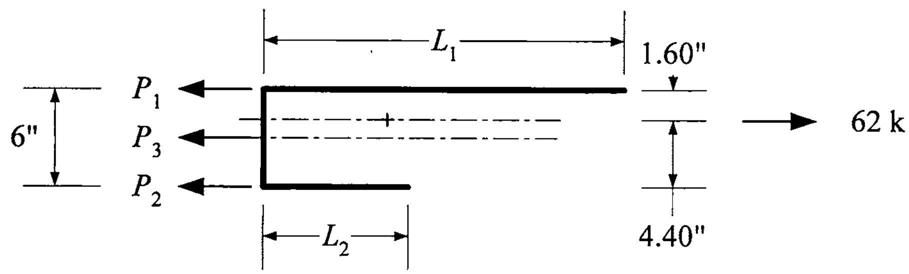
$$\text{Length of longitudinal welds} = \frac{62 - 4.176(6)}{2.366(2)} = 7.81 \text{ in.; use 8 in.}$$

The second options requires shorter longitudinal welds:

Use $\frac{3}{16}$-in. E70 fillet welds, across the end and 8 inches on each side

(b) Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$R_n/\Omega = 2.784$ kips/in.




$P_3 = 6(2.784) = 16.7$ kips

$\sum M_{L2} = 62(4.40) - 16.7\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 37.12$ kips

$\sum F = 62 - 16.7 - 37.12 - P_2 = 0 \quad \Rightarrow \quad P_2 = 8.18$ kips

$L_1 = \frac{P_1}{2.784} = \frac{37.12}{2.784} = 13.3$ in. Use 13½ in.

$L_2 = \frac{P_2}{2.784} = \frac{8.18}{2.784} = 2.94$ in. Use 3 in.

For the second option, the strength of the longitudinal welds is

$0.85(2.784) = 2.366$ kips/in.

and the strength of the transverse weld is $1.5(2.784) = 4.176$ kips/in.

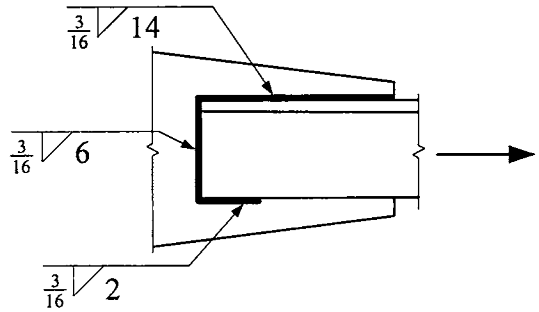
$P_3 = 6(4.176) = 25.06$ kips

$\sum M_{L2} = 62(4.40) - 25.06\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 32.94$ kips

$\sum F = 62 - 25.06 - 32.94 - P_2 = 0 \quad \Rightarrow \quad P_2 = 4.00$ kips

$L_1 = \frac{P_1}{2.366} = \frac{32.94}{2.366} = 13.9$ in. Use 14 in.

$L_2 = \frac{P_2}{2.366} = \frac{4.00}{2.366} = 1.69$ in. Use 2 in. Use the weld shown.




## 8.4-17

(a) $P_u = 1.2D + 1.6L = 1.2(40) + 1.6(80) = 176.0$ kips

For $t$ = 3/8 in., min. $w$ = 3/16 in. and max $w$ = 5/8 – 1/16 = 9/16 in.

Use E70 fillet welds. Try $w$ = 3/16 in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(3) = 4.176 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(3/8) = 8.1$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(3/8) = 9.788$ kips/in.

The weld shear strength controls. Total length required = $\dfrac{176}{4.176} = 42.15$ in.

Length of longitudinal welds = $\dfrac{42.15 - 5}{2} = 18.6$ in.; use 19 in.

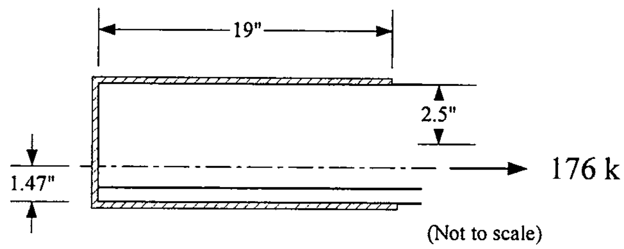
For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

Length of longitudinal welds = $\dfrac{176 - 6.264(5)}{3.550(2)} = 20.4$ in.; use 20½ in.

<u>Use 3/16-in. E70 fillet welds, across the end and 19 inches on each side</u>

(b)




$$f_{1x} = \frac{176}{2(19)+5} = 4.093 \text{ kips/in.} \rightarrow$$

Locate centroid with respect to left side.

$$\bar{x} = \frac{19(9.5)\times 2}{2(19)+5} = 8.395 \text{ in.}$$

$$I_x = 19(2.5)^2(2) + \frac{(5)^3}{12} = 247.9 \text{ in.}^4$$

$$I_y = 2\left(\frac{(19)^3}{12} + 19(9.5-8.395)^2\right) + 5(8.395)^2 = 1542 \text{ in.}^4$$

$$J = I_x + I_y = 247.9 + 1542 = 1790 \text{ in.}^4$$

$$M = 176(2.5-1.47) = 181.3 \text{ in.-kips}$$

Stress at upper right:

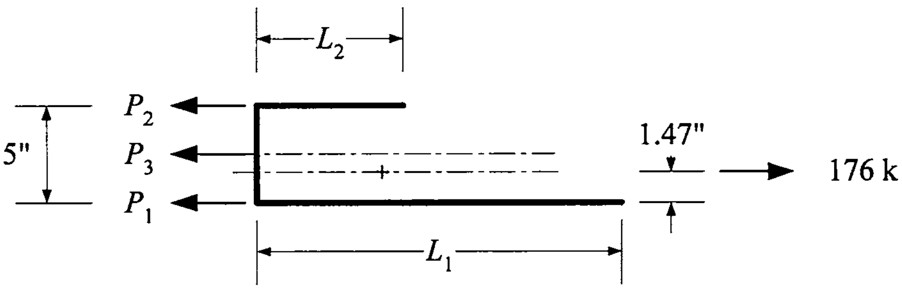
$$f_{2x} = \frac{My}{J} = \frac{181.3(2.5)}{1790} = 0.2532 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{181.3(19-8.395)}{1790} = 1.074 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(4.093+0.2532)^2 + (1.074)^2} = 4.477 \text{ kips/in.}$$

For a 3/16-inch weld (from part a), $\phi R_n = 4.176$ kips/in. < 4.477 kips/in. (N.G.)

Use $w = 3/16$ inch and balance the welds.




Option 1: $P_3 = 5(4.176) = 20.88$ kips

$\sum M_{L1} = 176(1.47) - 20.88\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 41.3$ kips

$\sum F = 176 - 20.88 - 41.3 - P_1 = 0 \quad \Rightarrow \quad P_1 = 113.8$ kips

$L_1 = \frac{P_1}{4.176} = \frac{113.8}{4.176} = 27.3$ in. Use 27½ in.

$L_2 = \frac{P_2}{4.176} = \frac{41.3}{4.176} = 9.89$ in. Use 10 in.

Option 2: The strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

$P_3 = 5(6.264) = 31.32$ kips

$\sum M_{L1} = 176(1.47) - 31.32\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 36.08$ kips

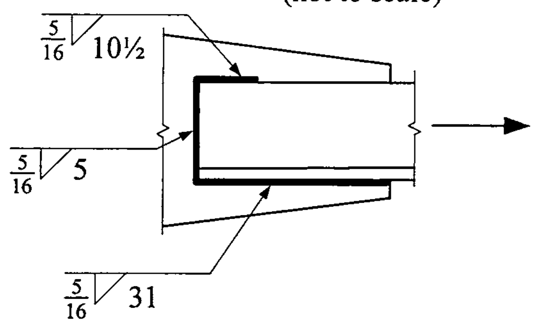
$\sum F = 176 - 31.32 - 36.08 - P_1 = 0 \quad \Rightarrow \quad P_1 = 108.6$ kips

$L_1 = \frac{P_1}{3.550} = \frac{108.6}{3.550} = 30.6$ in. Use 31 in.

$L_2 = \frac{P_2}{3.550} = \frac{36.08}{3.550} = 10.2$ in. Use 10½ in.

Option 1 results in a shorter connection. Use the connection shown

(not to scale)




Note that, if a larger weld is used, the connection will be shorter.

Although the problem statement requires that the minimum weld size be used, as was done above, an alternate solution with a larger weld will be presented.

**Alternate Solution**

(a) Try a 5/16-in. fillet weld. First, assuming the same strength for both the longitudinal and transverse welds,

$\phi R_n = 1.392D = 1.392(5) = 6.96$ kips/in. (controls over base metal shear strength)

Total length required = $\dfrac{176}{6.96} = 25.29$ in.

Length of longitudinal welds = $\dfrac{25.29-5}{2} = 10.2$ in.; use 10½ in.

For the second option, the strength of the longitudinal welds is

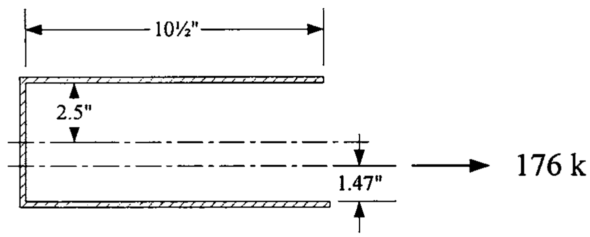
$0.85(6.96) = 5.916$ kips/in.

and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

Length of longitudinal welds = $\dfrac{176-10.44(5)}{5.916(2)} = 10.5$ in.; use 10½ in.

Use 5/16-in. E70 fillet welds, across the end and 10½ inches on each side

(b)




$$f_{1x} = \frac{176}{2(10.5)+5} = 6.769 \text{ kips/in.} \rightarrow$$

Locate centroid with respect to left side.

$$\bar{x} = \frac{10.5(5.25)\times 2}{2(10.5)+5} = 4.24 \text{ in.}$$

$$I_x = 10.5(2.5)^2(2) + \frac{(5)^3}{12} = 141.7 \text{ in.}^4$$

$$I_y = 2\left(\frac{(10.5)^3}{12} + 10.5(5.25-4.24)^2\right) + 5(4.24)^2 = 304.2 \text{ in.}^4$$

$$J = I_x + I_y = 141.7 + 304.2 = 445.9 \text{ in.}^4$$

$$M = 176(2.5-1.47) = 181.3 \text{ in.-kips}$$

Stress at upper right:

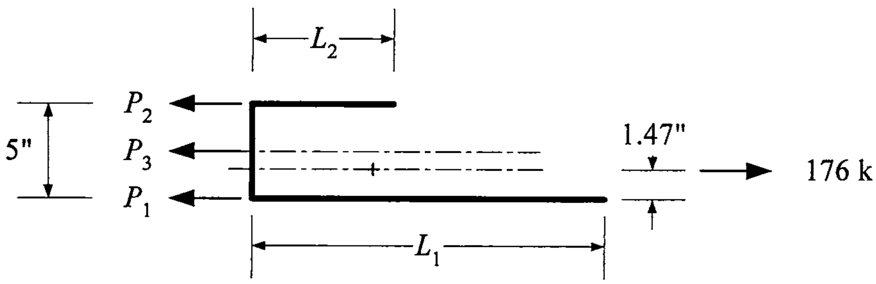
$$f_{2x} = \frac{My}{J} = \frac{181.3(2.5)}{445.9} = 1.016 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{181.3(10.5-4.24)}{445.9} = 2.545 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(6.769+1.016)^2 + (2.545)^2} = 8.19 \text{ kips/in.}$$

For a 5/16-inch weld (from part a), $\phi R_n = 6.96$ kips/in. < 8.19 kips/in. (N.G.)

Use $w = 5/16$ inch and balance the welds.




Option 1: $P_3 = 5(6.96) = 34.8$ kips

$\sum M_{L1} = 176(1.47) - 34.8\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 34.34$ kips

$\sum F = 176 - 34.8 - 34.34 - P_1 = 0 \quad \Rightarrow \quad P_1 = 106.9$ kips

$L_1 = \frac{P_1}{6.96} = \frac{106.9}{6.96} = 15.4$ in. Use 15½ in.

$L_2 = \frac{P_2}{6.96} = \frac{34.34}{6.96} = 4.93$ in. Use 5 in.

Option 2: The strength of the longitudinal welds is $0.85(6.96) = 5.916$ kips/in.

and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

$P_3 = 5(10.44) = 52.2$ kips

$\sum M_{L1} = 176(1.47) - 52.2\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 25.64$ kips

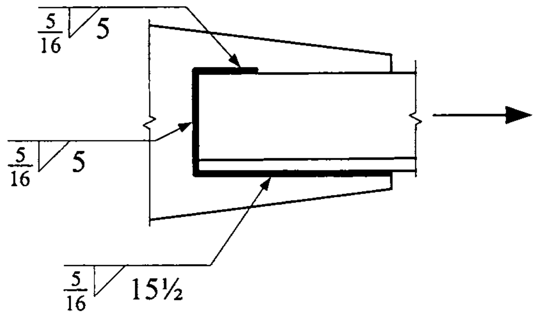
$\sum F = 176 - 52.2 - 25.64 - P_1 = 0 \quad \Rightarrow \quad P_1 = 98.16$ kips

$L_1 = \frac{P_1}{5.916} = \frac{98.16}{5.916} = 16.6$ in. Use 17 in.

$L_2 = \frac{P_2}{5.916} = \frac{25.64}{5.916} = 4.33$ in. Use 4½ in.

Option 1 results in a shorter connection. Use the connection shown

(not to scale)

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**8.4-18**

(a) $P_a = D + L = 40 + 80 = 120$ kips

For $t = 3/8$ in., min. $w = 3/16$ in. and max $w = 5/8 - 1/16 = 9/16$ in.

Use E70 fillet welds. Try $w = 3/16$ in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\frac{R_n}{\Omega} = 0.9279D = 0.9279(3) = 2.784 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $\frac{R_n}{\Omega} = 0.4F_yt = 0.4(36)(3/8) = 5.4$ kips/in.

Rupture: $\frac{R_n}{\Omega} = 0.3F_ut = 0.3(58)(3/8) = 6.525$ kips/in.

The weld shear strength controls. Total length required = $\frac{120}{2.784} = 43.1$ in.

Length of longitudinal welds = $\frac{43.1 - 5}{2} = 19.1$ in.; use 19½ in.

For the second option, the strength of the longitudinal welds is

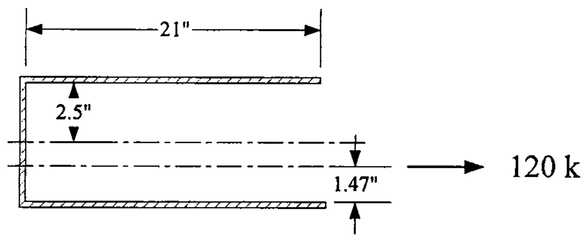
$$0.85(2.784) = 2.366 \text{ kips/in.}$$

and the strength of the transverse weld is $1.5(2.784) = 4.176$ kips/in.

Length of longitudinal welds = $\dfrac{120 - 4.176(5)}{2.366(2)} = 21.0$ in.; use 21in.

Use 3/16-in. E70 fillet welds, across the end and 21 inches on each side

(b)




$$f_{1x} = \frac{120}{2(21)+5} = 2.553 \text{ kips/in.} \rightarrow$$

Locate centroid with respect to left side.

$$\bar{x} = \frac{21(10.5)\times 2}{2(21)+5} = 9.383 \text{ in.}$$

$$I_x = 21(2.5)^2(2) + \frac{(5)^3}{12} = 272.9 \text{ in.}^4$$

$$I_y = 2\left(\frac{(21)^3}{12} + 21(10.5 - 9.383)^2\right) + 5(9.383)^2 = 2036 \text{ in.}^4$$

$$J = I_x + I_y = 272.9 + 2036 = 2309 \text{ in.}^4$$

$$M = 120(2.5 - 1.47) = 123.6 \text{ in.-kips}$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{123.6(2.5)}{2309} = 0.1338 \text{ kips/in.} \rightarrow$$

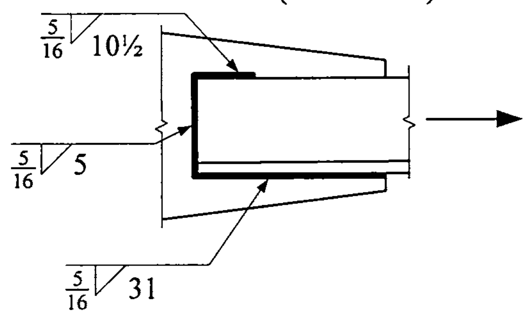
$$f_{2y} = \frac{Mx}{J} = \frac{123.6(21 - 9.383)}{2309} = 0.6219 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(2.553 + 0.1338)^2 + (0.619)^2} = 2.757 \text{ kips/in.}$$

For a 3/16-inch weld (from part a), $\dfrac{R_n}{\Omega} = 2.784$ kips/in. > 2.757 kips/in. (OK)

No revision required. Use the connection shown

(not to scale)




## 8.4-19

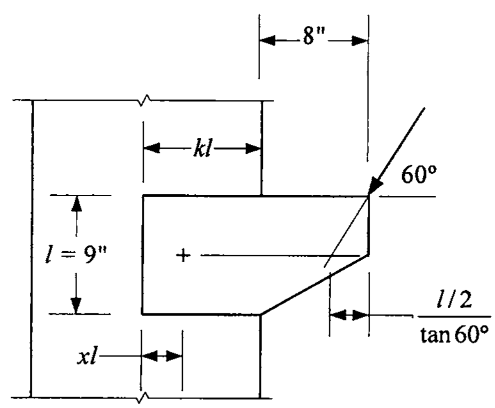
(a) $P_u = 1.2D + 1.6L = 1.2(6) + 1.6(30) = 55.2$ kips

For $t = 3/8$ in., min. $w = 3/16$ in. and max $w = 3/8 - 1/16 = 5/16$ in.

Use E70 electrodes, ultimate strength analysis, and the tables in Part 8 of the Manual.

For $D = 3$ ($w = 3/16$ in.), $C_{\text{min}} = \dfrac{P_u}{\phi C_1 D \ell} = \dfrac{55.2}{0.75(1.0)(3)(9)} = 2.73$

Try a C-shaped weld. Use Table 8-8, Angle = 30°




$$e_x = 8 - \frac{9/2}{\tan 60°} + k\ell - xl = 5.402 + k\ell - xl$$

$$= 5.402 + 9k - 9x$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 9k - 9x}{9} = 0.6002 + k - x$$

Try $k = 1.0$.

$$x = 0.333, \qquad a = 0.6002 + k - x = 0.6002 + 1.0 - 0.333 = 1.267$$

$$C = 2.77 > 2.73 \qquad \text{(OK)}$$

(This will be the shortest total length that will work for the minimum weld size.)

$$\text{Volume of weld metal} = \frac{1}{2}\left(\frac{3}{16}\right)^2(9 + 9 + 9) = 0.4746 \text{ in.}^3$$

Try an L-shaped weld. Use table 8-10, Angle = 30°.

$$e_x = 8 - \frac{y\ell}{\tan 60°} + k\ell - xl = 8.0 - 0.5774y\ell + k\ell - xl$$

$$= 8.0 - 0.5774y(9) + k(9) - x(9) = 8.0 - 5.197y + 9.0k - 9.0x$$

$$a = \frac{e_x}{\ell} = \frac{8.0 - 5.197y + 9.0k - 9.0x}{9} = 0.8889 - 0.5774y + k - x$$

Try $k = 1.0$.

$$x = 0.250, \qquad y = 0.250$$

$$a = 0.8889 - 0.5774(0.250) + 1.0 - 0.250 = 1.495$$

$$C = 1.38 < 2.73 \qquad \text{(N.G. for } D = 3\text{)}$$

For $D = 5$ (maximum weld size),

$$C_{\min} = \frac{P_u}{\phi C_1 D\ell} = \frac{55.2}{0.75(1.0)(5)(9)} = 1.64 > 1.38 \qquad \text{(N.G.)}$$

This configuration will not work.

Try two parallel welds. Use table 8-5, Angle = 30°.

$$k\ell = 9 \text{ in.}, \qquad \ell = \frac{9}{k}$$

$$e_x = 8 - \frac{9/2}{\tan 60°} + \frac{\ell}{2} = 8 - \frac{9/2}{\tan 60°} + \frac{9/k}{2} = 5.402 + \frac{4.5}{k}$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 4.5/k}{\ell} = \frac{5.402 + 4.5/k}{9} = 0.6002 + \frac{0.5}{k}$$

Try $D = 3$ ($C_{\min} = 2.73$) and $k = 1.0$ :

$a = 0.6002 + \frac{0.5}{1.0} = 1.1$

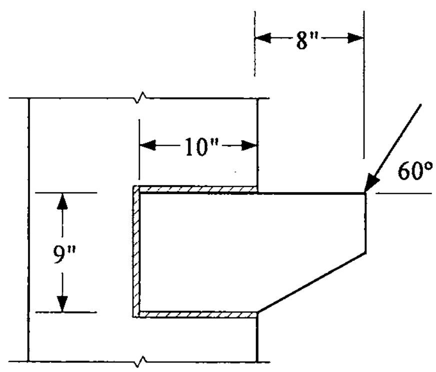
$C = 2.05 < 2.73$ (N.G.)

Try $D = 4$, $C_{\min} = \frac{P_u}{\phi C_1 D \ell} = \frac{55.2}{0.75(1.0)(4)(9)} = 2.04 < 2.05$ (OK)

Volume of weld metal $= \frac{1}{2}\left(\frac{1}{4}\right)^2 (9+9) = 0.5625 \text{ in.}^3$

This is larger than that required for the C-shaped weld with $w = \frac{3}{16}$ inch.

Use a $\frac{3}{16}$-in. fillet weld as shown.




(b) This weld uses the smallest volume of weld that will work for the available length.

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**8.4-20**

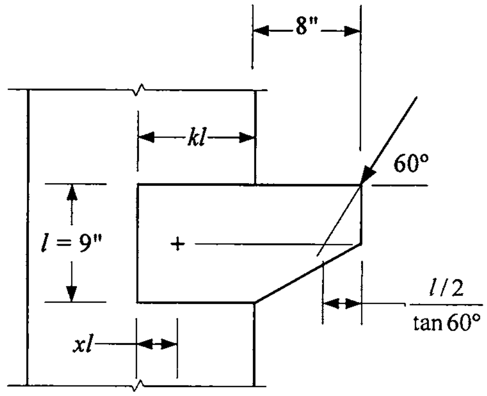
(a) $P_a = D + L = 6 + 30 = 36$ kips

For $t = 3/8$ in., min. $w = 3/16$ in. and max $w = 3/8 - 1/16 = 5/16$ in.

Use E70 electrodes, ultimate strength analysis, and the tables in Part 8 of the Manual.

For $D = 3$ ($w = 3/16$ in.), $C_{\min} = \frac{\Omega P_a}{C_1 D \ell} = \frac{2.00(36)}{1.0(3)(9)} = 2.67$

Try a C-shaped weld. Use Table 8-8, Angle = 30°




$$e_x = 8 - \frac{9/2}{\tan 60°} + k\ell - xl = 5.402 + k\ell - xl$$

$$= 5.402 + 9k - 9x$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 9k - 9x}{9} = 0.6002 + k - x$$

Try $k = 1.0$.

$$x = 0.333, \qquad a = 0.6002 + k - x = 0.6002 + 1.0 - 0.333 = 1.267$$

$$C = 2.77 > 2.67 \qquad \text{(OK)}$$

(This will be the shortest total length that will work for the minimum weld size.)

$$\text{Volume of weld metal} = \frac{1}{2}\left(\frac{3}{16}\right)^2 (9 + 9 + 9) = 0.4746 \text{ in.}^3$$

Try an L-shaped weld. Use table 8-10, Angle = 30°.

$$e_x = 8 - \frac{y\ell}{\tan 60°} + k\ell - xl = 8.0 - 0.5774y\ell + k\ell - xl$$

$$= 8.0 - 0.5774y(9) + k(9) - x(9) = 8.0 - 5.197y + 9.0k - 9.0x$$

$$a = \frac{e_x}{\ell} = \frac{8.0 - 5.197y + 9.0k - 9.0x}{9} = 0.8889 - 0.5774y + k - x$$

Try $k = 1.0$.

$$x = 0.250, \qquad y = 0.250$$

$$a = 0.8889 - 0.5774(0.250) + 1.0 - 0.250 = 1.50$$

$C = 1.45 < 2.73$ (N.G. for $D = 3$)

For $D = 5$ (maximum weld size),

$$C_{\min} = \frac{\Omega P_a}{C_1 D\ell} = \frac{2.00(36)}{1.0(5)(9)} = 1.6 > 1.45 \quad \text{(N.G.)}$$

This configuration will not work.

Try two parallel welds. Use table 8-5, Angle = 30°.

$$k\ell = 9 \text{ in.}, \quad \ell = \frac{9}{k}$$

$$e_x = 8 - \frac{9/2}{\tan 60°} + \frac{\ell}{2} = 8 - \frac{9/2}{\tan 60°} + \frac{9/k}{2} = 5.402 + \frac{4.5}{k}$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 4.5/k}{\ell} = \frac{5.402 + 4.5/k}{9} = 0.6002 + \frac{0.5}{k}$$

Try $D = 3$ ($C_{\min} = 2.67$) and $k = 1.0$ :

$$a = 0.6002 + \frac{0.5}{1.0} = 1.1$$

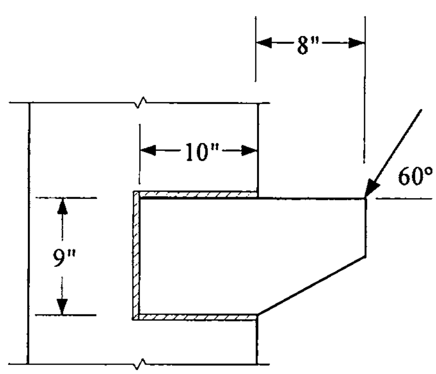
$$C = 2.05 < 2.73 \quad \text{(N.G.)}$$

Try $D = 4$, $C_{\min} = \frac{\Omega P_a}{C_1 D\ell} = \frac{2.00(36)}{1.0(4)(9)} = 2.0 < 2.05$ (OK)

Volume of weld metal $= \frac{1}{2}\left(\frac{1}{4}\right)^2(9 + 9) = 0.5625 \text{ in.}^3$

This is larger than that required for the C-shaped weld with $w = \frac{3}{16}$ inch.

Use a $\frac{3}{16}$-in. fillet weld as shown.



(b) This weld uses the smallest volume of weld that will work for the available length.

---

**8.5-1**

Direct shear: $f_v = \dfrac{20}{2(2+10)} = 0.8333$ kips/in.

Tension: Locate neutral axis with respect to top.

$$\bar{y} = \frac{2(10)(5)}{24} = 4.167 \text{ in.,} \qquad M = 20(4) = 80 \text{ in.-kips}$$

$$I = 2\left[\frac{(10)^3}{12} + 10(5-4.167)^2 + 2(4.167)^2\right] = 250.0 \text{ in.}^4$$

$$f_t = \frac{Mc}{I} = \frac{80(4.167)}{250} = 1.333 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.8333)^2 + (1.333)^2} = 1.57 \text{ kips/in.} \qquad \underline{f_r = 1.57 \text{ kips/in.}}$$

---

**8.5-2**

Direct shear: $f_v = \dfrac{20}{8+6(2)} = 1.0$ kips/in.

Tension: Locate neutral axis with respect to bottom.

$$\bar{y} = \frac{8(6) + 2(5)(2) + 2(1)(2)}{20} = 3.6 \text{ in.,} \qquad M = 20(3) = 60 \text{ in.-kips}$$

$$I = 8(6-3.6)^2 + 2\left[\frac{(2)^3}{12} + 2(5-3.6)^2\right] + 2\left[\frac{(2)^3}{12} + 2(2.6)^2 + 2(3.6)^2\right]$$

$$= 135.5 \text{ in.}^4$$

$$f_t = \frac{Mc}{I} = \frac{60(6-3.6)}{135.5} = 1.063 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(1.0)^2 + (1.063)^2} = 1.46 \text{ kips/in.} \qquad \underline{f_r = 1.46 \text{ kips/in.}}$$

---

**8.5-3**

Maximum weld size is $w = 5/8 - 1/16 = 9/16$ in

(a) LRFD solution

Weld strength: $\phi R_n = 1.392D = 1.392(9) = 12.53$ kips/in.

Base metal shear strength of the W14 × 48:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(50)(0.595) = 17.85$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(65)(0.595) = 17.40$ kips/in.

Base metal shear strength of the L6 × 6 × 5/8:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(5/8) = 13.50$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(5/8) = 16.31$ kips/in.

The weld shear strength controls; $\phi R_n = 12.53$ kips/in.

Direct shear: $f_v = \frac{R_u}{2(6)} = \frac{1}{12}R_u$ kips/in

Tension: $M = 4R_u$

$$I = \frac{(6)^3}{12} \times 2 = 36.0 \text{ in.}^4, \qquad f_t = \frac{Mc}{I} = \frac{4R_u(3)}{36} = \frac{1}{3}R_u \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{\left(\frac{R_u}{12}\right)^2 + \left(\frac{R_u}{3}\right)^2} = 0.3436R_u \text{ kips/in.}$$

From $0.3436R_u = 12.53$, $\underline{R_u = 36.5 \text{ kips}}$

(b) ASD solution

Weld strength: $\frac{R_n}{\Omega} = 0.9279D = 0.9279(9) = 8.351$ kips/in.

Base metal shear strength of the W14 × 48:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(50)(0.595)}{1.50} = 11.90$ kips/in

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(65)(0.595)}{2.00} = 11.60$ kips/in.

Base metal shear strength of the L6 × 6 × 5/8:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(36)(5/8)}{1.50} = 9.0$ kips/in

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(58)(5/8)}{2.00} = 10.88$ kips/in.

The weld shear strength controls; $\frac{R_n}{\Omega} = 8.351$ kips/in.

Direct shear: $f_v = \frac{R_a}{2(6)} = \frac{1}{12}R_a$ kips/in

Tension: $M = 4R_a$

$I = \frac{(6)^3}{12} \times 2 = 36.0 \text{ in.}^4, \quad f_t = \frac{Mc}{I} = \frac{4R_a(3)}{36} = \frac{1}{3}R_a$ kips/in.

$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{\left(\frac{R_a}{12}\right)^2 + \left(\frac{R_a}{3}\right)^2} = 0.3436R_a$ kips/in.

From $0.3436R_a = 8.351$, $\underline{R_a = 24.3 \text{ kips}}$

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**8.5-4**

(a) $P_u = 1.2D + 1.6L = 1.2(9) + 1.6(15) = 34.8$ kips

$P_{ux} = 34.8\cos 70° = 11.9$ kips, $\quad P_{uy} = 34.8\sin 70° = 32.7$ kips

Direct shear: $f_v = \frac{32.7}{2(12)} = 1.363$ kips/in.

Tension: $f_{t1} = \frac{11.9}{24} = 0.4958$ kips/in

$M = 11.9(6) + 32.7(6) = 267.6$ in.-kips, $\quad I = \frac{(12)^3}{12}(2) = 288.0 \text{ in.}^4$

$f_{t2} = \frac{Mc}{I} = \frac{267.6(6)}{288} = 5.575$ kips/in.

$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(1.363)^2 + (0.4958 + 5.575)^2} = 6.222$ kips/in.

$D = \frac{6.222}{1.392} = 4.47$ sixteenths. Try $w = \frac{5}{16}$ in.

Check base metal shear (bracket plate controls) $\quad f_v = \dfrac{32.7}{12} = 2.73$ kips/in.

Yielding: $\phi R_n = 0.6F_y t = 0.6(36)(3/8) = 8.1$ kips/in. > 2.73 kips/in. (OK)

Rupture: $\phi R_n = 0.45F_u t = 0.45(58)(3/8)$

$= 9.79$ kips/in. > 2.73 kips/in. (OK)

<u>Use a $\frac{5}{16}$-in. fillet weld.</u>

(b) $P_a = D + L = 9 + 15 = 24$ kips

$P_{ax} = 24\cos 70° = 8.208$ kips, $\quad P_{ay} = 24\sin 70° = 22.55$ kips

Direct shear: $\quad f_v = \dfrac{22.55}{2(12)} = 0.9396$ kips/in.

Tension: $\quad f_{t1} = \dfrac{8.208}{24} = 0.342$ kips/in

$M = 8.208(6) + 22.55(6) = 184.5$ in.-kips, $\quad I = \dfrac{(12)^3}{12}(2) = 288.0$ in.$^4$

$f_{t2} = \dfrac{Mc}{I} = \dfrac{184.5(6)}{288} = 3.844$ kips/in.

$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.9396)^2 + (0.342 + 3.844)^2} = 4.29$ kips/in.

$D = \dfrac{4.29}{0.9279} = 4.62$ sixteenths. Try $w = \frac{5}{16}$ in.

Check base metal shear (bracket plate controls)

$f_v = \dfrac{22.55}{12} = 2.73$ kips/in.

Yielding: $\dfrac{R_n}{\Omega} = \dfrac{0.6F_y t}{1.50} = \dfrac{0.6(36)(3/8)}{1.50}$

$= 5.4$ kips/in > 2.73 kips/in. (OK)

Rupture: $\dfrac{R_n}{\Omega} = \dfrac{0.6F_u t}{2.00} = \dfrac{0.6(58)(3/8)}{2.00}$

$= 6.53$ kips/in. > 2.73 kips/in. (OK)

<u>Use a $\frac{5}{16}$-in. fillet weld.</u>

**8.5-5**

(a) LRFD solution

Weld strength: $\phi R_n = 1.392D = 1.392(5) = 6.96$ kips/in.

Base metal shear strength (flange of WT controls):

Yielding: $\phi R_n = 0.6F_y t = 0.6(50)(0.855) = 25.65$ kips/in.

Rupture: $\phi R_n = 0.45F_u t = 0.45(65)(0.855) = 25.01$ kips/in.

The weld shear strength controls; $\phi R_n = 6.96$ kips/in.

Direct shear: $f_v = \dfrac{P_u}{2(16)} = 0.03125P_u$ kips/in

Tension: $M = 8P_u$

$$I = \frac{(16)^3}{12} \times 2 = 682.7 \text{ in.}^4,$$

$$f_t = \frac{Mc}{I} = \frac{8P_u(8)}{682.7} = 9.375 \times 10^{-2}P_u \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.03125P_u)^2 + (0.09375P_u)^2} = 9.882 \times 10^{-2}P_u \text{ kips/in.}$$

From $0.09882P_u = 6.96$, $\underline{P_u = 70.4 \text{ kips}}$

(b) ASD solution

Weld strength: $\dfrac{R_n}{\Omega} = 0.9279D = 0.9279(5) = 4.640$ kips/in.

Base metal shear strength (flange of WT controls):

Yielding: $\dfrac{R_n}{\Omega} = 0.4F_y t = 0.4(50)(0.855) = 17.1$ kips/in

Rupture: $\dfrac{R_n}{\Omega} = 0.3F_u t = 0.3(65)(0.855) = 16.67$ kips/in.

The weld shear strength controls; $\dfrac{R_n}{\Omega} = 4.640$ kips/in.

Direct shear: $f_v = \dfrac{P_a}{2(16)} = 0.03125P_a$ kips/in.

Tension: $M = 8P_a$

$$I = \frac{(16)^3}{12} \times 2 = 682.7 \text{ in.}^4$$

$$f_t = \frac{Mc}{I} = \frac{8P_a(8)}{682.7} = 9.375 \times 10^{-2} P_a \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.03125P_a)^2 + (0.09375P_a)^2} = 9.882 \times 10^{-2} P_a \text{ kips/in.}$$

From $0.09882P_a = 4.640$, $\underline{P_a = 47.0 \text{ kips}}$

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**8.6-1**

Nominal bolt shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

For Group A-N bolts,

$$R_n = F_{nv}A_b = 54(0.6013) = 32.47 \text{ kips/bolt}$$

(a) LRFD solution

$$\phi R_n = 0.75(32.47) = 24.35 \text{ kips/bolt}$$

For 8 bolts, $\phi R_n = 8(24.35) = 195$ kips

$$M_u = 1.2M_D + 1.6M_L = 1.2(45) + 1.6(135) = 270.0 \text{ ft-kips}$$

$$H = \frac{M_u}{d} = \frac{270(12)}{18.0} = 180 \text{ kips} < 195 \text{ kips} \quad \text{(OK)}$$

<u>Bolts have enough shear strength.</u>

(b) ASD solution

$$\frac{R_n}{\Omega} = \frac{32.47}{2.00} = 16.24 \text{ kips/bolt}$$

For 8 bolts, $\frac{R_n}{\Omega} = 8(16.24) = 130$ kips

$$H = \frac{M_a}{d} = \frac{180(12)}{18.0} = 120 \text{ kips} < 130 \text{ kips} \quad \text{(OK)}$$

<u>Bolts have enough shear strength.</u>

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**8.6-2**

(a) Web plate:

Check bolt shear. $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013$ in.$^2$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.6013) = 24.35$ kips/bolt

For 4 bolts, $4(24.35) = 97.4$ kips

Check bearing on plate (plate is thinner than beam web, and $F_u$ is smaller):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge, $\ell_e = 2.25$ in.

$$\ell_c = \ell_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.781)(5/16)(58) = 29.05 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(5/16)(58) = 28.55 \text{ kips} < 29.05 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 28.55 \text{ kips/bolt}$$

For other bolts, $s = 3$ in.

$$\ell_c = s - h = 3 - \frac{15}{16} = 2.062 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.062)(5/16)(58) = 33.64 \text{ kips}$$

$$\phi(2.4dtF_u) = 28.55 \text{ kips} < 33.64 \text{ kips} \;\therefore \text{ use } \phi R_n = 28.55 \text{ kips/bolt}$$

Therefore, shear controls for the bolts.

Check plate shear yielding strength:

$$\phi R_n = 1.00(0.60F_y A_g) = 1.00(0.6)(36)(13.5 \times 5/16) = 91.1 \text{ kips}$$

Check shear rupture strength. Use a hole diameter of $\frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{nv} = (13.5 - 4 \times 1.0)\left(\frac{5}{16}\right) = 2.969 \text{ in.}^2$$

$$\phi R_n = \phi(0.60F_u A_{nv}) = 0.75(0.60)(58)(2.969) = 77.5 \text{ kips}$$

Check block shear. The shear areas are

$$A_{gv} = \frac{5}{16}(11.25) = 3.516 \text{ in.}^2$$

$$A_{nv} = \frac{5}{16}\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 2.422 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{5}{16}\left[2.25 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.5469 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(2.422) + 1.0(58)(0.5469) = 116.0 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.516) + 1.0(58)(0.5469) = 107.7 \text{ kips}$$

The nominal block shear strength is therefore 107.7 kips. The design block shear strength is

$$\phi R_n = 0.75(107.7) = 80.8 \text{ kips}$$

Check connection of shear plate to column flange.

Weld strength = 1.392 × 4 sixteenths = 5.568 kips/in.

For two welds, use 2(5.568) = 11.14 kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n = 0.6F_yt = 0.6(36)(5/16) = 6.75$ kips/in.

Rupture: $\phi R_n = 0.45F_ut = 0.45(58)(5/16) = 8.156$ kips/in.

Plate shear yielding controls: Use $\phi R_n = 6.75$ kips/in. × 13.5 in. = 91.1 kips

Summary:

Shear/bearing strength = 86.6 kips
Plate shear yielding strength = 91.1 kips
Plate shear rupture strength = 77.5 kips
Plate block shear strength = 80.8 kips
Weld (base metal shear) strength = 91.1 kips.

Shear rupture controls: Shear design strength = 77.5 kips

(b) Top flange plate.

Tension on the gross section: $\phi_t P_n = 0.90(36)(5/8 \times 7) = 142$ kips

Tension on the net section: $A_n = (7 - 2 \times 1.0)(5/8) = 3.125 \text{ in.}^2$

$\phi_t P_n = 0.75(58)(3.125) = 135.9$ kips

Check bolt shear. From part (a), $\phi R_n = 24.35$ kips/bolt

For 8 bolts, $8(24.35) = 195$ kips

Check bearing.

For the hole nearest the edge, $\ell_e = 2.25$ in.

$$\ell_c = \ell_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.781)(5/8)(58) = 58.11$ kips

$\phi(2.4 d t F_u) = 0.75(2.4)(7/8)(5/8)(58) = 57.09 \text{ kips} < 58.11$ kips

$\therefore$ use $\phi R_n = 57.09$ kips/bolt

For other bolts, $s = 3$ in.

$$\ell_c = s - h = 3 - \frac{15}{16} = 2.062 \text{ in.}$$

$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(2.062)(5/8)(58) = 67.27$ kips

$\phi(2.4 d t F_u) = 57.09 \text{ kips} < 67.27$ kips $\therefore$ use $\phi R_n = 57.09$ kips/bolt

Therefore, the bolt strength is controlled by shear.

Check block shear on the plate.

Shear areas: $A_{gv} = \frac{5}{8}(11.25) \times 2 = 14.06 \text{ in.}^2$

$A_{nv} = \frac{5}{8}[11.25 - 3.5(1.0)] \times 2 = 9.688 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = \frac{5}{8}[2.5 - 1(1.0)] = 0.9375 \text{ in.}^2$ (using 0.5+0.5 hole diameters)

$$R_n = 0.6 F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(9.688) + 1.0(58)(0.9375) = 391.5 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(14.06) + 1.0(58)(0.9375) = 358.1 \text{ kips}$$

The nominal block shear strength is therefore 358.1 kips. The design block shear strength is

$$\phi R_n = 0.75(358.1) = 269 \text{ kips}$$

Check block shear on the beam flange:

Shear areas: $A_{gv} = 0.565(11.25) \times 2 = 12.71 \text{ in.}^2$

$A_{nv} = 0.565[11.25 - 3.5(1.0)] \times 2 = 8.758 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = 0.565[2.25 - 0.5(1.0)](2) = 1.978 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(65)(8.758) + 1.0(65)(1.978) = 470.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(12.71) + 1.0(65)(1.978) = 509.9 \text{ kips}$$

The nominal block shear strength is therefore 470.1 kips. The design block shear strength is

$$\phi R_n = 0.75(470.1) = 353 \text{ kips}$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(2.25 + 0.5) = 1.788 \text{ in.}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.}, \qquad \frac{KL}{r} = \frac{1.788}{0.1804} = 9.911$$

From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_yA_g$$

$$\therefore \ \phi_cP_n = \phi_cF_yA_g = 0.90(36)(7 \times 5/8) = 142 \text{ kips}$$

Summary:

Tension on gross section: 142 kips
Tension on net section: 135.9 kips
Bolt shear/bearing: 195 kips
Block shear on flange plate: 269 kips
Block shear on beam flange: 353 kips
Compression on bottom flange plate: 142 kips

Tension on net section controls. Use $H = 135.9$ kips and a moment arm equal to the beam depth (this is conservative).

$$\phi M_n = Hd = 135.9(16.1) = 2188 \text{ in.-kips} = 182 \text{ ft-kips}$$

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.565(7.04) = 3.978 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 3.978 - 0.565(2 \times 1) = 2.848 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.848) = 185.1 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(3.978) = 218.8 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{185.1}{3.978}(72.7) = 3383 \text{ in.-kips} = 281.9 \text{ ft-kips}$$

$\phi_b M_n = 0.90(281.9) = 254 \text{ ft-kips} > 182 \text{ ft-kips}$ $\qquad$ $\underline{\phi_b M_n = 182 \text{ ft-kips}}$

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**8.6-3**

(a) Web plate: The nominal shear strength of one bolt is

$$R_n = F_{nv} A_b = 54(0.6013) = 32.47 \text{ kips}$$

and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{32.47}{2.00} = 16.24 \text{ kips}$$

For 4 bolts, $R_n/\Omega = 4(16.24) = 64.96$ kips

Check bearing on plate (plate is thinner than beam web, and $F_u$ is smaller):

$$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in. For the hole nearest the edge,}$$

$$\ell_c = \ell_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c tF_u}{\Omega} = \frac{1.2(1.781)(5/16)(58)}{2.00} = 19.37 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(5/16)(58)}{2.00} = 19.03 \text{ kips} < 19.37 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.03 \text{ kips/bolt}$$

For the other bolts, $s = 3$ in.

$$\ell_c = s - h = 3 - 15/16 = 2.063 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c tF_u}{\Omega} = \frac{1.2(2.063)(5/16)(58)}{2.00} = 22.44 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = 19.03 \text{ kips} < 22.44 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.03 \text{ kips/bolt}$$

For the connection, shear controls over bearing.

Shear yielding: from AISC Equation J4-3,

$$R_n = 0.60F_yA_g = 0.60(36)\left(\frac{5}{16} \times 13.5\right) = 91.13 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{91.13}{1.50} = 60.8 \text{ kips}$$

For shear rupture, the net area is

$$A_{nv} = \left[13.5 - 4\left(\frac{7}{8} + \frac{1}{8}\right)\right](5/16) = 2.969 \text{ in.}^2$$

From AISC Equation J4-4,

$$R_n = 0.6F_uA_{nv} = 0.6(58)(2.969) = 103.3 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{103.3}{2.00} = 51.7 \text{ kips}$$

Connection to column flange:

Shear strength of weld = $0.9279D = 0.9279(4) = 3.712$ kips/in. For two welds, use

$$2(3.712) = 7.424 \text{ kips/in.}$$

Base metal shear strength:

yield strength: $\frac{R_n}{\Omega} = 0.4F_y t = 0.4(36)\left(\frac{5}{16}\right) = 4.5$ kips/in.

rupture strength: $\frac{R_n}{\Omega} = 0.3F_u t = 0.3(58)\left(\frac{5}{16}\right) = 5.438$ kips/in.

Base metal shear yield strength controls. Let

$$\frac{V_a}{13.5} = 4.5, \qquad V_a = 60.8 \text{ kips}$$

Check block shear. The shear areas are

$$A_{gv} = \frac{5}{16}(11.25) = 3.516 \text{ in.}^2$$

$$A_{nv} = \frac{5}{16}\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 2.422 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{5}{16}\left[2.25 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.5469 \text{ in.}^2$

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(2.422) + 1.0(58)(0.5469) = 116.0 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(3.516) + 1.0(58)(0.5469) = 107.7 \text{ kips}$$

The nominal block shear strength is therefore 107.7 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{107.7}{2.00} = 53.9 \text{ kips}$$

Summary:

Bolt shear strength = 65.0 kips
Plate shear yielding strength = 60.8 kips
Plate shear rupture strength = 51.7 kips
Weld (base metal) strength = 60.8 kips.
Plate block shear strength = 53.9 kips

Shear rupture controls: <u>Available shear strength = $V_a$ = 51.7 kips</u>

(b) Flange plate:

Bolt shear strength from part (a) is

$\frac{R_n}{\Omega} = 16.24$ kips. For 8 bolts, $R_n/\Omega = 8(16.24) = 129.9$ kips

Check bearing. Bearing strength is proportional to $t$ and $F_u$. For the plate, $tF_u = (5/8)(58) = 36.25$ kips. For the W16 × 45, $t_f F_u = 0.565(65) = 36.73$ kips. The plate bearing strength controls.

$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in. For the hole nearest the edge,

$$\ell_c = \ell_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c t F_u}{\Omega} = \frac{1.2(1.781)(5/8)(58)}{2.00} = 38.74 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(5/8)(58)}{2.00} = 38.06 \text{ kips} < 38.74 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.06 \text{ kips/bolt}$$

For the other bolts, $s = 3$ in.

$$\ell_c = s - h = 3 - 15/16 = 2.063 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2\ell_c t F_u}{\Omega} = \frac{1.2(2.063)(5/8)(58)}{2.00} = 44.87 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = 38.06 \text{ kips} < 44.87 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.06 \text{ kips/bolt}$$

For the connection, shear controls over bearing.

Tension on the gross section: $\frac{R_n}{\Omega} = \frac{F_y A_g}{\Omega} = \frac{36\left(\frac{5}{8} \times 7\right)}{1.67} = 94.31$ kips

Tension on the net section: $A_n = \frac{5}{8}(7 - 2 \times 1.0) = 3.125 \text{ in.}^2$

$$\frac{R_n}{\Omega} = \frac{F_u A_n}{\Omega} = \frac{58(3.125)}{2.00} = 90.63 \text{ kips}$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(2.25 + 0.5) = 1.788 \text{ in.}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.,} \qquad \frac{KL}{r} = \frac{1.788}{0.1804} = 9.911$$

From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_y A_g$$

$$\frac{P_n}{\Omega} = \frac{F_y A_g}{1.67} = \frac{36\left(\frac{5}{8} \times 7\right)}{1.67} = 94.31 \text{ kips}$$

Check block shear in the plate. The shear areas are

$$A_{gv} = \frac{5}{8}(11.25) \times 2 = 14.06 \text{ in.}^2$$

$$A_{nv} = \frac{5}{8}\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 9.688 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{5}{8}\left[2.5 - 1.0\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.9375 \text{ in.}^2$

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(9.688) + 1.0(58)(0.9375) = 391.5 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(14.06) + 1.0(58)(0.9375) = 358.1 \text{ kips}$$

The nominal block shear strength is therefore 358.1 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{358.1}{2.00} = 179.1 \text{ kips}$$

Check block shear in the beam flange. The shear areas are

$$A_{gv} = 0.565(11.25) \times 2 = 12.71 \text{ in.}^2$$

$$A_{nv} = 0.565\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 8.758 \text{ in.}^2$$

The tension area is $A_{nt} = 0.565\left[2.25 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 1.978 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(65)(8.758) + 1.0(65)(1.978) = 470.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(12.71) + 1.0(65)(1.978) = 509.9 \text{ kips}$$

The nominal block shear strength is therefore 470.1 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{470.1}{2.00} = 235.1 \text{ kips}$$

Summary:

Bolt shear: 129.9 kips
Tension on gross section: 94.31 kips
Tension on net section: 90.63 kips
Compression on bottom flange plate: 94.31 kips
Block shear in flange plate: 179.1 kips
Block shear in beam flange: 235.1 kips

Tension on the net section controls. Use $H = 90.63$ kips and a moment arm equal to the beam depth (this is conservative).

$$M_a = Hd = 90.63(16.1) = 1459 \text{ in.-kips} = 122 \text{ ft-kips}$$

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_fb_f = 0.565(7.04) = 3.978 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 3.978 - 0.565(2 \times 1) = 2.848 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.848) = 185.1 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(3.978) = 218.8 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{185.1}{3.978}(72.7) = 3383 \text{ in.-kips} = 281.9 \text{ ft-kips}$$

$$\frac{M_n}{\Omega} = \frac{281.9}{1.67} = 169 \text{ ft-kips} > 122 \text{ ft-kips} \quad \therefore \text{ tension on the net section controls}$$

$$\underline{M_a = 122 \text{ ft-kips}}$$

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**8.6-4**

LRFD solution

Reaction: $R_u = 1.2D + 1.6L = 1.2(8) + 1.6(21) = 43.2$ kips

Moment: $M_u = 1.2D + 1.6L = 1.2(42) + 1.6(104) = 216.8$ ft-kips

Web plate: neglect eccentricity.

Try $\frac{5}{8}$-in. diameter bolts. Assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(5/8)^2/4 = 0.3068 \text{ in.}^2$$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.3068) = 12.43$ kips/bolt

Number of bolts required is

$$\frac{43.2}{12.43} = 3.48 \text{ try 4 bolts}$$

Determine plate thickness required for bearing. Assume that

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(5/8)t(58) = 65.25t$$

Load resisted by each bolt $= \frac{43.2}{4} = 10.8$ kips.

Let $\phi R_n = 10.8 = 65.25t \quad \Rightarrow \quad t = 0.166$ in. Try $t = \frac{3}{16}$ in.

Determine whether plate or beam web controls bearing. For the plate,

$$tF_u = \frac{3}{16}(58) = 10.9 \text{ kips/in.}$$

For the beam web, $t_w F_u = 0.300(65) = 19.5$ kips/in. > 10.9 kips/in. ∴ plate controls.

Check bearing strength assumption. $h = \frac{5}{8} + \frac{1}{16} = \frac{11}{16}$ in.

For the hole nearest the edge, min. $\ell_e = \frac{7}{8}$ in. Try $1\frac{1}{2}$ in.

$$\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{11/16}{2} = 1.156 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.156)(3/16)(58) = 11.31 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(5/8)(3/16)(58) = 12.23 \text{ kips} > 11.31 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 11.31 \text{ kips/bolt}$$

For other bolts, min. $s = 2.667(5/8) = 1.667$ in. Use $s = 2\frac{1}{2}$ in.

$$\ell_c = s - h = 2.5 - \frac{11}{16} = 1.813 \text{ in.}$$

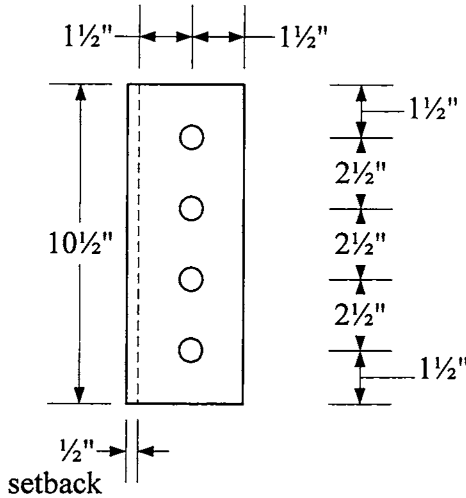
$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.813)(3/16)(58) = 17.74 \text{ kips}$$

$$\phi(2.4dtF_u) = 12.23 \text{ kips} < 17.74 \text{ kips} \quad \therefore \text{ use } \phi R_n = 12.23 \text{ kips/bolt}$$

Bearing controls over shear at each bolt location. Total strength is

$$11.31 + 3(12.23) = 48.0 \text{ kips} > 43.2 \text{ kips} \qquad \text{(OK)}$$

Use four $\frac{5}{8}$-in. diameter Group A bolts .




Determine plate thickness required for shear. Shear yielding strength is

$$\phi R_n = 1.00(0.60F_yA_{gv})$$

Let $43.2 = 1.00(0.60)(36)(10.5t) \quad \Rightarrow \quad t = 0.191$ in. Try $t = \frac{1}{4}$ in.

Check shear rupture strength. Use hole diameter $= \frac{5}{8} + \frac{1}{8} = \frac{3}{4}$ in.

$$A_{nv} = (10.5 - 4 \times 3/4)\left(\frac{1}{4}\right) = 1.875 \text{ in.}^2$$

$$\phi R_n = \phi(0.60F_uA_{nv}) = 0.75(0.60)(58)(1.875)$$

$$= 48.9 \text{ kips} > 43.2 \text{ kips} \qquad \text{(OK)}$$

Check block shear.

Shear areas: $A_{gv} = \frac{1}{4}(1.5 + 3 \times 2.5) = \frac{1}{4}(9) = 2.25 \text{ in.}^2$

$$A_{nv} = \frac{1}{4}[9 - 3.5(3/4)] = 1.594 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(3/4)] = 0.2813 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(1.594) + 1.0(58)(0.2813) = 71.79 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.25) + 1.0(58)(0.2813) = 64.92 \text{ kips}$$

The nominal block shear strength is therefore 64.92 kips. The design block shear strength is

$$\phi R_n = 0.75(64.92) = 48.7 \text{ kips} > 43.2 \text{ kips} \qquad \text{(OK)}$$

<u>Use a PL $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$ as shown above.</u>

Connection of shear plate to column flange: Use E70 electrodes

Minimum weld size, based on the plate thickness, is $\frac{1}{8}$ inch. Try $w = \frac{1}{8}$ in.

Weld strength = $1.392 \times 2 = 2.784$ kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n = 0.6F_yt = 0.6(36)(1/4) = 5.4$ kips/in. > 2.784 kips/in.

Rupture: $\phi R_n = 0.45F_ut = 0.45(58)(1/4) = 6.525$ kips/in. > 2.784 kips/in.

Total length required = $43.2/2.784 = 15.5$ in.

<u>Use a continuous $\frac{1}{8}$ in. fillet weld, both sides of plate.</u>

Flange plate: From $M = Hd$,

$$H = \frac{M}{d} = \frac{216.8(12)}{17.7} = 147.0 \text{ kips}$$

Try $\frac{7}{8}$ in. diameter bolts. Assume that threads are in the plane of shear.

$$A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2, \qquad \phi R_n = 0.75(54)(0.6013) = 24.35 \text{ kips/bolt}$$

Number of bolts required for shear is

$$\frac{147.0}{24.35} = 6.04 \text{ try 8 bolts (4 pair)}$$

Determine plate thickness required for bearing: $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in

Minimum $\ell_e = 1\frac{1}{8}$ inches. Use $1\frac{1}{2}$ inches. Minimum $s = 2.667(7/8) = 2.33$ in., use $2\frac{1}{2}$ in.

For the hole nearest the edge, $\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031$ in.

$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.031)t(58) = 53.82t$ kips

$\phi(2.4dtF_u) = 0.75(2.4)(7/8)t(58) = 91.35t$ kips > $53.82t$ kips

$\therefore$ use $\phi R_n = 53.82t$ kips/bolt

For other bolts, $\ell_c = s - h = 2.5 - \frac{15}{16} = 1.563$ in.

$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(1.563)t(58) = 81.59t$ kips

$\phi(2.4dtF_u) = 91.35t$ kips > $81.59t$ kips $\therefore$ use $\phi R_n = 81.59t$ kips/bolt

Total connection strength = $2(24.35t) + 6(24.35t) = 170.5t$ (shear controls)

Let $543.4t = 170.5$, Solution is: $\{t = 0.3138\}$ in.

Design top flange plate as a tension connection element (AISC J4.2).

Tension on gross area:

$$\text{Required } A_g = \frac{H}{0.90F_y} = \frac{147.0}{0.9(36)} = 4.537 \text{ in.}^2$$

Tension on net area:

$$\text{Required } A_e = \frac{H}{0.75F_u} = \frac{147.0}{0.75(58)} = 3.379 \text{ in.}^2$$

Try a plate width of $w_g = 7$ in.

For gross area requirement, $t = \frac{A_g}{w_g} = \frac{4.537}{7} = 0.648$ in.

For net area requirement, hole diameter = $\frac{7}{8} + \frac{1}{8} = 1$ in.

$$t = \frac{A_n}{w_n} = \frac{A_n}{w_g - \Sigma d_{hole}} = \frac{3.379}{7 - 2(7/8 + 1/8)} = 0.676 \text{ in. (controls)}$$

Try a plate $\frac{3}{4} \times 7$

$$A_n = [7 - 2(1.0)](3/4) = 3.75 \text{ in.}^2$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(1.5 + 0.5) = 1.3 \text{ in.}$$

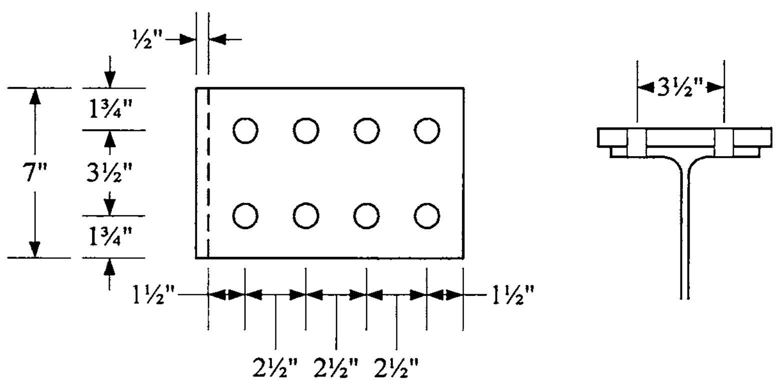
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.}, \qquad \frac{KL}{r} = \frac{1.3}{0.1804} = 7.21$$

From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_y A_g$$

$$\therefore \ \phi P_n = \phi F_y A_g = 0.90(36)(7 \times 3/4) = 170 \text{ kips} > 147.0 \text{ kips} \qquad \text{(OK)}$$

Check block shear on the plate using the dimensions and bolt layout shown. (Use the "Workable Gage" from Part 1 of the Manual.)




Shear areas: $A_{gv} = \frac{3}{4}(9) \times 2 = 13.5 \text{ in.}^2$

$A_{nv} = \frac{3}{4}[9 - 3.5(1.0)] \times 2 = 8.25 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = \frac{3}{4}[3.5 - 1(1.0)] = 1.875 \text{ in.}^2$ (using 0.5 + 0.5 hole diameters)

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(8.25) + 1.0(58)(1.875) = 395.9 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(13.5) + 1.0(58)(1.875) = 400.4 \text{ kips}$$

The nominal block shear strength is therefore 395.9 kips. The design block shear strength is

$$\phi R_n = 0.75(395.9) = 297 \text{ kips} > 147.0 \text{ kips} \qquad \text{(OK)}$$

Check block shear strength of beam flange.

Transverse spacing = gage distance = 3.5 in.

Transverse edge distance = $\frac{6-3.5}{2} = 1.25$ in.

Longitudinal spacing and edge distance same as for plate.

Shear areas: $A_{gv} = 0.425(9) \times 2 = 7.65 \text{ in.}^2$

$$A_{nv} = 0.425[9 - 3.5(1.0)] \times 2 = 4.675 \text{ in.}^2$$

Tension area: $A_{nt} = 0.425[2.5 - 1(1.0)] = 0.6375 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(4.675) + 1.0(58)(0.6375) = 199.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(7.65) + 1.0(58)(0.6375) = 202.2 \text{ kips}$$

The nominal block shear strength is therefore 199.7 kips. The design block shear strength is

$$\phi R_n = 0.75(199.7) = 149.8 \text{ kips} > 147.0 \text{ kips} \quad \text{(OK)}$$

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.425(6.00) = 2.55 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.55 - 0.425(2 \times 1) = 1.7 \text{ in.}^2$$

$$F_uA_{fn} = 65(1.7) = 110.5 \text{ kips}$$

$$Y_tF_yA_{fg} = 1.1(50)(2.55) = 140.3 \text{ kips}$$

Since $F_uA_{fn} < Y_tF_yA_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x = \frac{110.5}{2.55}(57.6) = 2496 \text{ in.-kips} = 208 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(208) = 187 \text{ ft-kips} < 216.8 \text{ ft-kips} \quad \text{(N.G.)}$$

Try a smaller diameter bolt. Try ½-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(1/2)^2/4 = 0.1963 \text{ in.}^2$$

Nominal shearing strength = $\phi R_n = 0.75(54)(0.1963) = 7.95$ kips/bolt

Number of bolts required for shear is

$$\frac{147.0}{7.95} = 18.49 \text{ try 20 bolts (10 pair)}$$

Bearing and block shear will be satisfactory.

Check reduction in beam flange area: Use a hole diameter = $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.55 - 0.425(2 \times 5/8) = 2.019 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.019) = 131.2 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(2.55) = 140.3 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

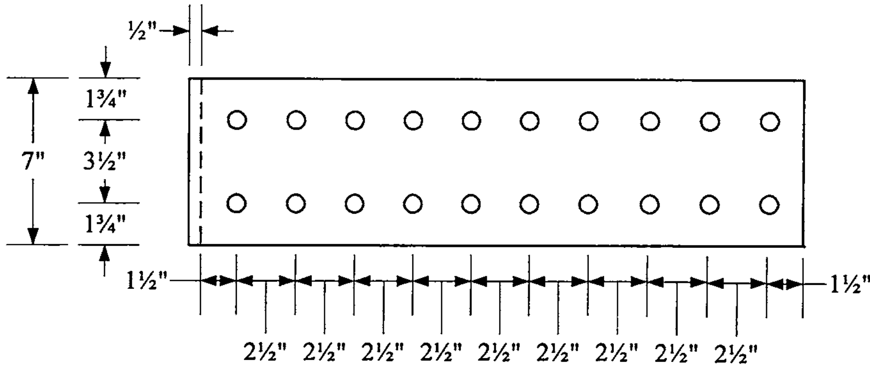
$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{131.2}{2.55}(57.6) = 2964 \text{ in.-kips} = 247 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(247) = 222.3 \text{ ft-kips} > 216.8 \text{ ft-kips required} \qquad \text{(OK)}$$

Plate length = $10(2.5) + 1.5 + 1.5 + 0.5 = 28.5$ in.

Use a PL $\frac{3}{4} \times 7 \times 2' - 4\frac{1}{2}''$ with 20 bolts

Summary: For the web plate, use a PL $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$ with four $\frac{5}{8}$-in. diameter Group A bolts as shown. Attach to column with a continuous $\frac{1}{8}$-in. fillet weld on both sides of the plate. For each flange plate, use a PL $\frac{3}{4} \times 7 \times 2' - 4\frac{1}{2}''$ with twenty $\frac{1}{2}$-in. diameter Group A bolts as shown.




**8.7-1**

W16 × 45 beam, W10 × 45 column, A992 steel. Flange plates are 5/8 × 7.

(a) LRFD solution

$$M_u = 1.2(0.3 \times 118) + 1.6(0.7 \times 118) = 174.6 \text{ ft-kips}$$

The force in the flange plate can be conservatively taken as

$$H = \frac{M_u}{d_b} = \frac{174.6(12)}{16.1} = 130.1 \text{ kips} = P_{bf}$$

Local flange bending: from AISC Eq. J10-1,

$$\phi R_n = \phi(6.25 F_{yf} t_f^2) = 0.90[6.25(50)(0.620)^2] = 108.1 \text{ kips} < 130.1 \text{ kips} \quad \text{(N.G.)}$$

Local web yielding:

$$\phi R_n = \phi[F_{yw} t_w (5k + \ell_b)] = 1.0(50)(0.350)[5(1.12) + 0.625)]$$

$$= 108.9 \text{ kips} < 130.1 \text{ kips} \qquad \text{(N.G.)}$$

Web crippling: from AISC Eq. J10-4,

$$\phi R_n = \phi \left\{ 0.80 t_w^2 \left[ 1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right] \sqrt{\frac{E F_y t_f}{t_w}} \right\}$$

$$= 0.75\left( 0.80(0.350)^2 \left[ 1 + 3\left(\frac{0.625}{10.1}\right)\left(\frac{0.350}{0.620}\right)^{1.5} \right] \sqrt{\frac{29000(50)(0.620)}{0.350}} \right)$$

$$= 127.1 \text{ kips} < 130.1 \text{ kips} \qquad \text{(N.G.)}$$

Local flange bending is the worst case. Required stiffener area is

$$A_{st} = \frac{P_{bf} - \phi R_{n\,\min}}{\phi_{st} F_{yst}} = \frac{130.1 - 108.1}{0.90(36)} = 0.679 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{7}{3} - \frac{0.350}{2} = 2.16 \text{ in.}$$

$$b \leq \frac{b_{f\,col} - t_w}{2} = \frac{8.02 - 0.350}{2} = 3.84 \text{ in.}$$

$$t_{st} \geq \frac{t_b}{2} = \frac{0.625}{2} = 0.313 \text{ in.}$$

Try a plate $\frac{1}{4} \times 3$

$$A_{st} = \tfrac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.679 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{b}{13} = \frac{3.5}{16} = 0.219 < 1/4 \qquad \text{(OK)}$$

Full-depth stiffeners are not required for this case. Use a depth of approximately

$$\frac{d}{2} = \frac{10.1}{2} = 5.05 \text{ in.}$$

Use 2 plates $\frac{1}{4} \times 3 \times 0'$- 5$''$.

Clip inside corners to avoid flange-to-web fillets.

(b) ASD solution

The flange plate force can be conservatively taken as

$$P_{bf} = \frac{M_a}{d_b} = \frac{118(12)}{16.1} = 87.95 \text{ kips}$$

Check local flange bending with AISC Equation J10-1:

$$\frac{R_n}{\Omega} = \frac{6.25 F_{yf} t_f^2}{\Omega}$$

$$= \frac{6.25(50)(0.620)^2}{1.67} = 71.93 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Check local web yielding with AISC Equation J10-2:

$$\frac{R_n}{\Omega} = \frac{F_{yw}t_w(5k+\ell_b)}{\Omega}$$

$$= \frac{50(0.350)[5(1.12)+0.625]}{1.50} = 72.63 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Check web crippling with AISC Equation J10-4:

$$\frac{R_n}{\Omega} = \frac{0.80t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}}{\Omega}$$

$$= \frac{0.80(0.350)^2\left[1+3\left(\frac{0.625}{10.1}\right)\left(\frac{0.350}{0.620}\right)^{1.5}\right]\sqrt{\frac{29000(50)0.620}{0.350}}}{2.00}$$

$$= 84.71 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Stiffeners are required. The smallest strength is 71.93 kips, for the limit state of local flange bending. The required stiffener area is

$$A_{st} = \frac{P_{bf}-(R_n/\Omega)_{\min}}{F_{yst}/\Omega} = \frac{87.95-71.93}{36/1.67} = 0.743 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{7}{3} - \frac{0.350}{2} = 2.16 \text{ in.}$$

$$b \leq \frac{8.02-0.350}{2} = 3.84 \text{ in.}$$

$$t_{st} \geq \frac{t_b}{2} = \frac{0.625}{2} = 0.313 \text{ in.}$$

Try a plate $\frac{1}{4} \times 3$

$$A_{st} = \frac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.743 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{b}{16} = \frac{3.5}{16} = 0.219 < 1/4 \qquad \text{(OK)}$$

Full-depth stiffeners are not required for this case. Use a depth of approximately

$$\frac{d}{2} = \frac{10.1}{2} = 5.05 \text{ in.}$$

<u>Use 2 plates $\frac{1}{4} \times 3 \times 0'$- $5''$.</u>

<u>Clip inside corners to avoid flange-to-web fillets.</u>

**8.7-2**

(a) LRFD solution

Force developed in the beam flange plate is

$H = F_y A = 36\left(\frac{11}{16} \times 9\right) = 223 \text{ kips} = P_{bf}$ (This is a *nominal* strength, so compare the effects of this with the nominal strengths of the column flange and web.)

Local flange bending: from AISC Eq. J10-1,

$$R_n = 6.25 F_{yf} t_f^2 = 6.25(50)(1.09)^2 = 371.3 \text{ kips} > 223 \text{ kips} \qquad \text{(OK)}$$

Local web yielding: from AISC Eq. J10-2,

$$R_n = F_{yw} t_w (5k + \ell_b) = (50)(0.680)(5 \times 1.69 + 11/16)$$

$$= 311 \text{ kips} > 223 \text{ kips} \qquad \text{(OK)}$$

Web crippling: from AISC Eq. J10-4,

$$R_n = 0.80 t_w^2 \left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_{yw} t_f}{t_w}}$$

$$= 0.80(0.680)^2 \left[1 + 3\left(\frac{11/16}{14.8}\right)\left(\frac{0.680}{1.09}\right)^{1.5}\right]\sqrt{\frac{29000(50)(1.09)}{0.680}}$$

$$= 603 \text{ kips} > 223 \text{ kips} \qquad \text{(OK)}$$

Stiffeners not required.

(b) ASD Solution

Same as LRFD Solution: Stiffeners not required.

**8.7-3**

(a) Web plate: neglect eccentricity.

Try $\frac{3}{4}$-in. diameter Group A bearing-type bolts. Assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) = 17.89$ kips/bolt

Number of bolts required is

$$\frac{45}{17.89} = 2.52 \text{ try 4 bolts}$$

Determine plate thickness required for bearing. Assume that

$$\phi R_n = \phi(2.4 d t F_u) = 0.75(2.4)(3/4)t(58) = 78.3t$$

Load resisted by each bolt = $\frac{45}{4} = 11.25$ kips. Let

$$\phi R_n = 11.25 = 78.3t \quad \Rightarrow \quad t = 0.144 \text{ in.} \qquad \text{Try } t = \tfrac{3}{16} \text{ in.}$$

Determine whether plate or beam web controls bearing. For the plate,

$$tF_u = \frac{3}{16}(58) = 10.9 \text{ kips/in.}$$

For the beam web, $t_w F_u = 0.300(65) = 19.5$ kips/in. > 10.9 kips/in. ∴ plate controls.

Check bearing strength assumption. $h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in.

For the hole nearest the edge, min. $\ell_e = 1$ in. Try $1\frac{1}{2}$ in.

$$\ell_c = \ell_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094 \text{ in.}$$

$$\phi R_n = \phi(1.2 \ell_c t F_u) = 0.75(1.2)(1.094)(3/16)(58) = 10.71 \text{ kips}$$

$$\phi(2.4 d t F_u) = 0.75(2.4)(3/4)(3/16)(58) = 14.68 \text{ kips} > 10.71 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 10.71 \text{ kips/bolt}$$

For other bolts, min. $s = 2.667(3/4) = 2.0$ in. Try $s = 2\frac{1}{2}$ in.

$$\ell_c = s - h = 2.5 - \frac{13}{16} = 1.688 \text{ in.}$$

$$\phi R_n = \phi(1.2 \ell_c t F_u) = 0.75(1.2)(1.688)(3/16)(58) = 16.52 \text{ kips}$$

$$\phi(2.4 d t F_u) = 14.68 \text{ kips} < 16.52 \text{ kips} \therefore \text{ use } \phi R_n = 14.68 \text{ kips/bolt}$$

Total shear/bearing strength (bearing controls at each bolt location):

$$\phi R_n = \phi R_n\ 10.71 + 3(14.68) = 54.8 \text{ kips} > 45 \text{ kips} \qquad \text{(OK)}$$

Total length of connection $= 2(1.5) + 3(2.5) = 10.5$ in.

Use four $\frac{3}{4}$-in. diameter Group A bolts .

Determine plate thickness required for shear. Shear yielding strength is

$$\phi R_n = 1.00(0.60F_yA_g)$$

Let $45 = 1.00(0.60)(36)(10.5t) \quad \Rightarrow \quad t = 0.198$ in. Try $t = \frac{1}{4}$ in.

Check shear rupture strength. Use hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_{nv} = (10.5 - 4 \times 7/8)\left(\frac{1}{4}\right) = 1.75 \text{ in.}^2$$

$$\phi R_n = \phi(0.6A_{nv}F_u) = 0.75(0.6)(1.75)(58) = 45.7 \text{ kips} > 45 \text{ kips} \qquad \text{(OK)}$$

Check block shear.

Shear areas: $A_{gv} = \frac{1}{4}(1.5 + 3 \times 2.5) = \frac{1}{4}(9) = 2.25 \text{ in.}^2$

$$A_{nv} = \frac{1}{4}[9 - 3.5(7/8)] = 1.484 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(7/8)] = 0.2656 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(1.484) + 1.0(58)(0.2656) = 67.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.25) + 1.0(58)(0.2656) = 64.0 \text{ kips}$$

The nominal block shear strength is therefore 64.0 kips. The design block shear strength is

$$\phi R_n = 0.75(64.0) = 48.0 \text{ kips} > 45 \text{ kips (OK)}$$

Use a plate $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$.

Connection of shear plate to column flange: Use E70 electrodes.

Minimum weld size, based on the shear plate thickness of $\frac{1}{4}$ inch, is $\frac{1}{8}$ inch. Try $w = \frac{1}{8}$ in.

Weld strength $= 1.392 \times 2 = 2.784$ kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n = 0.6F_y t = 0.6(36)(1/4) = 5.4$ kips/in. > 2.784 kips/in.

Rupture: $\phi R_n = 0.45F_u t = 0.45(58)(1/4) = 6.525$ kips/in. > 2.784 kips/in.

Weld strength controls: Total length required = $45/2.784 = 16.16$ in.

Use a continuous $\frac{1}{8}$-in. fillet weld on each side of plate.

Check for column stiffener requirements. The force developed in the beam flange is

$$H = \frac{M_u}{d_b - t_b} = \frac{220(12)}{17.7 - 0.425} = 152.8 \text{ kips} = P_{bf}$$

Local flange bending: from AISC Eq. J10-1,

$$\phi R_n = \phi 6.25 F_{yf} t_f^2 = 0.90[6.25(50)(0.660)^2] = 122.5 \text{ kips} < 152.8 \text{ kips} \quad \text{(N.G.)}$$

Local web yielding: from AISC Eq. J10-2,

$$\phi R_n = \phi[F_{yw} t_w (5k + \ell_b)] = 1.00[(50)(0.370)(5 \times 1.25 + 0.425)]$$

$$= 123.5 \text{ kips} < 152.8 \text{ kips} \quad \text{(N.G.)}$$

Web crippling: from AISC Eq. J10-4,

$$\phi R_n = \phi\left\{0.80 t_w^2 \left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_{yw} t_f}{t_w}}\right\}$$

$$= 0.75\left(0.80(0.370)^2\left[1 + 3\left(\frac{0.425}{13.9}\right)\left(\frac{0.370}{0.660}\right)^{1.5}\right]\sqrt{\frac{29,000(50)(0.660)}{0.370}}\right)$$

$$= 137.2 \text{ kips} < 152.8 \text{ kips} \quad \text{(N.G.)}$$

Local flange bending is the worst case. Required stiffener area is

$$A_{st} = \frac{P_{bf} - \phi R_{n\,\min}}{\phi_{st} F_{yst}} = \frac{152.8 - 122.5}{0.90(36)} = 0.935 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{6.00}{3} - \frac{0.370}{2} = 1.82 \text{ in.}$$

$$b \leq \frac{b_{f\,col} - t_w}{2} = \frac{8.06 - 0.370}{2} = 3.85 \text{ in.}$$

$t_{st} \geq \frac{t_b}{2} = \frac{0.425}{2} = 0.213 \text{ in.}$

Try a plate $\frac{1}{4} \times 3$

$A_{st} = \frac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.935 \text{ in.}^2$ (OK)

Check for $t_{st} > b/16$ :

$\frac{b}{16} = \frac{3}{16} = 0.188 \text{ in.} < \frac{1}{4} \text{ in.}$ (OK)

Full-depth stiffeners are not required for this case. Use a depth of approximately

$\frac{d}{2} = \frac{13.9}{2} = 6.95 \text{ in.}$ Use 2 plates $\frac{1}{4} \times 3 \times 0'$- 7".

Clip inside corners $\frac{5}{8}$ in. to avoid flange-to-web fillets.

(b) Check panel zone reinforcement requirements.

$$P = \frac{(M_1 + M_2)}{d_m} - V = \frac{M_u}{d_b - t_b} - V_u = \frac{220(12)}{17.7 - 0.425} - 0 = 152.8 \text{ kips}$$

$P_u = 0.6P_y > 0.4P_y$, so use AISC Equation J10-10.

$$R_n = 0.60F_y d_c t_w\left(1.4 - \frac{P_r}{P_c}\right) = 0.60F_y d_c t_w\left(1.4 - \frac{P_u}{P_y}\right)$$

$$= 0.60(50)(13.9)(0.370)(1.4 - 0.6) = 123.4 \text{ kips}$$

$\phi R_n = 0.90(123.4) = 111.1 \text{ kips} < 152.8 \text{ kips}$ (N.G.)

Alternative 1: use a web doubler plate.

Use AISC Equation J10-10 to find the required doubler plate thickness. Multiplying both sides by $\phi$ and solving for $t_w$ gives

$$t_w = \frac{\phi R_n}{\phi(0.60F_y d_c)(1.4 - P_u/P_y)}$$

Substituting the plate thickness $t_d$ for $t_w$ and using the yield stress of the doubler plate, we get

$$t_d = \frac{\phi R_n}{\phi(0.60F_y d_c)(1.4 - P_u/P_y)}$$

$$= \frac{152.8 - 111.1}{0.90(0.60)(36)(13.9)(1.4 - 0.6)} = 0.193 \text{ in.}$$

where $152.8 - 111.1$ is the extra strength, in kips, to be furnished by the doubler plate.

Use a ¼-inch doubler plate.

(The welds will not be designed. For an explanation, see Example 8.12 in the textbook.)

Alternative 2: use a diagonal stiffener. With this alternative, use full-depth horizontal stiffeners.

The shear force to be resisted by the web reinforcement is $152.8 - 111.1 = 41.7$ kips. If this force is taken as the horizontal component of an axial compressive force $P$ in the stiffener,

$$P\cos\theta = 41.7 \text{ kips}$$

$$\theta = \tan^{-1}\left(\frac{d_b}{d_c}\right) = \tan^{-1}\left(\frac{17.7}{13.9}\right) = 51.86°$$

$$P = \frac{41.7}{\cos(51.86°)} = 67.52 \text{ kips}$$

Since the stiffener is continuously connected along its length, we will treat it as a compression member whose effective length *KL* is zero. From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_y A_g$$

$$\therefore\ \phi P_n = \phi F_y A_g = 0.90(36)A_{st}$$

Equating this strength to the required strength, we obtain the required area of stiffener:

$$0.90(36)A_{st} = 67.52 \qquad \Rightarrow \qquad A_{st} = 2.08 \text{ in.}^2$$

Try 2 stiffeners, $3 \times \frac{3}{8}$, one on each side of the web.

$$A_{st} \text{ provided} = 2\left(3 \times \frac{3}{8}\right) = 2.25 \text{ in.}^2 > 2.08 \text{ in.}^2 \text{ required (OK)}$$

Check for $t_{st} > b/16$ :

$$\frac{b}{16} = \frac{3}{16} = 0.188 \text{ in.} < \frac{3}{8} \text{ in.} \qquad \text{(OK)}$$

Design the welds. The approximate length of each diagonal stiffener is

$$L_{st} = \frac{d_c}{\cos\theta} = \frac{13.9}{\cos(51.86°)} = 22.51 \text{ in.}$$

If welds are used on both sides of the stiffeners, the available length for welding is

$$L = 22.51(4) = 90.04 \text{ in.}$$

The weld size, in sixteenths of an inch, required for strength is

$$D = \frac{P}{1.392L} = \frac{67.52}{1.392(90.04)} = 0.539 \text{ sixteenths}$$

Use the minimum size of $\frac{3}{16}$ inch (AISC Table J2.4).

Because of the small size required for strength, use intermittent welds. From AISC J2.2b,

Minimum length = $4w = 4\left(\frac{3}{16}\right) = 0.75$ in., but not less than 1.5 in. (1.5 in. controls)

For a group of four welds, the capacity is

$$4(1.392DL) = 4(1.392)(3)(1.5) = 25.06 \text{ kips}$$

Required weld capacity per inch = $\frac{P}{L_{st}} = \frac{67.52}{22.51} = 3.00$ kips/in.

Base metal shear strength of column web:

Yielding: $\phi R_n = 0.6F_y t_w \times 2 = 0.6(50)(0.370) \times 2$

$= 22.2$ kips/in. > 3.00 kips/in.

Rupture: $\phi R_n = 0.45F_u t_w \times 2 = 0.45(65)(0.370) \times 2$

$= 21.7$ kips/in. > 3.00 kips/in.

Base metal shear strength of stiffener:

Yielding: $\phi R_n = 0.6F_y t_{st} \times 2 = 0.6(36)(3/8) \times 2 = 16.2$ kips/in. > 3.00 kips/in.

Rupture: $\phi R_n = 0.45F_u t_{st} \times 2 = 0.45(58)(3/8) \times 2$

$= 19.6$ kips/in. > 3.00 kips/in.

Required spacing of welds = $\frac{25.06}{3} = 8.35$ in.

Use $\frac{3}{16}$-in. × $1\frac{1}{2}$-in. intermittent fillet welds spaced at 8 inches on center, on each side of each diagonal stiffener.

**8.8-1**

(a) LRFD solution

Factored shear and moment:

$$V_u = 1.2(0.25 \times 25) + 1.6(0.75 \times 25) = 37.5 \text{ kips}$$

$$M_u = 1.2(0.25 \times 65) + 1.6(0.75 \times 65) = 97.5 \text{ ft-kips}$$

Is required moment strength at least 60% of available moment strength?

$$\phi_b M_n = \phi_b M_p = 125 \text{ ft-kips}, \qquad 0.60(125) = 75.0 \text{ ft-kips}$$

Required moment strength = $M_u = 97.5$ ft-kips > 75.0 ft-kips (OK)

Flange force = $\dfrac{M_u}{d - t_f} = \dfrac{97.5(12)}{13.7 - 0.335} = 87.54$ kips

Tensile load per bolt = $87.54/4 = 21.89$ kips

Tensile strength: $A_b = \pi(0.75)^2/4 = 0.4418 \text{ in.}^2$

$$\phi R_n = \phi F_{nt} A_b = 0.75(90)(0.4418) = 29.82 \text{ kips/bolt} > 21.9 \text{ kips/bolt} \qquad \text{(OK)}$$

Shear load per bolt = $37.5/4 = 9.375$ kips (compression side bolts)

Shear strength:

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) = 17.9 \text{ kips/bolt} > 9.38 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength: Assume that the upper limit of $2.4dtF_u$ controls and that column flange thickness controls ($t_f = 0.435$ in.).

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.435)(65)$$

$$= 38.2 \text{ kips/bolt} > 9.38 \text{ kips/bolt} \qquad \text{(OK)}$$

Bolts are adequate

(b) ASD solution

Is required moment strength at least 60% of available moment strength?

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 82.8 \text{ ft-kips}, \qquad 0.60(82.8) = 49.7 \text{ ft-kips}$$

Required moment strength = $M_a$ = 65 ft-kips > 49.7 ft-kips (OK)

Flange force = $\frac{M_a}{d - t_f} = \frac{65(12)}{13.7 - 0.335} = 58.36$ kips

Tensile load per bolt = 58.36/4 = 14.59 kips

Tensile strength: $A_b = \pi(0.75)^2/4 = 0.4418$ in.$^2$

$$\frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90(0.4418)}{2.00} = 19.9 \text{ kips/bolt} > 14.6 \text{ kips/bolt} \quad \text{(OK)}$$

Shear load per bolt = 25/4 = 6.25 kips (compression side bolts)

Shear strength:

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{54(0.4418)}{2.00} = 11.9 \text{ kips/bolt} > 6.25 \text{ kips/bolt} \quad \text{(OK)}$$

Bearing strength: Assume that the upper limit of $2.4dtF_u$ controls and that column flange thickness controls ($t_f = 0.435$ in.).

$$\frac{R_n}{\Omega} = \frac{2.4dtF_u}{\Omega} = \frac{2.4(3/4)(0.435)(65)}{2.00}$$

$$= 25.5 \text{ kips/bolt} > 6.25 \text{ kips/bolt} \quad \text{(OK)}$$

Bolts are adequate

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**8.8-3**

LRFD solution

From the $Z_x$ tables, for a W18 × 40,

$$M_u = \phi_b M_{px} = 294 \text{ ft-kips}$$

$$V_u = \phi_v V_n = 169 \text{ kips}$$

From the dimensions and properties tables

$d = 17.9$ in., $t_w = 0.315$ in., $b_{fb} = 6.02$ in., $t_{fb} = 0.525$ in., workable gage = 3.50 in.

For the bolt pitch, try $p_{fo} = p_{fi} = 2$ in.

For the gage distance, use the workable gage $g = 3.50$ in.

Required bolt diameter:

$$h_0 = d - \frac{t_{fb}}{2} + p_{fo} = 17.9 - \frac{0.525}{2} + 2 = 19.64 \text{ in.}$$

$$h_1 = d - \frac{t_{fb}}{2} - t_{fb} - p_{fi} = 17.9 - \frac{0.525}{2} - 0.525 - 2 = 15.11 \text{ in.}$$

$$d_{b\text{Req'd}} = \sqrt{\frac{2M_u}{\pi\phi F_t(h_0 + h_1)}} = \sqrt{\frac{2(294 \times 12)}{\pi(0.75)(90)(19.64 + 15.11)}} = 0.979 \text{ in.}$$

Try $d_b = 1$ inch.

Moment strength based on bolt strength:

$$P_t = F_t A_b = 90\pi(1)^2/4 = 70.69 \text{ kips/bolt}$$

$$M_n = 2P_t(h_0 + h_1) = 2(70.69)(19.64 + 15.11) = 4913 \text{ in.-kips}$$

$$\phi M_n = 0.75(4913) = 3685 \text{ in-kips} = 307.1 \text{ ft-kips} > \phi_b M_{px} = 294 \text{ ft-kips}$$

$$\therefore \text{ use } \phi M_n = \phi_b M_{px} = 294 \text{ ft-kips} = 3528 \text{ in.-kips}$$

Determine end-plate width. From AISC Table J3.4,

minimum $\ell_e = 1\frac{1}{4}$ in.

The minimum plate width is $g + 2\ell_e = 3.50 + 2(1.25) = 6.0$ in.

but no less than the beam flange width of 6.02 in.

Maximum *effective* end-plate width $= b_{fb} + 1 = 6.02 + 1 = 7.02$ in.

Try $b_p = 7$ in. Compute the required plate thickness:

$$s = \frac{1}{2}\sqrt{b_p g} = \frac{1}{2}\sqrt{7(3.5)}$$

$$= 2.475 \text{ in.} > p_{fi} \;\therefore \text{ use the original value of } p_{fi} = 2.0 \text{ in.}$$

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \frac{1}{2}\right] + \frac{2}{g}[h_1(p_{fi} + s)]$$

$$= \frac{7}{2}\left[15.11\left(\frac{1}{2} + \frac{1}{2.475}\right) + 19.64\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \frac{2}{3.5}[15.11(2 + 2.475)]$$

$$= 119.1$$

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(3528)}{0.9(36)(119.1)}} = 1.01 \text{ in.}$$

Try $t_p = 1\frac{1}{4}$ inch.

Beam flange force:

$$F_{fu} = \frac{M_u}{d - t_{fb}} = \frac{294 \times 12}{17.9 - 0.525} = 203.1 \text{ kips}$$

$$\frac{F_{fu}}{2} = \frac{203.1}{2} = 101.6 \text{ kips}$$

The shear yield strength of the end plate is

$$\phi(0.6)F_y t_p b_p = 0.90(0.6)(36)(1.25)(7) = 170.1 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Shear rupture strength of end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = (1.25)\left[7 - 2\left(1 + \frac{1}{8}\right)\right] = 5.938 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.938) = 155.0 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Check bolt shear. The compression side bolts must be capable of resisting the entire vertical shear.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(1)^2}{4} = 0.7854 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.7854) = 31.81 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 31.81 = 127.2$ kips

$$V_u = 169 \text{ kips} > 127.2 \text{ kips} \qquad \text{(N.G.)}$$

Try $d_b = 1\frac{1}{4}$ inch.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(1.25)^2}{4} = 1.227 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(1.227) = 49.69 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 49.69 = 199$ kips > 169 kips (OK)

Recompute the plate dimensions. For 1¼-inch diameter bolts,

minimum $\ell_e = 1\frac{5}{8}$ in.

The minimum plate width is

$$g + 2\ell_e = 3.50 + 2(1.625) = 6.75 \text{ in.}$$

Maximum *effective* end-plate width = $b_{fb} + 1 = 6.02 + 1 = 7.02$ in.

Use the currently selected width of $b_p = 7$ in.

Because the bolt size has increased, check the shear rupture strength of the end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = (1.25)[7 - 2(1.25 + 1/8)] = 5.313 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.313) = 138.7 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the plate at the compression side bolts.

$$h = d + \frac{1}{16} = 1\frac{1}{4} + \frac{1}{16} = 1.313 \text{ in.}$$

For the outer bolts,

$$\ell_c = p_{fo} + t_{fb} + p_{fi} - h = 2 + 0.525 + 2 - 1.313 = 3.212 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(3.212)(1.25)(58) = 209.6 \text{ kips}$$

The upper limit is

$$\phi(2.4 d t F_u) = 0.75(2.4)(1.25)(1.25)(58) = 163.1 \text{ kips} < 209.6 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 163.1 \text{ kips/bolt}$$

Since the inner bolts are not near an edge or adjacent bolts, the outer bolts control. The total bearing strength is

$$4 \times 163.1 = 652 \text{ kips} > V_u = 169 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the column flange. Use $\ell_c = 3.212$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(3.212)(0.525)(65) = 98.65 \text{ kips}$$

The upper limit is

$$\phi(2.4 d t F_u) = 0.75(2.4)(1.25)(0.525)(65) = 76.78 \text{ kips} < 98.65 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 76.78 \text{ kips/bolt}$$

The total bearing strength is

$$4 \times 76.78 = 307 \text{ kips} > V_u = 169 \text{ kips} \qquad \text{(OK)}$$

The plate length, using detailing dimensions and the notation of Figure 8.51 in the textbook, is

$$d + 2p_{fo} + 2d_e = 17\tfrac{7}{8} + 2(2) + 2\left(1\tfrac{5}{8}\right) = 25\tfrac{1}{8} \text{ in.}$$

Use a PL $1\frac{1}{4} \times 7 \times 2' - 1\frac{1}{8}''$

and four 1¼-inch diameter Group A fully-tightened bolts at each flange.

Beam flange-to-plate weld design: The flange force is

$$F_{fu} = 203.1 \text{ kips}$$

AISC Design Guide 4 recommends that the minimum design flange force should be 60% of the flange yield strength:

$$\text{Min. } F_{fu} = 0.6F_y(b_{fb}t_{fb}) = 0.6(50)(6.02)(0.525) = 94.8 \text{ kips} < 203.1 \text{ kips}$$

Therefore, use the actual flange force of 203.1 kips. The flange weld length is

$$b_{fb} + (b_{fb} - t_w) = 6.02 + (6.02 - 0.315) = 11.73 \text{ in.}$$

The weld strength is

$$\phi R_n = 1.392D \times 11.73 \times 1.5$$

where $D$ is the weld size in sixteenths of an inch, and the factor of 1.5 accounts for the direction of the load on the weld. If we equate the weld strength to the flange force,

$$1.392D \times 11.73 \times 1.5 = 203.1, \qquad D = 8.29 \text{ sixteenths}$$

From AISC Table J2.4, the minimum weld size is ¼ in. (based on the thickness of the flange, which is the thinner connected part).

Use a $\frac{9}{16}$-inch fillet weld at each flange.

Beam web-to-plate weld design: To develop the yield stress in the web near the tension bolts, let

$$1.392D \times 2 = 0.6F_y t_w$$

for two welds, one on each side of the web. The required weld size is

$$D = \frac{0.6F_y t_w}{1.392(2)} = \frac{0.6(50)(0.315)}{1.392(2)} = 3.39 \text{ sixteenths}$$

Use a ¼-inch fillet weld on each side of the web in the tension region.

The applied shear of $V_u = 169$ kips must be resisted by welding a length of web equal to the smaller of the following two lengths:

1. from mid-depth to the compression flange:

$$L = \frac{d}{2} - t_{fb} = \frac{17.9}{2} - 0.525 = 8.425 \text{ in.}$$

2. from the inner row of tension bolts plus $2d_b$ to the compression flange:

$$L = d - 2t_{fb} - p_{fi} - 2d_b = 17.9 - 2(0.525) - 2.0 - 2(1.25) = 12.35 \text{ in.} > 8.425 \text{ in.}$$

Equating the weld strength to the required shear strength, we get

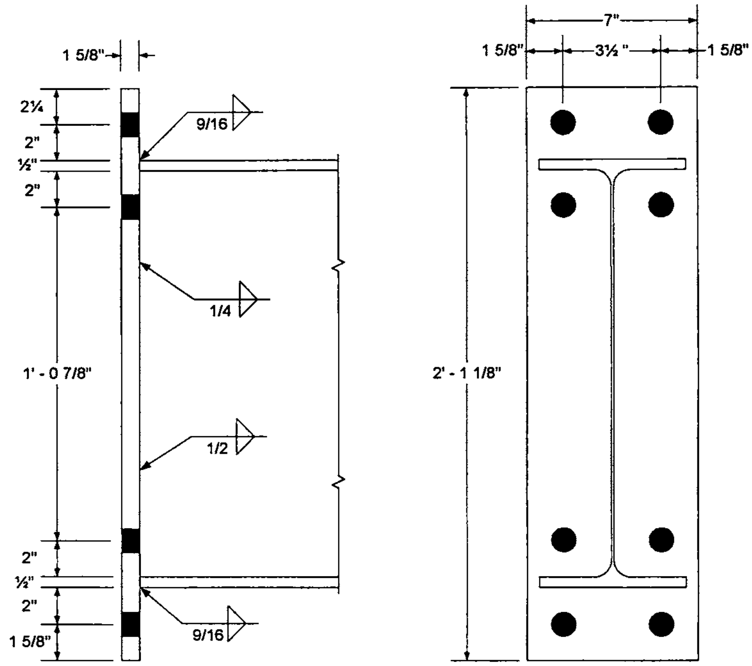
$$1.392D \times 8.425 \times 2 = 169, \qquad D = 7.21 \text{ sixteenths } (w = ½ \text{ in.})$$

From AISC Table J2.4, the minimum weld size is 3/16 in.

Use a ½-inch fillet weld on each side of the web

between mid-depth and the compression flange.

The design is summarized in the figure below.




Not to scale

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**8.8-4**

LRFD solution

Factored shear and moment:

$$V_u = 1.2(13) + 1.6(34) = 70.0 \text{ kips}$$

$$M_u = 1.2(20) + 1.6(48) = 100.8 \text{ ft-kips}$$

From the dimensions and properties tables

$d = 12.3$ in., $t_w = 0.260$ in., $b_{fb} = 6.52$ in., $t_{fb} = 0.440$ in.,

workable gage = 3.50 in.

For the bolt pitch, try $p_{fo} = p_{fi} = 2$ in.

For the gage distance, use the workable gage $g = 3.50$ in.

Required bolt diameter:

$$h_0 = d - \frac{t_{fb}}{2} + p_{fo} = 12.3 - \frac{0.440}{2} + 2 = 14.08 \text{ in.}$$

$$h_1 = d - \frac{t_{fb}}{2} - t_{fb} - p_{fi} = 12.3 - \frac{0.440}{2} - 0.440 - 2 = 9.64 \text{ in.}$$

$$d_{b\text{Req'd}} = \sqrt{\frac{2M_u}{\pi\phi F_t(h_0 + h_1)}} = \sqrt{\frac{2(100.8 \times 12)}{\pi(0.75)(90)(14.08 + 9.64)}} = 0.694 \text{ in.}$$

Try $d_b = \frac{3}{4}$ inch.

Moment strength based on bolt strength:

$$P_t = F_t A_b = 90\pi(3/4)^2/4 = 39.76 \text{ kips/bolt}$$

$$M_n = 2P_t(h_0 + h_1) = 2(39.76)(14.08 + 9.64) = 1886 \text{ in.-kips}$$

$$\phi M_n = 0.75(1886) = 1415 \text{ in-kips} = 117.9 \text{ ft-kips}$$

Determine end-plate width. From AISC Table J3.4,

minimum $\ell_e = 1$ in.

The minimum plate width is $g + 2\ell_e = 3.50 + 2(1) = 5.5$ in.

but no less than the beam flange width of 6.52 in.

Maximum *effective* end-plate width $= b_{fb} + 1 = 6.52 + 1 = 7.52$ in.

Try $b_p = 8$ in., with an effective $b_p = 7.52$ in. Compute the required plate thickness:

$$s = \frac{1}{2}\sqrt{b_p g} = \frac{1}{2}\sqrt{7.52(3.5)}$$

$= 2.565$ in. $> p_{fi}$ ∴ use the original value of $p_{fi} = 2.0$ in.

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \frac{1}{2}\right] + \frac{2}{g}[h_1(p_{fi} + s)]$$

$$= \frac{7.52}{2}\left[9.64\left(\frac{1}{2} + \frac{1}{2.565}\right) + 14.08\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \frac{2}{3.5}[9.64(2 + 2.565)]$$

$$= 81.99$$

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(1415)}{0.90(36)(81.99)}} = 0.769 \text{ in.}$$

Try $t_p = \frac{7}{8}$ inch.

Beam flange force:

$$F_{fu} = \frac{M_u}{d - t_{fb}} = \frac{100.8 \times 12}{12.3 - 0.440} = 102.0 \text{ kips}$$

$$\frac{F_{fu}}{2} = \frac{102.0}{2} = 51 \text{ kips}$$

The shear yield strength of the end plate is

$$\phi(0.6)F_y t_p b_p = 0.90(0.6)(36)(7/8)(7.52) = 128 \text{ kips} > 51 \text{ kips} \quad \text{(OK)}$$

Shear rupture strength of end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = \frac{7}{8}\left[7.52 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\right] = 5.049 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.049) = 132 \text{ kips} > 51 \text{ kips} \quad \text{(OK)}$$

Check bolt shear. The compression side bolts must be capable of resisting the entire vertical shear.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(54)(0.4418) = 17.89 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 17.89 = 71.6$ kips

$$V_u = 70 \text{ kips} < 71.6 \text{ kips} \quad \text{(OK)}$$

Check bearing in the plate at the compression side bolts.

$$h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For the outer bolts,

$$\ell_c = p_{fo} + t_{fb} + p_{fi} - h = 2 + 0.440 + 2 - 13/16 = 3.628 \text{ in.}$$

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(3.628)(7/8)(58) = 165.7 \text{ kips}$$

The upper limit is

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(7/8)(58) = 68.51 \text{ kips} < 165.2 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 68.51 \text{ kips/bolt}$$

Since the inner bolts are not near an edge or adjacent bolts, the outer bolts control. Shear controls overall.

Check bearing in the column flange. Use $\ell_c = 3.628$ in.

$$\phi R_n = \phi(1.2\ell_c t F_u) = 0.75(1.2)(3.628)(0.680)(65) = 144.3 \text{ kips}$$

The upper limit is

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.680)(65) = 59.67 \text{ kips} < 144.3 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 69.62 \text{ kips/bolt}$$

Shear controls overall.

The plate length, using detailing dimensions and the notation of Figure 8.51 in the textbook, is

$$d + 2p_{fo} + 2d_e = 12\tfrac{3}{8} + 2(2) + 2(1) = 18\tfrac{3}{8} \text{ in.}$$

Use a PL $\frac{7}{8} \times 8 \times 1' - 6\frac{3}{8}''$ and four $\frac{3}{4}$-inch diameter Group A fully-tightened bolts at each flange.

Beam flange-to-plate weld design: The flange force is

$$F_{fu} = 102.0 \text{ kips}$$

AISC Design Guide 4 recommends that the minimum design flange force should be 60% of the flange yield strength:

$$\text{Min. } F_{fu} = 0.6F_y(b_{fb}t_{fb}) = 0.6(50)(6.52)(0.440) = 86.06 \text{ kips} < 102.0 \text{ kips}$$

Therefore, use the actual flange force of 102.0 kips. The flange weld length is

$$b_{fb} + (b_{fb} - t_w) = 6.52 + (6.52 - 0.260) = 12.78 \text{ in.}$$

The weld strength is

$$\phi R_n = 1.392D \times 12.78 \times 1.5$$

where $D$ is the weld size in sixteenths of an inch, and the factor of 1.5 accounts for the direction of the load on the weld. If we equate the weld strength to the flange force,

$$1.392D \times 12.78 \times 1.5 = 102.0, \qquad D = 3.82 \text{ sixteenths}$$

From AISC Table J2.4, the minimum weld size is 3/16 in. (based on the thickness of the flange, which is the thinner connected part).

Use a ¼-inch fillet weld at each flange.

Beam web-to-plate weld design: To develop the yield stress in the web near the tension bolts, let

$$1.392D \times 2 = 0.6F_y t_w$$

for two welds, one on each side of the web. The required weld size is

$$D = \frac{0.6F_y t_w}{1.392(2)} = \frac{0.6(50)(0.260)}{1.392(2)} = 2.80 \text{ sixteenths}$$

Minimum size = 3/16 in., based on web thickness.

Use a 3/16-inch fillet weld on each side of the web in the tension region.

The applied shear of $V_u = 70$ kips must be resisted by welding a length of web equal to the smaller of the following two lengths:

1. from mid-depth to the compression flange:

$$L = \frac{d}{2} - t_{fb} = \frac{12.3}{2} - 0.440 = 5.71 \text{ in.}$$

2. from the inner row of tension bolts plus $2d_b$ to the compression flange:

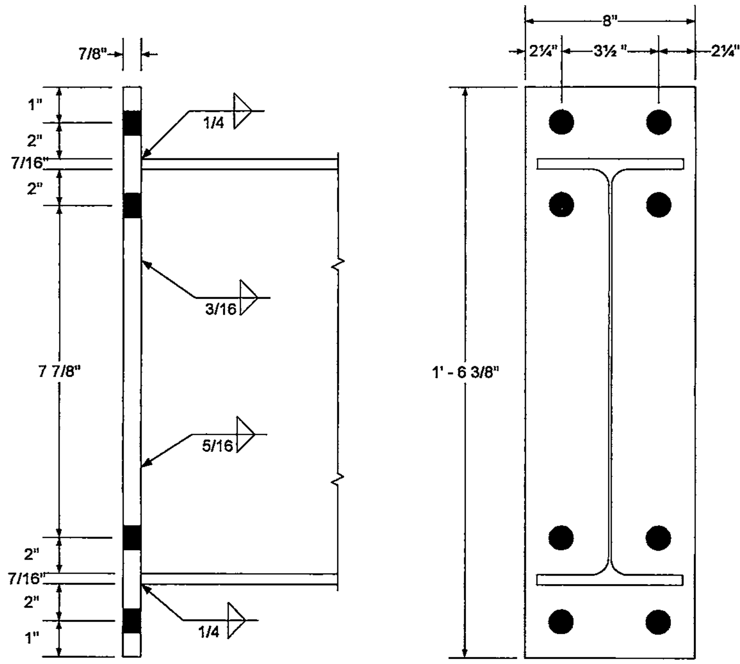
$$L = d - 2t_{fb} - p_{fi} - 2d_b = 12.3 - 2(0.440) - 2.0 - 2(7/8) = 7.67 \text{ in.} > 5.71 \text{ in.}$$

Use $L = 5.71$ inches. Equating the weld strength to the required shear strength, we get

$$1.392D \times 5.71 \times 2 = 70, \qquad D = 4.40 \text{ sixteenths}$$

Use a 5/16-inch fillet weld on each side of the web between mid-depth and the compression flange.

The design is summarized in the following figure .




Not to scale

# CHAPTER 9 - COMPOSITE CONSTRUCTION

**9.1-1**

(a) $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \dfrac{E_s}{E_c} = \dfrac{29,000}{3492} = 8.3 \quad \text{use } 8$

$$\frac{b}{n} = \frac{32}{8} = 4.0 \text{ in.}$$

For a W16 × 26, $A_s = 7.68 \text{ in.}^2, \; d = 15.7 \text{ in.}, I_x = 301 \text{ in.}^4$

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Slab | 16 | 2 | 32 | 21.33 | 3.194 | 184.6 |
| W | 7.68 | 11.85 | 91.01 | 301 | 6.656 | 641.2 |
| Sum | 23.68 | | 123.0 | | | 825.8 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{123.0}{23.68} = 5.194 \text{ in.}, \qquad I_{tr} = 825.8 \text{ in.}^4$$

(b) Top of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(150 \times 12)(5.194 - 4)}{825.8} = \underline{2.60 \text{ ksi (compression)}}$$

Bottom of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(150 \times 12)(4 + 15.7 - 5.194)}{825.8} = \underline{31.6 \text{ ksi (tension)}}$$

Top of slab:

$$f_c = \frac{M\bar{y}}{nI_{tr}} = \frac{(150 \times 12)(5.194)}{8(825.8)} = \underline{1.42 \text{ ksi (compression)}}$$

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**9.1-2**

(a) $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \dfrac{E_s}{E_c} = \dfrac{29,000}{3492} = 8.3 \quad \text{use } 8$

$$\frac{b}{n} = \frac{81}{8} = 10.13 \text{ in.}$$

For a W14 × 22, $A_s = 6.49 \text{ in.}^2, \; d = 13.7 \text{ in.}, I_x = 199 \text{ in.}^4$

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I}+Ad^2$ |
|---|---|---|---|---|---|---|
| Slab | 5.063E+01 | 2.500E+00 | 1.266E+02 | 1.055E+02 | 1.063E+00 | 1.627E+02 |
| W | 6.490E+00 | 1.185E+01 | 7.691E+01 | 1.990E+02 | 8.287E+00 | 6.447E+02 |
| Sum | 5.712E+01 | | 2.035E+02 | | | 8.074E+02 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{203.5}{57.12} = 3.563 \text{ in.,} \qquad I_{tr} = 807.4 \text{ in.}^4$$

(b) Top of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(135 \times 12)(5 - 3.563)}{807.4} = \qquad \underline{2.88 \text{ ksi (compression)}}$$

Bottom of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(135 \times 12)(5 + 13.7 - 3.563)}{825.8} = \qquad \underline{29.7 \text{ ksi (tension)}}$$

Top of slab:

$$f_c = \frac{M\bar{y}}{nI_{tr}} = \frac{(135 \times 12)(3.563)}{8(807.4)} = \qquad \underline{0.894 \text{ ksi (compression)}}$$

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**9.1-3**

(a) $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \frac{E_s}{E_c} = \frac{29,000}{3492} = 8.3 \quad \text{use } 8$

$$\frac{b}{n} = \frac{108}{8} = 13.5 \text{ in.}$$

For a W21 × 57, $A_s = 16.7 \text{ in.}^2$, $d = 21.1 \text{ in.}$, $I_x = 1170 \text{ in.}^4$

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I}+Ad^2$ |
|---|---|---|---|---|---|---|
| Slab | 81 | 3 | 243 | 243 | 2.316 | 677.5 |
| W | 16.7 | 16.55 | 276.4 | 1170 | 11.23 | 3276 |
| Sum | 97.7 | | 519.4 | | | 3954 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{519.4}{97.7} = 5.316 \text{ in.,} \qquad I_{tr} = 3954 \text{ in.}^4$$

(b) Top of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{M(5.316 - 6)}{3954} = -1.730 \times 10^{-4} M$$

Bottom of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{M(6 + 21.1 - 5.316)}{3954} = 5.509 \times 10^{-3}M$$

For $M$ in ft-kips, Max. $f_s = 5.509 \times 10^{-3}M \times 12 = 0.0661M$ ksi

Max $f_s = 0.0661M$ ksi

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**9.1-4**

Determine location of plastic neutral axis:

$$A_sF_y = 7.68(50) = 384.0 \text{ kips}, \quad 0.85\, f_c{'}bt = 0.85(4)(32)(4) = 435.2 \text{ kips}$$

Since 384.0 kips < 435.2 kips, PNA is in the slab and $C$ = 384 kips.

From $C = T$,

$$0.85f_c{'}ab = A_sF_y$$

$0.85(4)a(32) = 384$, Solution is: $\{a = 3.529\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.7}{2} + 4 - \frac{3.5298}{2} = 10.09 \text{ in.}$$

$M_n = Ty = 384(10.09) = 3875$ in.-kips = 323 ft-kips $\qquad$ $M_n = 323$ ft-kips

---

**9.1-5**

Determine location of plastic neutral axis:

$$A_sF_y = 6.49(50) = 324.5 \text{ kips}, \quad 0.85\, f_c{'}bt = 0.85(4)(81)(5) = 1377 \text{ kips}$$

Since 324.5 kips < 1377 kips, PNA is in the slab and $C$ = 324.5 kips.

From $C = T$,

$$0.85f_c{'}ab = A_sF_y$$

$0.85(4)a(81) = 324.5$, Solution is: $\{a = 1.178\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{13.7}{2} + 5 - \frac{1.178}{2} = 11.26 \text{ in.}$$

$M_n = Ty = 324.5(11.26) = 3654$ in.-kips = 304 ft-kips $\qquad$ $M_n = 304$ ft-kips

---

**9.1-6**

Determine location of plastic neutral axis:

$$A_sF_y = 16.7(50) = 835.0 \text{ kips}, \quad 0.85\, f_c{}'bt = 0.85(4)(108)(6) = 2203 \text{ kips}$$

Since 835.0 kips < 2203 kips, PNA is in the slab and $C$ = 835 kips.

From $C = T$,

$$0.85 f_c{}'ab = A_sF_y$$

$$0.85(4)a(108) = 835, \text{ Solution is: } \{a = 2.274\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{21.1}{2} + 6 - \frac{2.274}{2} = 15.41 \text{ in.}$$

$$M_n = Ty = 835(15.41) = 1.287 \times 10^4 \text{ in.-kips} = 1070 \text{ ft-kips}$$

$\underline{M_n = 1070 \text{ ft-kips}}$

---

**9.2-1**

Determine location of plastic neutral axis:

$$A_sF_y = 10.3(50) = 515.0 \text{ kips}, \quad 0.85\, f_c{}'bt = 0.85(4)(96)(4.5) = 1469 \text{ kips}$$

Since 515 kips < 1469 kips, PNA is in the slab and $C$ = 515 kips.

From $C = T$,

$$0.85 f_c{}'ab = A_sF_y$$

$$0.85(4)a(96) = 515, \text{ Solution is: } \{a = 1.578\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{17.7}{2} + 4.5 - \frac{1.578}{2} = 12.56 \text{ in.}$$

$$M_n = Ty = 515(12.56) = 6468 \text{ in.-kips}$$

Loads: before concrete cures:

$$\text{Slab: } \frac{4.5}{12}(150)(8) = 450.0 \text{ lb/ft}$$

$$w_D = 450 + 35 = 485 \text{ lb/ft}, \qquad w_L = 20(8) = 160 \text{ lb/ft}$$

After concrete cures:

$w_D = 485$ lb/ft, $w_L = 160(8) = 1280$ lb/ft

(a) LRFD: $\phi_b M_n = 0.90(6468)/12 = 485$ ft-kips

Before concrete cures:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.485) + 1.6(0.160) = 0.838 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.838)(35)^2 = 128 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_n = \phi_b M_p = 249$ ft-kips $> 128$ ft-kips (OK)

After concrete cures:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.485) + 1.6(1.280) = 2.63 \text{ k/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.63)(35)^2 = 403 \text{ ft-kips} < 485 \text{ ft-kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) ASD: $\frac{M_n}{\Omega_b} = \frac{6468}{1.67(12)} = 323$ ft-kips

Before concrete cures:

$$w_a = w_D + w_L = 0.485 + 0.160 = 0.645 \text{ k/ft}$$

$$M_a = \frac{1}{8}(0.645)(35)^2 = 98.8 \text{ ft-kips}$$

From the $Z_x$ table, $M_n/\Omega_b = M_p/\Omega_b = 166$ ft-kips $> 98.8$ ft-kips (OK)

After concrete cures:

$$w_a = w_D + w_L = 0.485 + 1.280 = 1.765 \text{ k/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.765)(35)^2 = 270 \text{ ft-kips} < 323 \text{ ft-kips} \qquad \text{(OK)}$$

Beam is satisfactory

---

**9.2-2**

Determine location of plastic neutral axis:

$$A_s F_y = 6.48(50) = 324.0 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(90)(4) = 1224 \text{ kips}$$

Since 324 kips < 1224 kips, PNA is in the slab and $C = 324$ kips.

From $C = T$,

$$0.85 f_c' ab = A_s F_y$$

$0.85(4)a(90) = 324$, Solution is: $\{a = 1.059\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{12.3}{2} + 4 - \frac{1.059}{2} = 9.621 \text{ in.}$$

$$M_n = Ty = 324(9.621) = 3117 \text{ in.-kips}$$

Loads: before concrete cures:

Slab: $\frac{4}{12}(150)(9) = 450.0$ lb/ft

$w_D = 450 + 22 = 472$ lb/ft, $\quad w_L = 20(9) = 180$ lb/ft

After concrete cures, $\quad w_D = 472$ lb/ft, $w_L = 100(9) = 900$ lb/ft

(a) LRFD: $\quad \phi_b M_n = 0.90(3117)/12 = 233.8$ ft-kips

Before concrete cures:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.472) + 1.6(0.180) = 0.8544 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.8544)(30)^2 = 96.1 \text{ ft-kips}$$

For $L_b = 30$ ft, $L_b > L_r = 9.13$ ft, so

$M_n = F_{cr} S_x \le M_p \qquad$ (elastic LTB)

where

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

From Figure 5.13, $C_b = 1.14$.

$$F_{cr} = \frac{1.14\pi^2(29000)}{\left(\frac{30 \times 12}{1.04}\right)^2} \sqrt{1 + 0.078 \frac{0.293(1.0)}{25.4(11.9)} \left(\frac{30 \times 12}{1.04}\right)^2} = 8.637 \text{ ksi}$$

So $M_n = F_{cr} S_x = 8.637(25.4) = 219.4$ in.-kips $= 18.28$ ft-kips

$$M_p = F_y Z_x = 50(29.3) = 1465 \text{ in.-kips}$$

$M_n < M_p \qquad$ (OK)

$$\phi_b M_n = 0.90(18.28) = 16.5 \text{ ft-kips} < 96.1 \text{ ft-kips} \qquad \text{(N.G.)}$$

Beam is unsatisfactory

(b) ASD:

Before concrete cures:

$$w_a = w_D + w_L = 0.472 + 0.180 = 0.652 \text{ k/ft}$$

$$M_a = \frac{1}{8}(0.652)(30)^2 = 73.4 \text{ ft-kips}$$

From part (a), $M_n = 18.28$ ft-kips

$$\frac{M_n}{\Omega} = \frac{18.28}{1.67} = 11.0 \text{ ft-kips} < 73.4 \text{ ft-kips} \qquad \text{(N.G.)}$$

Beam is unsatisfactory

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**9.3-1**

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{4}{12}\right)(150) = 50 \text{ psf}, \qquad 50(6) = 300 \text{ lb/ft}$$

$$w_D = 300 + 16 = 316 \text{ lb/ft}, \qquad w_L = 20(6) = 120 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 316 \text{ lb/ft}, \qquad w_L = (125 + 15)(6) = 840 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(25 \times 12)/4 = 75$ in. or $6(12) = 72$ in., use $b = 72$ in.

$$A_s F_y = 4.71(50) = 235.5 \text{ kips}, \qquad 0.85 f'_c A_c = 0.85(4)(4 \times 72) = 979.2 \text{ kips}$$

Use $C = 235.5$ kips.

$$a = \frac{C}{0.85 f'_c b} = \frac{235.5}{0.85(4)(72)} = 0.962 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{12.0}{2} + 4 - \frac{0.962}{2} = 9.519 \text{ in.}$$

$$M_n = Cy = 235.5(9.519) = 2242 \text{ in.-kips} = 186.8 \text{ ft-kips}$$

(a) LRFD solution

Before the concrete cures:

From the $Z_x$ table, $\phi_b M_n = \phi_b M_p = 75.4$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(316) + 1.6(120) = 571.2 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(0.5712)(25)^2 = 44.6 \text{ ft-kips} < 75.4 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures, $\phi_b M_n = 0.90(186.8) = 168.1$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(316) + 1.6(840) = 1723 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(1.723)(25)^2 = 135 \text{ ft-kips} < 168 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_v V_n = 79.1$ kips

$$V_u = \frac{w_u L}{2} = \frac{1.723(25)}{2} = 21.5 \text{ kips} < 79.1 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) ASD solution

Before the concrete cures:

From the $Z_x$ table, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 50.1$ ft-kips

$$w_a = w_D + w_L = 316 + 120 = 436 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.436)(25)^2 = 34.1 \text{ ft-kips} < 50.1 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{186.8}{1.67} = 112 \text{ ft-kips}$$

$$w_a = w_D + w_L = 316 + 840 = 1156 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(1.156)(25)^2 = 90.3 \text{ ft-kips} < 112 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\dfrac{V_n}{\Omega_v} = 52.8$ kips

$$V_a = \frac{w_a L}{2} = \frac{1.156(25)}{2} = 14.5 \text{ kips} < 52.8 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

---

**9.3-2**

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{5}{12}\right)(150) = 62.5 \text{ psf}, \qquad 62.5(8) = 500.0 \text{ lb/ft}$$

$$w_D = 500 + 40 = 540 \text{ lb/ft}, \qquad w_L = 20(8) = 160 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 540 + 10(8) = 620 \text{ lb/ft}, \qquad w_L = (150 + 20)(8) = 1360 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(40.67 \times 12)/4 = 122.0$ in. or $8(12) = 96$ in., use $b = 96$ in.

$$A_s F_y = 11.8(50) = 590.0 \text{ kips}, \qquad 0.85 f'_c A_c = 0.85(4)(5 \times 96) = 1632 \text{ kips}$$

Use $C = 590$ kips.

$$a = \frac{C}{0.85 f'_c b} = \frac{590}{0.85(4)(96)} = 1.808 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{17.9}{2} + 5 - \frac{1.808}{2} = 13.05 \text{ in.}$$

$$M_n = Cy = 590(13.05) = 7700 \text{ in.-kips} = 641.7 \text{ ft-kips}$$

(a) LRFD solution

Before the concrete cures:

From the $Z_x$ table, $\phi_b M_n = \phi_b M_p = 294$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(540) + 1.6(160) = 904.0 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(0.9040)(40.67)^2 = 187 \text{ ft-kips} < 294 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_b M_n = 0.90(641.7) = 578 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(620) + 1.6(1360) = 2920 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(2.920)(40.67)^2 = 604 \text{ ft-kips} > 578 \text{ ft-kips} \qquad \text{(N.G.)}$$

Beam is unsatisfactory

(b) ASD solution

Before the concrete cures:

From the $Z_x$ table, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 196$ ft-kips

$$w_a = w_D + w_L = 540 + 160 = 700 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(0.700)(40.67)^2 = 145 \text{ ft-kips} < 196 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{641.7}{1.67} = 384 \text{ ft-kips}$$

$$w_a = w_D + w_L = 620 + 1360 = 1980 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(1.980)(40.67)^2 = 409 \text{ ft-kips} > 384 \text{ ft-kips} \qquad \text{(N.G.)}$$

Beam is unsatisfactory

---

**9.4-1**

Loads applied before the concrete cures:

slab weight = $\left(\frac{6}{12}\right)(150) = 75$ psf, $\quad 75(9) = 675$ lb/ft

$$w_D = 675 + 57 = 732 \text{ lb/ft}, \qquad w_L = 20(9) = 180 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 732 \text{ lb/ft}, \qquad w_L = 250(9) = 2250 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(40 \times 12)/4 = 120$ in. or $9(12) = 108$ in., use $b = 108$ in.

$$A_s F_y = 16.7(50) = 835.0 \text{ kips}, \qquad 0.85 f_c' A_c = 0.85(4)(6 \times 108) = 2203 \text{ kips}$$

Use $C = 835$ kips.

$$a = \frac{C}{0.85 f_c' b} = \frac{835}{0.85(4)(108)} = 2.274 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{21.1}{2} + 6 - \frac{2.274}{2} = 15.41 \text{ in.}$$

$$M_n = Cy = 835(15.41) = 12{,}870 \text{ in.-kips} = 1073 \text{ ft-kips}$$

(a) Before the concrete cures:

From the $Z_x$ table, $\phi_b M_n = \phi_b M_p = 484$ ft-kips

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(732) + 1.6(180) = 1166 \text{ lb/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(1.166)(40)^2 = 233 \text{ ft-kips} < 484 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_b M_n = 0.90(1073) = 966 \text{ ft-kips}$$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(732) + 1.6(2250) = 4478 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(4.478)(40)^2 = 896 \text{ ft-kips} < 966 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_v V_n = 256$ kips

$$V_u = \frac{w_u L}{2} = \frac{4.478(40)}{2} = 89.6 \text{ kips} < 256 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) Before the concrete cures:

From the $Z_x$ table, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 322$ ft-kips

$w_a = w_D + w_L = 732 + 180 = 912$ lb/ft

$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.912)(40)^2 = 182$ ft-kips $< 322$ ft-kips (OK)

After the concrete cures,

$\frac{M_n}{\Omega_b} = \frac{1073}{1.67} = 643$ ft-kips

$w_a = w_D + w_L = 732 + 2250 = 2982$ lb/ft

$M_a = \frac{1}{8}(2.982)(40)^2 = 596$ ft-kips $< 643$ ft-kips (OK)

Shear: $\frac{V_n}{\Omega_v} = 171$ kips

$V_a = \frac{w_a L}{2} = \frac{2.982(40)}{2} = 59.6$ kips $< 171$ kips (OK)

Beam is satisfactory

(c) $A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418$ in.$^2$, $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492$ ksi

$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$

$= 0.5(0.4418)\sqrt{4(3492)} = 26.11$ kips

$R_g R_p A_{sa} F_u = 1.0(0.75)(0.4418)(65) = 21.54$

$= 21.54$ kips $< 26.11$ kips $\therefore$ use $Q_n = 21.54$ kips

$N_1 = \frac{V'}{Q_n} = \frac{835}{21.54} = 38.8$, round up to 39.

total number $= 2(39) = 78$ Use 78 studs

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**9.4-2**

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{4}{12}\right)(150) = 50.0 \text{ psf}, \qquad 50(8) = 400 \text{ lb/ft}$$

$$w_D = 400 + 22 = 422 \text{ lb/ft}, \qquad w_L = 20(8) = 160 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 422 \text{ lb/ft}, \qquad w_L = (120 + 20)(8) = 1120 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(27 \times 12)/4 = 81.0$ in. or $8(12) = 96$ in., use $b = 81$ in.

$$A_sF_y = 6.49(50) = 324.5 \text{ kips}, \qquad 0.85f'_cA_c = 0.85(4)(4 \times 81) = 1102 \text{ kips}$$

Use $C = 324.5$ kips.

$$a = \frac{C}{0.85f'_cb} = \frac{324.5}{0.85(4)(81)} = 1.178 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{13.7}{2} + 4 - \frac{1.178}{2} = 10.26 \text{ in.}$$

$$M_n = Cy = 324.5(10.26) = 3329 \text{ in.-kips} = 277.4 \text{ ft-kips}$$

(a) Before the concrete cures:

From the $Z_x$ table, $\phi_bM_n = \phi_bM_p = 125$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(422) + 1.6(160) = 762.4 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(0.7624)(27)^2 = 69.47 \text{ ft-kips} < 125 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_bM_n = 0.90(277.4) = 250 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(422) + 1.6(1120) = 2298 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(2.298)(27)^2 = 209 \text{ ft-kips} < 250 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_vV_n = 94.5$ kips

$$V_u = \frac{w_u L}{2} = \frac{2.298(27)}{2} = 31.0 \text{ kips} < 94.5 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) Before the concrete cures:

From the $Z_x$ table, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 82.8$ ft-kips

$$w_a = w_D + w_L = 422 + 160 = 582 \text{ lb/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(0.582)(27)^2 = 53.0 \text{ ft-kips} < 82.8 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{277.4}{1.67} = 166 \text{ ft-kips}$$

$$w_a = w_D + w_L = 422 + 1120 = 1542 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(1.542)(27)^2 = 141 \text{ ft-kips} < 166 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\dfrac{V_n}{\Omega_v} = 63.0$ kips

$$V_a = \frac{w_a L}{2} = \frac{1.542(27)}{2} = 20.8 \text{ kips} < 63.0 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(c) $A_{sa} = \dfrac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$, $\qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492$ ksi

$$Q_n = 0.5 A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.4418)(65) = 21.54$$

$$= 21.54 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 21.54 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{324.5}{21.54} = 15.1, \text{ round up to 16.}$$

total number = 2(16) = 32 $\qquad$ Use 32 studs

## 9.4-3

$A_s F_y = 7.68(50) = 384.0$ kips, $\quad 0.85 f'_c A_c = 0.85(4)(4 \times 32) = 435.2$ kips

$C = V' = 384$ kips.

$$A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5 A_{sa}\sqrt{f'_c E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.4418)(65) = 21.54$$

$$= 21.54 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 21.54 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{384}{21.54} = 17.83, \text{ round up to 18.total number} = 2(18) = 36$$

Use 36 studs

## 9.4-4

$A_s F_y = 6.49(50) = 324.5$ kips,

$0.85 f'_c A_c = 0.85(4)(5 \times 81) = 1377$ kips

$C = V' = 324.5$ kips.

For $\frac{7}{8}$ in. $\times\ 3\frac{1}{2}$ in. studs,

$$A_{sa} = \frac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5 A_{sa}\sqrt{f'_c E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.603)\sqrt{4(3492)} = 35.63 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.6013)(65)$$

$$= 29.31 \text{ kips} < 35.63 \text{ kips} \qquad \therefore \text{ use } Q_n = 29.31 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{324.5}{29.31} = 11.07\text{, round up to 12.}$$

Total number = 2(12) = <u>24 studs</u>

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**<u>9.5-1</u>**

(a) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{4.5}{12}(150) = 56.25$ psf

$$w_D = 56.25(6.5) = 365.6 \text{ lb/ft}, \;\; w_L = 175(6.5) = 1138 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.3656) + 1.6(1.138) = 2.260 \text{ k/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.260)(36)^2 = 366.1 \text{ ft-kips}$$

Try a 16-in. deep beam. Selection of a trial shape:

$$w = \frac{3.4M_u}{\phi F_y(d/2 + t - a/2)} = \frac{3.4(366.1 \times 12)}{0.90(50)(16/2 + 4.5 - 0.5)} = 27.66 \text{ lb/ft}$$

Try a W16 × 31.

Determine location of plastic neutral axis.

Effective flange width = (36 × 12)/4 = 108 in. or 6.5(12) = 78 in. (controls)

$$A_sF_y = 9.13(50) = 456.5 \text{ kips}, \quad 0.85 f_c'bt = 0.85(4)(78)(4.5) = 1193 \text{ kips}$$

Since 456.5 kips < 1193 kips, PNA is in the slab and $C$ = 456.5 kips.

From $C = T$,

$$0.85f_c'ab = A_sF_y$$

$$0.85(4)a(78) = 456.5\text{, Solution is: } \{a = 1.721\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.9}{2} + 4.5 - \frac{1.721}{2} = 11.59 \text{ in.}$$

$$\phi_b M_n = \phi_b Ty = 0.90(456.5)(11.59) = 4762 \text{ in.-kips} = 397 \text{ ft-kips} > 366 \text{ ft-kips}$$

Check beam weight:

$$w_u = 1.2(0.3656 + 0.031) + 1.6(1.138) = 2.297 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(2.297)(36)^2 = 372 \text{ ft-kips} < 397 \text{ ft-kips (OK)}$$

Check shear. From the $Z_x$ tables, $\phi_v V_n = 131$ kips

$$V_u = \frac{w_u L}{2} = \frac{2.297(36)}{2} = 41.4 \text{ kips} < 131 \text{ kips (OK)}$$

Before concrete cures:

$$w_D = 365.6 + 31 = 396.6 \text{ lb/ft}, \qquad w_L = 20(6.5) = 130.0 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.3966) + 1.6(0.130) = 0.6839 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.6839)(36)^2 = 111 \text{ ft-kips}$$

$\phi_b M_n = \phi_b M_p = 203$ ft-kips >111 ft-kips (OK) <u>Use a W16 × 31</u>

(b) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{4.5}{12}(150) = 56.25$ psf

$$w_D = 56.25(6.5) = 365.6 \text{ lb/ft}, \quad w_L = 175(6.5) = 1138 \text{ lb/ft}$$

$$w_a = w_D + w_L = 365.6 + 1138 = 1504 \text{ k/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.504)(36)^2 = 243.6 \text{ ft-kips}$$

Try a 16-in. deep beam. Selection of a trial shape:

$$w = \frac{3.4\Omega_b M_a}{F_y(d/2 + t - a/2)} = \frac{3.4(1.67)(243.6 \times 12)}{50(16/2 + 4.5 - 0.5)} = 27.7 \text{ lb/ft}$$

Try a W16 × 31.

Determine location of plastic neutral axis.

Effective flange width = $(36 \times 12)/4 = 108$ in. or $6.5(12) = 78$ in. (controls)

$$A_s F_y = 9.13(50) = 456.5 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(78)(4.5) = 1193 \text{ kips}$$

Since 456.5 kips < 1193 kips, PNA is in the slab and $C$ = 456.5 kips.

From $C = T$,

$$0.85 f_c' ab = A_s F_y$$

$$0.85(4)a(78) = 456.5, \text{ Solution is: } \{a = 1.721\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.9}{2} + 4.5 - \frac{1.721}{2} = 11.59 \text{ in.}$$

$$\frac{M_n}{\Omega_b} = \frac{1}{\Omega_b} Ty = \frac{1}{1.67}(456.5)(11.59) = 3168 \text{ in.-kips} = 264 \text{ ft-kips} > 243.6$$
ft-kips

Check beam weight:

$$w_a = 1504 + 31 = 1535 \text{ lb/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.535)(36)^2 = 248.7 \text{ ft-kips} < 264 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear. From the $Z_x$ tables,

$$\frac{V_n}{\Omega_v} = 87.5 \text{ kips}$$

$$V_a = \frac{w_a L}{2} = \frac{1.535(36)}{2} = 27.6 \text{ kips} < 87.5 \text{ kips} \quad \text{(OK)}$$

Before concrete cures:

$$w_D = 365.6 + 31 = 396.6 \text{ lb/ft}, \qquad w_L = 20(6.5) = 130.0 \text{ lb/ft}$$

$$w_a = 396.6 + 130.0 = 526.6 \text{ lb//ft}$$

$$M_a = \frac{1}{8}(0.5266)(36)^2 = 85.3 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 135 \text{ ft-kips} > 85.3 \text{ ft-kips} \quad \text{(OK)}$$

Use a W16 × 31

(c) Max. stud diameter (for a W16 × 31) = $2.5t_f = 2.5(0.440) = 1.1$ in.

Try ½ in. × 2 in. studs.

$$A_{sa} = \frac{\pi(1/2)^2}{4} = 0.1963 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.1963)\sqrt{4(3492)} = 11.60 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.1963)(65)$$

$$= 9.570 \text{ kips} < 11.60 \text{ kips} \qquad \therefore \text{ use } Q_n = 9.570 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{456.5}{9.570} = 47.7\text{, round up to 48. total number} = 2(48) = 96$$

Min. longitudinal spacing = $6d = 6(0.5) = 3.0$ in.

Min. transverse spacing = $4d = 4(0.5) = 2.0$ in.

Max. longitudinal spacing = $8t = 8(4.5) = 36.0$ in. (upper limit = 36 in.)

For one stud at each section, the required spacing will be

$$s = \frac{\text{span}}{\text{no. studs}} = \frac{36(12)}{96} = 4.5 \text{ in.} \qquad \text{(OK)}$$

Use 96 studs, ½ in. × 2 in., spaced at 4½ in. on center

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**9.5-2**

(a) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{4}{12}(150) = 50.0$ psf

$$w_D = 50.0(5) = 250 \text{ lb/ft}, \quad w_L = (125 + 20)(5) = 725 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.250) + 1.6(0.725) = 1.46 \text{ k/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.46)(40)^2 = 292 \text{ ft-kips}$$

Try a 14-in. deep beam. Selection of a trial shape:

$$w = \frac{3.4M_u}{\phi F_y(d/2 + t - a/2)} = \frac{3.4(292 \times 12)}{0.90(50)(14/2 + 4 - 0.5)} = 25.2 \text{ lb/ft}$$

Try a W14 × 30.

Determine location of plastic neutral axis.

Effective flange width = $(40 \times 12)/4 = 120$ in. or $5(12) = 60$ in. (controls)

$$A_s F_y = 8.85(50) = 442.5 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(60)(4) = 816.0 \text{ kips}$$

Since 442.5 kips < 816.0 kips, PNA is in the slab and $C$ = 442.5 kips.

From $C = T$,

$$0.85 f_c' ab = A_s F_y$$

$0.85(4)a(60) = 442.5$, Solution is: $\{a = 2.169\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{13.8}{2} + 4 - \frac{2.169}{2} = 9.816 \text{ in.}$$

$\phi_b M_n = \phi_b Ty = 0.90(442.5)(9.816)/12 = 326$ ft-kips > 292 ft-kips (OK)

Check beam weight:

$w_u = 1.2(0.250 + 0.030) + 1.6(0.725) = 1.496$ lb/ft

$$M_u = \frac{1}{8}(1.496)(40)^2 = 299 \text{ ft-kips} < 326 \text{ ft-kips (OK)}$$

Check shear. From the $Z_x$ tables, $\phi_v V_n = 112$ kips

$$V_u = \frac{w_u L}{2} = \frac{1.496(40)}{2} = 29.9 \text{ kips} < 112 \text{ kips (OK)}$$

Before concrete cures:

$w_D = 250 + 30 = 280$ lb/ft, $w_L = 20(5) = 100$ lb/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(0.280) + 1.6(0.100) = 0.496$ k/ft

$$M_u = \frac{1}{8}(0.496)(40)^2 = 99.2 \text{ ft-kips}$$

$\phi_b M_n = \phi_b M_p = 177$ ft-kips > 99.2 ft-kips (OK) <u>Use a W14 × 30</u>

(b) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{4}{12}(150) = 50.0$ psf

$w_D = 50.0(5) = 250$ lb/ft, $w_L = (125 + 20)(5) = 725$ lb/ft

$w_a = w_D + w_L = 250 + 725 = 975$ k/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.975)(40)^2 = 195.0 \text{ ft-kips}$$

Try a 14-in. deep beam. Selection of a trial shape:

$$w = \frac{3.4\Omega_b M_a}{F_y(d/2 + t - a/2)} = \frac{3.4(1.67)(195.0 \times 12)}{50(14/2 + 4 - 0.5)} = 25.31 \text{ lb/ft}$$

Try a W14 × 30.

Determine location of plastic neutral axis.

Effective flange width = $(40 \times 12)/4 = 120$ in. or $5(12) = 60$ in. (controls)

$$A_sF_y = 8.85(50) = 442.5 \text{ kips}, \quad 0.85\, f_c'bt = 0.85(4)(60)(4) = 816.0 \text{ kips}$$

Since 442.5 kips < 816.0 kips, PNA is in the slab and $C$ = 442.5 kips.

From $C = T$,

$$0.85 f_c'ab = A_sF_y$$

$$0.85(4)a(60) = 442.5, \text{ Solution is: } \{a = 2.169\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{13.8}{2} + 4 - \frac{2.169}{2} = 9.816 \text{ in.}$$

$$\frac{M_n}{\Omega_b} = \frac{1}{\Omega_b}Ty = \frac{1}{1.67}(442.5)(9.816) = 2601 \text{ in.-kips} = 217 \text{ ft-kips} > 195 \text{ ft-kips}$$

Check beam weight:

$$w_a = 975 + 30 = 1005 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(1.005)(40)^2 = 201.0 \text{ ft-kips} < 217 \text{ ft-kips} \quad \text{(OK)}$$

Check shear. From the $Z_x$ tables,

$$\frac{V_n}{\Omega_v} = 74.5 \text{ kips}$$

$$V_a = \frac{w_aL}{2} = \frac{1.005(40)}{2} = 20.1 \text{ kips} < 74.5 \text{ kips (OK)}$$

Before concrete cures:

$$w_D = 250 + 30 = 280 \text{ lb/ft}, \quad w_L = 20(5) = 100 \text{ lb/ft}$$

$$w_a = 280 + 100 = 380 \text{ lb//ft}$$

$$M_a = \frac{1}{8}(0.380)(40)^2 = 76.0 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 118 \text{ ft-kips} > 76.0 \text{ ft-kips (OK)}$$

<u>Use a W14 × 30</u>

(c) Max. stud diameter (for a W14 × 30) = $2.5t_f = 2.5(0.385) = 0.963$ in.

Try ½ in. × 2 in. studs.

$$A_{sa} = \frac{\pi(1/2)^2}{4} = 0.1963 \text{ in.}^2, \quad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.1963)\sqrt{4(3492)} = 11.60 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.1963)(65)$$

$$= 9.570 \text{ kips} < 11.60 \text{ kips} \qquad \therefore \text{ use } Q_n = 9.570 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{442.5}{9.570} = 46.2\text{, round up to 47. total number} = 2(47) = 94$$

Min. longitudinal spacing = $6d = 6(0.5) = 3.0$ in.

Min. transverse spacing = $4d = 4(0.5) = 2.0$ in.

Max. longitudinal spacing = $8t = 8(4) = 32$ in. (upper limit = 36 in.)

For one stud at each section, the required spacing will be

$$s = \frac{\text{span}}{\text{no. studs}} = \frac{40(12)}{94} = 5.10 \text{ in.} \qquad \text{(OK)}$$

<u>Use 94 studs, ½ in. × 2 in., spaced at approximately 5 in. on center</u>

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**9.5-3**

(a) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{5}{12}(150) = 62.5$ psf

$$w_D = 62.5(7) = 437.5 \text{ lb/ft}, \quad w_L = 800(7) = 5600 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.4375) + 1.6(5.600) = 9.485 \text{ k/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(9.485)(30)^2 = 1067 \text{ ft-kips}$$

For a 16-in. deep beam,

$$w = \frac{3.4M_u}{\phi F_y(d/2 + t - a/2)} = \frac{3.4(1067 \times 12)}{0.90(50)(16/2 + 5 - 0.5)} = 77.39 \text{ lb/ft}$$

For an 18-in. deep beam,

$$w = \frac{3.4M_u}{\phi F_y(d/2 + t - a/2)} = \frac{3.4(1067 \times 12)}{0.90(50)(18/2 + 5 - 0.5)} = 71.66 \text{ lb/ft}$$

Try a W18 × 76.

Determine location of plastic neutral axis.

Effective flange width = (30 × 12)/4 = 90 in. or 7(12) = 84 in. (controls)

$$A_sF_y = 22.3(50) = 1115 \text{ kips}, \quad 0.85\, f_c'bt = 0.85(4)(84)(5) = 1428 \text{ kips}$$

Since 1115 kips < 1428 kips, PNA is in the slab and $C$ = 1115 kips.

From $C = T$,

$$0.85f_c'ab = A_sF_y$$

$$0.85(4)a(84) = 1115, \text{ Solution is: } \{a = 3.904\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{18.2}{2} + 5 - \frac{3.904}{2} = 12.15 \text{ in.}$$

$$\phi_bM_n = \phi_bTy = 0.90(1115)(12.15)/12 = 1016 \text{ ft-kips} < 1067 \text{ ft-kips} \qquad \text{(N.G.)}$$

Try a W18 × 86.

$$A_sF_y = 25.3(50) = 1265 \text{ kips}, \quad 0.85\, f_c'bt = 0.85(4)(84)(5) = 1428 \text{ kips}$$

Use $C$ = 1265 kips.

From $C = T$,

$$0.85f_c'ab = A_sF_y$$

$$0.85(4)a(84) = 1265, \text{ Solution is: } \{a = 4.429\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{18.4}{2} + 5 - \frac{4.429}{2} = 11.99 \text{ in.}$$

$$\phi_bM_n = \phi_bTy = 0.90(1265)(11.99)/12 = 1138 \text{ ft-kips} > 1067 \text{ ft-kips} \qquad \text{(OK)}$$

Check beam weight:

$$w_u = 1.2(0.4375 + 0.086) + 1.6(5.600) = 9.588 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(9.588)(30)^2 = 1079 \text{ ft-kips} < 1138 \text{ ft-kips (OK)}$$

Check shear. From the $Z_x$ tables, $\phi_vV_n = 265$ kips

$$V_u = \frac{w_uL}{2} = \frac{9.588(30)}{2} = 144 \text{ kips} < 265 \text{ kips (OK)}$$

Before concrete cures:

$$w_D = 437.5 + 86 = 523.5 \text{ lb/ft}, \qquad w_L = 20(7) = 140 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.5235) + 1.6(0.140) = 0.8522 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.8522)(30)^2 = 95.9 \text{ ft-kips}$$

$\phi_b M_n = \phi_b M_p = 698$ ft-kips > 95.9 ft-kips (OK) <u>Use a W18 × 86</u>

(b) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{5}{12}(150) = 62.5$ psf

$$w_D = 62.5(7) = 437.5 \text{ lb/ft}, \; w_L = 800(7) = 5600 \text{ lb/ft}$$

$$w_a = w_D + w_L = 437.5 + 5600 = 6038 \text{ k/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(6.038)(30)^2 = 679.3 \text{ ft-kips}$$

Try an 18-in. deep beam. Selection of a trial shape:

$$w = \frac{3.4\Omega_b M_a}{F_y(d/2 + t - a/2)} = \frac{3.4(1.67)(679.3 \times 12)}{50(18/2 + 5 - 0.5)} = 68.57 \text{ lb/ft}$$

Try a W18 × 86.

Determine location of plastic neutral axis.

Effective flange width = (30 × 12)/4 = 90 in. or 7(12) = 84 in. (controls)

$$A_s F_y = 25.3(50) = 1265 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(84)(5) = 1428 \text{ kips}$$

Use $C = 1265$ kips.

From $C = T$,

$$0.85 f_c' ab = A_s F_y$$

$0.85(4)a(84) = 1265$, Solution is: $\{a = 4.429\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{18.4}{2} + 5 - \frac{4.492}{2} = 11.95 \text{ in.}$$

$$\frac{M_n}{\Omega_b} = \frac{1}{\Omega_b}Ty = \frac{1}{1.67}(1265)(11.95)$$

$$= 9052 \text{ in.-kips} = 754 \text{ ft-kips} > 679.3 \text{ ft-kips}$$

Check beam weight:

$$w_a = 6038 + 86 = 6124 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(6.124)(30)^2 = 689 \text{ ft-kips} < 754 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear. From the $Z_x$ tables,

$$\frac{V_n}{\Omega_v} = 177 \text{ kips}$$

$$V_a = \frac{w_a L}{2} = \frac{6.124(30)}{2} = 91.9 \text{ kips} < 177 \text{ kips (OK)}$$

Before concrete cures:

$$w_D = 437.5 + 86 = 523.5 \text{ lb/ft}, \qquad w_L = 20(7) = 140 \text{ lb/ft}$$

$$w_a = 523.5 + 140 = 663.5 \text{ lb//ft}$$

$$M_a = \frac{1}{8}(0.6635)(30)^2 = 74.6 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 464 \text{ ft-kips} > 74.6 \text{ ft-kips (OK)}$$

Use a W18 × 86

(c) Max. stud diameter (for a W18 × 86) = $2.5t_f = 2.5(0.770) = 1.93$ in.

Try $\frac{5}{8}$ in. × $2\frac{1}{2}$ in. studs.

$$A_{sa} = \frac{\pi(5/8)^2}{4} = 0.3068 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.3068)\sqrt{4(3492)} = 18.13 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.3068)(65) = 14.96$$

$$= 14.96 \text{ kips} < 18.13 \text{ kips} \qquad \therefore \text{ use } Q_n = 14.96 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{1265}{14.96} = 84.56\text{, round up to 85. total number} = 2(85) = 170$$

Min. longitudinal spacing = $6d = 6(5/8) = 3.75$ in.

Min. transverse spacing = $4d = 4(5/8) = 2.5$ in.

Max. longitudinal spacing = $8t = 8(5) = 40$ in. (but upper limit = 36 in.)

For one stud at each section, the required spacing will be

$$s = \frac{\text{span}}{\text{no. studs}} = \frac{30(12)}{170} = 2.12 \text{ in.}$$

For two studs at each section, the required spacing will be $2 \times 2.12 = 4.24$ in.

Try ¾ in. × 3 in. studs.

$$A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.4418)(65)$$

$$= 21.54 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 21.54 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{1265}{21.54} = 58.73, \text{ round up to 59. total number} = 2(59) = 118$$

For one stud at each section, the required spacing will be

$$s = \frac{\text{span}}{\text{no. studs}} = \frac{30(12)}{118} = 3.051 \text{ in.}$$

For two studs at each section, the required spacing will be $2 \times 3.051 = 6.10$ in.

<u>Use 118 studs, ¾ in. × 3 in., spaced at approximately 6 in. on center</u>

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**9.6-1**

(a) Before concrete cures:

$$\text{Slab: } \frac{4.5}{12}(150)(8) = 450.0 \text{ lb/ft}$$

$$w_D = 450 + 35 = 485 \text{ lb/ft}, \qquad w_{const} = 20(8) = 160 \text{ lb/ft}, \qquad I_s = 510 \text{ in.}^4$$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.485/12)(35 \times 12)^4}{384(29000)(510)} = 1.107 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.160/12)(35 \times 12)^4}{384(29000)(510)} = 0.3653 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{const} = 1.107 + 0.3653 = 1.472 \text{ in.} \qquad \underline{\Delta = 1.47 \text{ in.}}$$

(b) After concrete has cured:

Compute the lower-bound moment of inertia.

Determine the compressive force, *C*.

$$A_s F_y = 10.3(50) = 515.0 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(96)(4.5) = 1469 \text{ kips}$$

Since 515 kips < 1469 kips, $C$ = 515 kips.

Area of transformed concrete = $A_c = \dfrac{C}{F_y} = \dfrac{515}{50} = 10.3 \text{ in.}^2$

$$a = \frac{C}{0.85 f_c' b} = \frac{515}{0.85(4)(96)} = 1.578 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4.5 - \frac{1.578}{2} = 3.711 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 1.030E+01 | 2.141E+01 | 2.205E+02 | 0.000E+00 | -6.281E+00 | 4.063E+02 |
| W18 x 35 | 1.030E+01 | 8.850E+00 | 9.116E+01 | 5.100E+02 | -6.281E+00 | 9.163E+02 |
| Sum | 2.060E+01 | | 3.117E+02 | | | 1.323E+03 |

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 1.513\text{E+01 in.}$$

$$I_{\ell b} = 1323 \text{ in.}^4$$

$$w_L = 160(8) = 1280 \text{ lb/ft}$$

$$\Delta = \Delta_D + \Delta_L = \Delta_D + \frac{5(w_L)L^4}{384EI_{\ell b}}$$

$$= 1.107 + \frac{5[1.280/12](35 \times 12)^4}{384(29000)(1323)} = 1.107 + 1.126 = 2.233 \text{ in.}$$

$$\underline{\Delta = 2.23 \text{ in.}}$$

---

**9.6-2**

(a) Before concrete cures:

Slab: $\frac{4}{12}(150)(9) = 450.0$ lb/ft

$w_D = 450 + 22 = 472$ lb/ft, $w_{const} = 20(9) = 180$ lb/ft, $I_s = 156$ in.$^4$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.472/12)(30 \times 12)^4}{384(29000)(156)} = 1.901 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const} L^4}{384EI_s} = \frac{5(0.180/12)(30 \times 12)^4}{384(29000)(156)} = 0.7251 \text{ in.}$$

$\Delta = \Delta_D + \Delta_{const} = 1.901 + 0.7251 = 2.626$ in. $\underline{\Delta = 2.63 \text{ in.}}$

(b) After concrete has cured:

Compute the lower-bound moment of inertia.

Determine the compressive force, $C$.

$A_s F_y = 6.48(50) = 324.0$ kips, $0.85 f_c' bt = 0.85(4)(90)(4) = 1224$ kips

Since 324 kips < 1224 kips, $C = 324$ kips.

Area of transformed concrete = $A_c = \frac{C}{F_y} = \frac{324}{50} = 6.48$ in.$^2$

$$a = \frac{C}{0.85 f_c' b} = \frac{324}{0.85(4)(90)} = 1.059 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4 - \frac{1.059}{2} = 3.471 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 6.480E+00 | 1.577E+01 | 1.022E+02 | 0.000E+00 | -4.811E+00 | 1.500E+02 |
| W24 × 55 | 6.480E+00 | 6.150E+00 | 3.985E+01 | 1.560E+02 | -4.811E+00 | 3.060E+02 |
| Sum | 1.296E+01 | | 1.420E+02 | | | 4.559E+02 |

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 1.096\text{E+01 in.}$$

$I_{\ell b} = 455.9$ in.$^4$, $w_L = 100(9) = 900$ lb/ft

$$\Delta = \Delta_D + \Delta_L = \Delta_D + \frac{5(w_L)L^4}{384EI_{\ell b}}$$

$$= 1.901 + \frac{5[0.900/12](30 \times 12)^4}{384(29000)(455.9)} = 1.901 + 1.241 = 3.142 \text{ in.}$$

$$\underline{\Delta = 3.14 \text{ in.}}$$

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**9.6-3**

(a) From Problem 9.3-1, a W12 × 16 is used, with $t = 4$ in., $s = 6$ ft, $L = 25$ ft, $q_{const} = 20$ psf, $q_{part} = 15$ psf, $q_L = 125$ psf, A992 steel and 4 ksi concrete.

Before concrete cures:

Slab: $\frac{4}{12}(150)(6) = 300$ lb/ft

$w_D = 300 + 16 = 316$ lb/ft, $w_{const} = 20(6) = 120$ lb/ft, $I_s = 103$ in.$^4$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.316/12)(25 \times 12)^4}{384(29000)(103)} = 0.9298 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.120/12)(25 \times 12)^4}{384(29000)(103)} = 0.3531 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{const} = 0.9298 + 0.3531 = 1.283 \text{ in.} \qquad \underline{\Delta = 1.28 \text{ in.}}$$

After concrete has cured:

$w_{part} = 15(6) = 90$ lb/ft, $w_L = 125(6) = 750$ lb/ft

Lower-bound moment of inertia:

Effective flange width = $(25 \times 12)/4 = 75$ in. or $6(12) = 72$ in., use $b = 72$ in.

For a W12 × 16, $A_s = 4.71$ in.$^2$, $d = 12.0$ in., $I_x = 103$ in.$^4$

Determine the compressive force, $C$.

$A_s F_y = 4.71(50) = 235.5$ kips, $0.85 f_c' bt = 0.85(4)(72)(4) = 979.2$ kips

Since 235.5 kips < 979.2 kips, $C$ = 235.5 kips.

Area of transformed concrete = $A_c = \frac{C}{F_y} = \frac{235.5}{50} = 4.71$ in.$^2$

$$a = \frac{C}{0.85 f_c' b} = \frac{235.5}{0.85(4)(72)} = 0.962 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4 - \frac{0.962}{2} = 3.519 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 4.71 | 15.519 | 73.094 | 0.00 | -4.760 | 106.69 |
| W12 x 16 | 4.71 | 6.00 | 28.26 | 103 | -4.760 | 209.7 |
| Sum | 9.42 | | 101.4 | | | 316.4 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{101.4}{9.42} = 10.76 \text{ in.}, \qquad I_{LB} = 316.4 \text{ in.}^4$$

$$\Delta_{part} = \frac{5w_{part}L^4}{384EI_{LB}} = \frac{5(0.090/12)(25 \times 12)^4}{384(29000)(316.4)} = 8.621 \times 10^{-2} \text{ in.}$$

$$\Delta_L = \frac{5w_L L^4}{384EI_{LB}} = \frac{5(0.750/12)(25 \times 12)^4}{384(29000)(316.4)} = 0.7184 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{part} + \Delta_L = 0.9298 + 0.08621 + 0.7184$$

$$= 1.73 \text{ in.} \qquad \underline{\Delta = 1.73 \text{ in.}}$$

(b) Maximum permissible $\Delta_L = \frac{L}{360} = \frac{25 \times 12}{360} = 0.833$ in. > 0.718 in. OK.

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**9.6-4**

(a) From Problem 9.4-1, a W21 × 57 is used, with $t = 6$ in., $s = 9$ ft, $L = 40$ ft, $q_{const} = 20$ psf, $q_L = 250$ psf, A992 steel and 4 ksi concrete.

Before concrete cures:

$$\text{Slab: } \frac{6}{12}(150)(9) = 675.0 \text{ lb/ft}$$

$$w_D = 675 + 57 = 732 \text{ lb/ft}, \qquad w_{const} = 20(9) = 180 \text{ lb/ft}, \qquad I_s = 1170 \text{ in.}^4$$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.732/12)(40 \times 12)^4}{384(29000)(1170)} = 1.243 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.180/12)(40 \times 12)^4}{384(29000)(1170)} = 0.3056 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{const} = 1.243 + 0.3056 = 1.549 \text{ in.} \qquad \underline{\Delta = 1.55 \text{ in.}}$$

After concrete has cured:

$$w_L = 250(9) = 2250 \text{ lb/ft}$$

Lower-bound moment of inertia:

Effective flange width = $(40 \times 12)/4 = 120$ in. or $9(12) = 108$ in.,

use $b = 108$ in.

For a W21 × 57, $A_s = 16.7 \text{ in.}^2$, $d = 21.1$ in., $I_x = 1170 \text{ in.}^4$

Determine the compressive force, $C$.

$$A_s F_y = 16.7(50) = 835.0 \text{ kips}, \quad 0.85 f_c' bt = 0.85(4)(108)(6) = 2203 \text{ kips}$$

Use $C = 835$ kips.

Area of transformed concrete = $A_c = \dfrac{C}{F_y} = \dfrac{835}{50} = 16.7 \text{ in.}^2$

$$a = \frac{C}{0.85 f_c' b} = \frac{835}{0.85(4)(108)} = 2.274 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6 - \frac{2.274}{2} = 4.863 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 1.670E+01 | 2.596E+01 | 4.336E+02 | 0.000E+00 | -7.707E+00 | 9.918E+02 |
| W12 × 22 | 1.670E+01 | 1.055E+01 | 1.762E+02 | 1.170E+03 | -7.707E+00 | 2.162E+03 |
| Sum | 3.340E+01 | | 6.098E+02 | | | 3.154E+03 |

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 1.826\text{E+01 in.}$$

$$I_{LB} = 3154 \text{ in.}^4$$

$$\Delta_L = \frac{5 w_L L^4}{384 E I_{LB}} = \frac{5(2.250/12)(40 \times 12)^4}{384(29000)(3154)} = 1.417 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_L = 1.243 + 1.417 = 2.66 \text{ in.} \qquad \underline{\Delta = 2.66 \text{ in.}}$$

Maximum permissible $\Delta = \dfrac{L}{240} = \dfrac{40 \times 12}{240} = 2.0 \text{ in.} < 2.66$ in. (N.G.)

(b) Try a W24 × 55. USe LRFD.

Determine location of plastic neutral axis.

$$A_sF_y = 16.2(50) = 810.0 \text{ kips}, \quad 0.85\, f_c'bt = 0.85(4)(108)(6) = 2203 \text{ kips}$$

Use $C = 810$ kips.

From $C = T$,

$$0.85f_c'ab = A_sF_y$$

$$0.85(4)a(108) = 810, \text{ Solution is: } \{a = 2.206\} \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{23.6}{2} + 6 - \frac{2.206}{2} = 16.70 \text{ in.}$$

$$\phi_b M_n = \phi_b Ty = 0.90(810)(16.70) = 1.217 \times 10^4 \text{ in.-kips} = 1014 \text{ ft-kips}$$

Loads:

$$w_D = \frac{6}{12}(150)(9) + 55 = 730.0 \text{ lb/ft}, \qquad w_L = 250(9) = 2250 \text{ lb/ft}$$

$$w_u = 1.2(730) + 1.6(2250) = 4476 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(4.476)(40)^2 = 895 \text{ ft-kips} < 1014 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear. From the $Z_x$ tables, $\phi_v V_n = 252$ kips

$$V_u = \frac{w_u L}{2} = \frac{4.476(40)}{2} = 89.5 \text{ kips} < 252 \text{ kips} \quad \text{(OK)}$$

Before concrete cures:

$$w_D = 730 \text{ lb/ft}, \qquad w_L = 20(9) = 180 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.730) + 1.6(0.180) = 1.164 \text{ k/ft}$$

$$M_u = \frac{1}{8}(1.164)(40)^2 = 233 \text{ ft-kips}$$

$$\phi_b M_n = \phi_b M_p = 503 \text{ ft-kips} > 233 \text{ ft-kips} \quad \text{(OK)}$$

Check deflection. Before concrete cures:

$$\text{Slab: } \frac{6}{12}(150)(9) = 675.0 \text{ lb/ft}$$

$$w_D = 675 + 55 = 730 \text{ lb/ft}, \qquad w_{const} = 20(9) = 180 \text{ lb/ft}, \qquad I_s = 1350 \text{ in.}^4$$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.730/12)(40 \times 12)^4}{384(29000)(1350)} = 1.074 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.180/12)(40 \times 12)^4}{384(29000)(1350)} = 0.2648 \text{ in.}$$

$\Delta = \Delta_D + \Delta_{const} = 1.074 + 0.2648 = 1.339$ in. $\underline{\Delta = 1.34 \text{ in.}}$

After concrete has cured:

$w_L = 250(9) = 2250$ lb/ft

Lower-bound moment of inertia:

Effective flange width = $(40 \times 12)/4 = 120$ in. or $9(12) = 108$ in., use $b = 108$ in.

For a W24 × 55, $A_s = 16.2 \text{ in.}^2$, $d = 23.6$ in., $I_x = 1350 \text{ in.}^4$

Determine the compressive force, $C$.

$A_sF_y = 16.2(50) = 810.0$ kips, $0.85 f_c' bt = 0.85(4)(108)(6) = 2203$ kips

Use $C = 810$ kips.

Area of transformed concrete = $A_c = \dfrac{C}{F_y} = \dfrac{810}{50} = 16.2 \text{ in.}^2$

$$a = \frac{C}{0.85f_c'b} = \frac{810}{0.85(4)(108)} = 2.206 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6 - \frac{2.206}{2} = 4.897 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 1.620E+01 | 2.850E+01 | 4.617E+02 | 0.000E+00 | -8.349E+00 | 1.129E+03 |
| W24 × 55 | 1.620E+01 | 1.180E+01 | 1.912E+02 | 1.350E+03 | -8.349E+00 | 2.479E+03 |
| Sum | 3.240E+01 | | 6.528E+02 | | | 3.608E+03 |

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 2.015\text{E+01 in.}$$

$I_{LB} = 3608 \text{ in.}^4$

$$\Delta_L = \frac{5w_LL^4}{384EI_{LB}} = \frac{5(2.250/12)(40 \times 12)^4}{384(29000)(3608)} = 1.239 \text{ in.}$$

$\Delta = \Delta_D + \Delta_L = 1.34 + 1.239 = 2.58$ in. $\underline{\Delta = 2.97 \text{ in.}}$

Maximum permissible $\Delta = \dfrac{L}{240} = \dfrac{40 \times 12}{240} = 2.0 \text{ in.} < 2.58 \text{ in.}$ (N.G.)

The largest component is before the concrete cures. The maximum permissible dead load deflection is

$$2.0 - 1.239 = 0.761 \text{ in.}$$

Required $I_s = \dfrac{5(0.730/12(40 \times 12)^4}{384(29000)(0.761)} = 1905 \text{ in.}^4$

Try a W24 × 76, with $I_x = 2100 \text{ in.}^4$

The strength and lower bound moment of inertia will be larger than before, so this shape will be adequate.

Use a W24 × 76

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**9.6-5**

(a) From Problem 9.4-2, a W14 × 22 is used, with $t = 4$ in., $s = 8$ ft, $L = 27$ ft, $q_{const} = 20$ psf, $q_{part} = 20$ psf, $q_L = 120$ psf, A992 steel and 4 ksi concrete.

Before concrete cures:

Slab: $\dfrac{4}{12}(150)(8) = 400 \text{ lb/ft}$

$w_D = 400 + 22 = 422 \text{ lb/ft}, \qquad w_{const} = 20(8) = 160 \text{ lb/ft}, \qquad I_s = 199 \text{ in.}^4$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.422/12)(27 \times 12)^4}{384(29000)(199)} = 0.8744 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const} L^4}{384EI_s} = \frac{5(0.160/12)(27 \times 12)^4}{384(29000)(199)} = 0.3315 \text{ in.}$$

$\Delta = \Delta_D + \Delta_{const} = 0.8744 + 0.3315 = 1.206 \text{ in.}$ $\qquad$ $\Delta = 1.21 \text{ in.}$

After concrete has cured:

$w_{part} = 20(8) = 160 \text{ lb/ft}, \qquad w_L = 120(8) = 960 \text{ lb/ft}$

Lower-bound moment of inertia:

Effective flange width = $(27 \times 12)/4 = 81$ in. or $8(12) = 96$ in., use $b = 81$ in.

For a W14 × 22, $A_s = 6.49 \text{ in.}^2$, $d = 13.7$ in., $I_x = 199 \text{ in.}^4$

Determine the compressive force, $C$.

$A_sF_y = 6.49(50) = 324.5$ kips, $\quad 0.85 f_c'bt = 0.85(4)(81)(4) = 1102$ kips

Since 1102 kips > 324.5 kips, $C = 324.5$ kips.

Area of transformed concrete = $A_c = \dfrac{C}{F_y} = \dfrac{324.5}{50} = 6.49\text{ in.}^2$

$$a = \frac{C}{0.85f_c'b} = \frac{324.5}{0.85(4)(81)} = 1.178\text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4 - \frac{1.178}{2} = 3.411\text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 6.49 | 17.11 | 111.1 | 0.00 | -5.131 | 170.8 |
| W14 x 22 | 6.49 | 6.85 | 44.46 | 199 | -5.131 | 369.8 |
| Sum | 12.98 | | 155.5 | | | 540.7 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{155.5}{12.98} = 11.98\text{ in.}, \qquad I_{LB} = 540.7\text{ in.}^4$$

$$\Delta_{part} = \frac{5w_{part}L^4}{384EI_{LB}} = \frac{5(0.160/12)(27 \times 12)^4}{384(29000)(540.7)} = 0.122\text{ in.}$$

$$\Delta_L = \frac{5w_L L^4}{384EI_{LB}} = \frac{5(0.960/12)(27 \times 12)^4}{384(29000)(540.7)} = 0.7321\text{ in.}$$

$$\Delta = \Delta_D + \Delta_{part} + \Delta_L = 0.8744 + 0.122 + 0.7321 = 1.73\text{ in.} \qquad \underline{\Delta = 1.73\text{ in.}}$$

(b) Maximum permissible $\Delta_L = \dfrac{L}{360} = \dfrac{27 \times 12}{360} = 0.900$ in. > 0.732 in. (OK)

---

**9.7-1**

(a) Lower-bound moment of inertia:

Effective flange width = $(43 \times 12)/4 = 129.0$ in. or $(28/3)(12) = 112.0$ in. (controls).

For a W27 × 84, $A_s = 24.7\text{ in.}^2$, $d = 26.7$ in., $I_x = 2850\text{ in.}^4$

Determine the compressive force, $C$.

$A_sF_y = 24.7(50) = 1235$ kips, $\quad 0.85 f_c'bt = 0.85(4)(112)(4) = 1523$ kips

Since 1235 kips < 1523 kips, $C = 1235$ kips.

Area of transformed concrete = $A_c = \dfrac{C}{F_y} = \dfrac{1235}{50} = 24.7 \text{ in.}^2$

$$a = \frac{C}{0.85f_c'b} = \frac{1235}{0.85(4)(112)} = 3.243 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6.5 - \frac{3.243}{2} = 4.879 \text{ in.}$$

Taking moments about the bottom of the steel, we get

| Component | $A$ | $y$ | $Ay$ | $\bar{I}$ | $d$ | $\bar{I} + Ad^2$ |
|---|---|---|---|---|---|---|
| Concrete | 24.70 | 31.58 | 780.0 | 0.00 | -9.115 | 2052 |
| W27 x 84 | 24.70 | 13.35 | 329.7 | 2850 | -9.115 | 4902 |
| Sum | 49.40 | | 1109.7 | | | 6954 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{1110}{49.40} = 22.47 \text{ in.} \qquad \underline{I_{LB} = 6954 \text{ in.}^4}$$

$$\Delta_L = \frac{5w_L L^4}{384EI_{tr}} = \frac{5(1.0/12)(43 \times 12)^4}{384(29000)(6954)} = 0.381 \text{ in.} \qquad \underline{\Delta_L = 0.381 \text{ in.}}$$

(b) $A_sF_y = 24.7(50) = 1235$ kips

$0.85\, f_c{'}bt = 0.85(4)(112)(6.5 - 3) = 1333$ kips

Use $C = 1235$ kips.

$0.85f_c{'}ab = A_sF_y$

$0.85(4)a(112) = 1235$, Solution is: $\{a = 3.243\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{26.7}{2} + 6.5 - \frac{3.243}{2} = 18.23 \text{ in.}$$

$M_n = Ty = 1235(18.23) = 2.251 \times 10^4$ in.-kips = 1880 ft-kips

$\underline{M_n = 1880 \text{ ft-kips}}$

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**9.7-2**

Steel headed stud anchors:

Maximum diameter = $2.5t_f = 2.5(0.615) = 1.54$ in. > 3/4 in. (OK)

Maximum diameter = 3/4 in. with formed steel deck. (OK)

$$A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g = 0.85 \text{ for two studs per rib}$$

$$R_g R_p A_{sa} F_u = 0.85(0.60)(0.4418)(65)$$

$$= 14.65 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 14.65 \text{ kips}$$

$$N_1 = \text{total no. of studs} \div 2 = \frac{30(12)}{6} \times 2 \times \frac{1}{2} = 60$$

$$\sum Q_n = Q_n \times N_1 = 14.65(60) = 879.0 \text{ kips}$$

$$A_s F_y = 18.3(50) = 915.0 \text{ kips}$$

$$0.85 f_c' bt = 0.85(4)(90)(4.5 - 2) = 765.0 \text{ kips}$$

Since $0.85 f_c' bt$ is the smallest of the three possibilities, $C = 765$ kips, and there is partial composite action.

$$C + C_s - T = 0$$

$$C + F_y b_f t' - F_y(A_s - b_f t') = 0$$

$$765 + 50(8.24)t' - 50(18.3 - 8.24t') = 0, \text{ Solution is: } \{t' = 0.182\}$$

Since $t_f = 0.615$ in., the PNA is in the flange.

$$C_s = b_f t' F_y = 8.24(0.182)(50) = 74.98 \text{ kips}$$

Compute $\bar{y}$, the distance from the top of the steel to the centroid of the area below the PNA.

| Component | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| W21 x 62 | 1.830E+01 | 1.050E+01 | 1.922E+02 |
| Flange segment | -1.500E+00 | 9.100E-02 | -1.365E-01 |
| Sum | 1.680E+01 | | 1.921E+02 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{192.1}{16.80} = 11.43 \text{ in.}$$

Location of concrete compressive force:

$$a = \frac{C}{0.85 f_c' b} = \frac{765}{0.85(4)(90)} = 2.5 \text{ in.}$$

Moment arm for concrete compressive force is

$$\bar{y} + t - \frac{a}{2} = 11.43 + 4.5 - \frac{2.5}{2} = 14.68 \text{ in.}$$

Moment arm for compressive force in the steel is

$$\bar{y} - \frac{t'}{2} = 11.43 - \frac{0.182}{2} = 11.34 \text{ in.}$$

$$M_n = \sum M_T = C(14.65) + C_s(11.34)$$

$$= 765(14.68) + 74.98(11.34) = 1.208 \times 10^4 \text{ in.-kips} = 1010 \text{ ft-kips}$$

$\therefore$ with 2 studs per rib, $\underline{M_n = 1010 \text{ ft-kips}}$

---

**9.7-3**

Steel headed stud anchors:

Maximum diameter = $2.5t_f = 2.5(0.420) = 1.05$ in. > 3/4 in. (OK)

Maximum diameter = 3/4 in. with formed steel deck. (OK)

$$A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sa} F_u = 1.0(0.60)(0.4418)(65) = 17.23$$

$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

$$N_1 = \text{total no. of studs} \div 2 = \frac{(30 \times 12)/(3 \times 6)}{2} = 10.0$$

$$\sum Q_n = Q_n \times N_1 = 17.23(10) = 172.3 \text{ kips}$$

$$A_s F_y = 7.69(50) = 384.5 \text{ kips}$$

$$0.85 f_c' bt = 0.85(4)(66)(4.5 - 1.5) = 673.2 \text{ kips}$$

Since $\sum Q_n$ is the smallest of the three possibilities, $C = 173.2$ kips, there is partial composite action, and the PNA is in the steel section. Determine whether the PNA is in the top flange or the web:

$$C + C_s - T = 0$$

$$173.2 + F_y b_f t' - F_y(A_s - b_f t') = 0$$

$$= 173.2 + 50(5.03)t' - 50(7.69 - 5.03t') = 0, \text{ Solution is: } \{t' = 0.4201\}$$

Since $t_f = 0.420$ in., the PNA is at the bottom of the flange.

$$C_s = b_f\, t' F_y = 5.03(0.420)(50) = 105.6 \text{ kips}$$

Compute $\bar{y}$, the distance from the top of the steel to the centroid of the area below the PNA.

| Component | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| W14 x 26 | 7.690E+00 | 6.950E+00 | 5.345E+01 |
| Flange segment | -2.113E+00 | 2.100E-01 | -4.437E-01 |
| Sum | 5.577E+00 | | 5.301E+01 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{53.01}{5.577} = 9.505 \text{ in.}$$

Location of concrete compressive force:

$$a = \frac{C}{0.85 f_c' b} = \frac{173.2}{0.85(4)(66)} = 0.7718 \text{ in.}$$

Moment arm for concrete compressive force is

$$\bar{y} + t - \frac{a}{2} = 9.505 + 4.5 - \frac{0.7718}{2} = 13.62 \text{ in.}$$

Moment arm for compressive force in the steel is

$$\bar{y} - \frac{t_f}{2} = 9.505 - \frac{0.420}{2} = 9.295 \text{ in.}$$

$$M_n = \sum M_T = C(13.62) + C_s(9.295)$$

$$= 173.1(13.62) + 105.6(9.295) = 3339 \text{ in.-kips} = 278 \text{ ft-kips}$$

$\underline{M_n = 278 \text{ ft-kips}}$

---

**9.7-4**

Steel headed stud anchors:

Maximum diameter = $2.5t_f = 2.5(0.440) = 1.1$ in. > 3/4 in. (OK)

Maximum diameter = 3/4 in. with formed steel deck. (OK)

$$A_{sa} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

Approximate spacing with one stud at each location is

$$s = \frac{40(12)}{34} = 14.1 \text{ in.}$$

$\therefore$ one stud at each location will work. Use

$$R_g = 1.0,\ R_p = 0.6$$

$$R_g R_p A_{sa} F_u = 1.0(0.60)(0.4418)(65) = 17.23$$

$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

$N_1$ = total no. of studs ÷ 2 = 34/2 = 17 studs

$$\sum Q_n = Q_n \times N_1 = 17.23(17) = 292.9 \text{ kips}$$

$$A_s F_y = 9.13(50) = 456.5 \text{ kips}$$

$$b = \frac{40(12)}{4} = 120 \text{ in. or } 10(12) = 120 \text{ in.}$$

The beam is a W16 × 31.

$$0.85 f_c' bt = 0.85(4)(120)(4.5 - 1.5) = 1224 \text{ kips}$$

Since $\sum Q_n$ is the smallest of the three possibilities, $C = 292.9$ kips, there is partial composite action, and the PNA is in the steel section. Determine whether the PNA is in the top flange or the web:

$$C + C_s - T = 0$$

$$C + F_y b_f t' - F_y(A_s - b_f t') = 0$$

$$= 292.9 + 50(5.53)t' - 50(9.13 - 5.53t') = 0, \text{ Solution is: } \{t' = 0.2958\} \text{ in.}$$

Since $t_f = 0.440$ in., the PNA is in the flange.

$$C_s = b_f t' F_y = 5.53(0.2958)(50) = 81.79 \text{ kips}$$

Compute $\bar{y}$, the distance from the top of the steel to the centroid of the area below the PNA.

| Component | $A$ | $y$ | $Ay$ |
|---|---|---|---|
| W16 x 31 | 9.130E+00 | 7.950E+00 | 7.258E+01 |
| Flange segment | -1.636E+00 | 9.100E-02 | -1.489E-01 |
| Sum | 7.494E+00 | | 7.243E+01 |

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{72.43}{7.494} = 9.665 \text{ in.}$$

Location of concrete compressive force:

$$a = \frac{C}{0.85 f'_c b} = \frac{292.9}{0.85(4)(120)} = 0.7179 \text{ in.}$$

Moment arm for concrete compressive force is

$$\bar{y} + t - \frac{a}{2} = 9.665 + 4.5 - \frac{0.7179}{2} = 13.81 \text{ in.}$$

Moment arm for compressive force in the steel is

$$\bar{y} - \frac{t'}{2} = 9.665 - \frac{0.2958}{2} = 9.517 \text{ in.}$$

$$M_n = \sum M_T = C(13.81) + C_s(9.517)$$

$$= 292.9(13.81) + 81.79(9.517) = 4823 \text{ in.-kips} = 401.9 \text{ ft-kips}$$

Loads: Before the concrete cures,

$$w_D = \frac{4.5}{12}(115)(10) + 31 = 462.3 \text{ lb/ft} \qquad w_L = 20(10) = 200 \text{ lb/ft}$$

After the concrete cures,

$$w_D = 462.3 + (5 + 5)(10) = 562.3 \text{ lb/ft} \qquad w_L = (120 + 20)(10) = 1400 \text{ lb/ft}$$

(a) LRFD Solution

Before the concrete cures,

$$w_u = 1.2(0.4623) + 1.6(0.200) = 0.8748 \text{ kips/ft}$$

$$M_u = \frac{1}{8}(0.8748)(40)^2 = 175 \text{ ft-kips}$$

$$\phi_b M_{nx} = \phi_b M_{px} = 203 \text{ ft-kips} > 175 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$w_u = 1.2(0.5623) + 1.6(1.400) = 2.915 \text{ kips/ft}$$

$$M_u = \frac{1}{8}(2.915)(40)^2 = 583.0 \text{ ft-kips}$$

$$\phi_b M_{nx} = 0.90(401.9) = 362 \text{ ft-kips} < 583 \text{ ft-kips} \qquad \text{(N.G.)}$$

<u>The strength is not adequate.</u>

(b) ASD Solution

Before the concrete cures,

$$w_a = 0.4623 + 0.200 = 0.6623 \text{ kips/ft}$$

$$M_a = \frac{1}{8}(0.6623)(40)^2 = 133 \text{ ft-kips}$$

$$\frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 135 \text{ ft-kips} > 133 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$w_a = 0.5623 + 1.400 = 1.962 \text{ kips/ft}$$

$$M_a = \frac{1}{8}(1.962)(40)^2 = 392 \text{ ft-kips}$$

$$\frac{M_{nx}}{\Omega_b} = \frac{401.9}{1.67} = 241 \text{ ft-kips} < 392 \text{ ft-kips} \qquad \text{(N.G.)}$$

<u>The strength is not adequate.</u>

---

**<u>9.8-1</u>**

From the solution to problem 9.7-3, for ¾-in. studs and $f_c' = 4$ ksi, $Q_n = 17.23$ kips

$$N_1 = \frac{30 \times 12}{18(2)} = 10.0$$

$$\sum Q_n = N_1 Q_n = 10(17.23) = 172.3 \text{ kips}$$

$$A_s F_y = 7.69(50) = 384.5 \text{ kips}$$

$0.85\, f_c{'}bt = 0.85(4)(66)(4.5 - 1.5) = 673.2$ kips

The smallest of these three controls; $\therefore\ C = 173.2$ kips (this is denoted as $\sum Q_n$ in the tables)

$$a = \frac{C}{0.85 f_c{'} b} = \frac{173.2}{0.85(4)(66)} = 0.7718 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4.5 - \frac{0.7718}{2} = 4.114 \text{ in.}$$

Since we are seeking a nominal strength, we can use either the LRFD value or the ASD value from the table.

We will use the LRFD value. For $\Sigma Q_n = 173$ kips, the interpolated value of $\phi_b M_n$ is

$$\phi_b M_n = 249 + 0.114(256 - 249) = 249.8 \text{ ft-kips}$$

and $M_n = \dfrac{\phi_b M_n}{\phi_b} = \dfrac{249.8}{0.90} = 278$ ft-kips (same as the solution of problem 9.7-3)

$\underline{M_n = 278 \text{ ft-kips}}$

---

**9.8-2**

(a) For 44 studs per beam,

$$N_1 = \frac{44}{2} = 22$$

Assuming 1 stud at each location, $Q_n = 17.2$ kips (*Manual* Table 3-21)

$$\sum Q_n = N_1 Q_n = 22(17.2) = 378.4 \text{ kips}$$

$$A_s F_y = 10.6(50) = 530.0 \text{ kips}$$

$$0.85\, f_c{'}bt = 0.85(4)(90)(5 - 2) = 918.0 \text{ kips}$$

The smallest of these three controls; $\therefore\ C = 378.4$ kips (this is denoted as $\sum Q_n$ in the tables)

$$a = \frac{C}{0.85 f_c{'} b} = \frac{378.4}{0.85(4)(90)} = 1.237 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{1.237}{2} = 4.382 \text{ in.}$$

LRFD Solution:

Interpolate in Table 3-19. First, interpolate vertically (create an intermediate row), then horizontally.

| $\sum Q_n$ | $Y2$ | | |
|---|---|---|---|
| | 4 | **4.382** | 4.5 |
| 380 | 429 | | 443 |
| **378.4** | **428** | **439** | **442** |
| 305 | 405 | | 416 |

$\phi_b M_n = 439$ ft-kips

ASD Solution:

Interpolate in Table 3-19. First, interpolate vertically (create an intermediate row), then horizontally.

| $\sum Q_n$ | $Y2$ | | |
|---|---|---|---|
| | 4 | **4.382** | 4.5 |
| 380 | 285 | | 295 |
| **378.4** | **285** | **292** | **295** |
| 305 | 269 | | 277 |

$R_n/\Omega_b = 292$ ft-kips

(b) For 20 studs per beam,

$$N_1 = \frac{20}{2} = 10$$

Assuming 1 stud at each location, $Q_n = 17.2$ kips (*Manual* Table 3-21)

$$\sum Q_n = N_1 Q_n = 10(17.2) = 172 \text{ kips}$$

$$A_s F_y = 10.6(50) = 530.0 \text{ kips}$$

$$0.85\, f_c{}' bt = 0.85(4)(90)(5-2) = 918.0 \text{ kips}$$

The smallest of these three controls; $\therefore\ C = 172$ kips (this is denoted as $\sum Q_n$ in the tables)

$$a = \frac{C}{0.85 f_c{}' b} = \frac{172}{0.85(4)(90)} = 0.5621 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{0.5621}{2} = 4.719 \text{ in}$$

LRFD Solution:

Interpolate in Table 3-19. First, interpolate vertically (create an intermediate row), then horizontally.

| $\sum Q_n$ | $Y2$ | | |
|---|---|---|---|
| | 4.5 | **4.719** | 5 |
| 181 | 368 | | 375 |
| **172** | **363** | **366** | **370** |
| 133 | 343 | | 348 |

$\phi_b M_n = 366$ ft-kips

ASD Solution:

Interpolate in Table 3-19. First, interpolate vertically (create an intermediate row), then horizontally.

| $\sum Q_n$ | $Y2$ | | |
|---|---|---|---|
| | 4.5 | **4.719** | 5 |
| 181 | 245 | | 250 |
| **172** | **242** | **244** | **246** |
| 133 | 228 | | 231 |

$R_n/\Omega_b = 244$ ft-kips

---

**9.8-3**

(a) $w_D = (51 + 10)(10) = 610$ lb/ft (neglect beam wt. and check it later.)

$w_L = (80 + 20)(10) = 1000$ lb/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(0.610) + 1.6(1.000) = 2.332$ lb/ft

$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.332)(35)^2 = 357$ ft-kips

Try a W21 × 48, $\phi_b M_n = 398$ ft-kips > 357 ft-kips (OK)

(continuous lateral support)

Check beam weight:

$M_u = 357 + \frac{1}{8}(1.2 \times 0.048)(35)^2 = 366$ ft-kips < 398 ft-kips (OK)

Check shear: From the $Z_x$ table, $\phi_v V_n = 216$ kips

$$V_u \approx \frac{2.332(35)}{2} = 40.8 \text{ kips} < 216 \text{ kips} \qquad \text{(OK)}$$

<u>Use a W21 × 48</u>

Compute total deflection: $w = 610 + 48 + 1000 = 1658$ lb/ft.

From the dimensions and properties table, $I_x = 959$ in.$^4$

$$\Delta = \frac{5wL^4}{384EI_x} = \frac{5(1.658/12)(35 \times 12)^4}{384(29000)(959)} = 2.01 \text{ in.}$$

<u>$\Delta = 2.01$ in.</u>

(b) Effective flange width = $(35 \times 12)/4 = 105$ in. or $10(12) = 120$ in., use $b = 105$ in.

Total load to be supported by the composite section (neglecting beam weight): from Part (a),

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.332)(35)^2 = 357 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \frac{a}{2} = 5 - \frac{1}{2} = 4.5$ in.

Try a W16 × 31. For PNA location 4, $\sum Q_n = 274$ kips, $\phi_b M_n = 362$ ft-kips

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{274}{0.85(4)(105)} = 0.7675 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{0.7675}{2} = 4.616 \text{ in.}$$

By interpolation, $\phi_b M_n = 365$ ft-kips > 357 ft-kips (OK)

Adjust for beam weight: $\quad w_u = 2.332 + 1.2(0.031) = 2.369$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.369)(35)^2 = 363 \text{ ft-kips} < 365 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 131$ kips

$$V_u = \frac{2.369(35)}{2} = 41.5 \text{ kips} < 131 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 51(10) + 31 = 541 \text{ lb/ft}, \qquad w_L = 20(10) = 200 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.541) + 1.6(0.200) = 0.9692 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.9692)(35)^2 = 148 \text{ ft-kips}$$

From Table 3-19, $\phi_b M_p = 203$ ft-kips > 148 ft-kips (OK) <u>Use a W16 × 31</u>

Stud anchors:

Maximum stud diameter = $2.5t_f = 2.5(0.440) = 1.1$ in.

But maximum diameter with formed steel deck = $\frac{3}{4}$ in. (controls)

Minimum height of stud above top of deck = $1\frac{1}{2}$ in. Use $h_s = 2 + 1\frac{1}{2} = 3\frac{1}{2}$ in.

Try $\frac{3}{4}$-in. × $3\frac{1}{2}$-in. studs. Assume one stud at each beam location. For lightweight concrete and $f_c{}' = 4$ ksi, $Q_n = 17.2$ kips (Manual Table 3-21)

$$N_1 = \frac{\sum Q_n}{Q_n} = \frac{274}{17.2} = 15.9, \text{ use 16 (32 per beam)}$$

(Actual $\sum Q_n = 16(17.2) = 275$ kips)

The approximate spacing is $\dfrac{L}{\text{no. studs}} = \dfrac{35(12)}{32} = 13.1$ in. (the exact spacing will depend on the the deck rib spacing).

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Max. longitudinal spacing = $8t = 8(5) = 40$ in. (upper limit = 36 in.)

∴ 32 studs OK. <u>Use 32 studs, $\frac{3}{4}$-in. × $3\frac{1}{2}$-in</u>

Compute total deflection.

Before concrete cures,

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.541/12)(35 \times 12)^4}{384(29,000)(375)} = 1.680 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const} L^4}{384EI_s} = \frac{5(0.200/12)(35 \times 12)^4}{384(29,000)(375)} = 0.6209 \text{ in.}$$

Maximum deflection before concrete cures is $\Delta_D + \Delta_{const} = 1.680 + 0.6209 = 2.30$ in.

After concrete cures:

Loads applied after concrete cures:

$$w = w_L + w_{part} + w_{misc} = (80 + 20 + 10)(10) = 1100 \text{ lb/ft}$$

From Manual Table 3-20, for PNA 4 and Y2 = 4.616 in., $I_{LB} = 916.0 \text{ in.}^4$

After the concrete cures, $\Delta_{L+P+misc} = \frac{5wL^4}{384EI_{LB}} = \frac{5(1.100/12)(35 \times 12)^4}{384(29000)(916.0)}$

$= 1.398$ in.

Maximum total deflection $= \Delta_D + \Delta_{L+P+misc} = 1.680 + 1.398 =$ 3.08 in.

---

**9.8-4**

(a) $w_D = (51 + 10)(10) = 610$ lb/ft (neglect beam wt. and check it later.)

$w_L = (80 + 20)(10) = 1000$ lb/ft

$w_a = w_D + w_L = 610 + 1000 = 1610$ lb/ft

$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.610)(35)^2 = 247$ ft-kips

Try a W21 × 48, $\frac{M_n}{\Omega_b} = 265$ ft-kips > 247 ft-kips (OK)

(continuous lateral support)

Check beam weight:

$M_a = 247 + \frac{1}{8}(0.048)(35)^2 = 254$ ft-kips < 265 ft-kips (OK)

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 144$ kips

$V_a \approx \frac{1.610(35)}{2} = 28.2$ kips < 144 kips (OK) Use a W21 × 48

Compute total deflection: $w = 610 + 48 + 1000 = 1658$ lb/ft.

From the dimensions and properties table, $I_x = 959$ in.$^4$

$\Delta = \frac{5wL^4}{384EI_x} = \frac{5(1.658/12)(35 \times 12)^4}{384(29000)(959)} = 2.01$ in. $\Delta = 2.01$ in.

(b) Effective flange width $= (35 \times 12)/4 = 105$ in. or $10(12) = 120$ in.,

use $b = 105$ in.

Total load to be supported by the composite section (neglecting beam weight): from

Part (a),

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.610)(35)^2 = 247 \text{ ft-kips}$$

Assume $a = 1$ in.: $Y2 = t - \frac{a}{2} = 5 - \frac{1}{2} = 4.5$ in.

Try a W16 × 31. For PNA location 3, $\sum Q_n = 335$ kips, $\frac{M_n}{\Omega_b} = 256$ ft-kips

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{335}{0.85(4)(105)} = 0.9384 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{0.9384}{2} = 4.53 \text{ in.}$$

For $Y2 = 4.5$ in. (conservatively) , $\frac{M_n}{\Omega_b} = 256$ ft-kips > 247 ft-kips (OK)

Adjust for beam weight: $w_a = 1.610 + 0.031 = 1.641$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.641)(35)^2 = 251 \text{ ft-kips} < 256 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 87.5$ kips

$$V_u = \frac{1.641(35)}{2} = 28.7 \text{ kips} < 87.5 \text{ kips} \quad \text{(OK)}$$

Before concrete cures:

$$w_D = 51(10) + 31 = 541 \text{ lb/ft}, \quad w_L = 20(10) = 200 \text{ lb/ft}$$

$$w_a = w_D + w_L = 0.541 + 0.200 = 0.741 \text{ kips/ft}$$

$$M_a = \frac{1}{8}(0.741)(35)^2 = 114 \text{ ft-kips}$$

From Table 3-19, $\frac{M_p}{\Omega_b} = 135$ ft-kips > 114 ft-kips (OK) <u>Use a W16 × 31</u>

Stud anchors:

Maximum stud diameter = $2.5 t_f = 2.5(0.440) = 1.1$ in.

But maximum diameter with formed steel deck = $\frac{3}{4}$ in. (controls)

Minimum height of stud above top of deck = $1\frac{1}{2}$ in. Use $h_s = 2 + 1\frac{1}{2} = 3\frac{1}{2}$ in.

Try $\frac{3}{4}$-in. × $3\frac{1}{2}$-in. studs. Assume one stud at each beam location. For lightweight concrete and $f_c' = 4$ ksi, $Q_n = 17.2$ kips (Manual Table 3-21)

$$N_1 = \frac{\sum Q_n}{Q_n} = \frac{335}{17.2} = 19.48\text{, use 20 (40 per beam)}$$

(Actual $\sum Q_n = 20(17.2) = 344.0$ kips)

The approximate spacing is $\dfrac{L}{\text{no. studs}} = \dfrac{35(12)}{40} = 10.5$ in. (the exact spacing will depend on the the deck rib spacing).

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Max. longitudinal spacing = $8t = 8(5) = 40$ in. (upper limit = 36 in.)

$\therefore$ 40 studs OK. Use 40 studs, $\frac{3}{4}$-in. × $3\frac{1}{2}$-in

Compute total deflection. Before concrete cures,

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.541/12)(35 \times 12)^4}{384(29,000)(375)} = 1.680 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const} L^4}{384EI_s} = \frac{5(0.200/12)(35 \times 12)^4}{384(29,000)(375)} = 0.6209 \text{ in.}$$

Maximum deflection before concrete cures is

$$\Delta_D + \Delta_{const} = 1.680 + 0.6209 = 2.30 \text{ in.}$$

After concrete cures:

Loads applied after concrete cures:

$$w = w_L + w_{part} + w_{misc} = (80 + 20 + 10)(10) = 1100 \text{ lb/ft}$$

From Manual Table 3-20, for PNA 3 and Y2 = 4.53 in., $I_{LB} = 976.8$ in.$^4$

After the concrete cures,

$$\Delta_{L+P+misc} = \frac{5wL^4}{384EI_{LB}} = \frac{5(1.100/12)(35 \times 12)^4}{384(29000)(976.8)} = 1.311 \text{ in.}$$

Maximum total deflection = $\Delta_D + \Delta_{L+P+misc} = 1.680 + 1.311 =$ 2.99 in.

**9.8-5**

(a) Total load to be supported by the composite section (neglecting beam weight):

Slab: $\frac{4}{12}(150) = 50.0$ psf

$$w_D = (50 + 20 + 8)(8) = 624 \text{ lb/ft}, \qquad w_L = 100(8) = 800 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(624) + 1.6(800) = 2029 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(2.029)(36)^2 = 329 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \frac{a}{2} = 4 - \frac{1}{2} = 3.5$ in.

Try a W16 × 31. For PNA location 4, $\sum Q_n = 274$ kips and $\phi_b M_n = 342$ ft-kips

Effective flange width = (36 × 12)/4 = 108 in. or 8(12) = 96 in. (controls)

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{274}{0.85(4)(96)} = 0.8395 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4 - \frac{0.8395}{2} = 3.58 \text{ in. Use 3.5 in. (conservatively)}$$

$$\phi_b M_n = 342 \text{ ft-kips} > 329 \text{ ft-kips} \qquad \text{(OK)}$$

Adjust for beam weight: $\quad w_u = 2.029 + 1.2(0.036) = 2.072$ kips/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(2.072)(36)^2 = 336 \text{ ft-kips} < 342 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 131$ kips

$$V_u = \frac{2.072(36)}{2} = 37.3 \text{ kips} < 131 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 50(8) + 36 = 436 \text{ lb/ft}, \qquad w_L = 20(8) = 160 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.436) + 1.6(0.160) = 0.7792 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.7792)(36)^2 = 126 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_p = 203$ ft-kips > 126 ft-kips (OK) <u>Use a W16 × 31</u>

(b) Stud anchors: For $\frac{3}{4}$-in. studs, normal weight concrete, and $f_c' = 4$ ksi; $Q_n = 17.2$

kips (Table 3-21, assuming one stud at each beam location)

$$N_1 = \frac{\sum Q_n}{Q_n} = \frac{274}{17.2} = 15.9\text{, use 16 (32 per beam)}$$

<u>32 shear studs required (assuming one stud at each beam location)</u>

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**<u>9.8-6</u>**

(a) Total load to be supported by the composite section (neglecting beam weight):

Slab: $\frac{4}{12}(150) = 50.0$ psf

$$w_D = (50 + 20 + 8)(8) = 624 \text{ lb/ft}, \qquad w_L = 100(8) = 800 \text{ lb/ft}$$

$$w_a = w_D + w_L = 624 + 800 = 1424 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.424)(36)^2 = 231 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \frac{a}{2} = 4 - \frac{1}{2} = 3.5$ in.

Try a W16 × 31. For PNA location 3, $\sum Q_n = 335$ kips and $\frac{M_n}{\Omega_b} = 239$ ft-kips

Effective flange width = $(36 \times 12)/4 = 108$ in. or $8(12) = 96$ in. (controls)

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{335}{0.85(4)(96)} = 1.026 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4 - \frac{1.026}{2} = 3.49 \text{ in. Use 3.5 in.}$$

$= 239$ ft-kips ft-kips > 231 ft-kips (OK)

Adjust for beam weight: $\quad w_a = 1424 + 31 = 1455$ lb/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.455)(36)^2 = 236 \text{ ft-kips} < 239 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 87.5$ kips

$$V_a = \frac{1.455(36)}{2} = 26.2 \text{ kips} < 87.5 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 50(8) + 36 = 436 \text{ lb/ft}, \qquad w_L = 20(8) = 160 \text{ lb/ft}$$

$$w_u = w_D + w_L = 0.426 + 0.160 = 0.586 \text{ kips/ft}$$

$$M_a = \frac{1}{8}(0.586)(36)^2 = 94.9 \text{ ft-kips}$$

From the $Z_x$ table, $\dfrac{M_p}{\Omega_b} = 135$ ft-kips > 94.9 ft-kips (OK) Use a W16 × 31

(b) Stud anchors:

For $\frac{3}{4}$-in. studs, normal weight concrete, and $f_c{'} = 4$ ksi; $Q_n = 17.2$ kips (Table 3-21, assuming one stud at each beam location)

$$N_1 = \frac{\sum Q_n}{Q_n} = \frac{335}{17.2} = 19.48, \text{ use 20 (40 per beam)}$$

40 shear studs required (assuming one stud at each beam location)

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**9.8-7**

(a) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\frac{5}{12}(150) = 62.5$ psf

$$w_D = 62.5(7) = 437.5 \text{ lb/ft}, \; w_L = 800(7) = 5600 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.4375) + 1.6(5.600) = 9.485 \text{ k/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(9.485)(30)^2 = 1067 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \dfrac{a}{2} = 5 - \dfrac{1}{2} = 4.5$ in.

Try a W24 × 76. For PNA location 6, $\sum Q_n = 394$ kips and $\phi_b M_n = 1110$ ft-kips

Effective flange width = (30 × 12)/4 = 90 in. or 7(12) = 84 in. (controls)

$$a = \frac{\sum Q_n}{0.85 f_c{'} b} = \frac{394}{0.85(4)(84)} = 1.380 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{1.380}{2} = 4.31 \text{ in.}$$

$$\phi_b M_n = 1102 \text{ ft-kips} > 1067 \text{ ft-kips} \qquad \text{(OK)}$$

Adjust for beam weight: $w_u = 9.485 + 1.2(0.076) = 9.576$ kips/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(9.576)(30)^2 = 1077 \text{ ft-kips} < 1110 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 315$ kips

$$V_u = \frac{9.576(30)}{2} = 144 \text{ kips} < 315 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 62.5(7) + 76 = 513.5 \text{ lb/ft}, \qquad w_L = 20(7) = 140 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.5135) + 1.6(0.140) = 0.8402 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.8402)(30)^2 = 94.5 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_p = 315$ ft-kips > 94.5 ft-kips (OK)

Check live load deflection. $\Delta_{max} = L/360 = \dfrac{30(12)}{360} = 1.0$ in.

From Manual Table 3-20, for a W24 × 76, PNA location 6, $\sum Q_n = 394$ kips, and $Y2 = 4.31$ in., $I_{LB} = 3642$ in.$^4$

$$\Delta_L = \frac{5w_LL^4}{384EI_{LB}} = \frac{5(5.600/12)(30 \times 12)^4}{384(29000)(3642)} = 0.966 \text{ in.} < 1.0 \text{ in.} \qquad \text{(OK)}$$

<u>Use a W24 × 76</u>

(b) Total load to be supported by the composite section (omit beam weight; check it later):

Slab: $\dfrac{5}{12}(150) = 62.5$ psf

$$w_D = 62.5(7) = 437.5 \text{ lb/ft}, \quad w_L = 800(7) = 5600 \text{ lb/ft}$$

$$w_a = w_D + w_L = 437.5 + 5600 = 6038 \text{ k/ft}$$

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(6.038)(30)^2 = 679 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \dfrac{a}{2} = 5 - \dfrac{1}{2} = 4.5$ in.

Try a W24 × 76. For PNA location 6, $\sum Q_n = 394$ kips and $\dfrac{M_n}{\Omega_b} = 736$ ft-kips

Effective flange width = (30 × 12)/4 = 90 in. or 7(12) = 84 in. (controls)

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{394}{0.85(4)(84)} = 1.380 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5 - \frac{1.380}{2} = 4.31 \text{ in.}$$

$$\frac{M_n}{\Omega_b} = 732 \text{ ft-kips} > 679 \text{ ft-kips} \qquad \text{(OK)}$$

Check beam weight:

$$w_a = 6038 + 76 = 6114 \text{ lb/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(6.114)(30)^2 = 688 \text{ ft-kips} < 732 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear. From the $Z_x$ tables,

$$\frac{V_n}{\Omega_v} = 210 \text{ kips}$$

$$V_a = \frac{w_a L}{2} = \frac{6.114(30)}{2} = 91.7 \text{ kips} < 210 \text{ kips} \quad \text{(OK)}$$

Before concrete cures:

$$w_D = 437.5 + 76 = 513.5 \text{ lb/ft}, \qquad w_L = 20(7) = 140 \text{ lb/ft}$$

$$w_a = 513.5 + 140 = 653.5 \text{ lb//ft}$$

$$M_a = \frac{1}{8}(0.6535)(30)^2 = 73.5 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 499 \text{ ft-kips} > 73.5 \text{ ft-kips} \quad \text{(OK)}$$

Check live load deflection. $\Delta_{\max} = L/360 = \dfrac{30(12)}{360} = 1.0$ in.

From Manual Table 3-20, for a W24 × 76, PNA location 6, $\sum Q_n = 394$ kips, and $Y2 = 4.31$ in., $I_{LB} = 3642 \text{ in.}^4$

$$\Delta_L = \frac{5 w_L L^4}{384 E I_{LB}} = \frac{5(5.600/12)(30 \times 12)^4}{384(29000)(3642)} = 0.966 \text{ in.} < 1.0 \text{ in.} \qquad \text{(OK)}$$

<u>Use a W24 × 76</u>

(c) Max. stud diameter (for a W24 × 76) = $2.5 t_f = 2.5(0.680) = 1.7$ in.

Try $\frac{5}{8}$ in. × $2\frac{1}{2}$ in. studs.

From Manual Table 3-21, $Q_n = 15.0$ kips

$$N_1 = \frac{V'}{Q_n} = \frac{394}{15.0} = 26.3\text{, round up to 27. total number} = 2(27) = 54$$

Min. longitudinal spacing = $6d = 6(5/8) = 3.75$ in.

Min. transverse spacing = $4d = 4(5/8) = 2.5$ in.

Max. longitudinal spacing = $8t = 8(5) = 40$ in. (but upper limit = 36 in.)

For one stud at each section, the approximate spacing will be

$$s = \frac{\text{span}}{\text{no. studs}} = \frac{30(12)}{54} = 6.67 \text{ in.} \qquad \text{(OK)}$$

Use 54 studs, $\frac{5}{8}$ in. × $2\frac{1}{2}$ in., spaced at approximately $6\frac{1}{2}$ in. on center

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**9.8-8**

(a) Total load to be supported by the composite section (neglecting beam weight):

Deck and slab: 53 psf

$$w_D = (53 + 10)(12) = 756 \text{ lb/ft}, \qquad w_L = (160 + 20)(12) = 2160 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(756) + 1.6(2160) = 4363 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(4.363)(40)^2 = 873 \text{ ft-kips}$$

Assume $a = 1$ in.: $\qquad Y2 = t - \frac{a}{2} = 6.5 - \frac{1}{2} = 6.0$ in.

Try a W21 × 62. For PNA location BFL, $\sum Q_n = 408$ kips and $\phi_b M_n = 893$ ft-kips

Effective flange width = $(40 \times 12)/4 = 120$ in. or $12(12) = 144$ in. Use $b = 120$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{408}{0.85(4)(120)} = 1.0 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6.5 - \frac{1.0}{2} = 6.0 \text{ in.}$$

$$\phi_b M_n = 893 \text{ ft-kips} > 873 \text{ ft-kips} \qquad \text{(OK)}$$

Adjust for beam weight: $\qquad w_u = 4.363 + 1.2(0.062) = 4.437$ kips/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(4.437)(40)^2 = 887 \text{ ft-kips} < 893 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 252$ kips

$$V_u = \frac{4.437(40)}{2} = 88.7 \text{ kips} < 252 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 53(12) + 62 = 698 \text{ lb/ft}, \qquad w_L = 20(12) = 240 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.698) + 1.6(0.240) = 1.222 \text{ k/ft}$$

$$M_u = \frac{1}{8}(1.222)(40)^2 = 244 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_p = 252$ ft-kips > 244 ft-kips (OK)

Check live load deflection. $\Delta_{max} = L/360 = \frac{40(12)}{360} = 1.33$ in.

From Manual Table 3-20, for a W21 × 62, PNA location BFL, $\sum Q_n = 408$ kips, and $Y2 = 6.0$ in., $I_{LB} = 2870$ in.$^4$

$$\Delta_L = \frac{5w_L L^4}{384EI_{LB}} = \frac{5(2.160/12)(40 \times 12)^4}{384(29000)(2870)} = 1.50 \text{ in.} > 1.33 \text{ in.} \qquad \text{(N.G.)}$$

Determine required $I_{LB}$.

$$\text{Req'd } I_{LB} = \frac{5(2.160/12)(40 \times 12)^4}{384(29000)(1.33)} = 3226 \text{ in.}^4$$

Try a W21 × 62, PNA location 3, $\sum Q_n = 662$ kips.

$$a = \frac{\sum Q_n}{0.85f_c'b} = \frac{662}{0.85(4)(120)} = 1.623 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6.5 - \frac{1.623}{2} = 5.689 \text{ in.}$$

From Manual Table 3-20, $I_{LB} = 3345$ in.$^4$ > 3226 in.$^4$ (OK)

Stud anchors:

Max. stud diameter (for a W21 × 62) = $2.5t_f = 2.5(0.615) = 1.54$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{3}{4}$ in. × $4\frac{1}{2}$ in. studs. Height above deck = $4.5 - 3 = 1.5$ in. (OK). Cover = $6.5 - 4.5 = 2.0$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(6.5) = 52.0$ in. (upper limit = 36 in.)

With one stud at each location, $Q_n = 17.2$ kips.

$$N_1 = \frac{\text{Span}}{\text{rib sp.}} \div 2 = \frac{40(12)}{12(2)} = 20$$

$$\sum Q_n = N_1 Q_n = 20(17.2) = 344 \text{ kips} < 662 \text{ kips} \qquad \text{(N.G.)}$$

With two studs at each location, $Q_n = 14.6$ kips.

$$N_1 = 2(20) = 40$$

$$\sum Q_n = N_1 Q_n = 40(14.6) = 584.0 \text{ kips} < 662 \text{ kips} \qquad \text{(N.G.)}$$

The beam flange will not accomodate 3 studs. Reduce the $\sum Q_n$ requirement in order to increase the number of studs required. Do this by using a larger shape.

For a required $I_{LB} = 3226$ in.$^4$ and $Y2 = 5.5$ in., Try a W24 × 76, PNA 7, $\sum Q_n = 280$ kips, $I_{LB} = 3460$ in.$^4$, $\phi_b M_n = 1050$ ft-kips.

With one stud at each location,

$$Q_n = 17.2 \text{ kips.}$$

$$N_1 = \frac{\text{Span}}{\text{rib sp.}} \div 2 = \frac{40(12)}{12(2)} = 20$$

$$\sum Q_n = N_1 Q_n = 20(17.2) = 344 \text{ kips} > 280 \text{ kips} \qquad \text{(OK)}$$

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{344}{0.85(4)(120)} = 0.8431 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6.5 - \frac{0.8431}{2} = 6.08 \text{ inches. Use 6 in.}$$

$$I_{LB} = 3540 \text{ in.}^4, \quad \phi_b M_n = 1060 \text{ ft-kips.}$$

Use a W24 × 76, with 80 studs, $\frac{3}{4}$ in. × $4\frac{1}{2}$ in., one per rib

(b) Total load to be supported by the composite section (neglecting beam weight):

Deck and slab: 53 psf

$$w_D = (53 + 10)(12) = 756 \text{ lb/ft}, \qquad w_L = (160 + 20)(12) = 2160 \text{ lb/ft}$$

$$w_a = w_D + w_L = 756 + 2160 = 2916 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(2.916)(40)^2 = 583 \text{ ft-kips}$$

Assume $a = 1$ in.: $\quad Y2 = t - \frac{a}{2} = 6.5 - \frac{1}{2} = 6.0$ in.

Deflection control: Check live load deflection.

$$\Delta_{max} = L/360 = \frac{40(12)}{360} = 1.33 \text{ in.}$$

$$\text{Req'd } I_{LB} = \frac{5(2.160/12)(40 \times 12)^4}{384(29000)(1.33)} = 3226 \text{ in.}^4$$

Try a W24 × 76. For PNA location 7 and $Y2 = 6.0$ in., $\sum Q_n = 280$ kips, $I_{LB} = 3540$ in.$^4$, $\frac{M_n}{\Omega_b} = 1060$ ft-kips (deflection controls)

Effective flange width = $(40 \times 12)/4 = 120$ in. or $12(12) = 144$ in. Use $b = 120$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{280}{0.85(4)(120)} = 0.6863 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 6.5 - \frac{0.6863}{2} = 6.157 \text{ in. (use } Y2 = 6.0 \text{ inches, conservatively)}$$

$$\therefore \frac{M_n}{\Omega_b} = 1060 \text{ ft-kips} > 583 \text{ ft-kips} \qquad \text{(OK)}$$

Beam weight is OK.

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 210$ kips

$$V_a \approx \frac{2.916(40)}{2} = 58.3 \text{ kips} < 210 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 53(12) + 62 = 698 \text{ lb/ft}, \qquad w_L = 20(12) = 240 \text{ lb/ft}$$

$$w_a = w_D + w_L = 698 + 240 = 938 \text{ k/ft}$$

$$M_a = \frac{1}{8}(0.938)(40)^2 = 188 \text{ ft-kips}$$

From the $Z_x$ table, $\frac{M_p}{\Omega_b} = 499$ ft-kips > 188 ft-kips (OK)

Stud anchors:

Max. stud diameter (for a W24 × 76) = $2.5t_f = 2.5(0.680) = 1.7$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{3}{4}$ in. × $4\frac{1}{2}$ in. studs. Height above deck = $4.5 - 3 = 1.5$ in. (OK). Cover = $6.5 - 4.5 = 2.0$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(6.5) = 52.0$ in. (upper limit = 36 in.)

With one stud at each location, $Q_n = 17.2$ kips.

$$N_1 = \frac{\text{Span}}{\text{rib sp.}} \div 2 = \frac{40(12)}{12(2)} = 20$$

$$\sum Q_n = N_1 Q_n = 20(17.2) = 344 \text{ kips} > 280 \text{ kips required} \qquad \text{(OK)}$$

<u>Use a W24 × 76, with 80 studs, $\frac{3}{4}$ in. × $4\frac{1}{2}$ in., one per rib</u>

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**9.8-9**

(a) Total load to be supported by the composite section (neglecting beam weight):

Deck and slab: 57 psf

$$w_D = (57)(9) = 513 \text{ lb/ft}, \qquad w_L = (225 + 20)(9) = 2205 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(513) + 1.6(2205) = 4144 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(4.144)(18.5)^2 = 177 \text{ ft-kips}$$

Deflection control: Check live load deflection requirement.

$$\Delta_{\max} = L/360 = \frac{18.5(12)}{360} = 0.617 \text{ in.}$$

$$\text{Req'd } I_{LB} = \frac{5(2.205/12)(18.5 \times 12)^4}{384(29000)(0.617)} = 325 \text{ in.}^4$$

Assume $a = 1$ in.: $\quad Y2 = t - \dfrac{a}{2} = 5.5 - \dfrac{1}{2} = 5.0$ in.

Try a W12 × 19. For PNA location 4 and $Y2 = 5.0$ in., $\sum Q_n = 173$ kips, $\phi_b M_n = 191$ ft-kips, $I_{LB} = 393$ in.$^4$

Effective flange width = $(18.5 \times 12)/4 = 55.5$ in. or $9(12) = 108$ in.

Use $b = 55.5$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{173}{0.85(4)(55.5)} = 0.9168 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5.5 - \frac{0.9168}{2} = 5.04 \text{ in., use 5.0 in.}$$

$\therefore\ \phi_b M_n = 191$ ft-kips > 177 ft-kips, and $I_{LB} = 393 \text{ in.}^4 > 325 \text{ in.}^4$ (OK)

Adjust for beam weight: $w_u = 4.144 + 1.2(0.019) = 4.167$ kips/ft

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(4.167)(18.5)^2 = 178 \text{ ft-kips} < 191 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 86.0$ kips

$$V_u = \frac{4.167(18.5)}{2} = 38.5 \text{ kips} < 86.0 \text{ kips} \quad \text{(OK)}$$

Before concrete cures:

$$w_D = 57(9) + 19 = 532 \text{ lb/ft}, \qquad w_L = 20(9) = 180 \text{ lb/ft}$$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(0.532) + 1.6(0.180) = 0.9264 \text{ k/ft}$$

$$M_u = \frac{1}{8}(0.9264)(18.5)^2 = 39.6 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_p = 92.6$ ft-kips > 39.6 ft-kips (OK)

Stud anchors:

Max. stud diameter (for a W12 × 19) = $2.5 t_f = 2.5(0.350) = 0.875$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{3}{4}$ in. × 3 in. studs. Height above deck = $3 - 1.5 = 1.5$ in. (OK). Cover = $5.5 - 3 = 2.5$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(5.5) = 44.0$ in. (upper limit = 36 in.)

With one stud at each location, $Q_n = 17.2$ kips.

$$N_1 = \frac{\text{Span}}{\text{rib sp.}} \div 2 = \frac{18.5(12)}{6(2)} = 18.5 \text{ say } 18$$

$$\sum Q_n = N_1 Q_n = 18(17.2) = 309.6 \text{ kips} > 173 \text{ kips} \qquad \text{(OK)}$$

Try two $\frac{5}{8}$ in. × 3 in. studs in every other rib. $\quad Q_n = 10.2$ kips

$$N_1 = \frac{\text{Span}}{\text{spacing}} \times 2 \div 2 = \frac{18.5(12)}{2 \times 6} = 18.5 \text{ say } 18$$

$$\text{Total no.} = 18.5(2) = 37$$

$$\sum Q_n = N_1 Q_n = 18(10.2) = 184 \text{ kips} > 173 \text{ kips} \qquad \text{(OK)}$$

Use a W12 × 19, with 37 studs, $\frac{5}{8}$ in. × 3 in., two in every other rib.

(b) Total load to be supported by the composite section (neglecting beam weight):

Deck and slab: 57 psf

$$w_D = (57)(9) = 513 \text{ lb/ft}, \qquad w_L = (225 + 20)(9) = 2205 \text{ lb/ft}$$

$$w_a = w_D + w_L = 513 + 2205 = 2718 \text{ lb/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(2.718)(18.5)^2 = 116 \text{ ft-kips}$$

Deflection control: Check live load deflection requirement.

$$\Delta_{\max} = L/360 = \frac{18.5(12)}{360} = 0.617 \text{ in.}$$

$$\text{Req'd } I_{LB} = \frac{5(2.205/12)(18.5 \times 12)^4}{384(29000)(0.617)} = 325 \text{ in.}^4$$

Assume $a = 1$ in.: $\quad Y2 = t - \frac{a}{2} = 5.5 - \frac{1}{2} = 5.0$ in.

Try a W12 × 19. For PNA location 4 and $Y2 = 5.0$ in., $\sum Q_n = 173$ kips,

$I_{LB} = 393$ in.$^4$, $\frac{M_n}{\Omega_b} = 127$ ft-kips

Effective flange width $= (18.5 \times 12)/4 = 55.5$ in. or $9(12) = 108$ in.

Use $b = 55.5$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{173}{0.85(4)(55.5)} = 0.9168 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5.5 - \frac{0.9168}{2} = 5.04 \text{ in., use 5.0 in.}$$

$$\therefore \ \frac{M_n}{\Omega_b} = 127 \text{ ft-kips} > 116 \text{ ft-kips}, \qquad I_{LB} = 393 \text{ in.}^4 > 325 \text{ in.}^4 \qquad \text{(OK)}$$

Beam weight is OK.

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 57.3$ kips

$$V_a \approx \frac{2.718(18.5)}{2} = 25.1 \text{ kips} < 57.3 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 57(9) + 19 = 532 \text{ lb/ft}, \qquad w_L = 20(9) = 180 \text{ lb/ft}$$

$$w_a = w_D + w_L = 532 + 180 = 712 \text{ k/ft}$$

$$M_a = \frac{1}{8}(0.712)(18.5)^2 = 30.5 \text{ ft-kips}$$

From the $Z_x$ table, $\frac{M_p}{\Omega_b} = 61.6$ ft-kips > 30.5 ft-kips (OK)

Stud anchors:

Max. stud diameter (for a W12 × 19) = $2.5t_f = 2.5(0.350) = 0.875$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{3}{4}$ in. × 3 in. studs. Height above deck = $3 - 1.5 = 1.5$ in. (OK).
Cover = $5.5 - 3 = 2.5$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(5.5) = 44.0$ in. (upper limit = 36 in.)

With one stud at each location, $Q_n = 17.2$ kips.

$$N_1 = \frac{\text{Span}}{\text{rib sp.}} \div 2 = \frac{18.5(12)}{6(2)} = 18.5 \text{ say } 18$$

$$\sum Q_n = N_1 Q_n = 18(17.2) = 309.6 \text{ kips} > 173 \text{ kips} \qquad \text{(OK)}$$

Try two $\frac{5}{8}$ in. × 3 in. studs in every other rib. $\qquad Q_n = 10.2$ kips

$$N_1 = \frac{\text{Span}}{\text{spacing}} \times 2 \div 2 = \frac{18.5(12)}{2 \times 6} = 18.5 \text{ say } 18$$

$$\text{Total no.} = 18.5(2) = 37$$

$$\sum Q_n = N_1 Q_n = 18(10.2) = 184 \text{ kips} > 173 \text{ kips} \qquad \text{(OK)}$$

<u>Use a W12 × 19, with 37 studs, $\frac{5}{8}$ in. × 3 in., two in every other rib.</u>

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**<u>9.8-10</u>**

(a) Total load to be supported by the composite section (neglecting beam weight):

$$\text{Slab: } \frac{5.5}{12}(115) = 52.71 \text{ psf}$$

$$w_D = 57.21(12) = 686.5 \text{ lb/ft}, \qquad w_L = (100 + 15)(12) = 1380 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(686.5) + 1.6(1380) = 3032 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(3.032)(35)^2 = 464 \text{ ft-kips}$$

Deflection control: Check live load deflection requirement.

$$\Delta_{\max} = L/360 = \frac{35(12)}{360} = 1.167 \text{ in.}$$

$$\text{Req'd } I_{LB} = \frac{5(1.380/12)(35 \times 12)^4}{384(29000)(1.167)} = 1377 \text{ in.}^4$$

Assume $a = 1$ in.: $\qquad Y2 = t - \frac{a}{2} = 5.5 - \frac{1}{2} = 5.0$ in.

Try a W16 × 45. For PNA location 4 and $Y2 = 5.0$ in., $\sum Q_n = 367$ kips, $\phi_b M_n = 534$ ft-kips, $I_{LB} = 1390$ in.$^4$ (deflection controls)

Effective flange width = $(35 \times 12)/4 = 105$ in. or $12(12) = 144$ in. Use $b = 105$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{367}{0.85(4)(105)} = 1.028 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5.5 - \frac{1.028}{2} = 4.986 \text{ in., use 5.0 in.}$$

$$\therefore \ \phi_b M_n = 534 \text{ ft-kips} > 464 \text{ ft-kips, and } I_{LB} = 1390 \text{ in.}^4 > 1377 \text{ in.}^4 \qquad \text{(OK)}$$

Adjust for beam weight: $\qquad w_u = 3.032 + 1.2(0.045) = 3.086$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(3.086)(35)^2 = 473 \text{ ft-kips} < 534 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the $Z_x$ table, $\phi_v V_n = 167$ kips

$$V_u = \frac{3.086(35)}{2} = 54.0 \text{ kips} < 167 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 686.5 + 45 = 731.5 \text{ lb/ft}, \qquad w_L = 0$$

$$w_u = 1.4w_D = 1.4(0.7315) = 1.024 \text{ k/ft}$$

$$M_u = \frac{1}{8}(1.024)(35)^2 = 157 \text{ ft-kips}$$

From the $Z_x$ table, $\phi_b M_p = 309$ ft-kips > 157 ft-kips (OK)

Stud anchors:

Max. stud diameter (for a W16 × 45) = $2.5t_f = 2.5(0.565) = 1.41$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{5}{8}$ in. × $3\frac{1}{2}$ in. studs. Height above deck = $3.5 - 2 = 1.5$ in. (OK). Cover = $5.5 - 3.5 = 2.0$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(5.5) = 44.0$ in. (upper limit = 36 in.)

With one stud every rib, $Q_n = 12.0$ kips.

$$N_1 = \frac{\text{Span}}{\text{spacing}} \div 2 = \frac{35(12)}{6} \div 2 = 35$$

Total no. = 70

$$\sum Q_n = N_1 Q_n = 35(12.0) = 420 \text{ kips} > 367 \text{ kips required} \qquad \text{(OK)}$$

<u>Use a W16 × 45 with 70 studs, $\frac{5}{8}$ in. × $3\frac{1}{2}$ in., one in each rib.</u>

(b) Total load to be supported by the composite section (neglecting beam weight):

Slab: $\frac{5.5}{12}(115) = 52.71$ psf

$$w_D = 57.21(12) = 686.5 \text{ lb/ft}, \qquad w_L = (100 + 15)(12) = 1380 \text{ lb/ft}$$

$$w_a = w_D + w_L = 686.5 + 1380 = 2067 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(2.067)(36)^2 = 335 \text{ ft-kips}$$

Deflection control: Check live load deflection requirement.

$$\Delta_{\text{max}} = L/360 = \frac{35(12)}{360} = 1.167 \text{ in.}$$

$$\text{Req'd } I_{LB} = \frac{5(1.380/12)(35 \times 12)^4}{384(29000)(1.167)} = 1377 \text{ in.}^4$$

Assume $a = 1$ in.: $\qquad Y2 = t - \frac{a}{2} = 5.5 - \frac{1}{2} = 5.0$ in.

Try a W16 × 45. For PNA location 4 and $Y2 = 5.0$ in., $\sum Q_n = 367$ kips, $\frac{M_n}{\Omega_b} = 355$ ft-kips, $I_{LB} = 1390$ in.$^4$ (deflection controls)

Effective flange width = (35 × 12)/4 = 105 in. or 12(12) = 144 in. Use $b = 105$ in.

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{367}{0.85(4)(105)} = 1.028 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 5.5 - \frac{1.028}{2} = 4.986 \text{ in., use 5.0 in.}$$

$$\therefore \frac{M_n}{\Omega_b} = 355 \text{ ft-kips} > 116 \text{ ft-kips}, \qquad I_{LB} = 1390 \text{ in.}^4 > 1377 \text{ in.}^4 \qquad \text{(OK)}$$

Beam weight is OK.

Check shear: From the $Z_x$ table, $\frac{V_n}{\Omega_v} = 111$ kips

$$V_a \approx \frac{2.067(35)}{2} = 36.2 \text{ kips} < 111 \text{ kips} \qquad \text{(OK)}$$

Before concrete cures:

$$w_D = 686.5 + 45 = 731.5 \text{ lb/ft}, \qquad w_L = 0$$

$$w_a = w_D + w_L = 731.5 \text{ k/ft}$$

$$M_a = \frac{1}{8}(0.7315)(35)^2 = 112 \text{ ft-kips}$$

From the $Z_x$ table, $\frac{M_p}{\Omega_b} = 205$ ft-kips > 112 ft-kips (OK)

Stud anchors:

Max. stud diameter (for a W16 × 45) = $2.5t_f = 2.5(0.565) = 1.41$ in.

But with deck, maximum diameter is 3/4 inch. Minimum stud height above deck = 1.5 inches. Minimum cover over top of stud = 1/2 inch.

Try $\frac{5}{8}$ in. × $3\frac{1}{2}$ in. studs. Height above deck = $3.5 - 2 = 1.5$ in. (OK).
Cover = $5.5 - 3.5 = 2.0$ in. (OK)

Min. longitudinal spacing = $6d = 6(3/4) = 4.5$ in.

Min. transverse spacing = $4d = 4(3/4) = 3.0$ in.

Max. longitudinal spacing = $8t = 8(5.5) = 44.0$ in. (upper limit = 36 in.)

With one stud every rib, $Q_n = 12.0$ kips.

$$N_1 = \frac{\text{Span}}{\text{spacing}} \div 2 = \frac{35(12)}{6} \div 2 = 35$$

Total no. = 70

$$\sum Q_n = N_1 Q_n = 35(12.0) = 420 \text{ kips} > 367 \text{ kips required} \qquad \text{(OK)}$$

Use a W16 × 45 with 70 studs, $\frac{5}{8}$ in. × $3\frac{1}{2}$ in., one in each rib.

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**9.10-1**

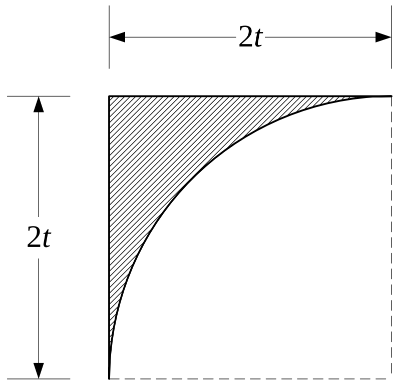
The following dimensions and properties from Part 1 of the Manual will be needed: For an HSS12 × 8 × 3/16, $A_s = 6.76$ in.$^2$, design wall thickness = $t_{des}$ = 0.174 in., and $I_y = 75.7$ in.$^4$

$$E_c = w^{1.5}\sqrt{f'_c} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

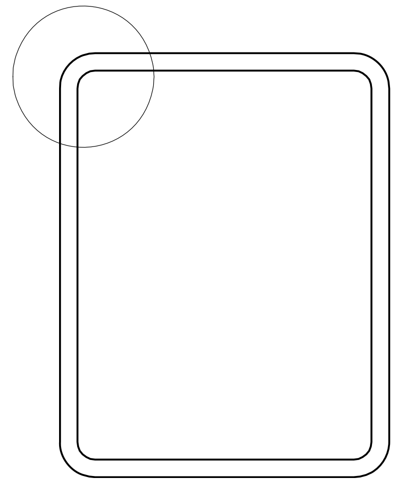
The area of concrete $A_c$ can be estimated as

$$bd - A_s = 8(12) - 6.76 = 89.24 \text{ in.}^2$$

or more accurately as follows:




Detail



The corner radius is given in Part 1 of the Manual as $2t_{des}$. The area of an quarter-circle spandrel with a radius $r$ is

$$r^2 - \frac{1}{4}\pi r^2 = (2t)^2 - \frac{1}{4}\pi(2t)^2 = t^2(4-\pi) = (0.174)^2(4-\pi)$$

$$= 2.599 \times 10^{-2} \text{ in.}^2$$

and since there are four of these segments to be deducted from the area $bd - A_s$,

$$A_c = bd - A_s - 4(0.02599) = 8(12) - 6.76 - 4(0.02599) = 89.14 \text{ in.}^2$$

For computing the moment of inertia of the concrete, the moment of inertia of the spandrel about an axis parallel to the 12-inch side through the point of tangency is

$$I = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 = \left(\frac{1}{3} - \frac{\pi}{16}\right)(2t)^4 = \left(\frac{16}{3} - \pi\right)t^4$$

$$= \left(\frac{16}{3} - \pi\right)(0.174)^4 = 2.009 \times 10^{-3} \text{ in.}^4$$

The distance to the centroid of the spandrel from this axis is

$$\bar{x} = \frac{2r}{3(4-\pi)} = \frac{2(2 \times 0.002009)}{3(4-\pi)} = 3.121 \times 10^{-3} \text{ in.}$$

From the parallel axis theorem, the moment of inertia of the spandrel about a centroidal axis parallel to the 12-inch side is

$$\bar{I} = I - A\bar{x}^2 = 0.002009 - 0.002599(0.003121)^2 = 2.009 \times 10^{-3} \text{ in.}^4$$

Use the parallel-axis theorem and the following table to obtain the moment of inertia of the concrete:

| Segment | A | Ibar | d | Ibar +Ad2 |
|---|---|---|---|---|
| Outer rectangle | 9.600E+01 | 5.120E+02 | 0.000E+00 | 5.120E+02 |
| Steel shape | -6.760E+00 | -7.570E+01 | 0.000E+00 | -7.57E+01 |
| spandrel | -2.599E-02 | -2.009E-03 | 3.481E+00 | -3.169E-01 |
| spandrel | -2.599E-02 | -2.009E-03 | 3.481E+00 | -3.169E-01 |
| spandrel | -2.599E-02 | -2.009E-03 | 3.481E+00 | -3.169E-01 |
| spandrel | -2.599E-02 | -2.009E-03 | 3.481E+00 | -3.169E-01 |
| | | | | |
| | | | Sum | 4.350E+02 |

$$I_c = 435.0 \text{ in.}^4$$

Note that an approximate solution can be obtained by using

$$I_c = \frac{hb^3}{12} - I_s = \frac{12(8)^3}{12} - 75.7 = 436.3 \text{ in.}^4$$

For the remainder of the solution, we will use the approximate values

$$A_c = 89.24 \text{ in.}^2 \qquad \text{and} \qquad I_c = 436.3 \text{ in.}^4$$

From AISC Equation I2-4,

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + f'_c A_c = 46(6.76) + 0 + 4(89.24) = 667.9 \text{ kips}$$

From AISC Equation I2-7,

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3$$

$$= 0.1 + 2\left(\frac{6.76}{89.24 + 6.76}\right) = 0.2408 < 0.3 \quad \therefore \text{ use } C_1 = 0.2408$$

From AISC Equation I2-6,

$$(EI)_{eff} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c$$

$$= 29000(75.7) + 0 + 0.2408(3492)(436.32) = 2.562 \times 10^6 \text{ kip-in.}^2$$

From AISC Equation I2-5,

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} = \frac{\pi^2 (2.562 \times 10^6)}{(0.65 \times 20 \times 12)^2} = 1039 \text{ kips}$$

Determine which nominal strength equation to use:

$$\frac{P_{no}}{P_e} = \frac{667.9}{1039} = 0.6428 < 2.25 \quad \therefore \text{ use AISC Equation I2-2:}$$

$$P_n = P_{no}\left[0.658^{\left(\frac{P_{no}}{P_e}\right)}\right] = 667.5[0.658^{(0.6428)}] = 510 \text{ kips}$$

$\underline{P_n = 510 \text{ kips}}$

Alternate solution using more accurate values of $A_c$ and $I_c$:

$$A_c = 89.14 \text{ in.}^2 \qquad \text{and} \qquad I_c = 435.0 \text{ in.}^4$$

From AISC Equation I2-4,

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + f'_c A_c = 46(6.76) + 0 + 4(89.14) = 667.5 \text{ kips}$$

From AISC Equation I2-7,

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3$$

$$= 0.1 + 2\left(\frac{6.76}{89.14 + 6.76}\right) = 0.2410 < 0.3 \quad \therefore \text{ use } C_1 = 0.2410$$

From AISC Equation I2-6,

$$(EI)_{eff} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c$$

$$= 29000(75.7) + 0 + 0.2410(3492)(435.0) = 2.561 \times 10^6 \text{ kip-in.}^2$$

From AISC Equation I2-5,

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} = \frac{\pi^2 (2.561 \times 10^6)}{(0.65 \times 20 \times 12)^2} = 1039 \text{ kips}$$

Determine which nominal strength equation to use:

$$\frac{P_{no}}{P_e} = \frac{667.5}{1039} = 0.6424 < 2.25 \quad \therefore \text{ use AISC Equation I2-2:}$$

$$P_n = P_{no}\left[0.658^{\left(\frac{P_{no}}{P_e}\right)}\right] = 667.5[0.658^{(0.6424)}] = 510 \text{ kips} \quad \text{(no difference)}$$

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**9.10-2**

For a W12 x 96, $A_s = 28.2 \text{ in.}^2,\ I_x = 833 \text{ in.}^4,\ I_y = 270 \text{ in.}^4$

$$A_{sr} = 4(1.00) = 4.0 \text{ in.}^2$$

$$I_{sr} = \sum Ad^2 = 4 \times 1.00\left(\frac{20 - 2 \times 2.5}{2}\right)^2 = 225.0 \text{ in.}^4$$

$$A_c = 20(20) - A_s - A_{sr} = 400 - 28.2 - 4.0 = 367.8 \text{ in.}^2$$

$$E_c = w^{1.5}\sqrt{f'_c} = (145)^{1.5}\sqrt{8} = 4939 \text{ ksi}$$

$$I_c = \frac{1}{12}(20)(20)^3 = 13,330 \text{ in.}^4$$

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f'_c A_c = 50(28.2) + 60(4.0) + 0.85(8)(367.8)$$

$$= 4151 \text{ kips}$$

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.3$$

$$= 0.1 + 2\left(\frac{28.2}{367.8 + 28.2}\right) = 0.2424 < 0.3$$

$$(EI)_{eff} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c$$

For the y axis,

$$(EI)_{eff} = 29000(270) + 0.5(29000)(225.0) + 0.2424(4939)(13330)$$

$$= 2.705 \times 10^7 \text{ kip-in.}^2$$

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} = \frac{\pi^2 (2.705 \times 10^7)}{(13 \times 12)^2} = 1.097 \times 10^4 \text{ kips}$$

For the x axis,

$$(EI)_{eff} = 29000(833) + 0.5(29000)(225.0) + 0.2424(4939)(13330)$$

$$= 4.338 \times 10^7 \text{ kip-in.}^2$$

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} = \frac{\pi^2 (4.338 \times 10^7)}{(26 \times 12)^2} = 4398 \text{ kips} \Rightarrow \text{ x axis controls.}$$

$$\frac{P_{no}}{P_e} = \frac{4151}{4398} = 0.9438 < 2.25 \therefore \text{ use AISC Eq. I2-2.}$$

$$P_n = P_{no}\left[0.658^{\left(\frac{P_{no}}{P_e}\right)}\right] = 4151(0.658)^{0.9438} = 2796 \text{ kips}$$

$P_n = 2800$ kips

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**9.10-3**

(a) $P_u = 1.2(65) + 1.6(130) = 286.0$ kips

$K_x L = K_y L = 0.80(18) = 14.4$ ft

Trial shapes:

| | | |
|---|---|---|
| HSS7.500 × 0.500 | 294 kips < $\phi_c P_n$ < 308 kips | $w$ = 37.4 lb/ft |
| HSS8.625 x 0.322 | 297 kips < $\phi_c P_n$ < 308 kips | $w$ = 28.6 lb/ft |
| HSS10 x 0.188 | 299 kips < $\phi_c P_n$ < 308 kips | $w$ = 19.7 lb/ft |

An HSS10 x 0.188 is the lightest. Use an HSS10 x 0.188

(b) $P_a = 65 + 130 = 195$ kips

$K_x L = K_y L = 0.80(18) = 14.4$ ft

Trial shapes:

| | | |
|---|---|---|
| HSS7.500 × 0.500 | 196 kips < $P_n/\Omega_c$ < 205 kips | $w$ = 37.4 lb/ft |
| HSS8.625 x 0.322 | 198 kips < $P_n/\Omega_c$ < 205 kips | $w$ = 28.6 lb/ft |
| HSS10 x 0.188 | 200 kips < $P_n/\Omega_c$ < 206 kips | $w$ = 19.7 lb/ft |

An HSS10 x 0.188 is the lightest. Use an HSS10 x 0.188

**9.10-4**

(a) $P_u = 1.2(200) + 1.6(500) = 1040$ kips

$K_xL = 36$ ft, $K_yL = 12$ ft

Strong axis buckling will control when

$$\frac{K_xL}{r_{mx}/r_{my}} > K_yL, \text{ or } r_{mx}/r_{my} < \frac{K_xL}{K_yL} = \frac{36}{12} = 3$$

From Table 4-14, $r_{mx}/r_{my}$ is always less than 3, so $\frac{K_xL}{r_{mx}/r_{my}}$ will always control.

Trial shapes:

| Approximate $r_x/r_y$ | $K_xL/(r_x/r_y)$ (ft) | Shape | Actual $r_x/r_y$ | Actual $K_xL/(r_x/r_y)$ (ft) | $\phi_c P_n$ (kips) | Weight (lb/ft) |
|---|---|---|---|---|---|---|
| 1 | 36.00 | * | | | | |
| 1.2 | 30.00 | * | | | | |
| 1.3 | 27.69 | HSS16 ×12 × 5/8 | 1.27 | 28.35 | 1050 | 110 |
| 1.4 | 25.70 | * | | | | |
| 1.5 | 24.00 | * | | | | |
| 1.6 | 22.50 | HSS20 ×12 × 3/8 | 1.56 | 23.08 | > 1070 | 78.5 |
| 1.7 | 21.18 | * | | | | |

* No shapes with a design strength > 1040 kips

An HSS20 x 12 × $\frac{3}{8}$ is the lightest. <u>Use an HSS20 x 12 × $\frac{3}{8}$</u>

(b) $P_a = 200 + 500 = 700$ kips

$K_xL = 36$ ft, $K_yL = 12$ ft

Strong axis buckling will control when

$$\frac{K_xL}{r_{mx}/r_{my}} > K_yL, \text{ or } r_{mx}/r_{my} < \frac{K_xL}{K_yL} = \frac{36}{12} = 3$$

From Table 4-14, $r_{mx}/r_{my}$ is always less than 3, so $\dfrac{K_xL}{r_{mx}/r_{my}}$ will always control. Trial shapes:

| Approximate $r_x/r_y$ | $K_xL/(r_x/r_y)$ (ft) | Shape | Actual $r_x/r_y$ | Actual $K_xL/(r_x/r_y)$ (ft) | $P_n/\Omega_c$ (kips) | Weight (lb/ft) |
|---|---|---|---|---|---|---|
| 1 | 36.00 | * | | | | |
| 1.2 | 30.00 | * | | | | |
| 1.3 | 27.69 | HSS16 ×12 × 5/8 | 1.27 | 28.35 | > 706 | 110 |
| 1.4 | 25.70 | * | | | | |
| 1.5 | 24.00 | * | | | | |
| 1.6 | 22.50 | HSS20 ×12 × 3/8 | 1.56 | 23.08 | > 731 | 78.5 |
| 1.7 | 21.18 | * | | | | |

* No shapes with an allowable strength > 700 kips

An HSS20 x 12 × $\frac{3}{8}$ is the lightest. <u>Use an HSS20 x 12 × $\frac{3}{8}$</u>

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# CHAPTER 10 - PLATE GIRDERS

## 10.4-1

Check classification of shape.

$$\frac{h}{t_w} = \frac{78}{0.5} = 156, \qquad 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

Since $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$, the web is slender and AISC Section F5 applies.

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3\times 26)\left(\frac{78+3}{2}\right)^2$$

$$= 2.757\times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{275,700}{(78/2+3)} = 6564 \text{ in.}^3$$

Tension flange: $\quad M_n = F_y S_x = 50(6564) = 3.282\times 10^5$ in.-kips

Compression flange: LTB is not a factor in this problem. Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{26}{2(3)} = 4.333 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$\therefore F_{cr} = F_y = 50$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200+300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc}t_{fc}} = \frac{78(0.5)}{26(3)} = 0.5 < 10$$

$$R_{PG} = 1 - \frac{0.5}{1200+300(0.5)}\left(156 - 5.70\sqrt{\frac{29,000}{50}}\right) = 0.9931 < 1.0$$

$$M_n = R_{pg}F_{cr}S_x = 0.9931(50)6564 = 3.259\times 10^5 \text{ in.-kips}$$

Compression flange strength controls. $M_n = 325900/12 = 2.716\times 10^4$ ft-kips

$\underline{\phi_b M_n = 27,200 \text{ ft-kips}}$

**10.4-2**

Check classification of shape.

$$\lambda = \frac{h}{t_w} = \frac{45}{3/8} = 120, \qquad \lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

Since $\lambda < \lambda_r$, the web is not slender.

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29000}{50}} = 90.55$$

Since , $\lambda_p < \lambda < \lambda_r$, the web is noncompact.

Flange: $\lambda = \frac{b_f}{2t_f} = \frac{10}{2(1)} = 5 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15$

$\therefore$ flange is compact. Since the flange is compact and the web is noncompact, AISC F4 applies (Table User Note F1.1), but AISC F5 may be conservatively used (F4 User Note).

Compression flange strength (because of symmetry, tension yielding will not control):

$$M_n = R_{pg}F_{cr}S_{xc}$$

Since the flange is compact, $F_{cr} = F_y = 50$ ksi, and LTB is not a factor in this problem.

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc}t_{fc}} = \frac{45(3/8)}{10(1)} = 1.688 < 10$$

$$R_{PG} = 1 - \frac{1.688}{1200 + 300(1.688)}\left(120 - 5.7\sqrt{\frac{29000}{50}}\right) = 1.017 > 1.0 \therefore \text{ use } 1.0$$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(3/8)(45)^3 + 2(10)\left(\frac{45 + 1}{2}\right)^2 = 13,430 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{13,430}{(45/2 + 1)} = 571.5 \text{ in.}^3$$

$$M_n = R_{pg}F_{cr}S_x = 1.0(50)(571.5) = 28,580 \text{ in.-kips} = 2380 \text{ ft-kips}$$

$$\underline{M_n = 2380 \text{ ft-kips}}$$

---

**10.4-3**

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{60}{3/8} = 160, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(60)^3 + 2\left(\frac{7}{8}\times 12\right)\left(\frac{60+7/8}{2}\right)^2$$

$$= 2.621 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{26210}{(60/2+7/8)} = 848.9 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(848.9) = 4.245 \times 10^4 \text{ in.-kips} = 3538 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg}F_{cr}S_{xc}$$

where the critical stress $F_{cr}$ is based on either flange local buckling or yielding. For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{12}{2(7/8)} = 6.857, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, there is no flange local buckling. The compression flange strength is therefore based on yielding, and $F_{cr} = F_y = 50$ ksi.

To compute the bending strength reduction factor $R_{pg}$, the value of $a_w$ will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{60(3/8)}{12(7/8)} = 2.143 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$= 1 - \frac{2.143}{1200 + 300(2.143)}\left(160 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9736$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9736(50)(848.9) = 4.132 \times 10^4 \text{ in.-kips} = 3443 \text{ ft-kips}$$

Check lateral-torsional buckling.

$$\frac{h}{6} = \frac{60}{6} = 10 \text{ in.}, \qquad I = \frac{1}{12}(10)(3/8)^3 + \frac{1}{12}(7/8)(12)^3 = 126.0 \text{ in.}^4$$

$$A = 10(3/8) + 12(7/8) = 14.25 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{126}{14.25}} = 2.974 \text{ in.}$$

$$L_b = 40/2 = 20 \text{ ft}$$

$$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} = 1.1(2.974)\sqrt{\frac{29000}{50}} = 78.79 \text{ in.} = 6.566 \text{ ft}$$

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} = \pi(2.974)\sqrt{\frac{29000}{0.7(50)}} = 268.9 \text{ in.} = 22.40 \text{ ft}$$

Since $L_p < L_b < L_r$, the girder is subject to inelastic lateral-torsional buckling. From AISC Equation F5-3,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq F_y$$

$$= 1.30\left[50 - (0.3 \times 50)\left(\frac{20 - 6.566}{22.40 - 6.566}\right)\right] = 48.46 \text{ ksi} \leq 50 \text{ ksi}$$

where $C_b = 1.30$ is from Figure 5.15 in the textbook. LTB has the lowest critical stress and controls.

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9736(48.46)(848.9) = 4.005 \times 10^4 \text{ in.-kips} = 3338 \text{ ft-kips}$$

$$\underline{M_n = 3340 \text{ ft-kips}}$$

## 10.4-4

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{52}{1/4} = 208, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(1/4)(52)^3 + 2\left(\frac{3}{4}\times 18\right)\left(\frac{52+3/4}{2}\right)^2$$

$$= 2.171 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{21710}{(52/2 + 3/4)} = 811.6 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(811.6) = 4.058 \times 10^4 \text{ in.-kips} = 3382 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg} F_{cr} S_{xc}$$

where the critical stress $F_{cr}$ is based on either flange local buckling or yielding.

To compute the bending strength reduction factor $R_{pg}$, the value of $a_w$ will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{52(1/4)}{18(3/4)} = 0.9630 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$= 1 - \frac{0.9630}{1200 + 300(0.9630)}\left(208 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9543$$

For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{18}{2(3/4)} = 12.0, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}}$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{52/0.25}} = 0.2774 < 0.35 \ \therefore \text{ use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35.0 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{0.35(29000)}{35.0}} = 16.18$$

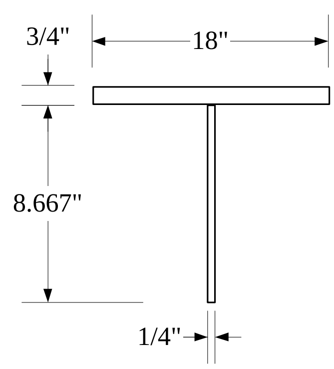
Since $\lambda_p < \lambda < \lambda_r$, the flange is noncompact, and FLB must be investigated.

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \text{ (AISC Eq. F5-8)}$$

$$= \left[50 - 0.3(50)\left(\frac{12.0 - 9.152}{16.18 - 9.152}\right)\right] = 43.92 \text{ ksi}$$

Check lateral-torsional buckling.

$$\frac{h}{6} = \frac{52}{6} = 8.667 \text{ in.}, \qquad I = \frac{1}{12}(3/4)(18)^3 + \frac{1}{12}(8.667)(1/4)^3 = 364.5 \text{ in.}^4$$




(not to scale)

$$A = 8.667(1/4) + 18(3/4) = 15.67 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{364.5}{15.67}} = 4.823 \text{ in.}$$

$$L_b = 50/2 = 25.0 \text{ ft}$$

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} = 1.1(4.823)\sqrt{\frac{29000}{50}} = 127.8 \text{ in.} = 10.65 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(4.823)\sqrt{\frac{29000}{0.7(50)}} = 436.1 \text{ in.} = 36.34 \text{ ft}$$

Since $L_p < L_b < L_r$, the girder is subject to inelastic lateral-torsional buckling. From AISC Equation F5-3,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

$$= 1.30\left[50 - (0.3 \times 50)\left(\frac{25.0 - 10.65}{36.34 - 10.65}\right)\right] = 54.11 \text{ ksi} > 50 \text{ ksi} \therefore \text{ use 50}$$
ksi

where $C_b = 1.30$ is from Figure 5.15 in the textbook. FLB has the lowest critical stress and controls.

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9543(43.92)(811.6)/12 = 2835 \text{ ft-kips}$$

$$\underline{M_n = 2840 \text{ ft-kips}}$$

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**10.4-5**

Check classification of shape.

$$\frac{h}{t_w} = \frac{50}{0.25} \qquad 5.70\sqrt{\frac{E}{F_y}} = 1140.0\sqrt{\left(\frac{E}{F_y}\right)} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

Since $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$, the web is slender and the provisions of AISC F5 apply.

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3 \times 22)\left(\frac{78 + 3}{2}\right)^2$$

$$= 2.363 \times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{236300}{(78/2 + 3)} = 5626 \text{ in.}^3$$

Tension flange: $M_n = F_y S_{xt} = 50(5626) = 2.813 \times 10^5$ in.-kips

Compression flange: LTB is not a factor in this problem. Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{22}{2(3)} = 3.667 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$\therefore F_{cr} = F_y = 50$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{78(0.5)}{22(3)} = 0.5909 < 10$$

$$R_{pg} = 1 - \frac{0.5909}{1200 + 300(0.5909)}\left(156 - 5.7\sqrt{\frac{29,000}{50}}\right) = 0.9920 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9920(50)(5626) = 2.79 \times 10^5 \text{ in.-kips}$$

Compression flange strength controls. $M_n = 279000/12 = 2.325 \times 10^4$ ft-kips

(a) LRFD

$$\phi_b M_n = 0.90(23250) = 20,900 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) + 1.6(2) = 4.4 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(500) = 800.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(4.4)(80)^2 + \frac{800(80)}{4} = 19,500 \text{ ft-kips}$$

Since 19,500 ft-kips < 20,900 ft-kips, flexural strength is adequate

(b) ASD

$$\frac{M_n}{\Omega_b} = \frac{23250}{1.67} = 1.392 \times 10^4 \text{ ft-kips}$$

$$w_a = w_D + w_L = 1 + 2 = 3 \text{ kips/ft}$$

$$P_a = P_L = 500 \text{ kips}$$

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(3)(80)^2 + \frac{500(80)}{4} = 1.24 \times 10^4 \text{ ft-kips}$$

Since 12,400 ft-kips < 13,900 ft-kips, flexural strength is adequate

**10.5-1**

(a) $\frac{h}{t_w} = \frac{70}{1/2} = 140, \qquad \frac{a}{h} = \frac{70}{70} = 1 < 3$

$$\left(\frac{260}{h/t_w}\right)^2 = \left(\frac{260}{140}\right)^2 = 3.45 > \frac{a}{h}$$

From AISC Equation G2-6,

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.0)^2} = 10$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{10(29000)}{50}} = 83.77$$

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{10(29000)}{50}} = 104.3$$

Since $\frac{h}{t_w} > 104.3, \qquad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(10)(29000)}{(140)^2(50)} = 0.4468$

Tension field action cannot be used in an end panel:

$$V_n = 0.6 F_y A_w C_v = 0.6(50)(0.5 \times 70)(0.4468) = 469.1 \text{ kips}$$

$\underline{V_n = 469 \text{ kips}}$

(b) $\frac{a}{h} = \frac{200}{70} = 2.857 < \left(\frac{260}{h/t_w}\right)^2$ and $< 3$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(2.857)^2} = 5.613$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5.613(29000)}{50}} = 62.76$$

Since $\frac{h}{t_w} > 62.76,$ tension field action can be used, and

$$V_n = 0.6 F_y A_w \left( C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}} \right)$$

Compute $C_v$. $\quad 1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{5.613(29000)}{50}} = 78.17$

Since $\frac{h}{t_w} > 78.11$, $\quad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(5.613)(29000)}{(140)^2(50)} = 0.2508$

$$V_n = 0.6(50)(0.5 \times 70)\left(0.2508 + \frac{1 - 0.2508}{1.15\sqrt{1 + (2.857)^2}}\right) = 489.3 \text{ kips}$$

$\underline{V_n = 489 \text{ kips}}$

(c) If no intermediate stiffeners are used, $\frac{a}{h} > 3$, and $k_v = 5$ (no tension field permitted)

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5(29{,}000)}{50}} = 59.24$$

Since $\frac{h}{t_w} > 59.24$, $V_n = 0.6 F_y A_w C_v$. Compute $C_v$.

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{5(29{,}000)}{50}} = 73.78$$

Since $\frac{h}{t_w} > 73.78$, $\quad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(5)(29000)}{(140)^2(50)} = 0.2234$

$$V_n = 0.6 F_y A_w C_v = 0.6(50)(0.5 \times 70)(0.2234) = 234.6 \text{ kips}$$

$\underline{V_n = 235 \text{ kips}}$

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**10.5-2**

(a) $\frac{h}{t_w} = \frac{90}{9/16} = 160.0$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(4.2) + 1.6(5) = 13.04 \text{ kips/ft}$$

Reaction $= \frac{w_u L}{2} = \frac{13.04(75)}{2} = 489.0$ kips = required $\phi_v V_n$ in end panel

Required $V_n = \frac{489}{0.90} = 543.3$ kips

$$V_n = 0.6A_wF_yC_v = 0.6(90 \times 9/16)(50)C_v = 543.3, \text{ Solution is: } \{C_v = 0.3577\}$$

$$C_v = \frac{1.51k_vE}{(h/t_w)^2F_y} = \frac{1.51k_v(29000)}{(160)^2(50)} = 0.3577, \text{ Solution is: } \{k_v = 10.46\}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(a/h)^2} = 10.46 \qquad \Rightarrow \qquad a/h = 0.9569$$

$a = 0.9569h = 0.9569(90) = 86.12$ in. <u>Use $a = 86$ in</u>.

(b) $\dfrac{h}{t_w} = \dfrac{90}{9/16} = 160.0$

$$w_a = D + L = 4.2 + 5 = 9.2$$

Reaction = $\dfrac{w_aL}{2} = \dfrac{9.2(75)}{2} = 345.0$ kips = required $V_n/\Omega_v$ in end panel

Required $V_n = \Omega_v \dfrac{V_n}{\Omega_v} = 1.67(345) = 576.2$ kips

$$V_n = 0.6A_wF_yC_v = 0.6(90 \times 9/16)(50)C_v = 576.2, \text{ Solution is: } \{C_v = 0.3794\}$$

$$C_v = \frac{1.51k_vE}{(h/t_w)^2F_y} = \frac{1.51k_v(29000)}{(160)^2(50)} = 0.3794, \text{ Solution is: } \{k_v = 11.09\}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(a)^2} = 11.09 \qquad \Rightarrow \qquad a/h = 0.9061$$

$a = 0.9061h = 0.9061(90) = 81.55$ in. <u>Use $a = 81$ in</u>..

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**<u>10.5-3</u>**

Before developing the LRFD and ASD solutions, compute the nominal shear strength of each panel.

$$\frac{h}{t_w} = \frac{66}{5/16} = 211.2$$

End panel: $a = 6(12) + 2 = 74$ in., $\dfrac{a}{h} = \dfrac{74}{66} = 1.121 < 3$

$$\left(\frac{260}{h/t_w}\right)^2 = \left(\frac{260}{211.2}\right)^2 = 1.516 > \frac{a}{h}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.121)^2} = 8.979$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{8.979(29000)}{50}} = 79.38$$

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{8.979(29000)}{50}} = 98.87$$

Since $\frac{h}{t_w} > 98.87, \qquad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(8.979)(29000)}{(211.2)^2(50)} = 0.1763$

Tension field action cannot be used in an end panel:

$$V_n = 0.6 F_y A_w C_v = 0.6(50)(5/16 \times 66)(0.1763) = 109.1 \text{ kips}$$

Second panel:

$$a = 12(12) + 9 - 74 = 79 \text{ in.}$$

$$\frac{a}{h} = \frac{79}{66} = 1.197 < \left(\frac{260}{h/t_w}\right)^2 \text{ and} < 3$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.197)^2} = 8.490$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{8.490(29000)}{50}} = 77.19$$

Since $\frac{h}{t_w} > 77.19,$ tension field action can be used, and

$$V_n = 0.6 F_y A_w \left( C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}} \right)$$

Compute $C_v$. $\qquad 1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{8.490(29000)}{50}} = 96.14$

Since $\frac{h}{t_w} > 96.14, \qquad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(8.490)(29000)}{(211.2)^2(50)} = 0.1667$

$$V_n = 0.6(50)(5/16 \times 66)\left( 0.1667 + \frac{1 - 0.1667}{1.15\sqrt{1 + (1.197)^2}} \right) = 390.6 \text{ kips}$$

Middle panel:

$$a = 55(12) - 2(12 \times 12 + 9) = 354 \text{ in.},$$

$$\frac{a}{h} = \frac{354}{66} = 5.364 > 3 \quad \therefore\ k_v = 5 \text{ and tension-field action cannot be used.}$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5(29000)}{50}} = 59.24$$

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{5(29000)}{50}} = 73.78$$

Compute $C_v$.

Since $\frac{h}{t_w} > 73.78$, $\quad C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} = \frac{1.51(5)(29000)}{(211.2)^2(50)} = 9.817 \times 10^{-2}$

$$V_n = 0.6 F_y A_w C_v = 0.6(50)(5/16 \times 66)(0.09817) = 60.74 \text{ kips}$$

(a) LRFD solution

End panel: Design strength $= \phi_v V_n = 0.90(109.1) = 98.2$ kips

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(0.225) + 1.6(2.0) = 3.470 \text{ kips/ft}$$

$$\text{Left reaction} = V_L = \frac{3.470(55)}{2} = 95.43 \text{ kips} < 98.2 \text{ kips} \qquad \text{(OK)}$$

Second panel: Design strength $= \phi_v V_n = 0.90(390.6) = 352$ kips

$$V_u \text{ at beginning of panel} = V_L - w_u x = 95.43 - 3.470(6.167)$$

$$= 74.0 \text{ kips} < 352 \text{ kips} \qquad \text{(OK)}$$

Middle panel: Design strength $= \phi_v V_n = 0.90(60.74) = 54.7$ kips

$$V_u \text{ at beginning of panel} = V_L - w_u x = 95.43 - 3.470(12.75)$$

$$= 51.2 \text{ kips} < 54.7 \text{ kips} \qquad \text{(OK)}$$

Girder has enough shear strength

(b) ASD solution

End panel: Allowable strength $= \dfrac{V_n}{\Omega_v} = \dfrac{109.1}{1.67} = 65.3$ kips

$$w_a = w_D + w_L = 0.225 + 2.0 = 2.225 \text{ kips/ft}$$

Left reaction $= V_L = \dfrac{2.225(55)}{2} = 61.19$ kips < 65.3 kips (OK)

Second panel: Allowable strength $= \dfrac{V_n}{\Omega_v} = \dfrac{390.6}{1.67} = 234$ kips

$V_a$ at beginning of panel $= V_L - w_a x = 61.19 - 2.225(6.167)$

$= 47.5$ kips < 234 kips (OK)

Middle panel: Allowable strength $= \dfrac{V_n}{\Omega_v} = \dfrac{60.74}{1.67} = 36.4$ kips

$V_a$ at beginning of panel $= V_L - w_a x = 61.19 - 2.225(12.75)$

$= 32.8$ kips < 36.4 kips (OK)

Girder has enough shear strength

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**10.5-4**

Determine the nominal shear strength for each panel

$$\frac{h}{t_w} = \frac{78}{0.5} = 156$$

End panel: $\dfrac{a}{h} = \dfrac{48}{78} = 0.6154 < 3$

$$\left(\frac{260}{h/t_w}\right)^2 = \left(\frac{260}{211.2}\right)^2 = 1.516 > \frac{a}{h}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(0.6154)^2} = 18.2$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{18.2(29,000)}{50}} = 113.0$$

Since $\dfrac{h}{t_w} > 113.0$, $V_n = 0.6F_yA_wC_v$. Compute $C_v$.

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{18.2(29,000)}{50}} = 140.8$$

Since $\dfrac{h}{t_w} > 140.8$,

$$C_v = \frac{1.51k_vE}{(h/t_w)^2F_y} = \frac{1.51(18.2)(29,000)}{(156)^2(50)} = 0.6550$$

$$V_n = 0.6A_wF_yC_v = 0.6(78 \times 0.5)(50)(0.6550) = 766.4 \text{ kips}$$

For the 12-ft panels,

$$\frac{a}{h} = \frac{144}{78} = 1.846 > \left(\frac{260}{h/t_w}\right)^2 \therefore k_v = 5$$

Tension field action cannot be used.

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5(29,000)}{50}} = 59.24$$

Since $\dfrac{h}{t_w} > 59.24$, $V_n = 0.6A_wF_yC_v$

Compute $C_v$. $\quad 1.37\sqrt{\dfrac{k_vE}{F_y}} = 1.37\sqrt{\dfrac{5(29,000)}{50}} = 73.78$

Since $\dfrac{h}{t_w} > 73.78$, $\quad C_v = \dfrac{1.51k_vE}{(h/t_w)^2F_y} = \dfrac{1.51(5)(29,000)}{(156)^2(50)} = 0.1799$

$$V_n = 0.6(78 \times 0.5)(50)(0.1799) = 210.5 \text{ kips}$$

All 12-ft panels have the same shear strength.

(a) LRFD Solution

Compute the factored-load shear at the beginning of each panel (this will be the maximum shear in the panel).

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) + 1.6(2) = 4.4 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(500) = 800.0 \text{ kips}$$

$$\text{Left reaction} = V_L = \frac{w_u L + P_u}{2} = \frac{4.4(80) + 800}{2} = 576.0 \text{ kips}$$

At 4 ft, $V_u = V_L - 4.4x = 576 - 4.4(4) = 558$ kips

At 16 ft, $V_u = 576 - 4.4(16) = 506$ kips

At 28 ft, $V_u = 576 - 4.4(28) = 453$ kips

End panel:

$$\phi_v V_n = 0.90(766.4) = 690 \text{ kips} > 576 \text{ kips} \quad \text{(OK)}$$

First interior panel:

$$\phi_v V_n = 0.90(210.5) = 190 \text{ kips} < 558 \text{ kips} \quad \text{(N.G.)}$$

The shear strength of all of the 12-ft panels will be the same, and the shear is greater than 190 kips in each panel, therefore, there will not be enough shear strength in any of the 12-ft panels.

The girder does not have enough shear strength.

(b) ASD Solution

Compute the shear at the beginning of each panel (this will be the maximum shear in the panel).

$$w_a = D + L = 1 + 2 = 3 \text{ kips/ft}$$

$$P_a = P_L = 500 \text{ kips}$$

$$\text{Left reaction} = V_L = \frac{w_a L + P_a}{2} = \frac{3(80) + 500}{2} = 370 \text{ kips}$$

At 4 ft, $V_a = V_L - 3x = 370 - 3(4) = 358$ kips

At 16 ft, $V_a = 370 - 3(16) = 322$ kips

At 28 ft, $V_a = 370 - 3(28) = 286$ kips

End panel:

$$\frac{V_n}{\Omega_n} = \frac{766.4}{1.67} = 459 \text{ kips} > 370 \text{ kips} \quad \text{(OK)}$$

First interior panel:

$$\frac{V_n}{\Omega_n} = \frac{210.5}{1.67} = 126 \text{ kips} < 358 \text{ kips} \qquad \text{(N.G.)}$$

The shear strength of all of the 12-ft panels will be the same, and the shear is greater than 190 kips in each panel, therefore, there will not be enough shear strength in any of the 12-ft panels.

The girder does not have enough shear strength.

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**10.6-1**

Bearing strength: $A_{pb} = (6 - 0.5)(1/2) \times 2 = 5.5 \text{ in.}^2$

$$R_n = 1.8F_yA_{pb} = 1.8(50)(5.5) = 495.0 \text{ kips}$$

Compressive strength: Use a length of web equal to

$$25t_w = 25(5/16) = 7.813 \text{ in.}$$

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(7.813)(5/16)^3 + 2\left[\frac{1}{12}(1/2)(6)^3 + 6(1/2)(3 + 5/32)^2\right] = 77.79 \text{ in.}^4$$

$$A = 7.813(5/16) + 2(6)(1/2) = 8.442 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{77.79}{8.442}} = 3.036 \text{ in.}$$

The slenderness ratio is $\frac{KL}{r} = \frac{Kh}{r} = \frac{0.75(56)}{3.036} = 13.83 < 25$

$\therefore\ P_n = F_yA_g = 50(8.442) = 422.1 \text{ kips}$

(a) LRFD solution

Bearing strength $= \phi R_n = 0.75(422.1) = 317 \text{ kips}$

Compressive strength $= \phi P_n = 0.90(316.6) = 285 \text{ kips}$

Compression controls: Maximum factored concentrated load = 285 kips

(b) ASD solution

Bearing strength = $\dfrac{R_n}{\Omega} = \dfrac{422.1}{2.00} = 211$ kips

Compressive strength = $\dfrac{P_n}{\Omega} = \dfrac{316.6}{1.67} = 190$ kips

Compression controls: <u>Maximum service concentrated load = 190 kips</u>

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**<u>10.6-2</u>**

Bearing strength: $A_{pb} = (6 - 0.5)(9/16) \times 4 = 12.38 \text{ in.}^2$

$R_n = 1.8F_yA_{pb} = 1.8(36)(12.38) = 802.2 \text{ kips}$

Compressive strength: Use a length of web equal to

$12t_w = 12(3/16) = 2.25 \text{ in.}$

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(2.25)(3/16)^3 + 4\left[\frac{1}{12}(9/16)(6)^3 + 6(9/16)(3 + 3/32)^2\right] = 169.7 \text{ in.}^4$$

$$A = 2.25(3/16) + 4(6)(9/16) = 13.92 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{169.2}{13.92}} = 3.486 \text{ in.}$$

The slenderness ratio is $\dfrac{KL}{r} = \dfrac{Kh}{r} = \dfrac{0.75(34)}{3.486} = 7.315 < 25$

$\therefore\ P_n = F_yA_g = 36(13.92) = 501.1 \text{ kips}$

(a) LRFD solution

Bearing strength = $\phi R_n = 0.75(802.2) = 602$ kips

Compressive strength = $\phi P_n = 0.90(501.1) = 451$ kips

Compression controls: <u>Maximum factored concentrated load = 451 kips</u>

(b) ASD solution

Bearing strength = $\frac{R_n}{\Omega} = \frac{802.2}{2.00} = 401$ kips

Compressive strength = $\frac{P_n}{\Omega} = \frac{451}{1.67} = 270$ kips

Compression controls: Maximum service concentrated load = 270 kips

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**10.7-1**

(a) Try $t_f = ¾$ in., $h = 73 - 2(0.75) = 71.5$ in.

For a slender web, $\frac{h}{t_w} \geq 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$

$\therefore\ t_w \leq \frac{h}{137.3} = \frac{71.5}{137.3} = 0.5208$ in.

For $\frac{a}{h} \leq 1.5$, $\frac{h}{t_w} \leq 12.0\sqrt{\frac{E}{F_y}} = 12.0\sqrt{\frac{29000}{50}} = 289.0$

For $\frac{a}{h} > 1.5$, $\frac{h}{t_w} \leq \frac{0.4E}{F_y} = \frac{0.4(29000)}{50} = 232.0$

$t_w \geq \frac{h}{232.0} = \frac{71.5}{232.0} = 0.3082$ in.

Try a $\frac{5}{16}$ in. × $71\frac{1}{2}$ in. web.

$\frac{h}{t_w} = \frac{71.5}{5/16} = 228.8$, $A_w = (5/16)(71.5) = 22.34 \text{ in.}^2$

Estimate required flange size.

$M_u = 1.2M_D + 1.6M_L = 1.2(0.25 \times 2400) + 1.6(0.75 \times 2400) = 3600$ ft-kips

$A_f = \frac{M_u/\phi_b}{hF_y} - \frac{A_w}{6} = \frac{3600(12)/0.90}{71.5(50)} - \frac{22.34}{6} = 9.703 \text{ in.}^2$

$b_f \geq \frac{9.703}{0.75} = 12.94$ in.

Try a $\frac{3}{4}$-in. × 14-in. flange, $A_f = 0.75(14) = 10.5 \text{ in.}^2$

$$I_x = \frac{1}{12} t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(5/16)(71.5)^3 + 2(10.5)\left(\frac{71.5 + 0.75}{2}\right)^2$$

$$= 3.692 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{36,920}{(71.5/2 + 0.75)} = 1012 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{14}{2(0.75)} = 9.333, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{228.8}} = 0.264 < 0.35 \therefore \text{ use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}} = 0.95\sqrt{\frac{0.35(29000)}{35}} = 16.18$$

Since $\lambda_p < \lambda < \lambda_r$,

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$= \left[50 - 0.3(50)\left(\frac{9.333 - 9.152}{16.18 - 9.152}\right)\right] = 49.61 \text{ ksi}$$

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{71.5(5/16)}{14(3/4)} = 2.128 < 10$$

$$R_{pg} = 1 - \frac{2.128}{1200 + 300(2.128)}\left(228.8 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.8941 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.8941(50)(1012) = 4.524 \times 10^4 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(45240)/12 = 3390 \text{ ft-kips} < 3600 \text{ ft-kips} \qquad \text{(N.G.)}$$

Try a $\frac{3}{4}$-in. × $17\frac{1}{2}$-in. flange, $A_f = 0.75(17.5) = 13.13 \text{ in.}^2$

$$I_x = \frac{1}{12} t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(5/16)(71.5)^3 + 2(13.13)\left(\frac{71.5 + 0.75}{2}\right)^2$$

$$= 4.379 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{43{,}790}{(71.5/2 + 0.75)} = 1200 \text{ in.}^3$$

$$\lambda = \frac{b_f}{2t_f} = \frac{17.5}{2(0.75)} = 11.67, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{228.8}} = 0.264 < 0.35 \therefore \text{ use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}} = 0.95\sqrt{\frac{0.35(29000)}{35}} = 16.18$$

Since $\lambda_p < \lambda < \lambda_r$,

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$= \left[50 - 0.3(50)\left(\frac{11.67 - 9.152}{16.18 - 9.152}\right)\right] = 44.63 \text{ ksi}$$

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{71.5(5/16)}{17.5(3/4)} = 1.702 < 10$$

$$R_{pg} = 1 - \frac{1.702}{1200 + 300(1.702)}\left(228.8 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9089 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9089(50)(1200) = 5.453 \times 10^4 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(54530)/12 = 4090 \text{ ft-kips} > 3600 \qquad \text{(OK)}$$

Use a $\frac{5}{16} \times 71\frac{1}{2}$ web and $\frac{3}{4} \times 17\frac{1}{2}$ flanges

(b) Try $t_f = ¾$ in., $\quad h = 73 - 2(0.75) = 71.5$ in.

For a slender web, $\dfrac{h}{t_w} \ge 5.70\sqrt{\dfrac{E}{F_y}} = 5.70\sqrt{\dfrac{29{,}000}{50}} = 137.3$

$$\therefore\ t_w \le \frac{h}{137.3} = \frac{71.5}{137.3} = 0.5208 \text{ in.}$$

For $\dfrac{a}{h} \le 1.5,\quad \dfrac{h}{t_w} \le 12.0\sqrt{\dfrac{E}{F_y}} = 12.0\sqrt{\dfrac{29000}{50}} = 289.0$

For $\dfrac{a}{h} > 1.5,\quad \dfrac{h}{t_w} \le \dfrac{0.4E}{F_y} = \dfrac{0.4(29000)}{50} = 232.0$

$$t_w \ge \frac{h}{232.0} = \frac{71.5}{232.0} = 0.3082 \text{ in.}$$

Try a $\frac{5}{16}$ in. × $71\frac{1}{2}$ in. web.

$$\frac{h}{t_w} = \frac{71.5}{5/16} = 228.8, \qquad A_w = (5/16)(71.5) = 22.34 \text{ in.}^2$$

Estimate required flange size.

$$M_a = 2400 \text{ ft-kips}$$

$$A_f = \frac{\Omega_b M_a}{hF_y} - \frac{A_w}{6} = \frac{1.67(2400\times 12)}{71.5(50)} - \frac{22.34}{6} = 9.73 \text{ in.}^2$$

$$b_f \ge \frac{9.73}{0.75} = 12.97 \text{ in.}$$

Try a $\frac{3}{4}$-in. × 14-in. flange, $A_f = 0.75(14) = 10.5 \text{ in.}^2$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(5/16)(71.5)^3 + 2(10.5)\left(\frac{71.5+0.75}{2}\right)^2$$

$$= 3.692\times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{36{,}920}{(71.5/2+0.75)} = 1012 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{14}{2(0.75)} = 9.333, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{228.8}} = 0.264 < 0.35 \;\therefore\; \text{use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}} = 0.95\sqrt{\frac{0.35(29000)}{35}} = 16.18$$

Since $\lambda_p < \lambda < \lambda_r$,

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$= \left[50 - 0.3(50)\left(\frac{9.333 - 9.152}{16.18 - 9.152}\right)\right] = 49.61 \text{ ksi}$$

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{71.5(5/16)}{14(3/4)} = 2.128 < 10$$

$$R_{pg} = 1 - \frac{2.128}{1200 + 300(2.128)}\left(228.8 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.8941 < 1.0$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.8941(50)(1012) = 4.524 \times 10^4 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{45240}{1.67(12)} = 2257 \text{ ft-kips} < 2400 \text{ ft-kips} \qquad \text{(N.G.)}$$

Try a $\frac{3}{4}$-in. × $17\frac{1}{2}$-in. flange, $A_f = 0.75(17.5) = 13.13 \text{ in.}^2$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(5/16)(71.5)^3 + 2(13.13)\left(\frac{71.5 + 0.75}{2}\right)^2$$

$$= 4.379 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{43,790}{(71.5/2 + 0.75)} = 1200 \text{ in.}^3$$

$$\lambda = \frac{b_f}{2t_f} = \frac{17.5}{2(0.75)} = 11.67, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{228.8}} = 0.264 < 0.35 \therefore \text{ use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}} = 0.95\sqrt{\frac{0.35(29000)}{35}} = 16.18$$

Since $\lambda_p < \lambda < \lambda_r$,

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$= \left[50 - 0.3(50)\left(\frac{11.67 - 9.152}{16.18 - 9.152}\right)\right] = 44.63 \text{ ksi}$$

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{71.5(5/16)}{17.5(3/4)} = 1.702 < 10$$

$$R_{pg} = 1 - \frac{1.702}{1200 + 300(1.702)}\left(228.8 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9089 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9089(50)(1200) = 5.453 \times 10^4 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{54530}{1.67(12)} = 2720 \text{ ft-kips} > 2400 \qquad \text{(OK)}$$

Use a $\frac{5}{16} \times 71\frac{1}{2}$ web and $\frac{3}{4} \times 17\frac{1}{2}$ flanges

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**10.7-2**

(a) Try $t_f = 1.5$ in., $\quad h = 86 - 2(1.5) = 83.0$ in.

For a slender web, $\dfrac{h}{t_w} \geq 5.70\sqrt{\dfrac{E}{F_y}} = 5.70\sqrt{\dfrac{29,000}{50}} = 137.3$

$$\therefore\ t_w \leq \frac{h}{137.3} = \frac{83}{137.3} = 0.6045 \text{ in.}$$

For $\dfrac{a}{h} \leq 1.5$, $\dfrac{h}{t_w} \leq 12.0\sqrt{\dfrac{E}{F_y}} = 12.0\sqrt{\dfrac{29000}{50}} = 289.0$

For $\dfrac{a}{h} > 1.5$, $\dfrac{h}{t_w} \leq \dfrac{0.4E}{F_y} = \dfrac{0.4(29000)}{50} = 232.0$

$$t_w \geq \frac{h}{232.0} = \frac{83.0}{232.0} = 0.3578 \text{ in.}$$

Try a $\frac{3}{8}$ in. × 83 in. web.

$$\frac{h}{t_w} = \frac{83}{3/8} = 221.3, \qquad A_w = (3/8)(83) = 31.13 \text{ in.}^2$$

Estimate required flange size.

$$M_u = 1.2M_D + 1.6M_L = 1.2(1320) + 1.6(2700) = 5904 \text{ ft-kips}$$

$$A_f = \frac{M_u/\phi_b}{hF_y} - \frac{A_w}{6} = \frac{5904(12)/0.90}{83(50)} - \frac{31.13}{6} = 13.78 \text{ in.}^2$$

$$b_f \geq \frac{13.78}{1.5} = 9.187 \text{ in.}$$

Try a $1\frac{1}{2}$-in. × $11\frac{3}{4}$-in. flange. $A_f = 1.5(11.75) = 17.63 \text{ in.}^2$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(83)^3 + 2(17.63)\left(\frac{83+1.5}{2}\right)^2$$

$$= 8.0810 \times 10^4 \text{ in.}^4$$

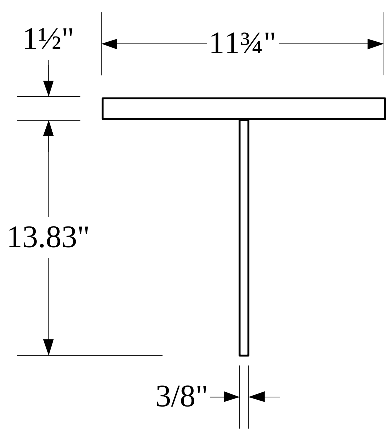
$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{80810}{(83/2 + 1.5)} = 1879 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{11.75}{2(1.5)} = 3.917, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, $F_{cr} = F_y = 50$ ksi

Check lateral-torsional buckling.




(not to scale)

$$\frac{h}{6} = \frac{83}{6} = 13.83 \text{ in.}$$

$$I = \frac{1}{12}(13.83)(3/8)^3 + \frac{1}{12}(1.5)(11.75)^3 = 202.8 \text{ in.}^4$$

$$A = 13.83(3/8) + 11.75(1.5) = 22.81 \text{ in.}^2$$

$$r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{202.8}{22.81}} = 2.982 \text{ in.}$$

$$L_b = 25 \text{ ft.}$$

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} = 1.1(2.982)\sqrt{\frac{29000}{50}} = 79.00 \text{ in.} = 6.583 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(2.982)\sqrt{\frac{29000}{0.7(50)}} = 269.7 = 22.48 \text{ ft}$$

Since $L_b > L_r$,

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_t}\right)^2} \leq F_y$$

$$= \frac{1.67\pi^2(29000)}{\left(\frac{25 \times 12}{2.982}\right)^2} = 47.23 \text{ ksi } < 50 \text{ ksi}$$

$\therefore$ LTB controls, and $F_{cr} = 47.23$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{83(3/8)}{11.75(1.5)} = 1.766 < 10$$

$$R_{pg} = 1 - \frac{1.766}{1200 + 300(1.766)}\left(221.3 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9142 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9142(50)(1879) = 8.589 \times 10^4 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(85890)/12 = 6442 \text{ ft-kips} > 5904 \text{ ft-kips} \qquad \text{(OK)}$$

Use a $\frac{3}{8} \times 83$ web and $1\frac{1}{2} \times 11\frac{3}{4}$ flanges

(b) Try $t_f = 1.5$ in., $h = 86 - 2(1.5) = 83.0$ in.

For a slender web, $\frac{h}{t_w} \geq 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$

$$\therefore \; t_w \leq \frac{h}{137.3} = \frac{83}{137.3} = 0.6045 \text{ in.}$$

For $\frac{a}{h} \leq 1.5$, $\frac{h}{t_w} \leq 12.0\sqrt{\frac{E}{F_y}} = 12.0\sqrt{\frac{29000}{50}} = 289.0$

For $\frac{a}{h} > 1.5$, $\frac{h}{t_w} \leq \frac{0.4E}{F_y} = \frac{0.4(29000)}{50} = 232.0$

$$t_w \geq \frac{h}{232.0} = \frac{83.0}{232.0} = 0.3578 \text{ in.}$$

Try a $\frac{3}{8}$ in. × 83 in. web.

$$\frac{h}{t_w} = \frac{83}{3/8} = 221.3, \qquad A_w = (3/8)(83) = 31.13 \text{ in.}^2$$

Estimate required flange size.

$$M_a = M_D + M_L = 1320 + 2700 = 4020 \text{ ft-kips}$$

$$A_f = \frac{\Omega_b M_a}{hF_y} - \frac{A_w}{6} = \frac{1.67(4020)(12)}{83(50)} - \frac{31.13}{6} = 14.22 \text{ in.}^2$$

$$b_f \geq \frac{14.22}{1.5} = 9.48 \text{ in.}$$

Try a $1\frac{1}{2}$-in. × $11\frac{3}{4}$-in. flange. $A_f = 1.5(11.75) = 17.63$ in.$^2$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(3/8)(83)^3 + 2(17.63)\left(\frac{83 + 1.5}{2}\right)^2$$

$$= 8.0810 \times 10^4 \text{ in.}^4$$

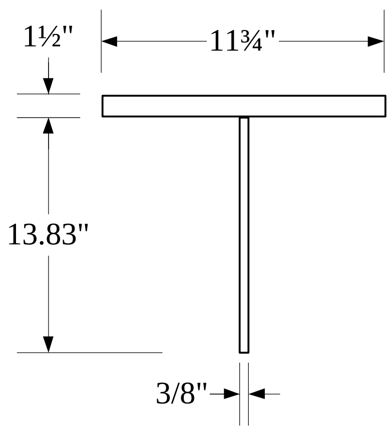
$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{80810}{(83/2 + 1.5)} = 1879 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{11.75}{2(1.5)} = 3.917, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

Since $\lambda < \lambda_p,\ F_{cr} = F_y = 50$ ksi

Check lateral-torsional buckling.




(not to scale)

$$\frac{h}{6} = \frac{83}{6} = 13.83 \text{ in.}$$

$$I = \frac{1}{12}(13.83)(3/8)^3 + \frac{1}{12}(1.5)(11.75)^3 = 202.8 \text{ in.}^4$$

$$A = 13.83(3/8) + 11.75(1.5) = 22.81 \text{ in.}^2$$

$$r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{202.8}{22.81}} = 2.982 \text{ in.}$$

$$L_b = 25 \text{ ft.}$$

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} = 1.1(2.982)\sqrt{\frac{29000}{50}} = 79.00 \text{ in.} = 6.583 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(2.982)\sqrt{\frac{29000}{0.7(50)}} = 269.7 = 22.48 \text{ ft}$$

Since $L_b > L_r$,

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_t}\right)^2} \le F_y$$

$$= \frac{1.67\pi^2(29000)}{\left(\frac{25 \times 12}{2.982}\right)^2} = 47.23 \text{ ksi } < 50 \text{ ksi}$$

$\therefore$ LTB controls, and $F_{cr} = 47.23$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{83(3/8)}{11.75(1.5)} = 1.766 < 10$$

$$R_{pg} = 1 - \frac{1.766}{1200 + 300(1.766)}\left(221.3 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9142 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9142(50)(1879) = 8.589 \times 10^4 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{85890}{1.67(12)} = 4286 \text{ ft-kips} > 4020 \text{ ft-kips} \qquad \text{(OK)}$$

Use a $\frac{3}{8} \times 83$ web and $1\frac{1}{2} \times 11\frac{3}{4}$ flanges

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**10.7-3**

Assume a girder weight of 160 lb/ft.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.5 + 0.160) = 0.792 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(125) = 200.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(0.792)(50)^2 + \frac{200(50)}{4} = 2748 \text{ ft-kips}$$

$$V_u = V_L = \frac{w_u L + P_u}{2} = \frac{0.792(50) + 200}{2} = 120 \text{ kips}$$

Total depth: $\frac{L}{10} = \frac{50(12)}{10} = 60$ in., $\frac{L}{12} = \frac{50(12)}{12} = 50$ in.

Try a total depth of 55 in. and $t_f = 1.5$ in.

$$h = 55 - 2(1.5) = 52 \text{ in.}$$

In order for the web to be slender,

$$\frac{h}{t_w} \geq 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

$$\therefore\ t_w \le \frac{h}{137.3} = \frac{55}{137.3} = 0.401\text{ in.}$$

For $\frac{a}{h} \le 1.5,\ \ \frac{h}{t_w} \le 12.0\sqrt{\frac{E}{F_y}} = 12.0\sqrt{\frac{29000}{50}} = 289.0$

For $\frac{a}{h} > 1.5,\ \ \frac{h}{t_w} \le \frac{0.4E}{F_y} = \frac{0.4(29000)}{50} = 232.0$

$$t_w \ge \frac{h}{232.0} = \frac{52}{232.0} = 0.2241\text{ in.}$$

In order for intermediate stiffeners to not be required, $a = 25$ ft.

Try a $\frac{3}{8}$-in. × 52-in. web, $\frac{h}{t_w} = \frac{52}{3/8} = 138.7,\quad A_w = 52(3/8) = 19.5\text{ in.}^2$

Check shear. $a = 25(12) = 300$ in.

$$\frac{a}{h} = \frac{300}{52} = 5.769 > 3$$

Since $a/h > 3,\ k_v = 5$ and tension-field action is not permitted.

Compute $C_v$. $1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5(29,000)}{50}} = 59.24$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5(29,000)}{50}} = 73.78$$

Since $\frac{h}{t_w} > 73.78$,

$$C_v = \frac{1.51k_vE}{(h/t_w)^2F_y} = \frac{1.51(5)(29,000)}{(138.7)^2(50)} = 0.2276$$

$$V_n = 0.6A_wF_yC_v = 0.6(19.5)(50)(0.2276) = 133.1\text{ kips}$$

Design strength $= \phi_vV_n = 0.90(133.1) = 120\text{ kips} = V_u$ (OK)

Estimate required flange size.

$$A_f = \frac{M_u/\phi_b}{hF_y} - \frac{A_w}{6} = \frac{2748(12/0.90}{52(50)} - \frac{19.5}{6} = 10.84\text{ in.}^2$$

$$b_f \ge \frac{10.84}{1.5} = 7.23\text{ in.}$$

Try a $1\frac{1}{2}$-in. × 9-in. flange, $A_f = 1.5(9) = 13.5 \text{ in.}^2$

$$\text{Girder weight} = [19.5 + 2(13.5)]\left(\frac{0.490}{144}\right)$$

$$= 0.158 \text{ kips/ft} < 0.160 \text{ kips/ft} \qquad \text{(OK)}$$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(52)^3 + 2(13.5)\left(\frac{52+1.5}{2}\right)^2$$

$$= 2.371 \times 10^4 \text{ in.}^4$$

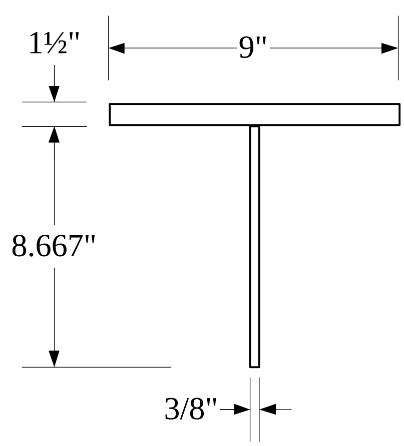
$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{23{,}710}{(52/2+1.5)} = 862.2 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{9}{2(1.5)} = 3.0, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, $\quad F_{cr} = F_y = 50 \text{ ksi}$

Check lateral-torsional buckling.




(not to scale)

$$\frac{h}{6} = \frac{52}{6} = 8.667 \text{ in.}, \qquad I = \frac{1}{12}(8.667)(3/8)^3 + \frac{1}{12}(1.5)(9)^3 = 91.16 \text{ in.}^4$$

$$A = 8.667(3/8) + 9(1.5) = 16.75 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{91.16}{16.75}} = 2.333 \text{ in.}$$

$$L_b = 10 \text{ ft.}$$

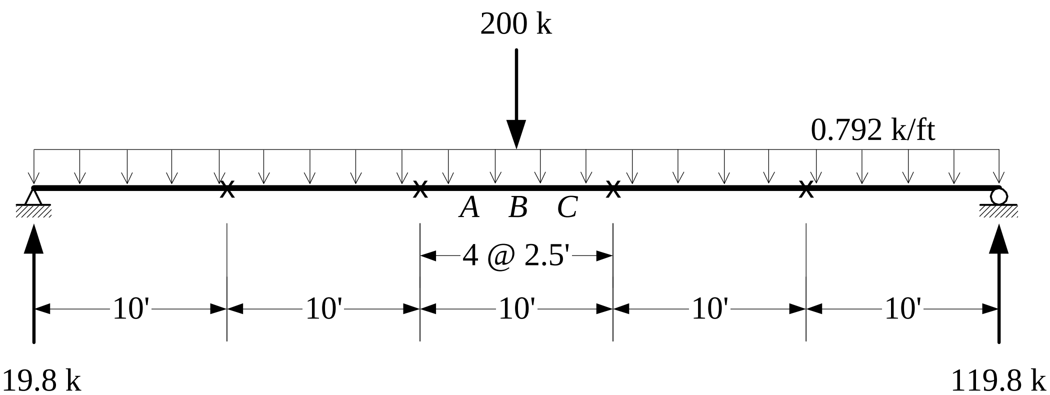
$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} = 1.1(2.333)\sqrt{\frac{29000}{50}} = 61.8 \text{ in.} = 5.15 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(2.333)\sqrt{\frac{29000}{0.7(50)}} = 211.0 \text{ in.} = 17.58 \text{ ft}$$

Since $L_p \le L_b < L_r$,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

Compute $C_b$. The central unbraced length is critical.




$$M_A = M_C = 119.8(22.5) - 0.792(22.5)^2/2 = 2495 \text{ ft-kips}$$

$$M_B = M_{\max} = 119.8(25) - 0.792(25)^2/2 = 2748 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(2748)}{2.5(2748) + 3(2495) + 4(2748) + 3(2495)} = 1.046$$

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y \; < 50 \text{ ksi}$$

$$= 1.046\left[50 - 0.3(50)\left(\frac{10 - 5.15}{17.58 - 5.15}\right)\right] = 46.18 \text{ ksi} < 50 \text{ ksi}$$

Compute the plate girder strength reduction factor.

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{52(3/8)}{9(1.5)} = 1.444 < 10$$

$$R_{pg} = 1 - \frac{1.444}{1200 + 300(1.444)}\left(138.7 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9987 < 1.0$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9987(50)(862.2) = 4.305 \times 10^4 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(43050)/12 = 3229 \text{ ft-kips} > 2748 \text{ ft-kips} \qquad \text{(OK)}$$

Use a $\frac{3}{8} \times 52$ web and $1\frac{1}{2} \times 9$ flanges

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**10.7-4**

At the support, $V_u = V_L = \frac{w_u L}{2} + P_u = \frac{2(48)}{2} + 120 = 168 \text{ kips}$

$$M_u = 168(24) - 2(24)^2/2 - 120(8) = 2496 \text{ ft-kips}$$

Try $t_f = 1.5$ in., $h = 48 - 2(1.5) = 45$ in.

For a slender web, $\frac{h}{t_w} \geq 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$

$$\therefore\ t_w \leq \frac{h}{137.3} = \frac{45}{137.3} = 0.328 \text{ in.}$$

For $\frac{a}{h} \leq 1.5$, $\frac{h}{t_w} \leq 12.0\sqrt{\frac{E}{F_y}} = 12.0\sqrt{\frac{29000}{50}} = 289.0$

For $\frac{a}{h} > 1.5$, $\frac{h}{t_w} \leq \frac{0.4E}{F_y} = \frac{0.4(29000)}{50} = 232.0$

$$t_w \geq \frac{h}{232.0} = \frac{45}{232.0} = 0.1940 \text{ in.}$$

Try a $\frac{3}{16}$ in. × 45 in. web.

$$\frac{h}{t_w} = \frac{45}{3/16} = 240.0, \qquad A_w = (3/16)(45) = 8.438 \text{ in.}^2$$

Estimate required flange size.

$$A_f = \frac{M_u/\phi_b}{hF_y} - \frac{A_w}{6} = \frac{2496(12)/0.90}{45(50)} - \frac{8.438}{6} = 13.38 \text{ in.}^2$$

$$b_f \geq \frac{13.38}{1.5} = 8.92 \text{ in.}$$

Try a $1\frac{1}{2}$-in. × 12-in. flange, $A_f = 1.5(12) = 18.0 \text{ in.}^2$

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/16)(45)^3 + 2(18)\left(\frac{45+1.5}{2}\right)^2$$

$$= 2.088 \times 10^4 \text{ in.}^4$$

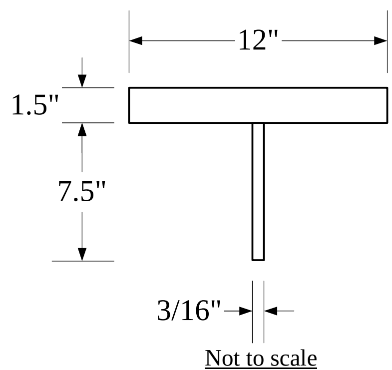
$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{2.088 \times 10^4}{(45/2+1.5)} = 870.0 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{12}{2(1.5)} = 4.0, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, $F_{cr} = F_y = 50$ ksi

Check lateral-torsional buckling.




$$\frac{h}{6} = \frac{45}{6} = 7.5 \text{ in.}, \qquad I = \frac{1}{12}(7.5)(3/16)^3 + \frac{1}{12}(1.5)(12)^3 = 216.0 \text{ in.}^4$$

$$A = 7.5(3/16) + 12(1.5) = 19.41 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{216.0}{19.41}} = 3.336 \text{ in.}$$

$$L_b = 16 \text{ ft.}$$

$$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} = 1.1(3.336)\sqrt{\frac{29000}{50}} = 88.38 \text{ in.} = 7.365 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(3.336)\sqrt{\frac{29000}{0.7(50)}} = 301.7 \text{ in.} = 25.14 \text{ ft}$$

Since $L_p \le L_b < L_r$,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

$$= 1.0\left[50 - 0.3(50)\left(\frac{16 - 7.365}{25.14 - 7.365}\right)\right] = 42.71 \text{ ksi (controls)}$$

($C_b = 1.0$ is a slightly conservative estimate.)

Compute the plate girder strength reduction factor.

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{45(3/16)}{12(1.5)} = 0.4688 < 10$$

$$R_{pg} = 1 - \frac{0.4688}{1200 + 300(0.4688)}\left(240.0 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9641 < 1.0$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9641(42.71)(870.0) = 3.582 \times 10^4 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(3.582 \times 10^4)/12 = 2687 \text{ ft-kips} > 2496 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: At left end,

$$\text{Required } \frac{\phi_v V_n}{A_w} = \frac{168}{8.438} = 19.91 \text{ ksi}$$

Determine the required $a/h$ manually (i.e., do not use the *Manual* Curves).

From $V_n = 0.6A_wF_yC_v$,

$$\text{Required } C_v = \frac{V_n}{0.6A_wF_y} = \frac{168/0.9}{0.6(8.438)(50)} = 0.7374$$

Determine required $k_v$. Assuming that AISC Eq. G2-5 controls,

$$C_v = \frac{1.5k_vE}{(h/t_w)^2F_y} \text{ and } k_v = \frac{C_v(h/t_w)^2F_y}{1.5E} = \frac{0.7374(240)^2(50)}{1.5(29,000)} = 48.82$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{48.82(29,000)}{50}} = 230.5 < \frac{h}{t_w} = 240$$

$\therefore$ Eq. G2-5 controls as assumed

From AISC Eq. G2-6, $\quad \frac{a}{h} = \sqrt{\frac{5}{k_v - 5}} = \sqrt{\frac{5}{48.82 - 5}} = 0.3378$

$$a = 0.3378h = 0.3378(45) = 15.2. \text{ Use } a = 15 \text{ in.}$$

At 15 in. from left end, $V_u = 168 - 2\left(\frac{15}{12}\right) = 165.5$ kips

$$\frac{\phi_v V_n}{A_w} = \frac{165.5}{8.438} = 19.61 \text{ ksi}$$

Use the curves in Table 3-17b with $h/t_w = 240$.

For $\frac{\phi_v V_n}{A_w} = 18$ ksi, $\frac{a}{h} = 1.05$ $\qquad$ For $\frac{\phi_v V_n}{A_w} = 21$ ksi, $\frac{a}{h} = 0.75$

For $\frac{\phi_v V_n}{A_w} = 19.91$ ksi, $\frac{a}{h} = 1.05 - \left(\frac{19.91 - 18}{21 - 18}\right)(1.05 - 0.75) = 0.859$

$$a = 0.859h = 0.859(45) = 38.66. \text{ Use } a = 38 \text{ in}$$

At $15 + 38 = 53$ in. from left end, $V_u = 168 - 2\left(\frac{53}{12}\right) = 159.2$ kips

$$\frac{\phi_v V_n}{A_w} = \frac{159.2}{8.438} = 18.87 \text{ ksi}$$

For $\frac{\phi_v V_n}{A_w} = 18$ ksi, $\frac{a}{h} = 1.05$ $\qquad$ For $\frac{\phi_v V_n}{A_w} = 21$ ksi, $\frac{a}{h} = 0.75$

For $\frac{\phi_v V_n}{A_w} = 18.87$ ksi, $\frac{a}{h} = 1.05 - \left(\frac{18.87 - 18}{21 - 18}\right)(1.05 - 0.75) = 0.963$

$$a = 0.963h = 0.963(45) = 43.34. \text{ Use } a = 43 \text{ in}$$

At $53 + 43 = 96$ in. from left end, $V_u = 168 - 2\left(\frac{96}{12}\right) = 152$ kips

$$\frac{\phi_v V_n}{A_w} = \frac{152}{8.438} = 18.01 \text{ ksi}, \qquad \frac{a}{h} = 1.05$$

$$a = 1.05(45) = 47.25 \text{ in.} \qquad \text{Use } a = 47 \text{ in.}$$

That puts the next stiffener at $96 + 47 = 143$ in. from the left end. Distance remaining to concentrated load is $16(12) - 143 = 49.0$ in. (This will be adequate).

At 16 ft from the left end (to the right of the concentrated load),

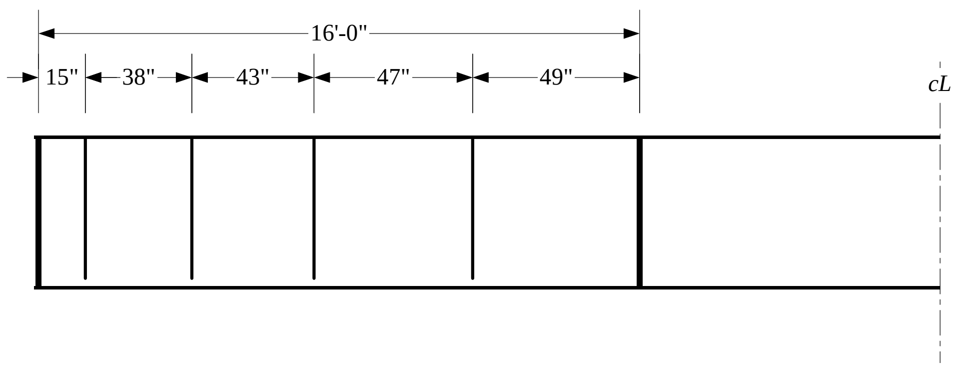
$$V_u = 168 - 2(16) - 120 = 16 \text{ kips}$$

$$\text{Required } \frac{\phi_v V_n}{A_w} = \frac{16}{8.438} = 1.896 \text{ ksi}$$

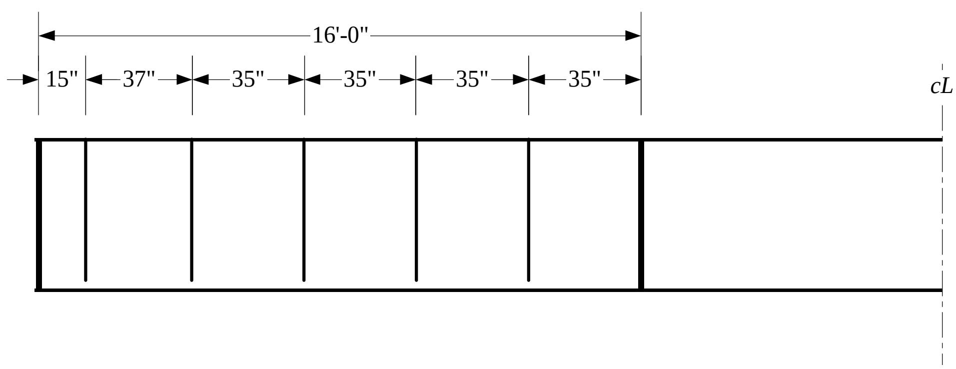
For $\frac{a}{h} = \frac{16 \times 12}{45} = 4.267 > 3$,

$$\frac{\phi_v V_n}{A_w} = 2.1 \text{ ksi}, \therefore \text{ stiffeners not needed in middle 1/3.}$$

The theoretical required stiffener spacing is shown below:




Use the stiffener spacing shown below.




Use a $\frac{3}{16}$ in. × 45 in. web and $1\frac{1}{2}$ in. × 12 in. flanges

Use the stiffener spacing shown above.

**10.7-5**

From Problem 10.8-4, use a $\frac{3}{16} \times 45$ web and $1\frac{1}{2} \times 12$ flanges. Reaction $= V_L = 168$ kips

Design the bearing stiffeners at the supports and use the same design for the interior stiffeners.

$$\text{Maximum stiffener width} = \frac{b_f - t_w}{2} = \frac{12 - 3/16}{2} = 5.906 \text{ in.}$$

Try $b = 5$ in.

For $\left(\frac{b}{t}\right)_{st} \le 0.56\sqrt{\frac{E}{F_{yst}}}$, $t \ge \dfrac{b}{0.56\sqrt{\frac{E}{F_{yst}}}} = \dfrac{5}{0.56\sqrt{\frac{29000}{50}}} = 0.371$ in.

Try two plates, $\frac{3}{8}$ in. × 5 in., with 1-in. cutouts.

Bearing strength: $A_{pb} = (5 - 1)(3/8) \times 2 = 3.0 \text{ in.}^2$

$$R_n = 1.8F_yA_{pb} = 1.8(50)(3) = 270.0 \text{ kips}$$

$$\phi R_n = 0.75(270) = 203 \text{ kips} > V_L = 168 \text{ kips} \qquad \text{(OK)}$$

Compressive strength: The maximum permissible length of web is

$$12t_w = 12(3/16) = 2.25 \text{ in.}$$

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(2.25)(3/16)^3 + 2\left[\frac{1}{12}(3/8)(5)^3 + 5(3/8)(2.5 + 3/32)^2\right] = 33.04 \text{ in.}^4$$

$$A = 2.25(3/16) + 2(5)(3/8) = 4.172 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{33.04}{4.172}} = 2.814 \text{ in.}$$

Compute the compressive strength:

$$\frac{KL}{r} = \frac{Kh}{r} = \frac{0.75(45)}{2.814} = 11.99 < 25$$

$F_{cr} = F_y = 50$ ksi

$\phi_c P_n = 0.90 A_g F_{cr} = 0.90(4.172)(50) = 188 \text{ kips} > 168 \text{ kips}$ (OK)

Use 2 PL $\frac{3}{8} \times 5$, with 1-in. cutouts.

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**10.7-6**

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(1.0) + 1.6(2) = 4.4$ kips/ft

$P_u = 1.6P_L = 1.6(500) = 800.0$ kips

Left reaction $= V_L = \dfrac{w_u L + P_u}{2} = \dfrac{4.4(80) + 800}{2} = 576.0$ kips

$$I_x = \frac{1}{12} t_w h^3 + 2A_f \left( \frac{h + t_f}{2} \right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3 \times 22)\left( \frac{78 + 3}{2} \right)^2$$

$$= 2.363 \times 10^5 \text{ in.}^4$$

The shear flow is

$$Q = A_f \left( \frac{h}{2} + \frac{t_f}{2} \right) = 3(22)\left( \frac{78}{2} + \frac{3}{2} \right) = 2673 \text{ in.}^3$$

At the support, $\dfrac{V_u Q}{I_x} = \dfrac{576(2673)}{236{,}300} = 6.516$ kips/in.

Minimum weld size = $\frac{3}{16}$ in. (AISC Table J2.4)

Minimum length = $4\left( \dfrac{3}{16} \right) = 0.75 \text{ in.} < 1.5 \text{ in.}$, use 1.5 in.

Use E70 electrodes, $\phi R_n = 1.392D$ kips/in., where $D$ = weld size in sixteenths.

Try $\frac{3}{16}$-in. × $1\frac{1}{2}$-in. intermittent fillet welds. For two welds,

Weld strength = $2 \times 1.392(3) = 8.352$ kips/in.

The base metal shear yield strength (web controls) is

$$0.6F_y t = 0.6(50)\left( \frac{1}{2} \right) = 15.0 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{1}{2}\right) = 14.63$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(8.352) = 12.53$ kips

Required spacing:

$$\frac{\phi R_n}{s} = \frac{V_u Q}{I_x} \quad \Rightarrow \quad \frac{12.53}{s} = 6.516 \quad \Rightarrow \quad s = 1.92 \text{ in.}$$

This spacing is too small for intermittant welds. Use a continuous weld.

Maximum clear spacing: From AISC E6,

$$d \le 0.75\sqrt{\frac{E}{F_y}}\, t_f = 0.75\sqrt{\frac{29{,}000}{50}}\,(3) = 54.2 \text{ in. (or 12 in.; 12 in.; controls.)}$$

Maximum $s$ = 12 + 1.5 in. = 13.5 in.

For $s$ = 13.5 in.,

$$\frac{\phi R_n}{s} = \frac{V_u Q}{I_x} \quad \Rightarrow \quad \frac{12.53}{13.5} = \frac{V_u(2673)}{236{,}300} \quad \Rightarrow \quad V_u = 81.95 \text{ kips}$$

Shear at mid-span, left of load, $= 576 - 4.4(40) = 400.0$ kips, so maximum spacing will never be used.

Spacing required at mid-span $= \dfrac{\phi R_n I_x}{V_u Q} = \dfrac{12.53(236300)}{400(2673)} = 2.77$ in.

This spacing is too small for intermittant welds.

<u>Use continuous $\frac{3}{16}$-in. E70 fillet welds.</u>

(b) $w_a = w_D + w_L = 1 + 2 = 3$ kips/ft

$P_a = 500$ kips

Left reaction $= V_L = \dfrac{w_a L + P_a}{2} = \dfrac{3(80) + 500}{2} = 370$ kips

$$I_x = \frac{1}{12} t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3 \times 22)\left(\frac{78 + 3}{2}\right)^2$$

$$= 2.363 \times 10^5 \text{ in.}^4$$

Shear flow:

$$Q = A_f\left(\frac{h}{2} + \frac{t_f}{2}\right) = 3(22)\left(\frac{78}{2} + \frac{3}{2}\right) = 2673 \text{ in.}^3$$

At the support, $\dfrac{V_a Q}{I_x} = \dfrac{370(2673)}{236300} = 4.185$ kips/in.

Minimum weld size = $\frac{3}{16}$ in. (AISC Table J2.4)

Minimum length = $4\left(\frac{3}{16}\right) = 0.75$ in. < 1.5 in., use 1.5 in.

Try $\frac{3}{16}$-in. × $1\frac{1}{2}$-in. intermittent fillet welds. For two welds and E70 electrodes,

weld strength = $2 \times 0.9279(3) = 5.567$ kips/in.

Base metal shear yield strength (web plate controls) is

$$0.4F_y t = 0.4(50)\left(\frac{1}{2}\right) = 10.0 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(65)\left(\frac{1}{2}\right) = 9.75$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(5.567) = 8.351$ kips

Required spacing:

$$\frac{R_n/\Omega}{s} = \frac{V_a Q}{I_x} \quad \Rightarrow \quad \frac{8.351}{s} = 4.185 \quad \Rightarrow \quad s = 2.00 \text{ in.}$$

This spacing is too small for intermittant welds. Use a continuous weld.

Maximum clear spacing: From AISC E6,

$$d \leq 0.75\sqrt{\frac{E}{F_y}}\, t_f = 0.75\sqrt{\frac{29{,}000}{50}}\,(3) = 54.2 \text{ in. (or 12 in.; 12 in.; controls.)}$$

Maximum $s$ = 12 + 1.5 in. = 13.5 in.

For $s$ = 13.5 in.,

$$\frac{R_n/\Omega}{s} = \frac{V_a Q}{I_x} \quad \Rightarrow \quad \frac{8.351}{13.5} = \frac{V_a(2673)}{236300} \quad \Rightarrow \quad V_a = 54.69 \text{ kips}$$

Shear at mid-span, left of load, $= 370 - 3(40) = 250$ kips, so maximum spacing will never be used.

Spacing required at mid-span = $\dfrac{(R_n/\Omega)I_x}{V_aQ} = \dfrac{8.351(236300)}{250(2673)} = 2.95$ in.

This spacing is too small for intermittant welds.

Use continuous $\frac{3}{16}$-in. E70 fillet welds.

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**10.7-7**

(a) Assume a girder weight of 400 lb/ft.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.400) + 1.6(4) = 6.88 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(175) = 280.0 \text{ kips}$$

$$V_u = V_L = \frac{w_uL}{2} + P_u = \frac{6.88(70)}{2} + 280 = 520.8 \text{ kips}$$

$$M_u = 520.8(35) - 6.88(35)^2/2 - 280(70/6) = 10,750 \text{ ft-kips}$$

Try $t_f = 1.5$ in. $\Rightarrow$ $h = 7(12) - 2(1.5) = 81$ in

For a slender web, $\dfrac{h}{t_w} \geq 5.70\sqrt{\dfrac{E}{F_y}} = 5.70\sqrt{\dfrac{29,000}{50}} = 137.3$

$$\therefore\ t_w \leq \frac{h}{137.3} = \frac{81}{137.3} = 0.5899 \text{ in.}$$

For $\dfrac{a}{h} \leq 1.5$, $\dfrac{h}{t_w} \leq 12.0\sqrt{\dfrac{E}{F_y}} = 12.0\sqrt{\dfrac{29000}{50}} = 289.0$

For $\dfrac{a}{h} > 1.5$, $\dfrac{h}{t_w} \leq \dfrac{0.4E}{F_y} = \dfrac{0.4(29000)}{50} = 232.0$

$$t_w \geq \frac{h}{232.0} = \frac{81}{232.0} = 0.3491 \text{ in.}$$

Try a $\frac{1}{2}$ in. × 81 in. web. $\dfrac{h}{t_w} = \dfrac{81}{0.5} = 162$, $A_w = 0.5(81) = 40.5 \text{ in.}^2$

Estimate required flange size.

$$A_f = \frac{M_u}{0.9hF_y} - \frac{A_w}{6} = \frac{10750(12)}{0.9(81)(50)} - \frac{40.5}{6} = 28.64 \text{ in.}^2$$

$$b_f \geq \frac{28.64}{1.5} = 19.09 \text{ in.}$$

Try a $1\frac{1}{2}$-in. × 24-in. flange, $A_f = 1.5(24) = 36.0 \text{ in.}^2$

Girder weight = $[40.5 + 2(36)]\left(\frac{0.490}{144}\right) = 0.3828$ kips/ft < 0.400 kips/ft estimate (OK)

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(0.5)(81)^3 + 2(36)\left(\frac{81+1.5}{2}\right)^2$$

$$= 1.447 \times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{144700}{(81/2+1.5)} = 3445 \text{ in.}^3$$

Compression flange: Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{24}{2(1.5)} = 8.0, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, $F_{cr} = F_y = 50$ ksi

Check lateral-torsional buckling.

$$\frac{h}{6} = \frac{81}{6} = 13.5 \text{ in.}, \qquad I = \frac{1}{12}(13.5)(0.5)^3 + \frac{1}{12}(1.5)(24)^3 = 1728 \text{in.}^4$$

$$A = 13.5(0.5) + 24(1.5) = 42.75 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{1728}{42.75}} = 6.358 \text{ in.}$$

$$L_b = 23.33 \text{ ft.}$$

$$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} = 1.1(6.358)\sqrt{\frac{29000}{50}} = 168.4 \text{ in.} = 14.03 \text{ ft}$$

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} = \pi(6.358)\sqrt{\frac{29000}{0.7(50)}} = 575.0 \text{ in.} = 47.92 \text{ ft}$$

Since $L_p < L_b < L_r$,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

$$= 1.0\left[50 - 0.3(50)\left(\frac{23.33 - 14.03}{47.92 - 14.03}\right)\right] = 45.88 \text{ ksi (controls)}$$

($C_b = 1.0$ is a slightly conservative estimate.)

Compute the plate girder strength reduction factor.

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{81(1/2)}{24(1.5)} = 1.125 < 10$$

$$R_{pg} = 1 - \frac{1.125}{1200 + 300(1.125)}\left(162 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9819 < 1.0$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9819(45.88)(3445) = 1.552 \times 10^5 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(1.552 \times 10^5)/12 = 11{,}640 \text{ ft-kips} > 10{,}750 \text{ ft-kips} \qquad \text{(OK)}$$

Try a $\frac{1}{2} \times 81$ web and $1\frac{1}{2} \times 24$ flanges

Shear: At left end (end panel),

$$\text{Required } \frac{\phi_v V_n}{A_w} = \frac{520.8}{40.5} = 12.86 \text{ ksi}, \qquad \frac{h}{t_w} = 162$$

From Table 3-17a in the *Manual*, $\frac{a}{h} \approx 0.72$ by interpolation.

$$a = 0.72h = 0.72(81) = 58.32. \text{ Use } a = 58 \text{ in.}$$

At 58 in. from left end, $V_u = 520.8 - 6.88\left(\frac{58}{12}\right) = 487.5$ kips

$$\frac{\phi_v V_n}{A_w} = \frac{487.5}{40.5} = 12.04 \text{ ksi}$$

From Table 3-17b, $\frac{a}{h} \approx 2.6$

$$a = 2.6(81) = 210.6. \text{ Use } a = 210 = 210.0 \text{ in.}$$

At $70/3 = 23.33$ ft from the left end (to the right of the concentrated load),

$V_u = 520.8 - 6.88(23.33) - 280 = 80.29$ kips

Required $\dfrac{\phi_v V_n}{A_w} = \dfrac{80.29}{40.5} = 1.982$ ksi

For $\dfrac{a}{h} = \dfrac{23.33 \times 12}{81} = 3.46 > 3$, $\dfrac{\phi_v V_n}{A_w} > 4.5$ ksi, $\therefore$ stiffeners not needed in middle 1/3.

Use 2 intermediate stiffeners spaced from each end as follows:

1 at 58 in. and 1 at 111 in.

Use a $\frac{1}{2} \times 81$ web and $1\frac{1}{2} \times 24$ flanges

(b) Intermediate stiffener size:

$$\left(\frac{b}{t}\right)_{st} \leq 0.56\sqrt{\frac{E}{F_y}}$$

Available width: $\dfrac{b_f - t_w}{2} = \dfrac{24 - 0.5}{2} = 11.75$ in. Try $b = 6$ in.

$$t \geq \frac{b}{0.56\sqrt{\dfrac{E}{F_y}}} = \frac{6}{0.56\sqrt{\dfrac{29000}{50}}} = 0.4449$$

$$I_{st} \geq I_{st1} + (I_{st2} - I_{st1})\left[\frac{V_r - V_{c1}}{V_{c2} - V_{c1}}\right]$$

$$I_{st1} = \min\left\{\begin{matrix} a \\ h \end{matrix}\right\} t_w^3 j$$

$$j = \frac{2.5}{(a/h)^2} - 2 = \frac{2.5}{(58/81)^2} - 2 = 2.876 > 0.5, \ \therefore \text{ use } j = 2.876$$

$$I_{st1} = \min\left\{\begin{matrix} 58 \\ 81 \end{matrix}\right\}(1/2)^3(2.876) = 20.85 \text{ in.}^4$$

$$I_{st2} = \frac{h^4 \rho_{st}^{1.3}}{40}\left(\frac{F_{yw}}{E}\right)^{1.5}, \qquad \rho_{st} = \max\left\{\begin{matrix} F_{yw}/F_{yst} \\ 1 \end{matrix}\right\} = 1$$

$$I_{st2} = \frac{(81)^4(1)^{1.3}}{40}\left(\frac{50}{29000}\right)^{1.5} = 77.04 \text{ in.}^4$$

For $h/t_w = 162$ and $a/h = 111/81 = 1.37$,

$$\frac{\phi_v V_n}{A_w} \approx 6.7 \text{ ksi for the no tension field case and Table 3-17a}$$

$$V_{c1} = \phi_v V_n = 6.7A_w = 6.7(81 \times 0.5) = 271.4 \text{ kips}$$

$$\frac{\phi_v V_n}{A_w} \approx 17 \text{ ksi for the tension field case and Table 3-17b}$$

$$V_{c2} = \phi_v V_n = 17A_w = 17(81 \times 0.5) = 688.5 \text{ kips}$$

$$V_r = 520.8 \text{ kips}$$

$$I_{st} \geq I_{st1} + (I_{st2} - I_{st1})\left[\frac{V_r - V_{c1}}{V_{c2} - V_{c1}}\right]$$

$$= 20.85 + (77.04 - 20.85)\left[\frac{520.8 - 271.4}{688.5 - 271.4}\right] = 54.5 \text{ in.}^4$$

Try two plates $\frac{1}{2} \times 6$

$$I_{st} \approx \frac{1}{12}(3/8)(6 + 0.5 + 6)^3 = 61.0 \text{ in.}^4 > 54.5 \text{ in}^4 \qquad \text{(OK)}$$

Length: From Figure 10.9 in the textbook,

$$c \geq 4t_w = 4(0.5) = 2 \text{ in., and } c \leq 6t_w = 6(0.5) = 3 \text{ in.}$$

Assume a flange-to-web weld size of $w = \frac{3}{16}$ in. (minimum size) and $c = 3$ in.

$$\text{Length} = h - w - c = 81 - \frac{3}{16} - 3 = 77.81 \text{ in., say 78 in.}$$

$$c = 81 - 78 - \frac{3}{16} = 2.813 \text{ in.} \qquad \text{OK)}$$

<u>Use two PL $\frac{1}{2} \times 6 \times 6'$-06'' for intermediate stiffeners.</u>

Design the bearing stiffeners at the supports for a load of $V_L = 520.8$ kips

$$\text{Maximum stiffener width} = \frac{b_f - t_w}{2} = \frac{24 - 0.5}{2} = 11.75 \text{ in.}$$

Try $b = 8$ in. and $t \geq \dfrac{b}{0.56\sqrt{\dfrac{E}{F_{yst}}}} = \dfrac{8}{0.56\sqrt{\dfrac{29000}{50}}} = 0.5932$ in.

Try two plates, $\frac{5}{8}$ in. × 8 in., with 1-in. cutouts.

Bearing strength: $A_{pb} = (8-1)(5/8) \times 2 = 8.75 \text{ in.}^2$

$R_n = 1.8F_yA_{pb} = 1.8(50)(8.75) = 787.5$ kips

$\phi R_n = 0.75(787.5) = 590.6 \text{ kips} > V_L = 520.8$ kips (OK)

Compressive strength: The maximum permissible length of web is

$12t_w = 12(0.5) = 6$ in.

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(6)(0.5)^3 + 2\left[\frac{1}{12}(5/8)(8)^3 + 8(5/8)(4+1/4)^2\right] = 234.0 \text{ in.}^4$$

$$A = 6(0.5) + 2(8)(5/8) = 13.0 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{234}{13}} = 4.243 \text{ in.}$$

Compute the compressive strength:

$$\frac{KL}{r} = \frac{0.75(81)}{4.243} = 14.32 < 25 \quad \therefore \; F_{cr} = F_y = 50 \text{ ksi}$$

$$\phi_c P_n = 0.90F_{cr}A_g = 0.90(50)(13) = 585.0 \text{ kips} > 519.1 \text{ kips} \qquad \text{(OK)}$$

Use 2 PL $\frac{5}{8}$ × 8 with 1-in. cutouts for bearing stiffeners at the supports.

Because there is a large difference between the reactions and the interior concentrated loads, use a different size for the interior bearing stiffeners. Design the interior bearing stiffeners for a load of $P_u = 280$ kips.

Try $b = 6$ in. and $t \geq \dfrac{b}{0.56\sqrt{\dfrac{E}{F_{yst}}}} = \dfrac{8}{0.56\sqrt{\dfrac{29000}{50}}} = 0.5932$ in.

Try two plates, $\frac{5}{8}$ in. × 6 in., with 1-in. cutouts.

Bearing strength: $A_{pb} = (6-1)(5/8) \times 2 = 6.25 \text{ in.}^2$

$R_n = 1.8F_yA_{pb} = 1.8(50)(6.25) = 562.5 \text{ kips}$

$\phi R_n = 0.75(562.5) = 422 \text{ kips} > P_u = 280 \text{ kips}$ (OK)

Compressive strength: The maximum permissible length of web is

$25t_w = 25(0.5) = 12.5 \text{ in.}$

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(12.5)(0.5)^3 + 2\left(\frac{1}{12}(5/8)(6)^3 + 6(5/8)(3+1/4)^2\right) = 101.8 \text{ in.}^4$$

$$A = 12.5(0.5) + 2(6)(5/8) = 13.75 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{101.8}{13.75}} = 2.721 \text{ in.}$$

Compute the compressive strength:

$$\frac{KL}{r} = \frac{0.75(81)}{2.721} = 22.33 < 25 \quad \therefore\ F_{cr} = F_y = 50 \text{ ksi}$$

$\phi_c P_n = 0.90F_{cr}A_g = 0.90(50)(13.75) = 618.8 \text{ kips} > 519.1 \text{ kips}$ (OK)

Use 2 PL $\frac{5}{8} \times 6$ with 1-in. cutouts for the interior bearing stiffeners.

(c) Design the flange-to-web welds.

The shear flow is $Q = A_f\left(\frac{h}{2} + \frac{t_f}{2}\right) = (1.5 \times 24)\left(\frac{81}{2} + \frac{1.5}{2}\right) = 1485 \text{ in.}^3$

At the support, $\frac{V_uQ}{I_x} = \frac{520.8(1485)}{144700} = 5.345 \text{ kips/in.}$

Minimum weld size = $\frac{3}{16}$ in. (AISC Table J2.4)

Minimum length = $4\left(\frac{3}{16}\right) = 0.75 \text{ in.} < 1.5 \text{ in.}$, use 1.5 in.

Use E70 electrodes, $\phi R_n = 1.392D$ kips/in., where $D$ = weld size in sixteenths.

Try $\frac{3}{16}$-in. $\times 1\frac{1}{2}$-in. intermittent fillet welds. For two welds,

Weld strength = $2 \times 1.392(3) = 8.352 \text{ kips/in.}$

Base metal shear yield strength (web plate controls) is

$$0.4F_y t = 0.4(50)\left(\frac{1}{2}\right) = 10.0 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(65)\left(\frac{1}{2}\right) = 9.75$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(8.352) = 12.53$ kips

Required spacing:

$$\frac{\phi R_n}{s} = \frac{V_u Q}{I_x} \quad \Rightarrow \quad \frac{12.53}{s} = 5.345 \quad \Rightarrow \quad s = 2.34 \text{ in.}$$

Since this is less than twice the length of the weld, use a continuous weld.

For $s = 2(1.5) = 3$ in., $V_u = \dfrac{\phi R_n I_x}{sQ} = \dfrac{12.53(144700)}{3(1485)} = 407$ kips

This occurs when $520.8 - 6.88x = 407$, Solution is: $\{x = 16.54\}$ ft

Maximum clear spacing: From AISC E6,

$$d \le 0.75\sqrt{\frac{E}{F_y}}\,t_f = 0.75\sqrt{\frac{29{,}000}{50}}(1.5) = 27.1 \text{ in. (or 12 in.; 12 in.; controls.)}$$

Maximum $s$ = 12 + 1.5 in. = 13.5 in.

For $s$ = 13.5 in.,

$$\frac{\phi R_n}{s} = \frac{V_u Q}{I_x} \quad \Rightarrow \quad \frac{12.53}{13.5} = \frac{V_u(1485)}{144700} \quad \Rightarrow \quad V_u = 90.4 \text{ kips}$$

Shear at first interior load, left of load, $= 520.8 - 6.88(23.33) = 360.3$ kips, so maximum spacing will not be used in the first third of the span.

Spacing required at left side of first interior load is

$$\frac{\phi R_n I_x}{V_u Q} = \frac{12.53(144700)}{360.3(1485)} = 3.389 \text{ in.}$$

Check middle third of span. Shear on right side of load $= 360.3 - 280 = 80.3$ kips

$$s = \frac{\phi R_n I_x}{V_u Q} = \frac{12.53(144700)}{80.3(1485)} = 15.2 \text{ in.} > 13.5 \text{ in. maximum} \therefore \text{ use } s = 13\tfrac{1}{2} \text{ in.}$$

Summary for flange-to-web welds:

Use $\frac{3}{16}$ in. continuous fillet welds for the first 17 feet.

Use $\frac{3}{16}$ in × $1\frac{1}{2}$ in. intermittent E70 fillet welds at 3 in. c.c. from 17 ft until the first interior bearing stiffener.

Use $\frac{3}{16}$ in × $1\frac{1}{2}$ in. intermittent E70 fillet welds at $13\frac{1}{2}$ in. c.c. between interior bearing stiffeners.

Welds for intermediate stiffeners ($\frac{1}{2} \times 6$ ):

Minimum weld size = $\frac{3}{16}$ in. (AISC Table J2.4)

Minimum length = $4\left(\frac{3}{16}\right) = 0.75$ in. < 1.5 in., use 1.5 in.

Use E70 electrodes, $\phi R_n = 1.392D$ kips/in., where $D$ = weld size in sixteenths.

Try $\frac{3}{16}$ in. × $1\frac{1}{2}$ in. intermittent fillet welds. For four welds, the weld strength is

$$4 \times 1.392(3) = 16.7 \text{ kips/in.}$$

The base metal shear yield strength is

$$0.6F_y t = 0.6(50)\left(\frac{1}{2}\right) \times 2 = 30.0 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{1}{2}\right) \times 2 = 29.25$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(16.7) = 25.05$ kips

From Equation 10.4, the shear to be transferred is

$$f = 0.045h\sqrt{\frac{F_y^3}{E}} = 0.045(81)\sqrt{\frac{(50)^3}{29,000}} = 7.568 \text{ kips/in.}$$

$$\frac{25.05}{s} = 7.568 \text{ kips/in.} \qquad \Rightarrow \qquad s = 3.31 \text{ in.}$$

A center-to-center spacing of 3 in. is equal to twice the length of the weld segment, so either a continuous weld or an intermittent weld can be used. Use intermittent welds.

Maximum clear spacing: From AISC E6,

$$d \le 0.75\sqrt{\frac{E}{F_y}}\,t_f = 0.75\sqrt{\frac{29,000}{50}}\,(1.5) = 27.1 \text{ in. (or 12 in.; 12 in.; controls.)}$$

Maximum $s$ = 12 + 1.5 in. = 13.5 in.

Use $\frac{3}{16}$ in. × $1\frac{1}{2}$ in. E70 fillet welds spaced at 3 in. c-c for the intermediate stiffeners.

Welds for bearing stiffeners at the supports (5/8 × 8):

Minimum weld size = $\frac{3}{16}$ in. (AISC Table J2.4, based on web thickness of 1/2 in.)

Minimum length = $4\left(\frac{3}{16}\right) = 0.75$ in. < 1.5 in., use 1.5 in.

Use E70 electrodes, $\phi R_n = 1.392D$ kips/in., where $D$ = weld size in sixteenths.

Try $\frac{3}{16}$ in. × $1\frac{1}{2}$ in. intermittent fillet welds. For four welds, the weld strength is

$$4 \times 1.392(3) = 16.7 \text{ kips/in.}$$

The base metal shear yield strength (web controls) is

$$0.6F_y t = 0.6(50)\left(\frac{1}{2}\right) \times 2 = 30.0 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{1}{2}\right) \times 2 = 29.25$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(16.7) = 25.05$ kips

The shear to be transferred is

$$\text{Reaction} \div \text{length available for weld} = \frac{520.8}{81 - 2(1.0)} = 6.592 \text{ kips/in.}$$

$$\frac{25.05}{s} = 6.592 \text{ kips/in.} \qquad \Rightarrow \qquad s = 3.80 \text{ in.}$$

Use $\frac{3}{16}$ in. × $1\frac{1}{2}$ in. E70 fillet welds spaced at $3\frac{1}{2}$ in. c-c for bearing stiffener at support.

Welds for interior bearing stiffeners ( $\frac{5}{8}$ × 6):

Minimum weld size = $\frac{3}{16}$ in. and minimum length = 1.5 in.

Try $\frac{1}{4}$ in. × $1\frac{1}{2}$ in. intermittent fillet welds. For four welds, the weld strength is

$4 \times 1.392(3) = 16.7$ kips/in.

The base metal shear yield strength (web controls) is

$$0.6F_y t = 0.6(50)\left(\frac{1}{2}\right) \times 2 = 30.0 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{1}{2}\right) \times 2 = 29.25$ kips/in.

Weld strength controls. For a 1.5-in. length, $\phi R_n = 1.5(16.7) = 25.05$ kips

The shear to be transferred is

$$P_u \div \text{length available for weld} = \frac{280}{81 - 2(1.0)} = 3.544 \text{ kips/in.}$$

$$\frac{25.05}{s} = 3.544 \text{ kips/in.} \quad \Rightarrow \quad s = 7.07 \text{ in.}$$

Use $\frac{3}{16}$ in. $\times\, 1\frac{1}{2}$ in. E70 fillet welds spaced at 7 in. c-c for interior bearing stiffeners.

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**10.7-8**

See solution to problem 10.7-7 for an example of the procedure.

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**10.7-9**

See solution to problem 10.7-7 for an example of the procedure.

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