SAMPLING DISTRIBUTION | CORRELATION & REGRESSION ANALYSIS

$(\bar{x})$ MEAN = $\frac{\text{DATA RANGE (MAXIMUM NUMBER + MINIMUM NUMBER)}}{2}$

STANDARD DEVIATION = $S = \sqrt{\frac{\Sigma x^2 - n(\bar{x})^2}{n-1}}$

SAMPLE ERROR = $X - \mu$

= MAXIMUM NUMBER AT DATA RANGE − MEAN

MEAN OF DISTRIBUTION = $\frac{\Sigma \text{ MEAN}}{\text{TOTAL SAMPLE SIZE}}$

SAMPLE MEAN = $\frac{\Sigma \text{ TOTAL}}{\text{NO. OF EVENTS}}$

Pb (28)

A P

TO BOT

ONE

IS SHO

(a) LIST A

(b) FOR EA

SAMPLE

(c) CALCUL

(d) GRAPH

...ION ANALYSIS

...INIMUM NUMBER)

Pb (28)

A POPULATION OF 5 SALES PERSONS SELLING CAR TELEPHONES TO BOTH PRIVATE AND COMMERCIAL CUSTOMERS OVER A PERIOD OF ONE MONTH. THE NUMBER OF TELEPHONES SOLD FOR EACH PERSON IS SHOWN IN FOLLOWING TABLE

SALES PERSON	PHONES SOLD
ADAM (A)	14 ✓
BAKER (B)	20 ✓
COLLINS (C)	12
DAVID (D)	8
EDWARDS (E)	16

(a) LIST ALL POSSIBLE SAMPLES of SIZE 2

(b) FOR EACH SAMPLE, CALCULATE DATA RANGE, SAMPLE MEAN SAMPLE STANDARD DEVIATION

(c) CALCULATE MEAN, FREQUENCY, RELATIVE FREQUENCY

(d) GRAPH.



SAMPLE COMBINATION

AB

AC

AD

AE

BC

BD

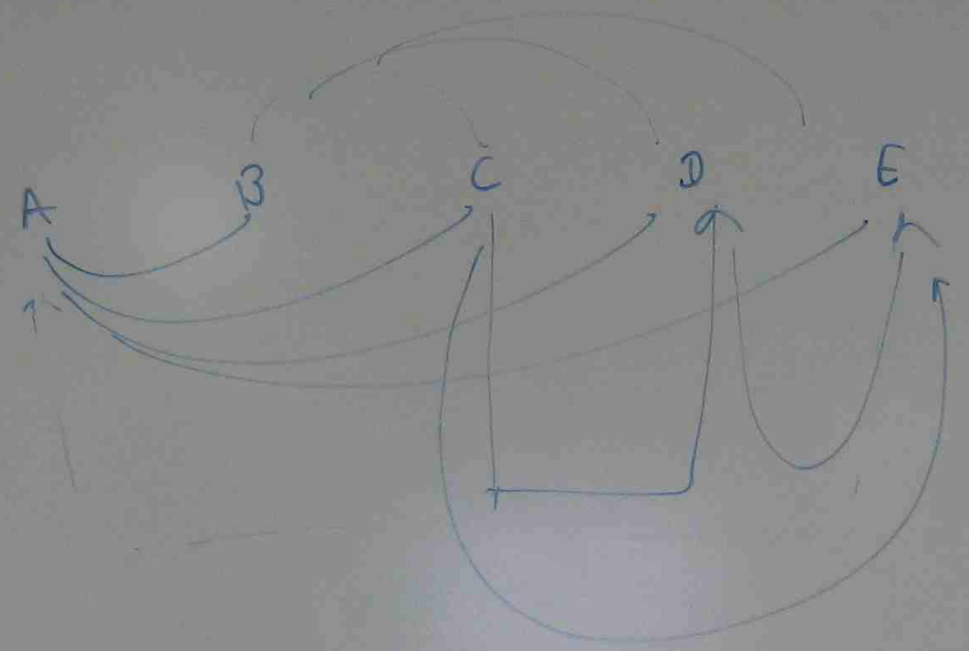
BE

CD

CE

DE

CAR TELEPHONES
OVER A PERIOD OF
SOLD FOR EACH PERSON



→ AB, AC, AD, AE, BC, BD, BE
CD, CE, DE

LAST – MEAN

LE MEAN

ENCY

SAMPLE COMBINATION	DATA RANGE	MEAN = $\frac{MAX + MIN}{2}$ $\bar{X}$	STANDARD DEVIATION	SAMPLE ERROR $\bar{X} - \mu$
AB	14 – 20	$\bar{X} = \frac{14+20}{2} = 17$	$\sqrt{\frac{14^2+20^2-2\times17^2}{2-1}} = 4.24$	20 – 17 = 3
AC	14 – 12	$\bar{X} = \frac{14+12}{2} = 13$	$\sqrt{\frac{14^2+12^2-2\times13^2}{2-1}} = 1.414$	12 – 13 = –1
AD	14 – 8	11	4.243	–3
AE	14 – 16	15	1.414	+1
BC	20 – 12	16	5.657	+2
BD	20 – 8	14 ←	8.485	0
BE	20 – 16	18	2.828	4
CD	12 – 8	10	2.828	–4
CE	12 – 16	14 ←	2.828	0
DE	8 – 16	12	5.657	–2

AE, BC, BD, BE

QE

LAST — MEAN

SAMPLE ERROR $\bar{x} - \mu$

$\sqrt{17^2}$ = 4.24 | 20 − 17 = 3

$\sqrt{x 13^2}$ = 1.414 | 12 − 13 = −1

3 | −3

+1

+2

0

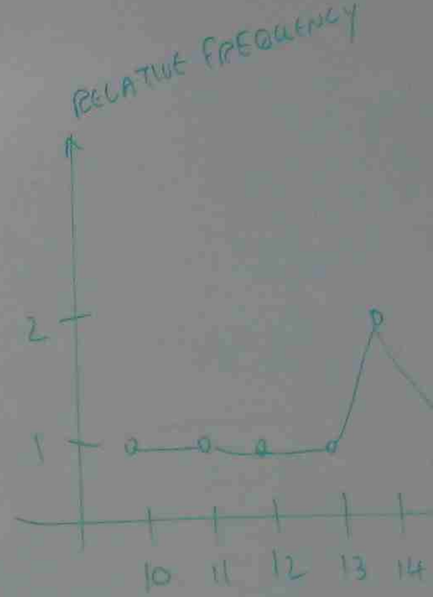
4

85

828

MEAN	FREQUENCY	RELATIVE FREQUENCY
10	1	$\frac{1}{\Sigma f} = \frac{1}{10} = 0.1$
11	1	$\frac{1}{10} = 0.1$
12	1	$\frac{1}{10} = 0.1$
13	1	$\frac{1}{10} = 0.1$
14	2	$\frac{2}{10} = 0.2$
15	1	$\frac{1}{10} = 0.1$
16	1	$\frac{1}{10} = 0.1$
17	1	$\frac{1}{10} = 0.1$
18	1	$\frac{1}{10} = 0.1$

$\Sigma f = 10$



ACCORDING TO FREQUENCY DISTRIBUTION, B+D, C+E CAN SELL MORE PRODUCTS.

Pg 29 IN ABOVE P

DISTRIBUTION

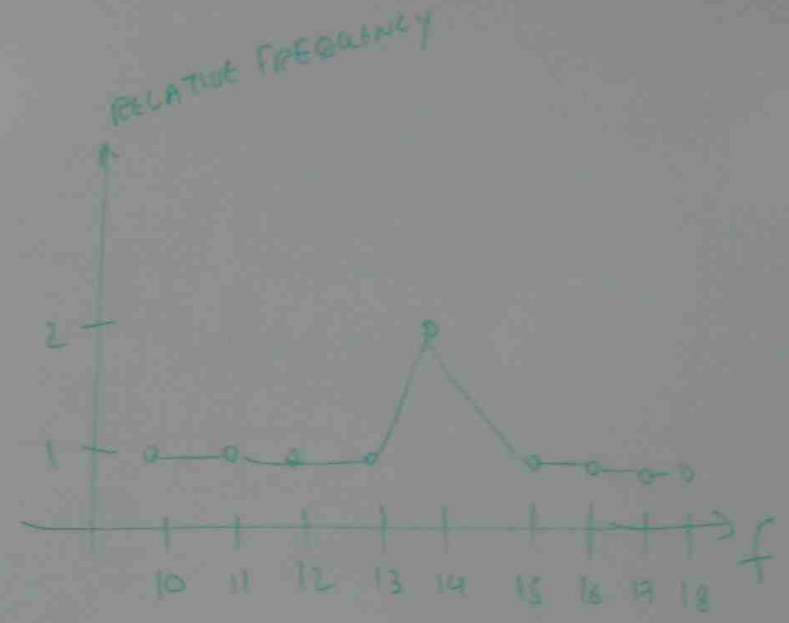
(i) MEAN OF DISTRIBUTION =

(ii) SAMPLE MEAN = Σ

AN	FREQUENCY	RELATIVE FREQUENCY
	1	$\frac{1}{\Sigma f} = \frac{1}{10} = 0.1$
	1	$\frac{1}{10} = 0.1$
	1	$\frac{1}{10} = 0.1$
	2	$\frac{2}{10} = 0.2$
	1	$\frac{1}{10} = 0.1$
	1	$\frac{1}{10} = 0.1$
	1	$\frac{1}{10} = 0.1$
	1	$\frac{1}{10} = 0.1$

$\Sigma f = 10$

TO FREQUENCY DISTRIBUTION,
E CAN SEE MORE PRODUCTS.



Pb (29) IN ABOVE PROBLEM, CALCULATE (i) MEAN OF DISTRIBUTION OF X, (ii) SAMPLE MEAN

(i) MEAN OF DISTRIBUTION $= \frac{\Sigma \text{MEAN}}{\text{TOTAL SAMPLE SIZE}} =$

$= \frac{17+13+11+15+16+14+18+10+14+12}{10}$

$= 14$

(ii) SAMPLE MEAN $= \frac{\Sigma \text{TOTAL EVENTS}}{\text{TOTAL PERSON}}$ ← SALES $= \frac{14+10+12+8+16}{5}$

$= 14$

STA

STAND

Pb (30)

WHAT IS

MEAN =

(μ)

STA

(a) &T

P(2

(b

STANDARD DISTRIBUTION of SELECTED SAMPLE

STANDARD DISTRIBUTION of SELECTED SAMPLE $\delta_{\bar{x}} = \dfrac{\text{PARENT STANDARD DISTRIBUTION}}{\sqrt{\text{NUMBER of SAMPLE}}}$

Pb (30) A CERTAIN BRAND of ROPE IS KNOWN TO HAVE A MEAN BREAKING STRENGTH 25 kg WITH A STANDARD DEVIATION of 0.5 kg. A RANDOM SAMPLE of ROPE IS TESTED FOR BREAKING STRENGTH. WHAT IS THE POSSIBILITY THAT SAMPLE MEAN BREAKING STRENGTH of 50 PIECES of ROPE WILL BE (i) BETWEEN 24.9 AND 25.1 kg (ii) LESS THAN 24.8 kg.

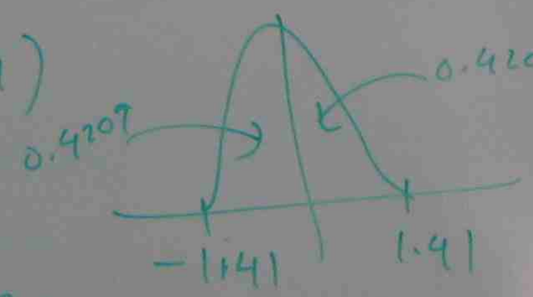
MEAN = 25 kg (μ) $\quad \delta = 0.5$ kg ← PARENT DISTRIBUTION

STANDARD DEVIATION of SELECTED SAMPLE $(\delta_{\bar{x}}) = \dfrac{\text{PARENT STANDARD DEVIATION}}{\sqrt{\text{No. of SAMPLE}}} = \dfrac{0.5}{\sqrt{50}} = 0.071$

(a) BETWEEN 24.9 AND 25.1 kg

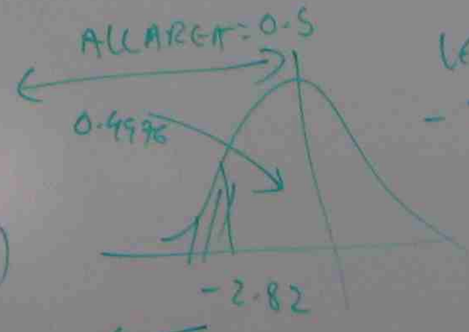
$$P(24.9 < \bar{x} < 25.1) \longrightarrow P\left(\frac{x_1-\mu}{\delta_{\bar{x}}} < \bar{x} < \frac{x_2-\mu}{\delta_{\bar{x}}}\right) = P\left(\frac{24.9-25}{0.071} < \bar{x} < \frac{25.1-25}{0.071}\right)$$

$$P(-1.41 < \bar{x} < 1.41)$$



TOTAL AREA = 0.4207 + 0.4207 = 0.8414

(b) $P\left(\dfrac{\bar{x}-\mu}{\delta_{\bar{x}}}\right) = P\left(\dfrac{24.8-25}{0.071}\right) = P(-2.82)$



LESS THAN = ALL − 2.82 AREA

$-2.82 \quad = 0.5 - 0.4976$

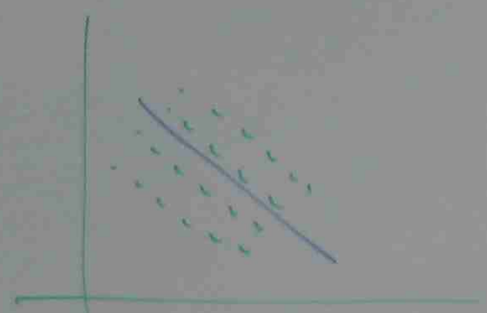
$= 0.0024$

CORRELATION AND REGRESSION ANALYSIS

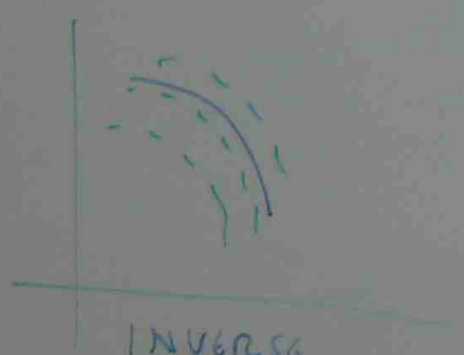
POSSIBLE RELATIONSHIPS BETWEEN X AND Y SCATTER PLOT



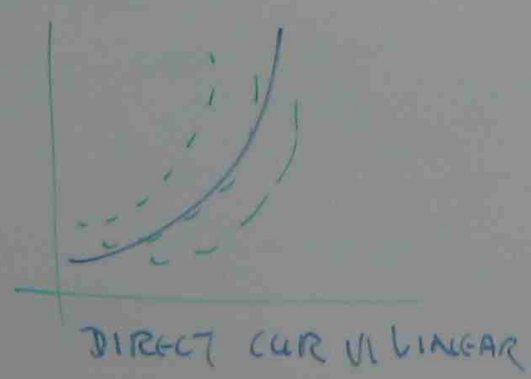
DIRECT LINEAR



INVERSE LINEAR



INVERSE CURVILINEAR



DIRECT CURVILINEAR



NON RELATIONSHIP

CORRELATION ANALYSIS ⟶ ACTIVITY & EFFECTIVENESS

INPUT

$$\bar{X} = \frac{X_1 + X_2 + \cdots}{n}$$

AVERAGE

STANDARD DEVIATION

$$S_x = \sqrt{\frac{X_1^2 + X_2^2 + \cdots - n(\bar{X})^2}{n-1}}$$

EFFECT / OUTPUT

$$\bar{Y}_{\text{AVERAGE}} = \frac{Y_1 + Y_2 + Y_3 + \cdots}{n}$$

$$S_y = \sqrt{\frac{Y_1^2 + Y_2^2 + \cdots - n(\bar{Y})^2}{n-1}}$$

$$\Sigma xy$$

$$S\ S_{xy} = \Sigma xy - n\,\bar{x}\,\bar{y}$$

$$S\ S_x = (n-1)\,S_x^2$$

$$S\ S_y = (n-1)\,S_y^2$$

$$\text{CORRELATION } \gamma = \frac{S\ S_{xy}}{\sqrt{SS_x \times S_{Sy}}}$$

IF γ IS POSITIVE
CLOSE TO +1
THEN INPUT IS STRONGLY
RELATED TO OUT PUT

IF γ IS NEGATIVE, INPUT
DOES NOT EFFECTED ON OUT PUT

pb 32

A CERTAIN LARGE COMPANY IS INTERESTED IN WHETHER OR NOT THERE IS A RELATIONSHIP BETWEEN THE AMOUNT SPENT ON ADVERTISING PER MONTH AND THE GROSS MONTHLY SALE VOLUME.
TO EXAMINE THIS PROBLEM, THE FOLLOWING SAMPLE OF ADVERTISING EXPENDITURES AND ASSOCIATED SALE VOLUMES.
VOLUMES ARE RANDAMLY SELECTED FOR MONTHLY BASIS

MONTH	ADVERTISING EXPENDITURE (x) $ 10000	SALE (Y) (x $ 10000
1	1.2	101
2	0.8	92
3	1.0	110
4	1.3	120
5	0.7	90
6	0.8	82
7	1.0	93
8	0.6	75
9	0.9	91
10	1.1	108

NOT
SING
NG

ADVERTISING

MEAN $\bar{x} = \dfrac{1.2+0.8+1.0+1.3+0.7+0.8+1.0+0.6+0.9+1.1}{10} = 0.94$

$S_x = \sqrt{\dfrac{1.2^2+0.8^2+1.0^2+1.3^2+0.7^2+0.8^2+1.0^2+0.6^2+0.9^2+1.1^2-10\,(0.94)^2}{10-1}} = 0.22211$

SALE

MEAN $= \bar{y} = \dfrac{101+92+110+120+90+82+93+75+91+108}{10} = 95.9$

$S_y = \sqrt{\dfrac{101^2+92^2+110^2+120^2+90^2+82^2+93^2+75^2+91^2+108^2-10\times 95.9^2}{10-1}} = 13.37$

$\sum xy = 1.2\times101+0.8\times92+1.0\times110+1.3\times120+0.7\times90+0.8\times82+1.0\times93+0.6\times75+0.9\times91+1.1\times108$

$= 924.8$

$SS_{xy} = \sum xy - n\bar{x}\bar{y} = 924.8 - 10\times0.94\times95.9 = 23.34$

$SS_x = (n-1)S_x^2 = (10-1)\times(0.22211)^2 = 0.444$

$SS_y = (n-1)S_y^2 = (10-1)\times(13.37)^2 = 1600.9$

$r = \dfrac{SS_{xy}}{\sqrt{SS_x \times SS_y}}$

$= \dfrac{23.34}{\sqrt{0.444\times1600.9}} = 0.875$

POSITIVE
CLOSE TO +1
THUS ADVERTISEMENT AFFECTS THE SALE

$\alpha = 0.05$

V_1 \ V_2	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								

CATEGORY	OBSERVED FREQUENCY (O)	E
A	48	CUST 200
B	98	200
C	54	200

$\chi^2_{\alpha, V} =$

CATEGORY	OBSERVED FREQUENCY (O)	EXPECTED FREQUENCY (E) CUSTOMER X % SHARE	O-E	$\frac{(O-E)^2}{E}$
A	48	$200 \times \frac{30}{100} = 60$	$O-E = 48-60 = -12$	$\frac{(-12)^2}{60} = 2.4$
B	98	$200 \times \frac{50}{100} = 100$	$O-E = 98-100 = -2$	$\frac{(-2)^2}{100} = 0.04$
C	54	$200 \times \frac{20}{100} = 40$	$54-40 = 14$	$\frac{(14)^2}{40} = 4.9$
			$\sum O-E = -12+(-2)+14 = 0$	$\sum \frac{(O-E)^2}{E} = 7.34$

$$\chi^2_{\alpha, V} = \chi^2_{0.05, (\text{NO. OF PRODUCTS} - 1)}$$

$$= \chi^2_{0.05, (3-1)} = \chi^2_{0.05, 2}$$

FIND FIRST NUMBER LESS THAN 7.34 IN χ^2 TABLE

V_2 $V_1 = 2$

1

4 — 6.94

REJECTION POINT

p632

A CER

THERE IS A

PER MONTH AN

TO EXAMINE

EXPENDITURES

SALE VOLUMES

MEN TH

1
2
3
4
5
6
7
8
9
10

STATISTICS

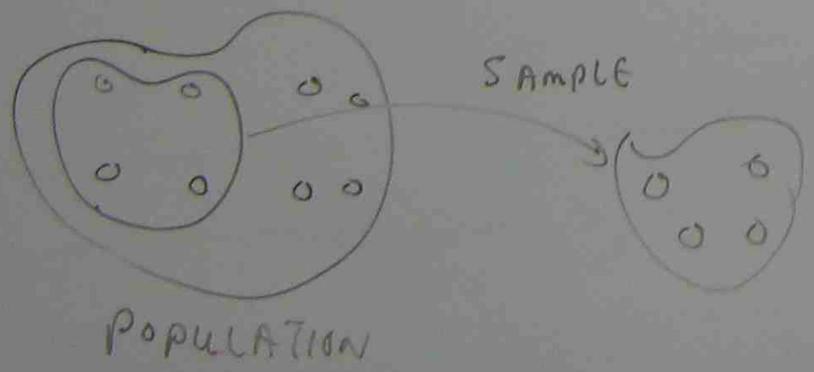
STATISTICS INVOLVES COLLECTING, SUMMARISING ANALYZING AND INTERPRETING DATA.

TYPES of DATA

CATEGORICAL DATA

NUMERICAL DATA

- DISCRETE
- CONTINUOUS



DISCRETE — COUNTABLE SUCH AS NUMBER OF DAYS.

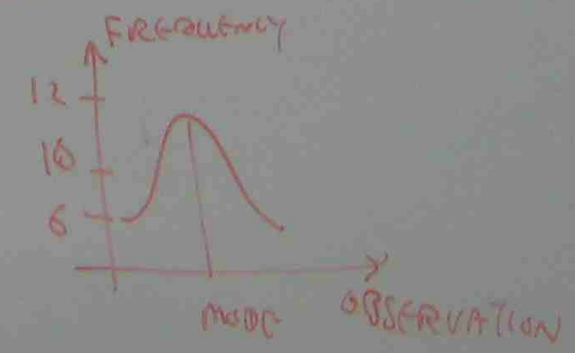
CONTINUOUS — INFINITE SUCH AS DAILY TEMPERATURE

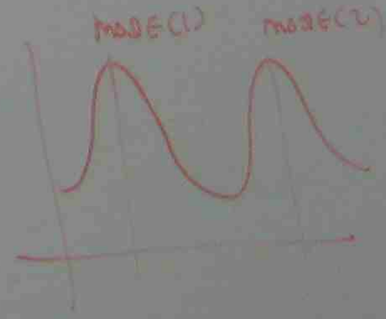
PARAMETER

A NUMERICAL MEASURE THAT DESCRIBES SOME CHARACTERISTICS OF POPULATION

STATISTICS - NUMERICAL MEASURE THE DESCRIBES SOME CHARACTERISTICS OF SAMPLE

MODE THE DATA SET THAT OCCURS MOST OFTEN





OF DAYS.

PERATURE

SOME

SOME

OFTEN

MODE(2)





NO MODE

Pb (1) FIND THE MODE OF THE FOLLOWING SAMPLES OF DATA

(a) 18, 19, 18, 20, 18, 18, 20, 21, 37, 18

(b) 27, 28, 29, 36, 14, 24

(c) 19, 18, 20, 39, 18, 18, 19, 46, 19

(a) MODE = 18

(b) NO MODE

(c) 19, 18 = MODES

18 = MODE 1

19 = MODE 2

MEDIAN

THE MIDDLE NUMBER IN AN ORDERED SET.

Pb (2) FIND THE MEDIAN OF EACH OF THE FOLLOWING SAMPLES OF DATA

(a) X: 7, 3, 2, 8, 11

(b) X: 18, 19, 18, 20, 18, 18, 20, 21, 37, 18

(a) 2, 3, 7, 8, 11

MEDIAN = 7

(b) X = 18, 18, 18, 18, 18, 19, 20, 20, 21, 37

MEDIAN = $\frac{18+19}{2}$

= 18.

MEAN

THE AR

MEASUREMENTS

BY ADDING TH

DIVIDING THIS

THE DATA S

$\bar{X}$ =

$$\text{MEDIAN} = \frac{18+19}{2}$$

$$= 18.5$$

MEAN THE ARITHMETIC AVERAGE of ALL MEASUREMENTS IN THE DATA SET OBTAINED BY ADDING THEM ALL UP AND DIVIDING THIS SUM BY THE SIZE of THE DATA SET.

$$\bar{X} = \frac{\Sigma X}{n}$$

21, 37

pb ③ FIND THE MEAN OF THE FOLLOWING SAMPLES OF DATA

(i) x: 11, 14, 10, 5

(ii) x: 2, 2, 4, 5, 5, 5, 6, 7, 8, 9, 11

(i) $\text{MEAN} = \bar{x} = \frac{11+14+10+5}{4} = \frac{40}{4} = 10$

(ii) $\text{MEAN} = \bar{X} = \frac{2+2+4+5+5+5+6+7+8+9+11}{11}$

$= \frac{64}{11} = 5.818$

pb ④ FIND THE MEAN OF THE FOLLOWING GROUP FREQUENCY DISTRIBUTIONS (GFD)

INTERVAL	FREQUENCY
0 → 9	1
10 → 19	2
20 → 29	4
30 → 39	8
40 → 49	4
50 → 59	1

INTERVAL	FREQUENCY
50 → 59	1

INTERVAL	MEAN	f	fX
0 ⟶ 9	$\frac{0+9}{2} = 4.5$	1	$4.5 \times 1 = 4.5$
10 ⟶ 19	$\frac{10+19}{2} = 14.5$	2	$14.5 \times 2 = 29$
20 ⟶ 29	$\frac{20+29}{2} = 24.5$	4	$24.5 \times 4 = 98$
30 ⟶ 39	$\frac{30+39}{2} = 34.5$	8	$34.5 \times 8 = 276$
40 ⟶ 49	$\frac{40+49}{2} = 44.5$	4	$44.5 \times 4 = 178$
50 ⟶ 59	$\frac{50+59}{2} = 54.5$	1	$54.5 \times 1 = 54.5$
		$\Sigma f = 20$	$\Sigma fx = 640$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{640}{20} = 32$$

+11

FREQUENCY

RANGE

THE DIFFERENCE BETWEEN THE LARGEST AND SMALLEST VALUES IN THE SAMPLE.

Pb (5) FIND THE RANGE OF THE FOLLOWING SAMPLE OF DATA.

(i) 18, 19, 18, 20, 18, 37, 21, 20, 18, 18

(ii) 18, 19, 18, 20, 18, 21, 20, 18, 18

(i) RANGE = LARGEST − SMALLEST

= 37 − 18 = 19

(ii) RANGE = LARGEST − SMALLEST

= 21 − 18 = 3

RESIDUAL

THE DIFFERENCE BETWEEN EACH SAMPLE X AND THE MEAN $\bar{X}$ IS CALLED THE DEVIATION FROM THE MEAN OR RESIDUAL.

$$\text{RESIDUAL} = X - \bar{X}$$

Pb (6) 10, 12, 15, 17, 21

CALCULATE (i) MEAN (ii) RESIDUAL (DEVIATION)

(i) $\bar{X} = \frac{10+12+15+17+21}{5} = \frac{75}{5} = 15$

(ii) RESIDUAL $= (X - \bar{X}) = (10-15), (12-15), (15-15), (17-15)$

$= -5, -3, 0, 2, 6$

VARIANCE

$$\text{VARIANCE} = \frac{\sum (X - \bar{X})^2}{n-1}$$

...CE BETWEEN EACH SAMPLE X AND THE MEAN $\bar{X}$

...THE DEVIATION FROM THE MEAN OR RESIDUAL.

$$\text{...IDUAL} = X - \bar{X}$$

..., 12, 15, 17, 21

(i) MEAN (ii) RESIDUAL (DEVIATION)

$$\frac{10+12+15+17+21}{5} = \frac{75}{5} = 15$$

$$\text{...AL} = (X - \bar{X}) = (10-15), (12-15), (15-15), (17-15), (21-15)$$

$$= -5, -3, 0, 2, 6$$

$$\text{VARIANCE} = \frac{\sum (X-\bar{X})^2}{n-1}$$

pb (7) CALCULATE VARIANCE OF 10, 12, 15, 17, 21

$$\bar{X} = \text{MEAN} = \frac{\sum X}{n} = \frac{10+12+15+17+21}{5} = 15$$

$$\text{VARIANCE} = s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{(10-15)^2 + (12-15)^2 + (15-15)^2 + (17-15)^2 + (21-15)^2}{5-1}$$

$$= \frac{(-5)^2 + (-3)^2 + (0)^2 + (2)^2 + (6)^2}{4}$$

$$= \frac{25+9+4+36}{4} = \frac{74}{4} = 18.5$$

STANDARD DEVIATION

THE STANDARD DEVIATION OF A SAMPLE OF DATA IS DEFINED TO THE POSITIVE SQUARE ROOT OF THE VARIANCE

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

APPROPRIATE
FOOTWEAR
THIS BUILDING

Pb (8) CALCULATE STANDARD DEVIATION OF 10, 12, 15, 17, 21

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(10-15)^2+(12-15)^2+(15-15)^2+(17-15)^2+(21-15)^2}{5-1}}$$

$$= \sqrt{18.5} = 4.3$$

$$\bar{x} = \frac{10+12+15+17+21}{5} = 15$$

CALCULATION OF S^2 (VARIANCE) AND S (STANDARD DEVIATION) FOR A SAMPLE OF RAW DATA

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}$$

Pb (9) CALCULATE THE VARIANCE AND STANDARD DEVIATION FOR THE FO SAMPLE OF DATA.

$$X: 2, 3, 5, 5, 9$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{2+3+5+5+9}{5} = \frac{24}{5} = 4.8$$

$$\Sigma x^2 = 2^2+3^2+5^2+5^2+9^2 = 144$$

$$n(\bar{x})^2 = 5 \times (4.8)^2 = 115.2$$

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}$$

$$S^2 = \frac{144-115.2}{5-1} = 7.2$$

$$S = \sqrt{7.2} = 2.68$$

ANCE AND STANDARD DEVIATION FOR THE FOLLOWING

$5, 5, 9$

$\frac{5+5+9}{} = \frac{24}{5} = 4.8$

$^2 + 5^2 + 9^2 = 144$

$)^2 = 115.2$

$= \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}$

$\frac{}{2} = 7.2$

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CALCULATION OF S^2 AND S FOR SAMPLE DATA PRESENTED AS G F D (GROUP FREQUENCY DISTRIBUTION).

VARIANCE: $S^2 = \frac{\Sigma f(x-\bar{x})^2}{n-1} = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f - 1} = \frac{\Sigma fx^2 - n(\bar{x})^2}{n-1}$

STANDARD DEVIATION = $S = \sqrt{\frac{\Sigma fx^2 - n(\bar{x})^2}{n-1}}$

Pb (10) FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING SAMPLE OF GROUPED DATA.

INTERVAL	FREQUENCY
10 – 14	4
15 – 19	7
20 – 24	5
25 – 29	3
30 – 34	1

INTERVAL	x (mid point)	f	fx	fx^2
10 – 14	$\frac{10+14}{2} = \frac{24}{2} = 12$	4	$4 \times 12 = 48$	$4 \times 12^2 = 576$
15 – 19	$\frac{15+19}{2} = 17$	7	$17 \times 7 = 119$	$7 \times 17^2 = 2023$
20 – 24	$\frac{20+24}{2} = 22$	5	$22 \times 5 = 110$	$5 \times 22^2 = 2420$
25 – 29	$\frac{25+29}{2} = 27$	3	$27 \times 3 = 81$	$3 \times 27^2 = 2187$
30 – 34	$\frac{30+34}{2} = 32$	1	$32 \times 1 = 32$	$1 \times 32^2 = 1024$
		$\Sigma f = 20$	$\Sigma fx = 390$	$\Sigma fx^2 = 8230$

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{390}{20} = 19.5$

$S^2 = \frac{\Sigma fx^2 - n(\bar{x})^2}{n-1} = \frac{8230 - 20 \times (19.5)^2}{20-1}$

$= \frac{8230 - 7605}{19} = 32.85$

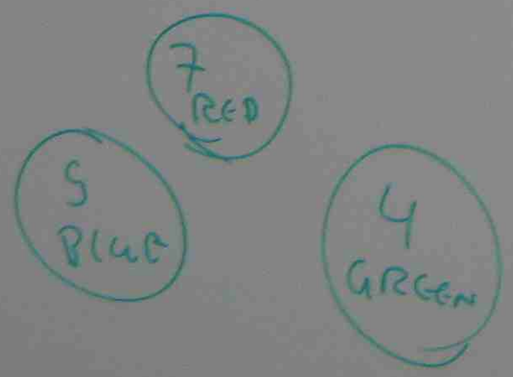
$S = \sqrt{32.85} = 5.734$

SET THEORY AND ELEMENTARY PROBABILITY

DEFINITION of PROBABILITY

$$P(A) = \frac{\text{NUMBER of TIMES A OCCUR}}{\text{TOTAL NUMBER of REPETITIONS of EXPERIMENT}} = \frac{m}{N}$$

Pb (11) A BAG CONTAINS 7 RED BALLS, 5 BLUE AND 4 GREEN BALLS. FIND THE PROBABILITY THAT IF ONE BALL ONLY IS DRAWN RANDOMLY, IT WILL BE GREEN.



N = TOTAL NUMBER OF BALLS = 7 + 5 + 4 = 16

n = NO. OF GREEN BALL = 4

$$P(\text{GREEN}) = \frac{n}{N} = \frac{4}{16} = \frac{1}{4}$$

Pb (12)

Pb (12) IF A SINGLE DRAW IS MADE FROM A STANDARD PACK OF CARDS FIND THE PROBABILITY OF DRAWING A JACK (OR) A QUEEN (OR) A KING.

1 PACK = 52 CARDS.

JACK = 4 CARDS

QUEEN = 4 CARDS

KING = 4 CARDS.

$P(A) = P(\text{JACK}) = \frac{4}{52} = \frac{1}{13}$

$P(B) = P(\text{QUEEN}) = \frac{4}{52} = \frac{1}{13}$

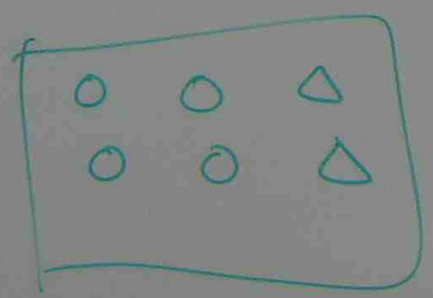
$P(C) = P(\text{KING}) = \frac{4}{52} = \frac{1}{13}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13}$

$= \frac{3}{13}$

$4 = 16$



$\frac{2}{6}$ $P(\Delta)$

$1 - \frac{2}{6} = \frac{4}{6}$ $1 - P(\Delta)$

$P(\bar{A}) = 1 - P(A)$

Pb (14) THERE ARE 52 CARDS IN A STANDARD PACK IN WHICH 39 CARDS CONTAIN HEART.

CALCULATE THE PROBABILITY THAT THE CARD CONTAINING HEART IS DRAWN AND PROBABILITY THAT THE CARD NOT CONTAINING HEART IS DRAWN.

$P(A) = P(\text{HEART}) = \frac{39}{52} = \frac{3}{4}$

$P(\bar{A}) = 1 - P(A)$

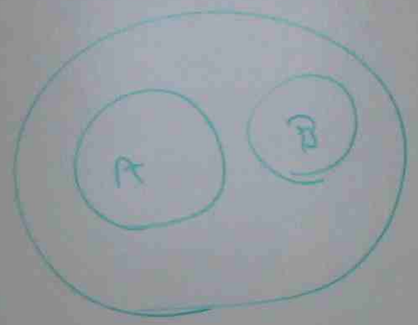
$= 1 - \frac{3}{4}$

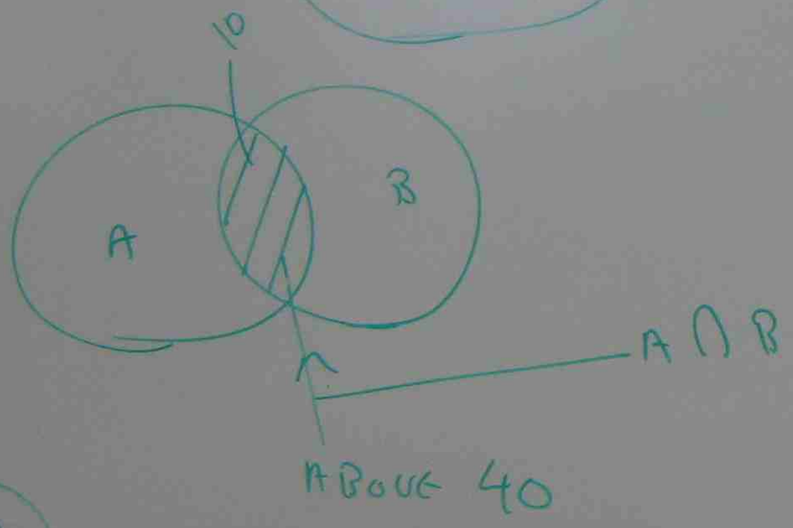
$= \frac{1}{4}$

UNION, INTERSECT, SUBSET



A ∪ B

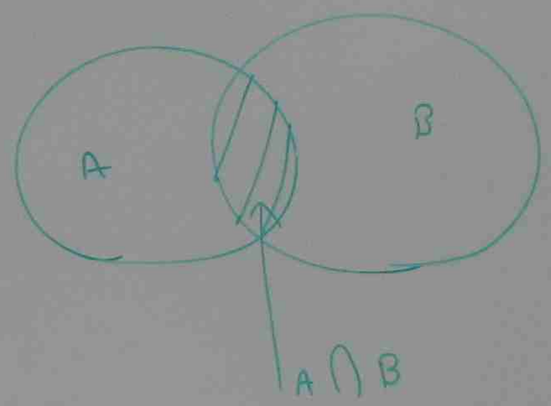




(A) 60 STUDENTS, 10 ABOVE AGE 40

(B) 50 STUDENTS, 10 ABOVE AGE 40

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(A) = \frac{60}{110}$$

$$P(B) = \frac{50}{110}$$

$$P(A \cap B) = \frac{10}{110}$$

$$P(A \cup B) = \frac{60}{110} + \frac{50}{110} - \frac{10}{110}$$

$$= \frac{100}{110}$$

Pb (15) $P(S) = 0.4$

FIND $P($

$P(S \cup T) = P(S)$

$= 0.4$

$=$



Pb (16) A SI

IS THE P

ON THE U

NUMBER

(18)

Pb (15) $P(S) = 0.4$, $P(T) = 0.5$, $P(S \cap T) = 0.15$

FIND $P(S \cup T)$

$$P(S \cup T) = P(S) + P(T) - P(S \cap T)$$

$$= 0.4 + 0.5 - 0.15$$

$$= 0.75$$

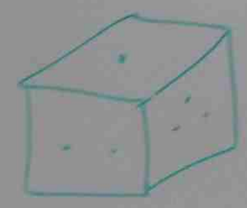
PROBABILITY OF SUBSET

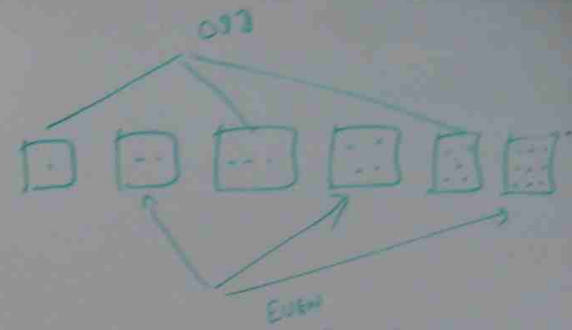


$A \subset B$, A IS SUB SET of B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Pb (16) A SINGLE DIE IS TOSSED. WHAT IS THE PROBABILITY OF A 6 APPEARING ON THE UPPER MOST FACE GIVEN THAT NUMBER SHOWING IS EVEN.





A = EVENT THAT "6" IS SHOWING

B = EVENT THAT EVEN NUMBER IS SHOWING



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\bar{A}) = 1 - P(A)$$

pb 17 SUPPOSE WE HAVE 3 EVENTS X, Y, Z

$P(X) = \frac{1}{2}$, $P(Y) = \frac{1}{4}$, $P(Z) = \frac{1}{8}$

FIND (a) $P(Z \cap X)$, (b) $P(Z \cup X)$

(c) $P(X \cup Y)$, (d) $P(X \cup \bar{Y})$

(a) $P(Z \cap X) = P(Z) \times P(X)$

$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

(b) $P(Z \cup X) = P(Z) + P(X) - P(Z \cap X)$

$= \frac{1}{8} + \frac{1}{2} - \frac{1}{16}$

$= \frac{2+8}{16} - \frac{1}{16}$

$= \frac{10}{16} - \frac{1}{16}$

$= \frac{10-1}{16} = \frac{9}{16}$

(c) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= P(X) + P(Y) - P(X) \times P(Y)$

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \times \frac{1}{4}$

$= \frac{4+2}{8} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

(d)

(d) $P(X \cup \bar{Y}) = P(X) + P(\bar{Y}) - P(X \cap \bar{Y})$

$= P(X) + [1 - P(Y)] - P(X) \times P(\bar{Y})$

$= P(X) + [1 - P(Y)] - P(X) \times [1 - P(Y)]$

$= \frac{1}{2} + \left[1 - \frac{1}{4}\right] - \frac{1}{2} \times \left[1 - \frac{1}{4}\right]$

$= \frac{1}{2} + \frac{3}{4} - \frac{1}{2} \times \frac{3}{4}$

$= \frac{4+6}{8} - \frac{3}{8}$

$= \frac{10}{8} - \frac{3}{8} = \frac{7}{8}$ //

Pb(18) SUPPOSE THAT WE KNOW FROM EXPERIMENT THAT 28% OF PEOPLE READ THE WOMAN'S DAY ARTICLE AND 45% OF PEOPLE READ THE NEW IDEA.

READING WOMEN'S DAY AND NEW IDEA ARE INDEPENDENT OF EACH OTHER.

TWO PEOPLE ARE SELECTED AT RANDOM.

FIND THE PROBABILITY THAT

(a) NEITHER READ NEW IDEA

(b) BOTH READ WOMEN'S DAY

(c) BOTH DO NOT READ WOMEN'S DAY

$P(W) = P(\text{WOMEN'S DAY}) = 0.28$

$P(\bar{W}) = P(\text{DO NOT READ WOMAN DAY}) = 1 - P(W)$

$= 1 - 0.28$

$= 0.72$

$P(N) = 0.45$

$P(\bar{N}) = 1 - P(N) = 1 - 0.45 = 0.55$

(a) NEITHER READS NEW IDEA (FROM TWO PEOPLE)

$P(\bar{N} \cap \bar{N}) = P(\bar{N}) \times P(\bar{N})$

$= 0.55 \times 0.55 = 0.3025$

(b) P

(c) P

Pb(19

6 A

OF

OF

FIND

(i) F

40

(

( )

$\frac{1}{8} = \frac{5}{8}$

(b) $P(W \cap W) = P(W) \times P(W)$

$= 0.28 \times 0.28 = 0.0784$

(c) $P(\bar{W} \cap \bar{W}) = P(\bar{W}) \times P(\bar{W})$

$= 0.72 \times 0.72 = 0.5184$

Pb (14) THERE ARE 10 PEOPLE TO BE CLASSIFIED ACCORDING TO SEX & AGE.

6 ARE FEMALES AND 4 ARE MALES.

OF 6 FEMALES, 4 ARE UNDER 40, 2 ARE 40 (OR) MORE

OF 4 MALES. 3 ARE UNDER 40, 1 IS 40 (OR) MORE

FIND THE PROBABILITY THAT PERSON IS

(i) FEMALE (ii) UNDER 40, (iii) FEMALE UNDER 40 (iv) UNDER 40 GIVEN THAT SHE IS FEMALE ⟵ AMONG THE FEMALES, FIND UNDER 40

(v) MALE AND 40 (OR) MORE, (vi) MALE

GIVEN THAT HE IS UNDER 40.

(i.) $P(F) = \frac{6}{10}$.

$P(A)$ UNDER 40

(ii) $P(A) = \frac{4+3}{10} = \frac{7}{10}$

$P(F)$, $P(A)$
FEMALE, UNDER 40

$P(F \cap A)$ = FEMALE UNDER 40 $= \frac{4}{10}$

$P(A \cap F) = P(F \cap A)$

(iv) $P(A/F) = \frac{P(A \cap F)}{P(F)} = \frac{P(F \cap A)}{P(F)} = \frac{\frac{4}{10}}{\frac{6}{10}}$

$= \frac{4}{10} \times \frac{10}{6}$

$= \frac{4}{6}$

(v)

(vi)

(v) $P(A)$ = UNDER 40

40 OR MORE = $P(\bar{A})$

$P(m)$ = MALE

MALE AND 40 (OR) MORE = $P(m \cap \bar{A}) = P(m) \times P(\bar{A})$

$= P(m) \times (1 - P(A))$

$= \frac{4}{10} \times \left(1 - \frac{7}{10}\right)$

$= \frac{4}{10} \times \frac{3}{10} = \frac{12}{100} \simeq \frac{1}{10}$

(vi) $P(\bar{A}/m) = \frac{P(m \cap A)}{P(m)}$

$= \frac{3/10}{4/10} = \frac{3}{4}$

$)=P(E \cap A)$

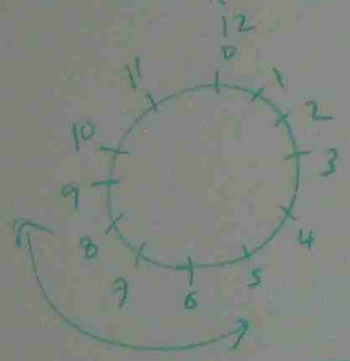
$A) = \frac{\frac{4}{10}}{\frac{6}{10}}$

$= \frac{4}{10} \times \frac{10}{6}$

$= \frac{4}{6}$

NORMAL PROBABILITY DISTRIBUTION

PROBABILITY – CHANCE (OR) LIKELIHOOD

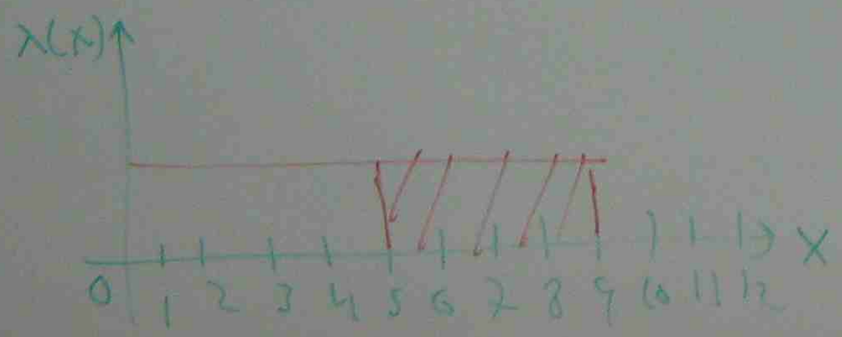


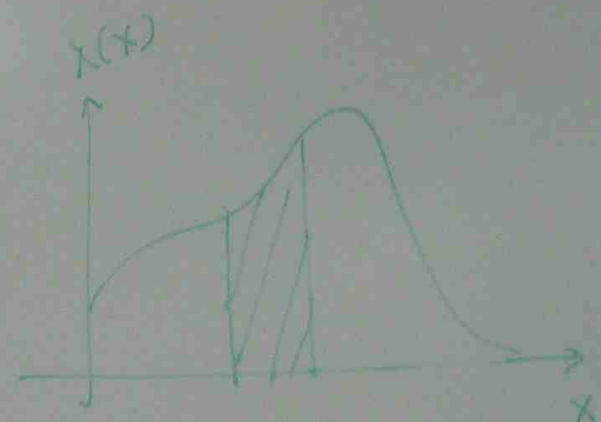
CHANCE TO STOP THE WATCH BETWEEN $5 \rightarrow 9$

TOTAL = 12 DIVISIONS

$5 \rightarrow 9$ = 4 DIVISIONS.

$$P(5 < x < 9) = \frac{4}{12} = \frac{1}{3}$$





PRACTICAL PROBABILITY GRAPH

NORMAL DISTRIBUTION

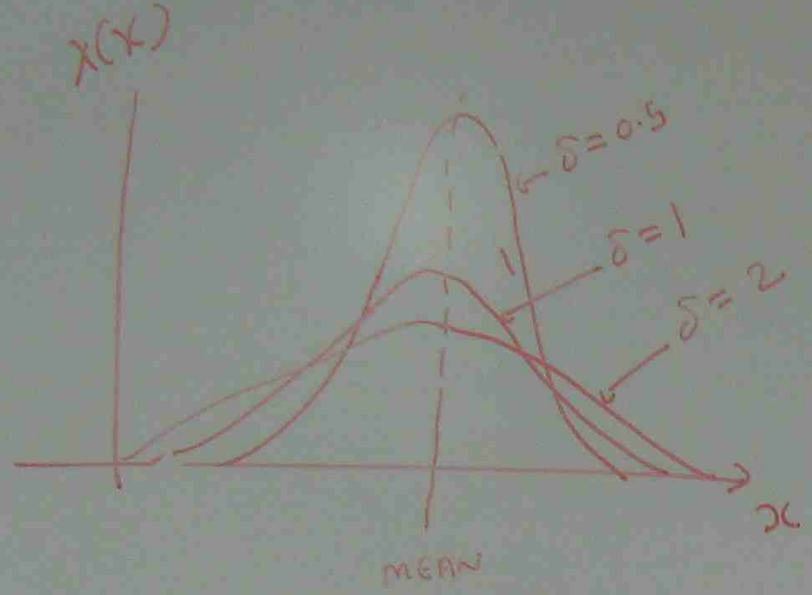
STANDARD DEVIATION = $\sigma = \sqrt{\frac{\Sigma f x^2 - n(\bar{x})^2}{n-1}}$

How much the number differs from average value

X	f
Event 1	5 Times
2	3 Times
3	4 Times.

$\bar{x}$ = MEAN = AVERAGE

n = TOTAL NUMBER OF TIMES.



SAME MEN $= \mu = \frac{\sum X_1}{n_1} = \frac{\sum X_2}{n_2} = \frac{\sum X_3}{n_3}$

NORMAL DISTRIBUTION { SAME MEAN, DIFFERENT STANDARD DEVIATION

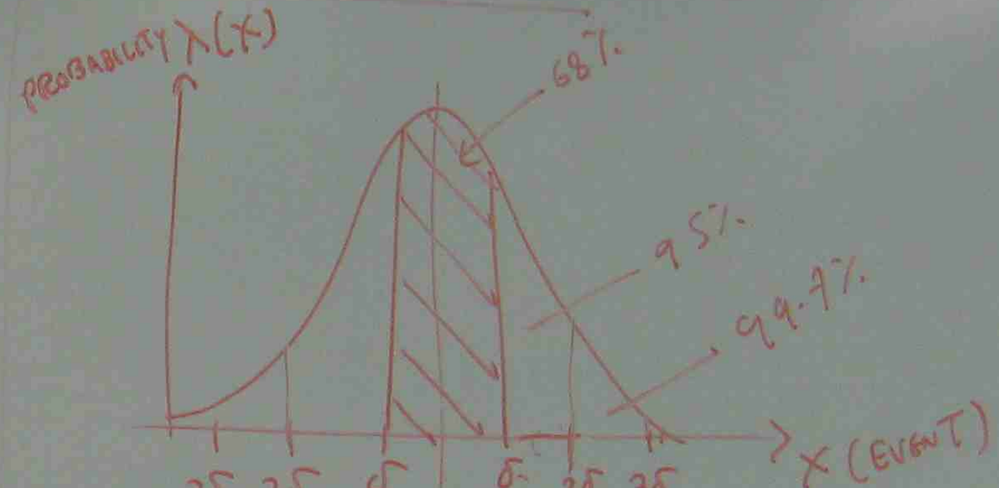
DIFFERENT MEAN, SAME STANDARD DEVIATION



5, δ

μ

NORMAL DISTRIBUTION



$P(\mu-\delta < x < \mu+\delta) = 68\% = 0.68$

$P(\mu-2\delta < x < \mu+2\delta) \Rightarrow 0.95$

$P(\mu-3\delta < x < \mu+3\delta) \longrightarrow 0.997$

51.

99.7%.

→ X (EVENT)

) = 68% = 0.68

) → 0.95

3δ) → 0.997

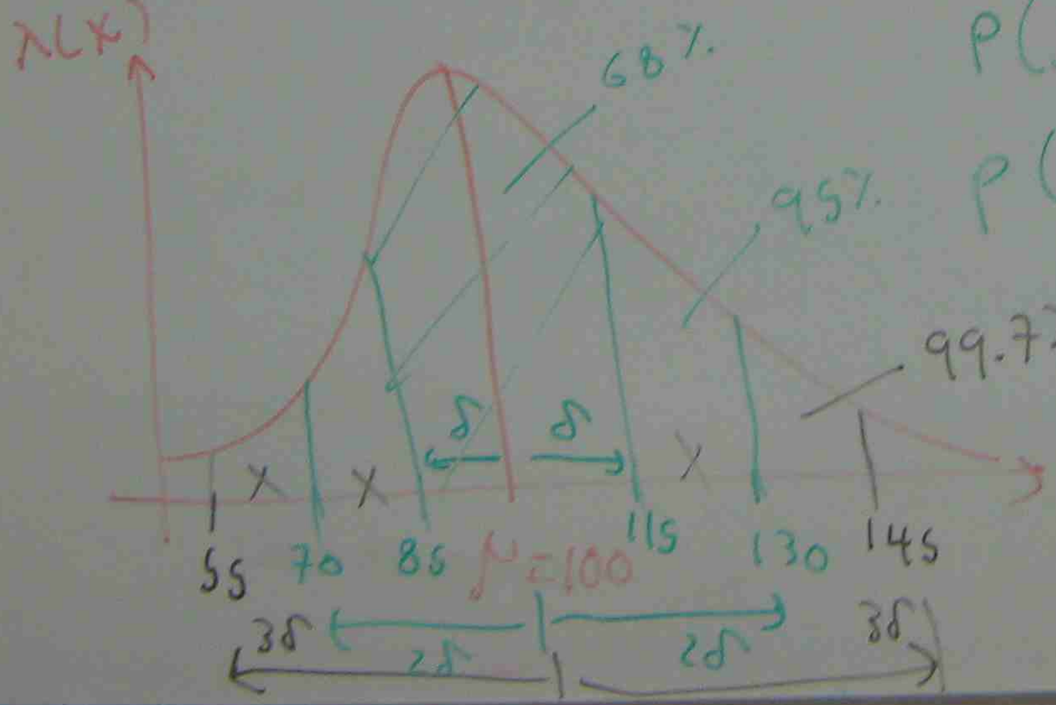
EX 20 SUPPOSE THAT IQ SCORES ARE NORMALLY DISTRIBUTED WITH A MEAN $\mu = 100$ AND A STANDARD DEVIATION $\delta = 15$.

INDICATE THE FOLLOWINGS ON THE GRAPH.

(a) APPROXIMATELY 68% OF PEOPLE HAVE IQ BETWEEN 85 AND 115

(b) APPROXIMATELY 95% OF PEOPLE HAVE IQ BETWEEN 70 AND 130

(c) ALMOST EVERY ONE (APPROXIMATELY 99.7%) HAVE IQ BETWEEN 55 AND 145.



$P(\mu-\delta < x < \mu+\delta) = 0.68$

$P(\mu-2\delta < x < \mu+2\delta) = 0.95$

$P(\mu-3\delta < x < \mu+3\delta) = 0.997$

STANDARD

STANDARD
(Z-SCORE

IF $x > \mu$
IF $x < \mu$
IF $x = \mu$

EX 21
HEIGHT
STAND
FIND
HEIGH
FEMA

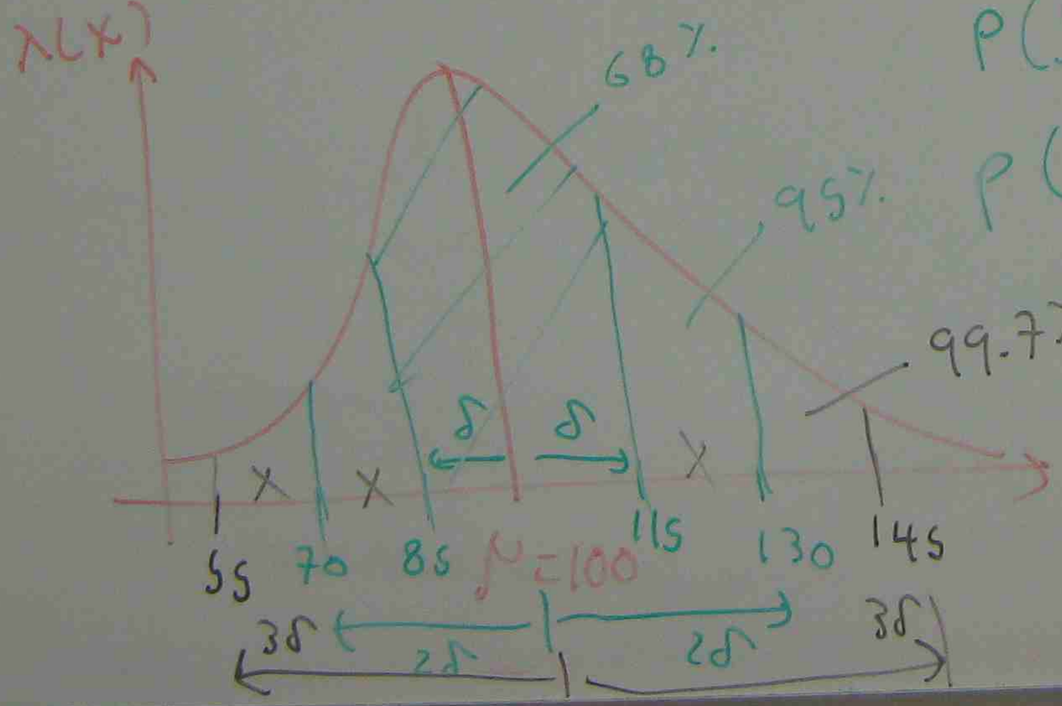
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$P(\mu-\delta < x < \mu+\delta) = 0.68$

$P(\mu-2\delta < x < \mu+2\delta) = 0.95$

$P(\mu-3\delta < x < \mu+3\delta) = 0.997$

STANDARD SCORE

STANDARD SCORE (Z-SCORE)

$z =$

IF $x > \mu$ $z =$

IF $x < \mu$ $z =$

IF $x = \mu$ $z =$

EX 21 THE HEIGHT HAS STANDARD FIND THE HEIGHT TO FEMALE HE

STANDARD SCORE (Z SCORE)

$$\text{STANDARD SCORE (Z-SCORE)} = \frac{\text{RAW DATA} - \text{MEAN}}{\text{STANDARD DEVIATION}}$$

$$z = \frac{x - \mu}{\delta}$$

IF $x > \mu$ $z = +$

IF $x < \mu$ $z = -$

IF $x = \mu$ $z = 0$ NO ERROR

EX(21) THE DISTRIBUTION OF ADULT FEMALE HEIGHT HAS A MEAN $\mu = 165.5$ cm, STANDARD DEVIATION $\delta = 8$ cm.

FIND THE STANDARD SCORE (RELATIVE HEIGHT TO MEAN) CORRESPONDING TO A FEMALE HEIGHT OF (a) 164cm (b) 178cm

(a) $z = \frac{x - \mu}{\delta} = \frac{164 - 165.5}{8} = -0.3$

(b) $z = \frac{x - \mu}{\delta} = \frac{178 - 165.5}{8} = 1.44$

EX(22) THE WEIGHT OF A CERTAIN SPECIES OF ELEPHANT ARE DISTRIBUTED WITH A MEAN OF 20 TONNES AND A STANDARD DEVIATION OF 2 TONNES. SUPPOSE ALSO THAT WEIGHTS OF GARDEN SNAILS ARE DISTRIBUTED WITH A MEAN OF 30 GRAM AND STANDARD DEVIATION 3 GRAM.

WHICH IS HEAVIER, AN ELEPHANT WEIGHING 23 TON (OR) A SNAIL WEIGHING 34.5 GRAMS.

ELEPHANT

$\mu =$

$x_1 =$

$z =$

SNAIL

$\mu_2 = 30$

$x_2 =$

$z =$

TWO AN

SAME RE

MEAN

ERATION

ADULT FEMALE

65.5 cm,

m.

(RELATIVE

ING TO A

m (b) 178cm

(a) $z = \frac{x-\mu}{\delta} = \frac{164-165.5}{8} = -0.3$

(b) $z = \frac{x-\mu}{\delta} = \frac{178-165.5}{8} = 1.44$

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ELEPHANT

$\mu_1 = 20 \quad \delta_1 = 2$

$X_1 = 23$

$z = \frac{X_1 - \mu_1}{\delta_1} = \frac{23-20}{2} = 1.5$

SNAIL

$\mu_2 = 30, \quad \delta_2 = 3$

$X_2 = 34.5$

$z = \frac{X_2 - \mu_2}{\delta_2} = \frac{34.5-30}{3}$

$z = 1.5$

TWO ANIMALS HAVE SAME RELATIVE WEIGHT.

EX (23)

IF A STUDENT SHOWS THE FOLLOWING RESULTS IN A SERIES OF TESTS. COMPARE THE OVERALL PERFORMANCE.

SUBJECT	STUDENT SCORE x	MEAN μ	STANDARD DEVIATION δ
MATHS	170	150	15
ENGLISH	120	120	10
HISTORY	188	200	12
SCIENCE	120	100	8

MATHS $z = \frac{x-\mu}{\delta} = \frac{170-150}{15} = 1.33$

ENGLISH $z = \frac{x-\mu}{\delta} = \frac{120-120}{10} = 0$

HISTORY $z = \frac{x-\mu}{\delta} = \frac{188-200}{12} = -1.0$

SCIENCE $z = \frac{x-\mu}{\delta} = \frac{120-100}{8} = 2.5$

PERFORMANCE

SCIENCE ⇒ BEST (2.5)

MATHS ⇒ 2nd BEST (1.33)

ENGLISH - NORMAL (0)

HISTORY : MOST POOR (-1.0)

READING z TA

z TABLE

z_0	0.00
0.0	
0.1	
0.2	
1.1	
3.0	

1.12 = (1.1)

0.3686

0.3690

S THE FOLLOWING RESULTS IN A
ARE THE OVERALL PERFORMANCE.

E	MEAN μ	STANDARD DEVIATION δ
	150	15
	120	10
	100	8

$\frac{150}{} = 1.33$

$\frac{120}{} = 0$

$\frac{-200}{12} = -1.0$

$\frac{120-100}{8} = 2.5$

PERFORMANCE

SCIENCE ⇒ BEST (2.5)

MATHS ⇒ 2nd BEST (1.33)

ENGLISH - NORMAL (0)

HISTORY = MOST POOR (−1.0)

READING Z TABLE

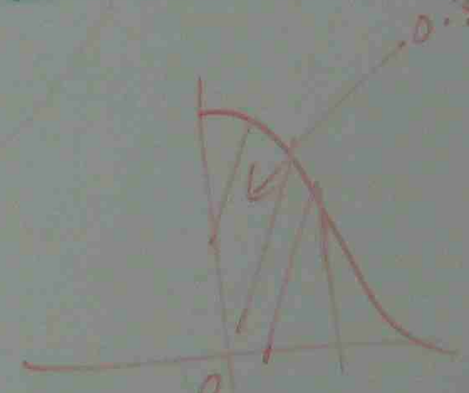
Z TABLE

z_0	0.00	0.01	0.02	0.03 - - - 0.04
0.0				
0.1				
0.2				
1.1			→ 0.3686	
3.0				

1.12 = 1.1 + .02

0.3686 ⟶ 1.12

0.3690 ⟶ 1.12



Ex 24

USE

NORMA

(a) BET

(b) BET

(c) BET

(d) To

(e) To

(f) BE

(a) P

(b) P

(c) P

(e)



0.03 - - 0.04

Ex (24) USE Z TABLE, CALCULATE THE AREA UNDER THE STANDARD NORMAL CURVE FOR THE FOLLOWING INTERVALS.

(a) BETWEEN $z=0$ AND $z=1.50$

(b) BETWEEN $z=0$ AND $z=-2.10$

(c) BETWEEN $z=-0.30$ AND $z=+2.25$

(d) TO THE RIGHT of $z=1.95$

(e) TO THE LEFT of $z=1.64$

(f) BETWEEN 0.60 AND 1.80

(a) $P(0<z<1.5) = 0.4332 - 0.0000$
$= 0.4332$

(b) $P(-2.1<z<0) = 0 - (-0.4821)$
$= 0.4821$

(c) $P(-0.30<z<2.25) = 0.4878 - (-0.1179)$
$= 0.4878 + 0.1179$
$= 0.6057$

(e) $P(z>1.95) =$ THE WHOLE AREA $- P(0<z<1.95)$
$= 0.5 - [0.4744 - 0]$
$= 0.5 - 0.4744 = 0.0256$

(e) $P(z<1.64)$
$=$ THE WHOLE AREA $+ (P[0<z<1.64])$
$= 0.5 + (0.4495 - 0)$
$= 0.5 + 0.4495$
$= 0.9495$

(f) $P(0.6<z<1.8)$
$= P(1.8) - P(0.6)$
$= 0.4641 - 0.2257$
$= 0.2384$

0.3684

0 1.12

σ

μ

EX (25)

(a) IF 37.7% OF POPULATION HAS A STANDARD SCORE BETWEEN THE MEAN ($z=1$) AND SOME POSITIVE z VALUE FIND THAT z VALUE

(b) FIND THE z VALUE CUTTING OFF TOP 5% OF POPULATION

(c) FIND THE z SCORE CUTTING OFF BOTTOM 10% OF POPULATION

(d) FIND THE VALUE OF z SO THAT 95% OF POPULATION HAS A SCORE BETWEEN $-z$ AND $+z$

(a) AREA = 37.7% $\Rightarrow$ 0.377 $\longrightarrow$ $z = ?$

0.06 | 1.1 — 0.3770

$z = 1.16$

(b) CUTTING OFF TOP 5% = THE WHOLE ARE − 5%

$= 0.5 - 0.05 = 0.45$

0.04 | 0.05

1.6 — 0.4495 - 0.4505

0.4495 ≈ 1.64

0.4505 ≈ 1.65

$0.4500 \approx \frac{1.64+1.65}{2}$

≈ 1.645

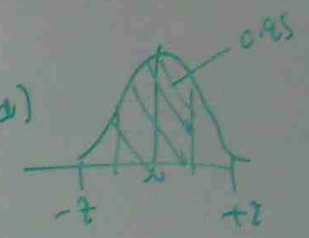
(c) CUTTING OFF BOTTOM 10% = THE WHOLE AREA − 10%

$= 0.5 - 0.1 = 0.4$

0.02 |

1.2 — 0.3997 0.3997 ≃ 0.4 ⇒ 1.28

(a)



2 AREA = 0.95

1 AREA = $\frac{0.95}{2} = 0.475$

0.475 $\longrightarrow$ $z = ?$

0.06 |

1.9 — 0.4713 $\Rightarrow$ 1.96

EX (26) THE HEIGHT OF ADULT MALES IN AUSTRALIA IS APPROXIMATELY NORMALLY DISTRIBUTED WITH MEAN $\mu = 170$cm AND STANDARD DEVIATION $\sigma = 7$cm.

(a) WHAT IS THE PROBABILITY THAT A MALE RANDOMLY SELECTED HAS HEIGHT (i) BETWEEN 160cm & 175 cm

(ii) GREATER THAN 188cm

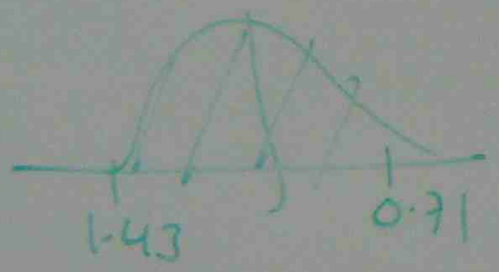
(b) WHAT IS THE GREATEST HEIGHT EXCEEDED BY 22% OF POPULATION.

(a) $P(160 < z < 175)$ = THE AREA BETWEEN
x_1 x_2

$z_1 = \frac{x_1 - \mu}{\sigma}$ AND $z_2 = \frac{x_2 - \mu}{\sigma}$

$z_1 = \frac{160 - 170}{7}$ AND $z_2 = \frac{175 - 170}{7}$

$z_1 = -1.43$ AND $z_2 = 0.71$



TOTAL = AREA (-1.43) + AREA (0.71)

0.03 | 1.4 — 0.4236

0.01 | 0.7 — 0.2611

$= 0.4236 + 0.2611$

$= 0.6847$

(ii) $P(X > 188)$

= TOTAL AREA − THE AREA $z = \frac{x - \mu}{\sigma}$

$= 0.5 - \left(z = \frac{188 - 170}{7} = 2.57\right)$

0.07 | 2.5 — 0.4949

$= 0.5 - 0.4949 = 0.0051$

(b) GREATEST HEIGHT EXCEED BY 22% OF POPULATION

= THE WHOLE AREA − 0.22

$= 0.5 - 0.22 = 0.28$ → FIND (z)

0.07 | 0.7 — 0.2794 ≃ 0.28

$\therefore z = 0.77$

DIFFERENTIATION

$$y = x^n$$

$$\frac{dy}{dx} = \frac{d}{dx} x^n$$

$$\boxed{\frac{dy}{dx} = n x^{n-1}}$$

$$y = a x^n$$

$$\frac{dy}{dx} = \frac{d}{dx}(a x^n)$$

$$= a \frac{d}{dx} x^n$$

$$\frac{dy}{dx} = an x^{n-1}$$

Ex $x^5 \rightarrow \frac{dy}{dx} = ?$

$$\frac{d}{dx} x^5 = 5 x^{5-1} = 5x^4$$

Ex $0.6 x^7$

$$\frac{d}{dx}(0.6 x^7) = 0.6 \frac{d}{dx} x^7$$

$$= 0.6 \times 7 \times x^{7-1}$$

$$= 4.2 x^6$$

EXERCISE FIND $\frac{dy}{dx}$ FOR

$$2x^{1.5}$$

DIFFERENTIATION OF A SUM OF FUNCTIONS

THE DIFFERENTIATION OF A SUM OF FUNCTIONS IS EQUAL TO THE SUM OF THE INDIVIDUAL DIFFERENTIATIONS OF THE FUNCTIONS.

$$y = f_1(x) + f_2(x) + f_3(x)$$

$$\frac{dy}{dx} = \frac{d}{dx} f_1(x) + \frac{d}{dx} f_2(x) + \frac{d}{dx} f_3(x)$$

Ex $y = 5x^3 + 6x^2 + 7$

FIND $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx} 5x^3 + \frac{d}{dx} 6x^2 + \frac{d}{dx} 7$$

$$= 5\frac{d}{dx}x^3 + 6\frac{d}{dx}x^2 + 0$$

$$= 5 \times 3x^{3-1} + 6 \times 2 \times x^{2-1}$$

$$= 15x^2 + 12x$$

Ex $y = \sin x + \cos x$

$\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{d}{dx}\sin x + \frac{d}{dx}\cos x$$

$$= \cos x + (-\sin x)$$

$$= \cos x - \sin x$$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

Ex $y = \sin 3x + \cos 5x$

$$\frac{dy}{dx} = \frac{d}{dx}\sin 3x + \frac{d}{dx}\cos 5x$$

$$= \cos 3x \times \frac{d3x}{dx} + (-\sin 5x)\frac{d}{dx}5x$$

$$= \cos x \; 3\frac{dx}{dx} - \sin 5x \; 5\frac{dx}{dx}$$

$$= 3\cos x - 5\sin 5x$$

DIFFERENTIATION OF PRODUCT FUNCTIONS

$$y = u(x) \times v(x)$$

$$\frac{dy}{dx} = u(x)\frac{dv(x)}{dx} + v(x) \times \frac{du(x)}{dx}$$

Ex $y = (x+1)(x+3)$

u v

$$\frac{dy}{dx} = u\frac{dv(x)}{dx} + v\frac{du(x)}{dx}$$

$$= (x+1)\frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x+1)$$

$$= (x+1)\left[\frac{dx}{dx} + \frac{d3}{dx}\right] + (x+3)\left[\frac{dx}{dx} + \frac{d1}{dx}\right]$$

$\frac{dx}{dx} \to 1$, $\frac{d3}{dx} \to 0$, $\frac{dx}{dx} \to 1$, $\frac{d1}{dx} \to 0$

$$= (x+1)(1) + (x+3)(1)$$

$$= x+1+x+3$$

$$= 2x+4$$

Ex

$\frac{dy}{dx} =$

Ex $y = (x+1)^2 (x+3)^3$

$u \qquad v$

$$\frac{dy}{dx} = (x+1)^2 \frac{d}{dx}(x+3)^3 + (x+3)^3 \frac{d}{dx}(x+1)^2$$

$$= (x+1)^2 \times 3(x+3)^{3-1} \frac{d}{dx}(x+3) + (x+3)^3 \times 2(x+1)^{2-1} \frac{d}{dx}(x+1)$$

$$= (x+1)^2 \times 3(x+3)^2 \left[\frac{d}{dx}x + \underbrace{\frac{d}{dx}3}_{0}\right] + (x+3)^3 \times 2(x+1) \left[\frac{d}{dx}x + \underbrace{\frac{d}{dx}1}_{0}\right]$$

$$= 3(x+1)^2 (x+3)^2 (1) + (x+3)^3 \times 2(x+1)(1)$$

$$= 3(x+1)^2 (x+3)^2 + 2(x+3)^3 (x+1)$$

$$= (x+1)(x+3)^2 \left[3(x+1) + 2(x+3)\right]$$

$$= (x+1)(x+3)^2 \left[3x+3+2x+6\right]$$

$$= (x+1)(x+3)^2 (5x+9)$$

DIFFERENTIATION OF QUOTIENT FUNCTIONS

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v(x)\frac{du(x)}{dx} - u(x)\frac{dv(x)}{dx}}{(v(x))^2}$$

Ex DIFFERENTIATE

$$y = \frac{x}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}x - x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1)\times 1 - x\left[\frac{d}{dx}x + \frac{d}{dx}1\right]}{(x+1)^2}$$

$$= \frac{x+1-x(1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$y = \frac{x^2+2x+1}{x^2-2x+1}$$

FIND $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(x^2-2x+1)\frac{d}{dx}(x^2+2x+1) - (x^2+2x+1)\frac{d}{dx}(x^2-2x+1)}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)\left[\frac{d}{dx}x^2 + \frac{d}{dx}2x + \frac{d}{dx}1\right] - (x^2+2x+1)\left[\frac{d}{dx}x^2 - \frac{d}{dx}2x + \frac{d}{dx}1\right]}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x^{2-1}+2+0) - (x^2+2x+1)(2x^{2-1}-2+0)}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x+2) - (x^2+2x+1)(2x-2)}{(x^2-2x+1)^2}$$

$$= \frac{(x^2-2x+1)\times 2(x+1) - (x^2+2x+1)2(x-1)}{(x^2-2x+1)^2}$$

$$\frac{2(x+1)^2(x+1) - 2(x+1)^2(x+1)}{[(x-1)^2]^2}$$

$$\frac{2(x-1)(x+1)[(x-1)-(x+1)]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[x-1-x-1]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[-2]}{(x-1)^4}$$

$$\frac{-4(x+1)}{(x-1)^3}$$ //

IN THIS BUILDING

DIFFERENTIATING TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}\sin(x) = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

DIFFERENTIATING EXPONENTIAL FUNCTION

$$y = e^x$$

$$\frac{dy}{dx} = \frac{de^x}{dx} = e^x$$

If $y = e^u$

$$\frac{dy}{dx} = \frac{d}{dx}e^u = e^u \frac{du}{dx}$$

Pb

DIFFERENTIATE (i) $y = e^{ax}$ (ii) $y = e^{\frac{1}{2}bx^2}$

(i) $y = e^{ax}$

$$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax}\frac{dax}{dx}$$

$\therefore e^{ax} \times a \frac{d}{}$

$= a e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\left(\frac{1}{2}bx^2 + x\right.}$$

$= e^{\frac{1}{2}bx^2 +}$

EN TIAL FUNCTION

e^x

$u \frac{du}{dx}$

pb DIFFERENTIATE (i) $y = e^{ax}$ (ii) $y = e^{\frac{1}{2}bx^2 + x}$

(i) $y = e^{ax}$

$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax} \frac{dax}{dx}$

$= e^{ax} \times a \frac{dx}{dx}$

$= a e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{1}{2}bx^2 + x}$

$= e^{(\frac{1}{2}bx^2 + x)} \frac{d}{dx}(\frac{1}{2}bx^2 + x)$

$= e^{\frac{1}{2}bx^2 + x} \left[\frac{d}{dx} \frac{1}{2} bx^2 + \frac{d}{dx} x \right]$

$= e^{\frac{1}{2}bx^2 + x} \left[\frac{b}{2} \frac{d}{dx} x^2 + 1 \right]$

$= e^{\frac{1}{2}bx^2 + x} \left[\frac{b}{2} \times 2 x^{2-1} + 1 \right]$

$= e^{\frac{1}{2}bx^2 + x} [bx + 1]$ //

DIFFERENTIATING LOGARITHMIC FUNCTION

$$y = \log_e u(x) = \ln u(x)$$

$$\frac{dy}{dx} = \frac{1}{u(x)} \frac{d}{dx} u(x)$$

DIFFERENTIATE

$\log_e x^3$

$\log_e (x^2-1)$

$\log_e \sin x$

(i) $\frac{d}{dx} \log_e x^2 = \frac{1}{x^2} \frac{d}{dx} x^2$

$= \frac{1}{x^2} \times 2x^{2-1}$

$= \frac{2x}{x^2}$

$= \frac{2}{x}$

(ii) $\frac{d}{dx} \log_e (x^2-1) = \frac{1}{(x^2-1)} \frac{d}{dx}(x^2-1)$

$= \frac{1}{(x^2-1)} \left[\frac{d}{dx} x^2 - \frac{d}{dx} 1\right]$

$= \frac{1}{(x^2-1)} [2x] = \frac{2x}{x^2-1}$

(iii) $\frac{d}{dx} \log_e \sin x = \frac{1}{\sin x} \frac{d}{dx} \sin x$

$= \frac{1}{\sin x} \cos x$

$= \frac{\cos x}{\sin x} = \cot x$

$\log_e u = \frac{\log_{10} u}{\log_{10} e} = \frac{\log_{10} u}{\log_{10} 2.718} = \frac{\log_{10} u}{0.434}$

$= 2.3 \log_{10} u$

$\log_{10} u = \frac{\log_e u}{2.3}$

$\frac{d}{dx} \log_{10} u = \frac{d}{dx}\left(\frac{\log_e u}{2.3}\right)$

$\frac{d}{dx} \log_{10} u = \frac{1}{2.3} \cdot \frac{d}{dx} \log_e u$

$= \frac{1}{2.3} \times \frac{1}{u} \frac{du}{dx}$

FIRST ORDER DERIVATIVE → $\frac{dy}{dx}$

SECOND ORDER DERIVATIVE → $\frac{d^2}{d}$

SUCCESSIVE DIFFERENTIATION

$$y = x^3 + 3x^2 + 4$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 + 3x^2 + 4\right]$$

$$= \frac{d}{dx}x^3 + \frac{d}{dx}3x^2 + \frac{d}{dx}4\Big]$$

$$= 3x^{3-1} + 3\times 2x^{2-1} + 0$$

$$= 3x^2 + 6x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3x^2 + 6x\right]$$

$$= \frac{d}{dx}3\left[x^2 + 2x\right]$$

$$\frac{d^2y}{dx^2} = 3\left[\frac{dx^2}{dx} + \frac{d2x}{dx}\right]$$

$$= 3\left[2x^{2-1} + 2\right]$$

$$= 3\left[2x + 2\right]$$

$$= 6(x+1)$$

pb FIND THE FIRST

$$y = \frac{1}{2x+1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-\right.$$

$$= -2x$$

$$= 4($$

$$= 8$$

pb FIND THE FIRST THREE DIFFERENTIAL COEFFICIENTS OF THE FUNCTION

$$y = \frac{1}{2x+1} = (2x+1)^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+1)^{-1} = -1(2x+1)^{-1-1}\frac{d}{dx}(2x+1)$$

$$= -(2x+1)^{-2} \times 2 = -2(2x+1)^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-2(2x+1)^{-2}\right] = -2\frac{d}{dx}(2x+1)^{-2}$$

$$= -2 \times (-2)(2x+1)^{-2-1}\frac{d}{dx}(2x+1)$$

$$= 4(2x+1)^{-3} \times 2$$

$$= 8(2x+1)^{-3}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx} 8(2x+1)^{-3}$$

$$= 8\frac{d}{dx}(2x+1)^{-3}$$

$$= 8(-3)(2x+1)^{-3-1}\frac{d}{dx}(2x+1)$$

$$= -24(2x+1)^{-4} \times 2$$

$$= -48(2x+1)^{-4}$$

$$= \frac{-48}{(2x+1)^4}$$ //

EXERCISES

① DIFFERENTIATE

(i) $y = x^{10}$

(ii) $y = x^3$

(iii) $y = mx^k$

(iv) $y = (x-5)^3$

(v) $y = 15(x+4)^7$

(vi) $y = 4x^{-3.2}$

(vii) $y = 3.2(2x+3)^{1/5}$

② DIFFERENTIATE

(a) $y = 2x^{3/2} + x^{-3/2}$

(b) $y = 7x^6 + 6x^5 + 4x^4 + 3x^2 + 2x + 1$

(c) $y = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$

(d) $y = -x^{-2} + \frac{1}{x} + x^2$

ARE NOT
APPROPRIATE
FOOTWEAR
IN THIS BUILDING

$+2x+1$

(3) THE LOSS IN ELECTRICAL MACHINE IS GIVEN BY

$$P = af + bf^2$$

WHERE P = POWER $\quad a, b$ = CONSTANT

f = FREQUENCY

FIND $\frac{dP}{df}$

(4) THE INDUCED EMF OF A DC MACHINE IS GIVEN BY

$$E = 0.58 + 681.5\, I_f - 461.8\, I_f^2 + 46.3\, I_f^3$$

I_f = FIELD EXCITATION CURRENT

FIND $\frac{dE}{df}$

(5) DIFFERENTIATE

(i) $(x+1)^{-\frac{1}{2}}(x-5)$

(ii) $(2x+7)^3(4x^2-5)^2$

(iii) $(5x^2+6x+3)(5x-1)$

(iv) $\frac{4-x}{x-x^2}$

(v) $\frac{x^{1/4}}{x^{1/2}-1}$

SUCCESSIVE DIFFERENT

$y = x^3 + 3$

$\frac{dy}{dx} = \frac{d}{dx}\left[x^3\right.$

$= \frac{d}{dx}x^3$

$= 3x^{3-1}$

$= 3x^2$

$\frac{d^2y}{dx^2} = \frac{d}{dx}$

$= \frac{d}{dx}$

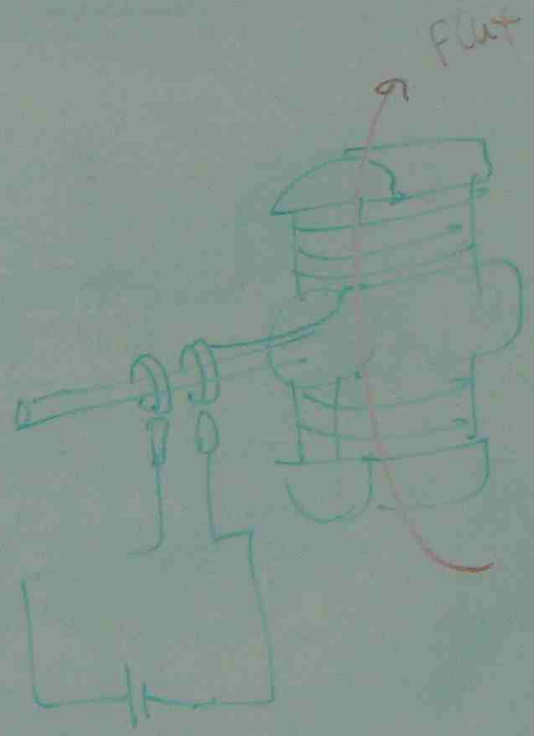
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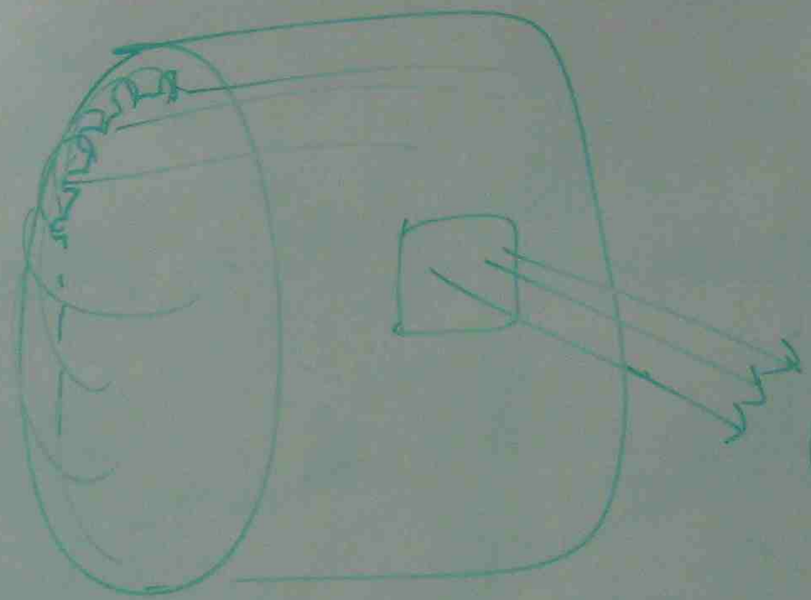
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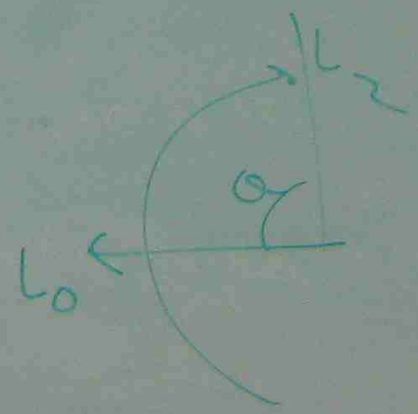
)

$-2L_2 \sin 2\theta$





$$N = \frac{120 f}{P}$$



Pb

DIFFERENTI

LOGARITHMIC

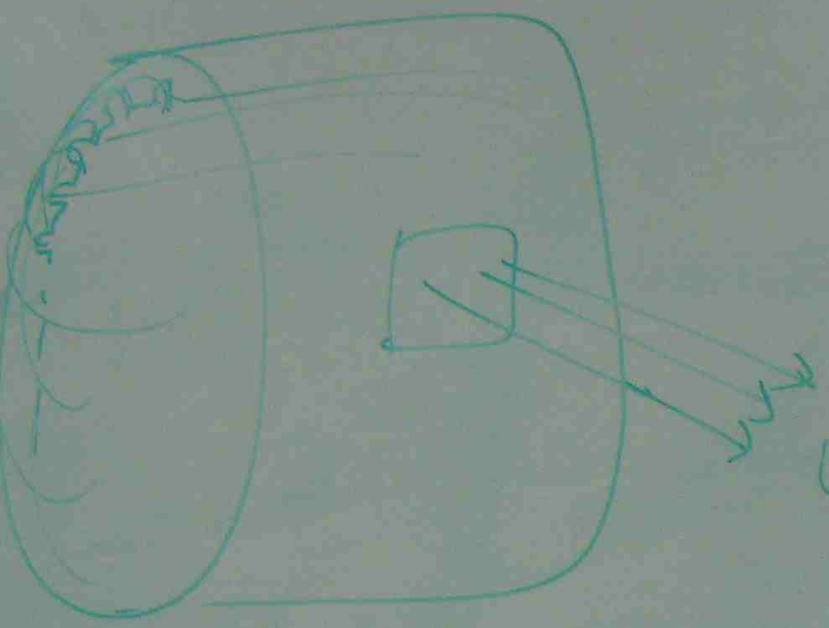
(i) e^{-ax}

(ii) $e^{(x^2+}$

(iii) $\log_e$

(iv) a^{x^2+}

(v) $\sin$ x



$= \frac{120 f}{p}$

L_2

Pb

DIFFERENTIATE THE FOLLOWING EXPONENTIAL, LOGARITHMIC AND POWER FUNCTIONS.

(i) e^{-ax}

(ii) $e^{(x^2+2x)}$

(iii) $\text{Log}_e (x^2+2x+3)$

(iv) a^{x^2+1}

(v) $x^{\sin x}$

(i) $\frac{d}{dx} e^{-ax}$

(ii) $\frac{d}{dx} e^{x^2}$

(iii) $\frac{d}{dx}$

CANVAS SHOES
ARE NOT
APPROPRIATE
FOOTWEAR
IN THIS BUILDING

$$\left[\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right]$$

$$= \frac{(x^2-1)\frac{d}{dx}2x - 2x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{2(x^2-1) - 2x \cdot 2x}{(x^2-1)^2}$$

$$= \frac{2x^2 - 2 - 4x^2}{(x^2-1)^2}$$

$$= \frac{-2(x^2+1)}{(x^2-1)^2}$$

(ii) $y = \sin^2 x$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^2 x = 2\sin^{2-1}x \frac{d\sin x}{dx} = 2\sin x\cos x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \cdot 2\sin x\cos x$$

$$= 2\left[\sin x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}\sin x\right]$$

$$= 2\left[\sin x(-\sin x) + \cos x \cdot \cos x\right]$$

$$= -2\left[\sin^2 x - \cos^2 x\right]$$

$$= 2\left[-\sin^2 x + \cos^2 x\right] \quad \Big| \; 2\left[1 - 2\sin^2 x\right]$$

$$= 2\left[-\sin^2 x + 1 - \sin^2 x\right]$$

(iii

(iii) $y = e^{\frac{x^2}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{x^2}{2}} = e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2$$

$$= e^{\frac{x^2}{2}} \times \frac{1}{2} \times 2x$$

$$= e^{x^2/2} \, x$$

$$= x e^{\frac{x^2}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} x e^{\frac{x^2}{2}}$$

$$= x \frac{d}{dx} e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}} \frac{dx}{dx}$$

$$= x \times e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2 + e^{\frac{x^2}{2}}$$

$$= x \, e^{x^2/2} \times 2x/2 + e^{x^2/2}$$

$$= e^{x^2/2} (x^2 + 1) \; //$$

(iv) $y = a^x$

$$\frac{dy}{dx} = \frac{d}{dx} a^x = x \, a^{x-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[x \cdot a^{x-1} \right]$$

$$= x \frac{d}{dx} a^{x-1} + a^{x-1} \frac{dx}{dx}$$

$$= x \times (x-1) \, a^{x-1-1} + a^{x-1}$$

$$= x(x-1) \, a^{x-2} + a^{x-1}$$

Ph THE CURRENT GROWTH IN A RESISTIVE-INDUCTIVE CIRCUIT FROM A SUDDENLY APPLIED BATTERY EMF IS

$$i = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

WHERE E, R, L ARE CONSTANTS, t IS TIME

FIND THE RATE OF CHANGE OF CURRENT WITH RESPECT TO TIME.

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right)\right]$$

$$= \frac{E}{R}\left[-\frac{d}{dt}e^{\frac{Rt}{L}}\right]$$

$$= -\frac{E}{R} \times e^{-Rt/L} \times \frac{d}{dt}\left(-\frac{Rt}{L}\right)$$

$$= -\frac{E}{R} \times e^{\frac{-Rt}{L}} \times \left(-\frac{R}{L}\right)$$

$$\left\{ \frac{di}{dt} = \frac{E}{L} \times e^{\frac{-Rt}{L}} \right. \quad *$$

DIFFERENTIATION OF A FUNCTION OF A FUNCTION

$$y = \sin^2(x^2+1)$$

Ex $y = \sin^3(2x^2-1)$ FIND y' $\left(\frac{dy}{dx} = ?\right)$

LET $u = 2x^2 - 1$

$$\frac{du}{dx} = \frac{d}{dx}(2x^2-1)$$

$$= 2x\,2x^{2-1}$$

$$= 4x$$

$$y = \sin^3 u$$

$$\frac{dy}{du} = 3\sin^{3-1} u \frac{d\sin u}{du}$$

$$= 3\sin^2 u \times \cos u$$

$\frac{dy}{dx}$

BUT u

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}}$$

$$= 4x \times 3\sin^2 u \cos u$$

But $u = 2x^2 - 1$

$$= 4x \; 3\sin^2(2x^2-1)\cos 2x^2 - 1$$

$$= 12x \sin^2 2x^2 - 1 \cos 2x^2 - 1 \; \#$$

DIFFERENTIATION OF IMPLICIT FUNCTION

Pb

$x^2 + y^2 = 9$ FIND $\frac{dy}{dx}$

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 9$

$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$

$2x^{2-1} \frac{dx}{dx} + 2y^{2-1} \frac{dy}{dx} = 0$

$2x + 2y \frac{dy}{dx} = 0$

$2y \times \frac{dy}{dx} = -2x$

E NOT
OPRIATE
WEAR
BUILDING

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Ob DIFFERENTIATE FOLLOWING FUNCTIONS WITH RESPECT TO X.

a) $x^2 + y^2 = 4$

b) $y \log_e x = 2$

(a) $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx} 4$

$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$

$2x^{2-1} + 2y \frac{dy}{dx} = 0$

$2x + 2y \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{y}$

(b) $y \log_e x = 2$

$\frac{d}{dx}\left[y \log_e x\right] = \frac{d}{dx} 2$

$\frac{d}{dx} u.v = u \frac{dv}{dx} + v \frac{du}{dx}$

$y \frac{d}{dx} \log_e x + \log_e x \frac{dy}{dx} = 0$

$y \times \frac{\frac{dx}{dx}}{x} + \log_e x \times \frac{dy}{dx} = 0$

$\frac{y}{x} + \log_e x \times \frac{dy}{dx} = 0$

$\frac{d}{dx} \log_a f(x) = \frac{f'(x)}{f(x)}$

$\log_e x \times \frac{dy}{dx}$

$\frac{dy}{dx} =$

$= \frac{d}{dx} 4$

0

$= 0$

(b) $y \log_e x = 2$

$\frac{d}{dx}\left[y \log_e x\right] = \frac{d}{dx} 2$

$\frac{d}{dx} u.v = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d}{dx} \log f(x) = \frac{f(x)'}{f(x)}$

$y \frac{d}{dx} \log_e x + \log_e x \frac{dy}{dx} = 0$

$y \times \frac{\frac{dx}{dx}}{x} + \log_e x \times \frac{dy}{dx} = 0$

$\frac{y}{x} + \log_e x \times \frac{dy}{dx} = 0$

$\log_e x \times \frac{dy}{dx} = -\frac{y}{x}$

$\frac{dy}{dx} = \frac{-y/x}{\log_e x}$

DIFFERENTIATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sin(10x+4) + \cos(7x+1)$

(ii) $\tan^2 3\theta$

(iii) $\sec x \tan x$

(iv) $\cos ec^4 (x^2+1)$

(v) $\cot 5x \sin 6x$

(vi) $1 - 10\sin 10t + 5\sin 20t + 2.5\sin 30t$

(i) $\frac{d}{dx}\left[\sin(10x+4) + \cos(7x+1)\right]$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + (-\sin(7x+1))\frac{d}{dx}(7x+1)$

$\cos(10x+4) \times 10 - \sin(7x+1) \times 7$

$10\cos(10x+4) - 7\sin(7x+1)$ ✗

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \quad \sec^2 3\theta \times$

$2\tan 3\theta \times \sec^2 3\theta$

$6\tan 3\theta \sec^2 3\theta$

NG TRIGONOMETRIC FUNCTIONS

+1)

$t + 2.9 \sin 3ot$

(i) $\frac{d}{dx}\left\{\sin(10x+4) + \cos(7x+1)\right\}$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + (-\sin(7x+1))\frac{d}{dx}(7x+1)$

$\cos(10x+4) \times 10 - \sin(7x+1) \times 7$

$10\cos(10x+4) - 7\sin(7x+1)$ //

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \sec^2 3\theta \times \frac{d}{d\theta}3\theta$

$2\tan 3\theta \times \sec^2 3\theta \times 3$

$6\tan 3\theta \sec^2 3\theta$ //

(iii) $\frac{d}{dx}\sec x \tan x = \sec x$

$= \sec x$

$=$

$=$

(iv) $\operatorname{cosec}^4(x^2+1)$

$u = x^2+1 \longrightarrow$ cosec

$\frac{d}{dx}\operatorname{cosec}^4 u = 4\operatorname{cosec}$ 4-

$= 4\operatorname{cosec}^3 u$

$= 4\operatorname{cosec}^3$

$= 8x$

$+\cos(7x+1)\}$

$= \cos(7x+1)$

$+ \{-\sin(7x+1)\} \frac{d}{dx}(7x+1)$

$\sin(7x+1) \times 7$

$\sin(7x+1)$ //

$\frac{u}{d\theta}$

$\frac{d}{d\theta} \tan 3\theta$

$2u \sec^2 3\theta \times \frac{d}{d\theta} 3\theta$

$2 \tan 3\theta \times \sec^2 3\theta \times 3$

$6 \tan 3\theta \sec^2 3\theta$ //

(iii) $\frac{d}{dx} \sec x \tan x = \sec x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \sec x$

$= \sec x \cdot \sec^2 x + \tan x \cdot \sec x \tan x$

$= \sec^3 x + \sec x \tan^2 x$

$= \sec x (\sec^2 x + \tan^2 x)$ //

(iv) $\operatorname{cosec}^4(x^2+1)$

$u = x^2+1 \longrightarrow \operatorname{cosec}^4 u$

$\frac{d}{dx} \operatorname{cosec}^4 u = 4 \operatorname{cosec}^{4-1} u \frac{du}{dx}$

$= 4 \operatorname{cosec}^3 u \times \frac{d}{dx}(x^2+1)$

$= 4 \operatorname{cosec}^3(x^2+1) \times 2x$

$= 8x \operatorname{cosec}^3(x^2+1)$

FOOTWEAR
IN THIS BUILDING

(v) $\cot 5x \sin 6x$

$$\frac{d}{dx}\cot 5x \sin 6x = \cot 5x \frac{d}{dx}\sin 6x + \sin 6x \frac{d}{dx}\cot 5x$$

$$= \cot 5x \cos 6x \frac{d}{dx}6x + \sin 6x \times (-\operatorname{Cosec}^2 5x)\frac{d}{dx}5x$$

$$= 6\cot 5x \cos 6x - 5\sin 6x \operatorname{cosec}^2 5x \quad //$$

(vi) $\frac{d}{dt}\left[1 - 10\sin 10t + 5\sin 20t + 2.5\sin 30t\right]$

$$\frac{d}{dt}1 - \frac{d}{dt}10\sin 10t + \frac{d}{dt}5\sin 20t + \frac{d}{dt}2.5\sin 30t$$

$$0 - 10\cos 10t \frac{d}{dt}10t + 5\cos 20t \frac{d}{dt}20t + 2.5\cos 30t \frac{d}{dt}30t$$

$$-100\cos 10t + 100\cos 20t + 75\cos 30t \quad //$$

---

pb / THE POTENTIAL DIFFER
OF SELF INDUCTANCE L

$$V_L = L\frac{di}{dt}$$

IF $i = I_0 \sin(314t +$

$$V_L = L\frac{d}{dt}[10\sin($$

$$= L \times 10 \cos(31$$

$$= 10L \cos(31$$

$$= 3140L \cos($$

$6x \frac{d}{dx} \cot 5x$

$\sin 6x + (-\operatorname{cosec}^2 5x) \frac{d}{dx} 5x$

$-5 \sin 6x \operatorname{cosec}^2 5x$ //

$\sin 30t$]

$.5 \sin 30t$ |

$2.5 \cos 30t \frac{d}{dt} 30t$

t //

pb THE POTENTIAL DIFFERENCE ACROSS AN INDUCTOR OF SELF INDUCTANCE L IS

$$V_L = L \frac{di}{dt}$$

IF $i = 10 \sin(314t + 60^\circ)$, FIND THE POTENTIAL DIFFERENCE V_L

$$V_L = L \frac{d}{dt} [10 \sin(314t + 60)]$$

$$= L \times 10 \cos(314t + 60) \times \frac{d}{dt}(314t + 60)$$

$$= 10L \cos(314t + 60) \times 314$$

$$= 3140L \cos(314t + 60)$$ //

Ph THE SELF INDUCTANCE OF A ROTOR WINDING OF A SALIENT POLE SYNCHRONOUS MACHINE IS

$$L = L_0 + L_2 \cos 2\theta$$

WHERE L_0 AND L_2 ARE CONSTANT AND θ IS THE ANGULAR POSITION OF THE ROTOR. FIND THE RATE OF CHANGE OF INDUCTANCE WITH ANGULAR POSITION.

$$\frac{dL}{d\theta} = \frac{d}{d\theta}\left(L_0 + L_2 \cos 2\theta\right)$$

$$= 0 + L_2 \frac{d}{d\theta} \cos 2\theta$$

$$= L_2(-\sin 2\theta) \times \frac{d}{d\theta} 2\theta$$

$$-2L_2 \sin 2\theta$$



(i) $\frac{d}{dx} e^{-ax} = e^{-ax} \frac{d}{dx}(-ax)$

$= -a e^{-ax}$

(ii) $\frac{d}{dx} e^{x^2+2x} = e^{x^2+2x} \frac{d}{dx}(x^2+2x)$

$= e^{x^2+2x} \left[\frac{d}{dx} x^2 + \frac{d}{dx} \cdot 2x\right]$

$= e^{x^2+2x} [2x+2]$

$= 2 e^{x^2+2x} [x+1]$

(iii) $\frac{d}{dx} \log_e (x^2+2x+3) = \frac{\frac{d}{dx}(x^2+2x+3)}{x^2+2x+3}$

$= \frac{2x+2}{x^2+2x+3} = \frac{2(x+1)}{x^2+2x+3}$

(iv) $\frac{d}{dx} a^{x^2+1}$ $= x^2+1$

$= (x^2+1$

$= 2x$

(v) $\frac{d}{dx} x^{\sin x}$ $=$

$=$

(iv) $\frac{d}{dx} a^{x^2+1} = a^{x^2+1} \log a \frac{d}{dx} x^2+1$

$= a^{x^2+1} \log a \, 2x$

$= 2x \, a^{x^2+1} \log a$

(v) $\frac{d}{dx} x^{\sin x} = \sin x \, x^{\sin x - 1} \frac{d}{dx} \sin x$

$= \sin x \, x^{\sin x - 1} \cos x$

$= \sin x \cos x \, x^{\sin x - 1}$

Ph FIND FIRST TWO DIFFERENTIAL COEFFICIENTS OF THE FOLLOWING FUNCTIONS

(i) $\log_e(x^2-1)$

(ii) $\sin^2 x$

(iii) $e^{\frac{x^2}{2}}$

(iv) a^x

(i) $y = \log_e(x^2-1)$

$$\frac{dy}{dx} = \frac{d}{dx}\log_e(x^2-1)$$

$$= \frac{\frac{d}{dx}(x^2-1)}{(x^2-1)}$$

$$= \frac{2x}{x^2-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{2x}{x^2-1}$$

$\frac{d}{dx}\frac{u}{v} =$

=

=

cosx $\frac{d}{dx}$ sinx

cosx * cosx

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

pb $\int \sin 3x \cos 4x \, dx = ?$

$$\sin 3x \cos 4x = \frac{1}{2}\left[\sin(3x+4x) + \sin(3x-4x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin(-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x - \sin x\right]$$

$$\int \frac{1}{2}\left[\sin 7x - \sin x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} - (-\cos x)\right]$$

$$\frac{1}{2}\left[\frac{(-\cos 7x)}{7} + \cos x\right] + C$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$$

$$-\frac{\cos 7x}{14} + \frac{\cos x}{2} + C$$

EXERCISE

INTEGRATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sec^2 x$

(ii) $\tan x$

(iii) $\cos 3x$

(iv) $\sin 6x$

(v) $\cot x$

(vi) $\operatorname{cosec} x \cot x$

(vii) $\sec x \tan x$

Ph $\int \frac{1}{5}\cos^2 5x\,dx = ?$

$\cos 2x = 2\cos^2 x - 1$

$\therefore \cos 10x = 2\cos^2 5x - 1$

$\cos^2 5x = \frac{\cos 10x + 1}{2}$

$\int \frac{1}{5}\left(\frac{\cos 10x + 1}{2}\right)dx$

$\frac{1}{10}\int (\cos 10x + 1)\,dx$

$\frac{1}{10}\left[\int \cos 10x\,dx + \int dx\right]$

$\frac{1}{10}\left[\right.$

$\frac{1}{10}\left[\right.$

sim

Ph $\int$

$\int \sec^2$

$\sec^2$

$+\cos(3x-5x)\Big]$

$]$

$]$

$x\,dx\Big]$

$\dfrac{s2x\,dx}{2}\Big]$

$+c$

c

ph $\int \sin x \sin 3x\,dx$

$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$

$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$

$= \frac{1}{2}\left[\cos 2x - \cos 4x\right]$

$\left(\int \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx\right.$

$\frac{1}{2}\left[\int \cos 2x\,dx - \int \cos 4x\,dx\right]$

$\frac{1}{2}\left[\int \frac{\cos 2x\,d2x}{2} - \int \frac{\cos 4x\,d4x}{4}\right]$

$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + c$ //

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

INTEGRATION of EXPONENTIAL FUNCTION

$$\int e^u \, du = e^u + c$$

ph $\int e^{3x} \, dx$

$$\int \frac{e^{3x} \, d3x}{3}$$

$$\frac{e^{3x}}{3} + c$$

ph $\int x e^{x^2} \, dx \longrightarrow \int e^{x^2} x \, dx$

$dx^2 = 2x^{2-1} \, dx$

$= 2x \, dx$

$\therefore x \, dx = \frac{dx^2}{2}$

$\int e^{x^2} \frac{dx^2}{2}$

$\frac{1}{2} \int e^{x^2} dx^2$

$\frac{1}{2} e^{x^2} + c$ XX

$e^{x^2} dx \longrightarrow \int e^{x^2} x\, dx$

$= 2x^{2-1} dx$

$= 2x\, dx$

$dx = \frac{dx^2}{2}$

$\int e^{x^2} \frac{dx^2}{2}$

$\frac{1}{2} \int e^{x^2} dx^2$

$\frac{1}{2} e^{x^2} + c$ ✗✗

EXERCISE

INTEGRATE THE FOLLOWING EXPONENTIAL FUNCTIONS

(i) e^{-3x}

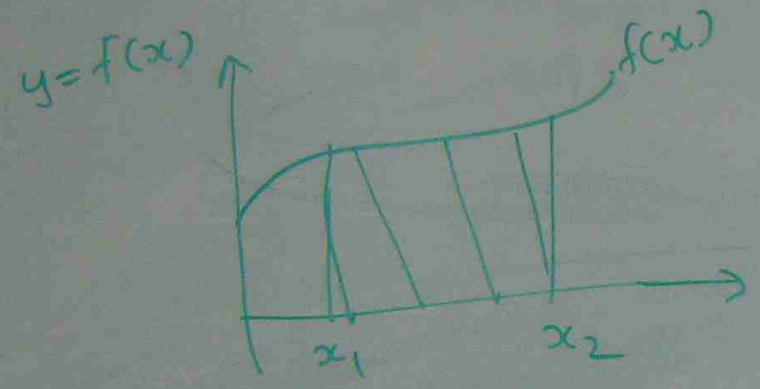
(ii) e^{4x}

(iii) $2e^{-\frac{1}{3}x}$

(iii) $5e$

(iv) $3xe^{x^2}$

INTEGRATION



AREA UNDER CURVE = $\int_{x_1}^{x_2} f(x)\,dx$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Ex $\int x^7\,dx = \frac{x^{7+1}}{7+1} + C$

$= \frac{x^8}{8} + C$

Ex $\int x^{\frac{1}{5}}\,dx = \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C$

$\frac{x^{6/5}}{6/5} + C$

$\frac{5}{6}x^{6/5} + C$

Ex $\int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + c$

$= \frac{x^{-2}}{-2} + c$

$= -\frac{1}{2} x^{-2} + c$

Ex $\int x^{-1/2} dx = \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + c$

$= \frac{x^{1/2}}{1/2} + c$

$= 2x^{1/2} + c$

$\int (x^n + x^m) dx = \int x^n dx + \int x^m dx$

$= \frac{x^{n+1}}{n+1} + \frac{x^{m+1}}{m+1} + c$

Ex $\int (x^4 + 2x^3) dx$

$\int x^4 dx + \int 2x^3 dx$

$\frac{x^{4+1}}{4+1} + 2\int x^3 dx$

$\frac{x^5}{5} + 2 \times \frac{x^{3+1}}{3+1} + c$

$\frac{x^5}{5} + 2\frac{x^4}{4} + c$

$$\frac{x^5}{5} + \frac{x^4}{2} + c \quad //$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c$$

$$\int (2x+3)^n \, d(2x+3) = \frac{(2x+3)^{n+1}}{n+1} + c$$

$$\int (2x+3)^n \, dx \neq \frac{(2x+3)^{n+1}}{n+1} + c$$

Ex $\int (2x+3)^3 \, dx = ?$

$$d(2x+3) = d2x + d3$$
$$= 2\,dx + 0$$
$$= 2\,dx$$

$$dx = \frac{d(2x+3)}{2}$$

$$\int (2x+3)^3 \, \frac{d(2x+3)}{2}$$

$$\frac{1}{2} \int (2x+3)^3 \, d(2x+3)$$

$$\frac{1}{2}\,\frac{(2x+3)^{3+1}}{3+1}+c$$

$$\frac{1}{2}\times\frac{(2x+3)^4}{4}+c$$

$$\frac{1}{8}(2x+3)^4+c$$

Ex $\int(-3x+2)^{-1/3}\,dx$

$d(-3x+2) = d(-3x)+d2$

$d(-3x+2) = -3\,dx$

$dx = -\frac{1}{3}\,d(-3x+2)$

$$\int(-3x+2)^{-\frac{1}{3}}\left(-\frac{1}{3}\,d(-3x+2)\right)$$

$$-\frac{1}{3}\int(-3x+2)^{-\frac{1}{3}}\,d(-3x+2)$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+c$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{2/3}}{2/3}+c$$

$$-\frac{1}{3}\times\frac{3}{2}(-3x+2)^{2/3}+c$$

$$-\frac{1}{2}(-3x+2)^{2/3}+c$$

EXERCISE

$$\int (5x+8)^{-2}\,dx$$

INTEGRATE THE FOLLOWINGS

(i) x^5

(ii) $\frac{1}{3}x^{-1/2}$

(iii) $6x^{-2}$

(iv) $3x^{1/5}$

(v) $\frac{1}{2}x^{-1/3} + x^2$

(vi) $(2x+8)^3$

(vii) $(1+x)^{-4}$

(viii) $(3-x)^{1/2}$

(ix) $(5x+6)^{-1/3}$

(x) $(3x-2)^{-3/2}$

INTEGRATION OF TR

$\frac{d}{dx}\sin x = \cos$

$\frac{d}{dx}\cos x = -\sin$

$\frac{d}{dx}\tan x = \sec$

$\frac{d}{dx}\cot x = -$

$\frac{d}{dx}\sec x = \sec$

$\frac{d}{dx}\operatorname{cosec} x = -$

$^{-4}$
$+x)$
$^{1/2}$
$3-x)$
$^{-1/3}$
$x+6)$
$^{-3/2}$
$,x-2)$

INTEGRATION OF TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}\sin x = \cos x \longleftrightarrow \int \sin x\,dx = -\cos x + c$

$\frac{d}{dx}\cos x = -\sin x \longleftrightarrow \int \cos x\,dx = \sin x + c$

$\frac{d}{dx}\tan x = \sec^2 x \longleftrightarrow \int \sec^2 x\,dx = \tan x + c$

$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x \longleftrightarrow \int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$\frac{d}{dx}\sec x = \sec x \tan x \longleftrightarrow \int \sec x \tan x = \sec x + c$

$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x \longleftrightarrow \int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + c$

$\frac{d}{dx}$

$x + c$

$x + c$

$\tan x + c$

$= -\cot x + c$

$x = \sec x + c$

$\cot x = -\operatorname{cosec} x + c$

$$\frac{d}{dx}\sin^2 x = 2\sin^{2-1} x \frac{d}{dx}\sin x = 2\sin x \cos x$$

$$\int \sin^2 x \, dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\int \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x \longrightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int \frac{1-\cos 2x}{2}\,dx = \int \frac{1}{2}\,dx - \int \frac{\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos 2x \,\frac{d2x}{2}$$

$$= \frac{x}{2} - \frac{1}{4}\int \cos 2x\,d2x$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

Ph $\int \cos^2 x\,dx = ?$

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = 1 + \cos 2x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$\int \frac{1+\cos 2x}{2}\,dx$$

$$\int \frac{1}{2}\,dx + \int \frac{\cos 2x\,dx}{2}$$

$$\frac{x}{2} + \int \frac{\cos 2x\,d2x}{2x2}$$

$$\frac{x}{2} + \frac{1}{4}\int \cos 2x \, dx$$

$$\frac{x}{2} + \frac{1}{4} \sin 2x + C$$

pb

$\int \tan^2 x \, dx = ?$

$\frac{dx}{2}$

$\frac{dx}{2x2}$

$\sec^2 x = \tan^2 x + 1$

$\operatorname{cosec}^2 x = \cot^2 x + 1$

$\sec^2 x = \tan^2 x + 1$

$\therefore \tan^2 x = \sec^2 x - 1$

$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$

$= \int \sec^2 x \, dx - \int dx$

$= \tan x - x + C$ ✗

$\sin 2x = 2\sin x \cos x$

Ph $\int \sin x \cos x \, dx = ?$

But $\sin 2x = 2\sin x \cos x$

$\therefore \sin x \cos x = \frac{\sin 2x}{2}$

$$\int \frac{\sin 2x}{2} dx = \int \frac{\sin 2x \, d2x}{2 \times 2}$$

$$= \frac{1}{4}\int \sin 2x \, d2x$$

$$= -\frac{\cos 2x}{4} + C$$

$$\frac{d}{dx}\sin x \cos x = \sin x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}\sin x$$

$$= \sin x(-\sin x) + \cos x \cdot \cos x$$

$$= -\sin^2 x + \cos^2 x$$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{1}{10}\left[\int \frac{\cos 10x \, d10x}{10} + x\right]$$

$$\frac{1}{10}\left[\frac{\sin 10x}{10} + x\right] + C$$

$$\frac{\sin 10x}{100} + \frac{x}{10} + C$$

Pb $\int \tan^2 3x \, dx = ?$

$\int \sec^2 x \, dx = \tan x + C$

$\sec^2 x = \tan^2 x + 1$

$\therefore \sec^2 3x = \tan^2 3x + 1$

$\tan^2 3x = \sec^2 3x - 1$

$\int (\sec^2 3x - 1) \, dx$

$\int \sec^2 3x \, dx - \int dx$

$\int \frac{\sec^2 3x \, d3x}{3} - x$

$\frac{\tan 3x}{3} - x + C$

Ph $\int \sin 6x \cos x \, dx$

$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$

$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$

$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$

$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$

$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$

Ph $\int \cos 3x \cos 5x \, dx = ?$

$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$

$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$

$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$

$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$

$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$

Ph $\int$ $\sin x$ $\frac{1}{2}$ $\frac{1}{2}$

ph $\int \sin x \sin 3x \, dx$

$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$

$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$

$= \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx$

$\frac{1}{2}\left[\int \cos 2x \, dx - \int \cos 4x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\cos 2x \, d2x}{2} - \int \frac{\cos 4x \, d4x}{4}\right]$

$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + c$ //

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Pb $\int \sin 6x \cos x \, dx$

$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$

$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$

$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$

$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$

$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$

Pb $\int \cos 3x \cos 5x \, dx = ?$

$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$

$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$

$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$

$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$

$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$

Pb

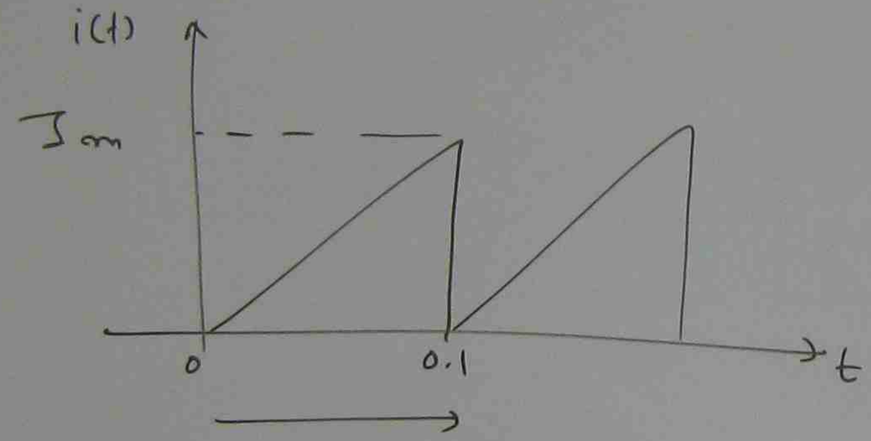
$\sin x$

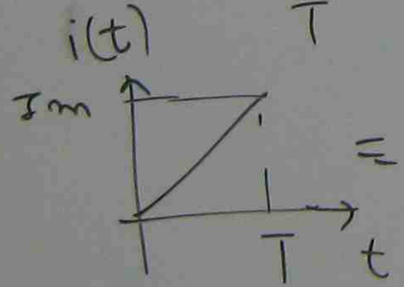
$\frac{1}{2}$

$\frac{1}{2}$

INTEGRATION OF ELECTRICAL WAVE FORMS

AVERAGE VALUE





$$= i(t) = \frac{I_m}{T} t$$

$$I_{AVE} = \frac{1}{T} \int_0^T i(t)\, dt$$

$$I_{AVE} = \frac{1}{T} \int_0^T \frac{I_m t}{T}\, dt$$

$$= \frac{I_m}{T^2} \int_0^T t\, dt$$

$$= \frac{I_m}{T^2} \left[\frac{t^2}{2}\right]_0^T$$

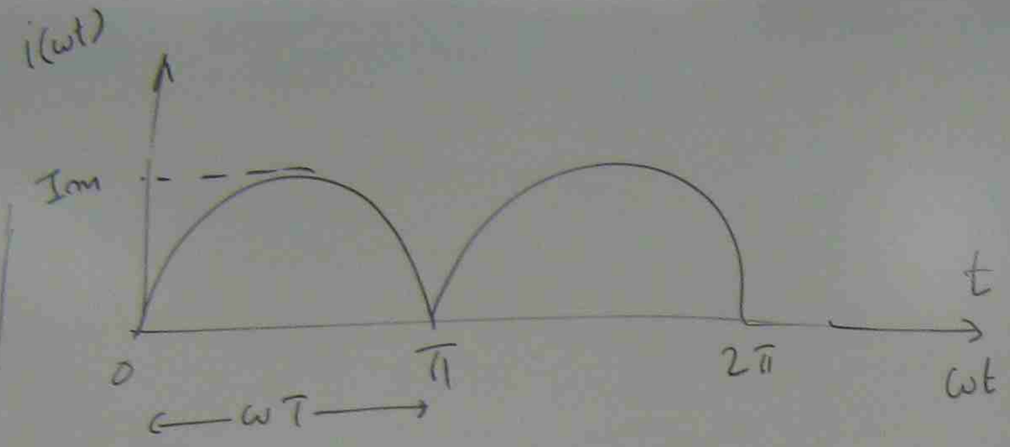
$$= \frac{I_m}{2T^2} \left[T^2 - 0\right]$$

$$= \frac{I_m}{2T^2} \times T^2$$

$$= \frac{I_m}{2}$$

i(ωt)

Im



$i(\omega t) = I_m \sin \omega t \quad 0 \leq \omega t \leq \pi$

$I_{AVE} = ?$

$$I_{AVE} = \frac{1}{T} \int_0^T i(\omega t)\, dt$$

$T = \pi$

$i(\omega t) = I_m \sin \omega t$

$$I_{AVE} = \frac{1}{\pi/\omega} \int_0^{\pi} I_m \sin \omega t \, dt$$

$$T = \frac{\pi}{\omega}$$

$$I_{AVE} = \frac{\omega}{\pi} \int_0^{\pi} I_m \sin \omega t \, dt$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi} \left[-\cos \omega t \right]_0^{\pi}$$

$$= \frac{I_m}{\pi} \left[\cos \pi - \cos 0 \right]$$

$$= \frac{I_m}{\pi} \times (-) \left[-1 - 1 \right]$$

$$= \frac{2 I_m}{\pi}$$

$$= 0.636\, I_m$$

$= I_m \sin\omega t \, d\omega t$

T

$= \int_0^{T/4} \sin\omega t \, d\omega t$

$= \left[-\cos\omega t \right]_0^{T/4}$

$= -\left[\cos\frac{T}{4} - \cos 0 \right]$

$= -(-)\left[-1 - 1 \right]$

$= \frac{2 I_m}{T/4}$

$\therefore I = 0.636 I_m$

RMS VALUE

$$I_{RMS} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 dt}$$

TRIANGULAR WAVE

$$i(t) = \frac{I_m}{T} t$$

$$I_{Rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2 t^2}{T^2} dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2} \int_0^T t^2 dt}$$

$$= \sqrt{\frac{I_m^2}{T^3} \left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3 - 0}{3}}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$= \sqrt{\frac{I_m^2}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

SINUSOIDAL WAVE

$i(t) = I_m \sin\omega t$

$T = \frac{\pi}{\omega}$

$I_{rms} = \sqrt{\frac{1}{T} \int dt}$

$= \sqrt{\frac{1}{\frac{\pi}{\omega}} \int dt}$

$= \sqrt{\frac{\omega}{\pi} \int dt}$

SINUSOIDAL WAVE

$$i(t) = I_m \sin \omega t$$

$$T = \frac{\pi}{\omega}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T (I_m \sin \omega t)^2 \, dt}$$

$$= \sqrt{\frac{1}{\frac{\pi}{\omega}} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{\omega}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \sin^2 \omega t \, d\omega t}$$

$$\cos 2\omega t = 1 - 2\sin^2 \omega t$$

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2} \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi}}$$

$$= \sqrt{\frac{I}{2}}$$

$$= \int$$

$$= \sqrt{\frac{I_m^2}{\pi \times 2} \int_0^{\pi} (1 - \cos 2\omega t)\, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\int_0^{\pi} d\omega t - \int_0^{\pi} \cos 2\omega t \, d\omega t \right]}$$

$\pi = 180^\circ$

$\sin 180^\circ = 0$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\omega t - \frac{1}{2} \sin \omega t \right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[(\pi - \tfrac{1}{2} \sin \pi) - (0 - \tfrac{1}{2} \sin 0) \right]}$$

($\sin \pi \to 0$, $\sin 0 \to 0$)

$$= \sqrt{\frac{I_m^2}{2\pi} \times \pi} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Rms value of Complex waveforms

Period $T = \frac{2\pi}{\omega}$

$$i(t) = I_0 + A_1 \cos\omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + \text{---} + B_1 \sin\omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t +$$

$$I_{Rms} = \sqrt{I_0^2 + \tfrac{1}{2}(A_1^2 + A_2^2 + A_3^2 + \text{---}) + \tfrac{1}{2}(B_1^2 + B_2^2 + B_3^2 + \text{--})}$$

$$= \sqrt{I_{DC}^2 + A_{1\,Rms}^2 + A_{2\,Rms}^2 + \text{--} + B_{1\,Rms}^2 + B_{2\,Rms}^2 + \text{--}}$$

$$= \sqrt{DC^2 + (\text{sum of Rms})^2}$$

sin 3ωt + - - -

Ph $i(t) = 10 + 15 \sin 100t + 5 \sin 200t$ Amp, FIND I_{Rms} =?

$$I_{Rms} = \sqrt{10^2 + \frac{1}{2}(15^2 + 5^2)} = 15 \text{ Amp}$$

FORM FACTOR

$$\text{FORM FACTOR} = \frac{\text{RMS VALUE}}{\text{AVERAGE VALUE}} = \frac{I_{Rms}}{I_{AVE}}$$

WAVE SHAPE	FORM FACTOR
RECTIFIED SQUARE WAVE	1
RECTIFIED SINE WAVE	1.11
TRIANGULAR WAVE	1.15

Ph

A VOLTAGE WAVE FORM IS REPRESENTED BY THE EQUATION

$V(t) = 10 - 10 \sin \omega t$ VOLT

DC + AC COMPONENT

COMPONENT

DETERMINE THE FOLLOWINGS

(a) AVERAGE VALUE

(b) RMS VALUE

(c) FORM FACTOR

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

(OR)

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$\omega = 2\pi f$

$\theta = \omega t$

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$\omega T = 2\pi$$

$$= \frac{1}{T}\int_0^T (10 - 10\sin\omega t)\,dt$$

$$= \frac{1}{T}\left[\int_0^T 10\,dt\right] - \int_0^T 10\sin\omega t\,dt$$

$$= \frac{1}{T}\left[10(t)_0^T - \int_0^T \frac{10\sin\omega t\,d\omega t}{\omega}\right]$$

$$= \frac{1}{T}\left[10T - \frac{10}{\omega}\int_0^T \sin\omega t\,d\omega t\right]$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}\left[\cos\omega t\right]_0^T\right]$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos\omega T - \cos 0)\right]$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos 2\pi - 1)\right]$$

$$\frac{1}{T}\left(10T + \frac{10}{\omega}(1-1)\right)$$

$$= 10\ V$$

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} V(\theta)\,d\theta$$

$$\frac{1}{2\pi}\int_0^{2\pi}(10 - 10\sin\theta)\,d\theta$$

$$\frac{1}{2\pi}\left[\int_0^{2\pi} 10\,d\theta - \int_0^{2\pi} 10\sin\theta\,d\theta\right]$$

$$\frac{1}{2\pi}\left[10(\theta)_0^{2\pi} + 10(\cos\theta)_0^{2\pi}\right]$$

$$\frac{1}{2\pi}\left[20\pi + 10(\cos 2\pi - \cos 0)\right]$$

$$= 10\ V$$

$$V_{Rms} =$$

$$V_{Rms} =$$

$$=$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2 dt}$$

(OR)

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [v(\theta)]^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [10 - 10\sin\theta]^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 10^2 (1-\sin\theta)^2 d\theta}$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1-\sin(\theta))^2 d\theta}$$

$$(1-x)^2 = 1 - 2x + x^2$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta) d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2(\cos\theta)_0^{2\pi} + \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2[\cos 2\pi - \cos 0] + \int_0^{2\pi} \frac{1}{2} d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2} d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[3\pi - \frac{\sin 4\pi}{4}\right.} \quad \sin 4\pi \to 0$$

$$= \sqrt{150} = 12.25 \text{ V}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\frac{\cos 2\theta\, d2}{4}$$

$$= \sqrt{\frac{50}{\pi} \int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{50}{\pi} \left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta \right.}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2\left(\cos\theta\right)_0^{2\pi} + \int_0^{2\pi} \left(1 - \frac{\cos 2\theta}{2}\right) d\theta \right.}$$

$\frac{\cos 2\theta\, d2\theta}{4}$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2\left[\cos 2\pi - \cos 0\right] + \int_0^{2\pi} \frac{1}{2} d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2} d\theta \right.}$$

$\cos 2\pi - \cos 0 \to 0$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi} \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[3\pi - \frac{\sin 4\pi}{4} \right.} \quad \frac{\sin 4\pi}{4} \to 0$$

$$= \sqrt{150} = 12.25 \text{ V}$$

$$= \sqrt{\frac{50}{\pi} \int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{50}{\pi} \left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2\left(\cos\theta\right)_0^{2\pi} + \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2\left[\cos 2\pi - \cos 0\right] + \int_0^{2\pi} \frac{1}{2}\, d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2}\, d\theta \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi} \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[3\pi - \frac{\sin 4\pi}{4} \right.} \qquad = \sqrt{150} = 12.25 \text{ V}$$

$\cos 0 \to 0$

$\sin 4\pi \to 0$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\frac{\cos 2\theta\, d2\theta}{4}$

(c) FORM FACTOR $= \frac{V_{rrms}}{V_{AVE}} = \frac{12.25}{10}$

$= 1.225$

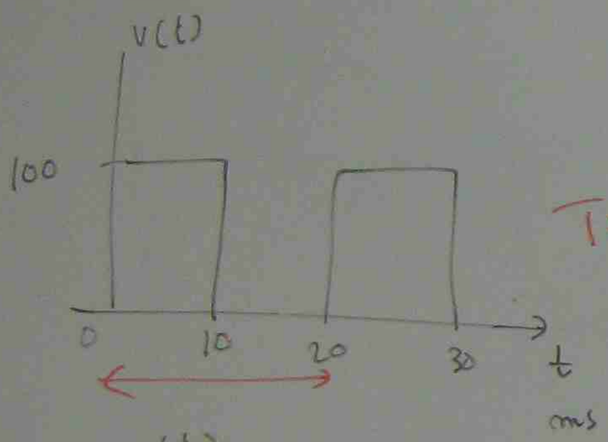
pb DETERMINE THE AVERAGE VALUE, RMS VALUE AND FORM FACTOR FOR EACH WAVE FORM SHOWN IN FIGURE

USE INTEGRAL CALCULUS

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt \qquad (a)$$

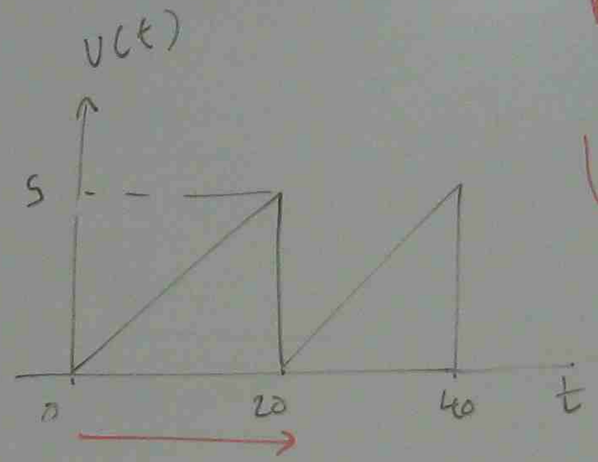
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

(a)



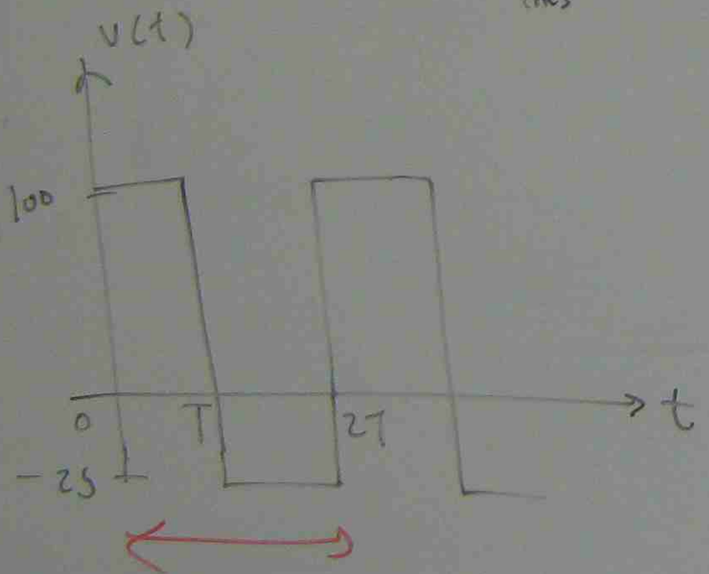
T = 20

(b)



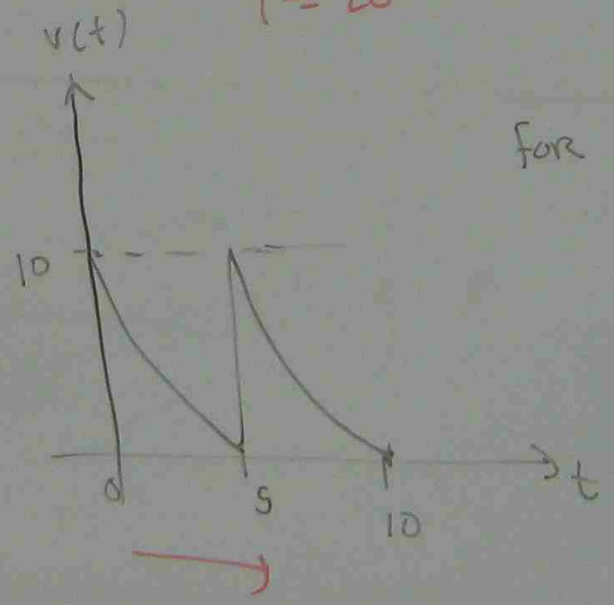
T = 20

(c)



T = 2T

(d)



T = 5

for $0 \le t \le 5ms$

$v(t) = 10e^{-1000t}$

V_{Rm}

(a) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{20}\left[\int_0^{10} 100\,dt + \int_{10}^{20} 0\,dt\right]$

$= \frac{1}{20}\left[100(t)_0^{10}\right]$

$= \frac{1}{20} \times 100 \times 10$

$= \frac{1000}{20} = 50\ V$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\left[\int_0^{10} 100^2\,dt + t\int_{10}^{20} 0^2\,dt\right]}$

$= \sqrt{\frac{1}{20} \times 10000(t)_0^{10}}$

$= \sqrt{\frac{1}{20} \times 100,00 \times 10}$

$= \sqrt{5000}$

$= 70.7\ V$

FORM FACTOR $= \frac{U_{Rms}}{U_{AVG}}$

$= \frac{70.7}{50} = 1.4142$

$V_{AVE} = \frac{1}{20}\int_0^{20} 0.25t\,dt$

$= \frac{1}{20} \times 0.25\left[\frac{t^2}{2}\right]_0^{20}$

$= \frac{1}{20} \times 0.25 \times \frac{20^2}{2}$

$= \frac{0.25 \times 20}{2} = 2.5\ V$

(b) $v(t) = \frac{V_m}{T} \times t$

$= \frac{5}{20}t = 0.25t$

$U_{AUG} = \frac{1}{T}\int_0^T v(t)\,dt$

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\int_0^{20} (0.25t)^2\,dt}$

$= \sqrt{\frac{1}{20} \times 0.0625 \times \left[\frac{t^3}{3}\right]_0^{20}}$

$$= \sqrt{\frac{1}{20} \times 0.0625 \times \frac{20^3}{3}}$$

$$= \sqrt{\frac{20^2 \times 0.0625}{3}}$$

$$= 0.25 \times 20 \times \frac{1}{\sqrt{3}}$$

$$= \frac{5}{1.7321} = 2.887 \text{ V}$$

$$\text{FORM FACTOR} = \frac{V_{RMS}}{V_{AVG}} = \frac{2.887}{2.5} = 1.155$$

(c) $$V_{AVE} = \frac{1}{T} \int_0^T v(t)\,dt$$

$$= \frac{1}{2T}\left[\int_0^T 100\,dt + \int_T^{2T} (-25)\,dt\right.$$

$$\frac{1}{2T}\left[\left[100\,(t)_0^T\right] - 25\,(t)_T^{2T}\right]$$

$$\frac{1}{2T}\left[100T - 25\left[2T - T\right]\right]$$

$$\frac{1}{2T}\left[100T - 25T\right]$$

$$\frac{75T}{2T} = 37.5 \text{ V}$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$= \sqrt{\frac{1}{2T}\left[\int_0^T 100^2\,dt + \int_T^{2T} (-25)^2\,dt\right.}$$

$$= \sqrt{\frac{1}{2T}\left[100{,}00T + 625\,(2T - T)\right]}$$

$$= \sqrt{\frac{1}{2T}\left[10000T + 6\right.}$$

$$= \sqrt{5312}$$

$$= 72.89 \text{ V}$$

$$\text{FORM FACTOR} = \frac{72.89}{37.5}$$

(d) $$V_{AVE} = \frac{1}{T}$$

$$= \frac{1}{5/1000}$$

$$= 200$$

$10000T + 625T$

12

.89 V

$= \frac{72.89}{37.5} = 1.94$

$= \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{5/1000}\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,dt$

$= 200\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,\frac{d(-1000t)}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$= -2\times\left[e^{-1000\times 5\times10^{-3}} - e^{0}\right]$

$= -2\times\left[e^{-5} - 1\right]$

$= -2\times\left[\frac{1}{148} - 1\right]$

$= -2\times -0.993$

$= 1.98\,V.$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$V_{Rms} = \sqrt{\frac{1}{\frac{5}{1000}}\int_0^{5\times10^{-3}}\left(10e^{-1000t}\right)^2 dt}$

$= \sqrt{\frac{1000}{5}\times10\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{2000\times\int_0^{5\times10^{-3}} \frac{e^{-2000t}\,d(-2000t)}{-2000}}$

$= \sqrt{-\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$= \sqrt{-\left(e^{-2000t}\right)_0^{5\times10^{-3}}}$

$\sqrt{\left[e^{-5} - 1\right]}$

$\sqrt{1 - e^{-5}}$

$0.995\,V$

$= \sqrt{\frac{1}{2T}\left[10000T + 625T\right]}$

$= \sqrt{5312.}$

$= 72.89\,V$

Form Factor $= \frac{72.89}{37.5} = 1.94$

(d) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{5/1000}\int_0^{5\times10^{-3}} 10e^{-1000t}\,dt$

$= 200\int_0^{5\times10^{-3}} \frac{10e^{-1000t}\,d(-1000)}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$= -2 \times \left[e^{-1000\times5\times10^{-3}} - e^{0}\right]$

$= -2 \times \left[e^{-5} - 1\right]$

$= -2 \times \left[\frac{1}{148} - 1\right]$

$= -2 \times -0.993$

$= 1.98\,V.$

$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$V_{RMS} = \sqrt{\frac{1}{\frac{5}{1000}}\int_0^{5\times10^{-3}}\left(10e^{-1000t}\right)^2\,dt}$

$= \sqrt{\frac{1000}{5}\times100\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{20000\times\int_0^{5\times10^{-3}} \frac{e^{-2000t}\,d(-2000t)}{-2000}}$

$= \sqrt{-10\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$= \sqrt{-10\left(e^{-2000t}\right)_0^{5\times10^{-3}}}$

$= \sqrt{-10\left[e^{-5} - 1\right]} \times 3.16$

$= \sqrt{1 - e^{-5}} \times 3.16$

$= 3.1$

$FF = \frac{3.1}{1.98} = 1.59$

Ph IF A 100 Ω RESISTOR DISSIPATES AN AVERAGE POWER OF 1000 WATT, DETERMINE

(a) RMS VALUE OF CURRENT

(b) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS SINUSOIDAL

(c) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS TRIANGULAR.

(a) $\text{POWER} = I^2 R$]rms

$I_{rms}^2 R = \text{POWER}$

$I_{rms}^2 \times 100 = 1000$

$I_{rms}^2 = 10$

$I_{rms} = \sqrt{10} = 3.16$ Amp.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2 dt}$$

SINUSOIDAL $\Rightarrow$ $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2 dt}$

(OR)

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I(\theta)^2 d\theta}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_{max}\sin\theta)^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi} \sin^2\theta \, d\theta}$$

$$\sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$I_m \sqrt{\frac{1}{2\pi}\left\{\left[\frac{1}{2}\theta\right]_0^{2\pi} - \left[\int_0^{2\pi} \frac{\cos 2\theta \, d2\theta}{4}\right]\right.}$$

$$\sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$I_m \sqrt{\frac{1}{2\pi}\left[\pi - (\sin 4\pi - \sin 0\right]}$$

4π → 2θ

$$I_m \times \frac{1}{\sqrt{2}} = 3.16$$

$$I_m = \sqrt{2} \times 3.16 = 4.46 \text{ Amp.}$$

$= 2\sin^2\theta$

$\theta = \frac{1-\cos 2\theta}{2}$

TRIANGULAR

$$i(t) = \frac{I_m t}{T}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2}\int_0^T t^2 dt}$$

$$= \sqrt{\frac{I_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

$$3.16 = \frac{I_m}{1.7321}$$

$$I_m = 3.16 \times 1.7321$$

$$= 5.47 \text{ Amp.}$$

$$= \sqrt{\frac{1}{2T}\left[10000\,T\right.}$$

$$= \sqrt{5312}$$

$$= 72.89\,V$$

Form Factor $= \frac{72.}{57}$

(d) $V_{AVE} = \frac{1}{}$

$= \frac{1}{5/1000}$

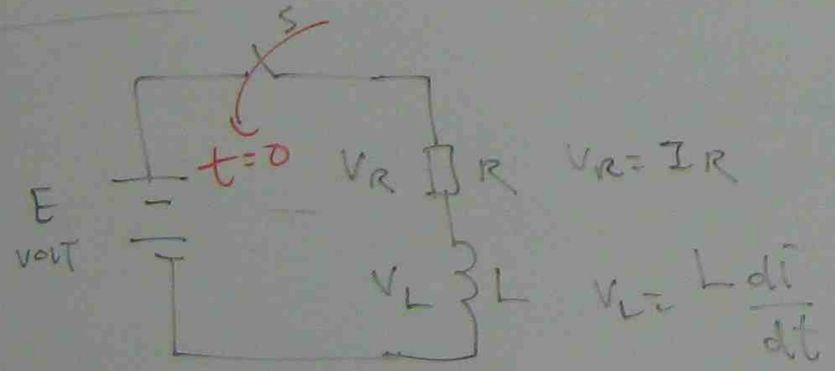
$= 200$

DIFFERENTIAL EQUATION & APPLICATION OF DIFFERENTIAL EQUATION IN ELECTRICAL CIRCUIT CALCULATION

$$\frac{dy}{dt} + 3y = 4 \quad \text{(FIRST ORDER)}$$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4 = 0 \quad \text{(SECOND ORDER)}$$

RL CIRCUIT



$V_R = IR$

$V_L = L\frac{di}{dt}$

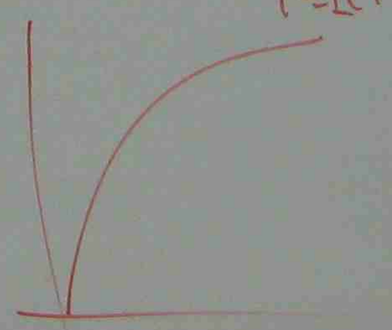
SWITCHING ON AT $t=0$

$$E = V_R + V_L$$

$$E = IR + L\frac{di}{dt}$$

$$i = I\left(1 - e^{-\frac{R}{L}t}\right)$$

VALUE OF CURRENT AT $t=0$



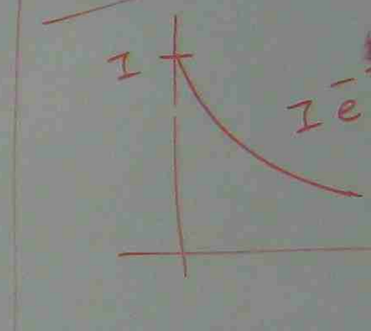
$$i = I\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$= I - Ie^{-\frac{Rt}{L}}$$

I — CONSTANT

$Ie^{-\frac{Rt}{L}}$ — VARIABLE WITH e FUNCTION

SWITCHING OFF



GENERALIZE

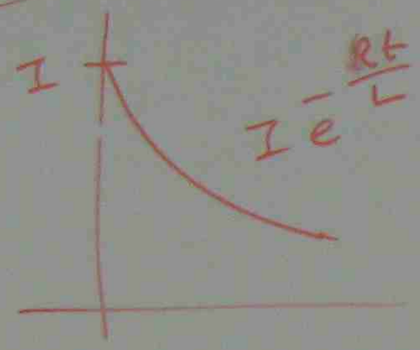
FOR ANY

THE EQU

L

i(t

SWITCHING OFF



GENERALIZED FORMAT

FOR ANSWER OF

THE EQUATION

$$L\frac{di}{dt} + IR = E$$

$$i(t) = I_p + Ae^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R}$$

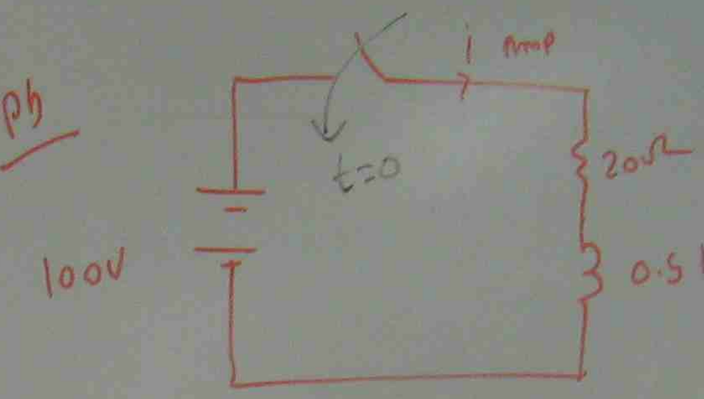
$(1-e^{-\frac{Rt}{L}})$

$I - Ie^{-\frac{Rt}{L}}$

I — VARIABLE WITH e FUNCTION

RESISTANT

Ph



FOR THE CIRCUIT SHOWN ABOVE, DETERMINE THE FOLLOWING VALUES AFTER THE SWITCH HAS BEEN CLOSED.

(a) THE FINAL VALUE OF CURRENT

(b) THE INITIAL VALUE OF CURRENT

(c) TIME CONSTANT OF THE CIRCUIT

(d) THE EQUATION OF THE CURRENT

(e) THE INITIAL RATE OF CHANGE OF CURRENT.

(a) SWITCH CLOSED, INDUCTOR IS SHORT CIRCUITED

$$I_{FINAL} = \frac{V}{R} = \frac{100}{20} = 5A$$

(b) $I_{INITIAL} = 0$

(c) TIME CONSTANT $(\tau) = \frac{L}{R} = \frac{0.5}{20} = 0.025 \text{ sec} = 25 \text{ ms}$

(d) $E = IR + L\frac{dI}{dt}$

$100 = 20I + 0.5\frac{dI}{dt}$ CIRCUIT EQUATION

$i(t) = \overset{\uparrow}{I}\left(1 - e^{-t/\tau}\right)$

FINAL CURRENT

$= 5\left(1 - e^{-\frac{t}{0.025}}\right)$

$i(t) = 5\left(1 - e^{-40t}\right)$

(e) $\frac{di(t)}{dt} = \frac{d}{dt} 5\left(1 - e^{-40t}\right)$

$= \frac{d}{dt}5 - \frac{d}{dt} 5e^{-40t}$ ($\frac{d}{dt}5 = 0$)

$= -5 \times (-40)\, e^{-40t}$

$= 200\, e^{-40t}$ A/s

RATE OF CHANGE OF CURRENT

ph

+

200V

DE TO

CUR R

SWIT

ASSU

HAD

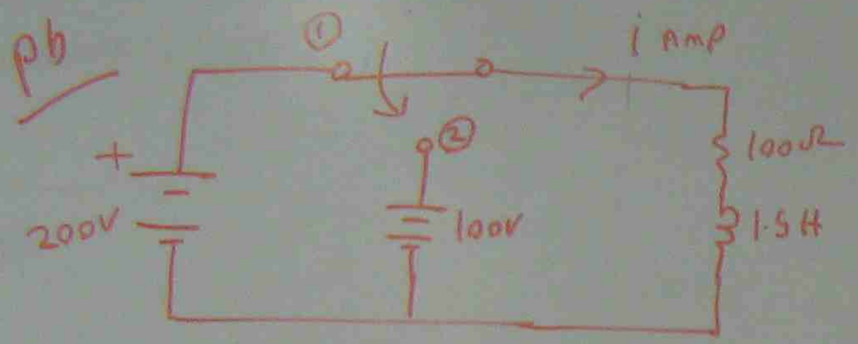
INITI

(OR

WHEN SWITCH

(2)

pb



DETERMINE THE EQUATION OF THE CURRENT IN ABOVE FIGURE AFTER SWITCHING TO POSITION ②.

ASSUME THAT STEADY STATE CURRENT HAD BEEN ATTAINED IN POSITION ①

INITIAL CURRENT $I(0) = \frac{200V}{100\Omega} = 2$ Amp.

(OR) $I(t)\Big|_{t=0} = 2$ Amp (NOT STEADY STATE)

$E = IR + L\frac{dI}{dt}$

$i(t) = I_p + A\,e^{-t/\tau}$

FINAL

$\tau = \frac{L}{R} = \frac{1.5}{100} = 0.015$

$I_p = 1$ Amp ← FINAL

$i(t) = 1 + A\,e^{\frac{-t}{0.015}}$

WHEN TIME ($t=0$), THE INITIAL CURRENT IS 2 AMP.

$i(t)\Big|_{t=0} = 2$

$2 = 1 + A\,e^{-\frac{0}{0.015}}$

$2 = 1 + A\,e^{0}$

$2 = 1 + A$

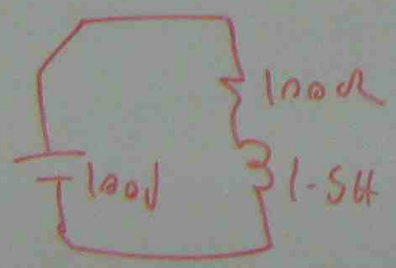
$\therefore A = 2 - 1 = 1$

$\therefore i(t) = 1 + 1\,e^{-\frac{t}{0.15}}$

$= 1 + e^{-t/0.15}$

$= 1 + e^{-6}$

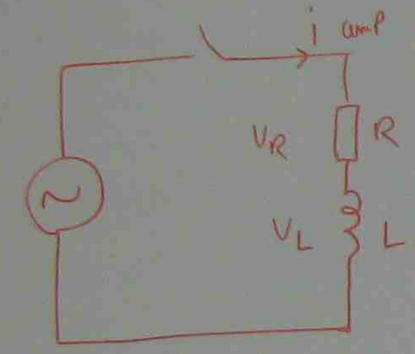
N SWITCH AT ②



FINAL CURRENT $= \frac{100}{100} = 1$ Amp.

↑ STEADY STATE CONSTANT VALUE

RESPONSE of RL AND RC CIRCUITS TO AC VOLTAGE



$e = E_m \sin(\omega t + \theta)$

$V_R = iR$

$V_L = L\frac{di}{dt}$

$V_R + V_L = e$

$iR + L\frac{di}{dt} = E_m \sin(\omega t + \theta)$

$i = i_c + i_p$

i_c ↑ COMPLEMENTARY COMPONENT

i_p ↑ PARTICULAR COMPONENT

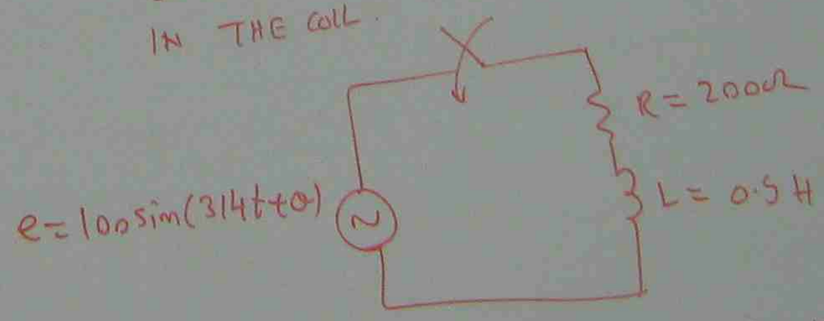
GENERAL SOLUTION

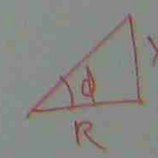
$$i = A e^{-\frac{t}{\tau}} + I_p \sin(\omega t + \alpha - \phi)$$

$i =$

$I =$

pb AN E.M.F $e = 100 \sin(314t + \theta)$ volt IS APPLIED TO A COIL OF RESISTANCE 200 Ω AND INDUCTANCE 0.5 HENRY WHEN θ is 30°. DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL.





$= 314 \times 0.$

$= 157$

$Z = R + jX_L = 200 + j157$

$\phi = \tan^{-1}\frac{157}{200} = 38.$

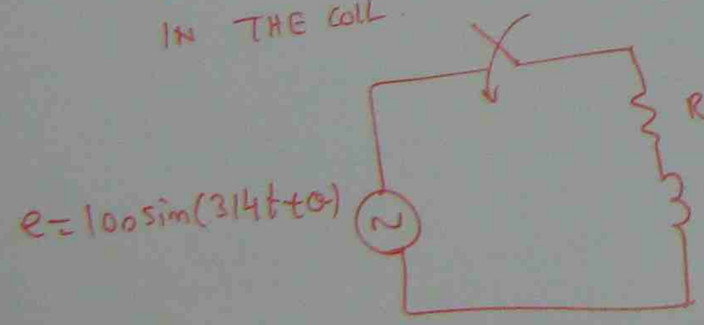
θ =

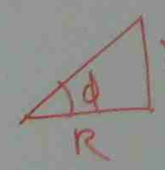
C VOLTAGE

GENERAL SOLUTION

$$i = A e^{-\frac{t}{\tau}} + I_p \sin(\omega t + \alpha - \phi)$$

AN E.M.F $e = 100 \sin(314t + \alpha)$ VOLT IS APPLIED TO A COIL OF RESISTANCE 200 Ω AND INDUCTANCE 0.5 HENRY WHEN α IS 30°. DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL.





$= 314 \times 0.5$

$= 157$

$Z = R + jX_L = 200 + j157$

$\phi = \tan^{-1} \frac{157}{200} = 38.13°$

$$i = A e^{-t/\tau} + I_p \sin(\omega t + \alpha - \phi)$$

$$\tau = \frac{L}{R} = \frac{0.5}{200} = 0.0025 = 2.5 \text{ ms}$$

$\theta - \phi = 30 - 38.13 = -8.13$

$$i = A e^{-\frac{t}{2.5 \times 10^{-3}}} + I_p \sin(314t - 8.13)$$

$$I_p = \frac{E}{Z} = \frac{100}{\sqrt{R^2 + X_L^2}} = \frac{100}{\sqrt{200^2 + 157^2}} = 0.392$$

$$i = A e^{-400t} + 0.392 \sin(314t - 8.13)$$

$t = 0 \rightarrow i = 0$

IN THIS BUILDING

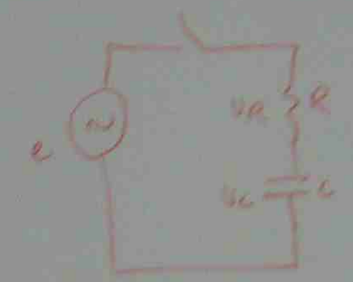
$$-400\times0$$
$$0 = A\,e \quad + 0.392\sin(314\times0 - 2.13)$$
$$0 = A\,e^{0} + 0.392\sin(-8.13)$$
$$0 = A - 0.392\sin 8.13$$
$$A = 0.0556$$
$$i = 0.0556\,e^{-400t} + 0.392\sin(314t - 8.13)$$

$$\frac{}{} = \frac{100}{\sqrt{200^2+157^2}} = 0.3932$$

$$2\sin(314t - 8.13)$$

$$(314t - 8.13)$$

RC CIRCUIT



$e = E_m \sin(\omega t + \alpha)$

$V_R = iR$

$V_C = \frac{1}{c}\int i\,dt$

$V_R + V_C = e$

$$iR + \frac{1}{c}\int i\,dt = e = E_m\sin(\omega t+\alpha)$$

$$\frac{d}{dt}(iR) + \frac{d}{dt}\left[\frac{1}{c}\int i\,dt\right] = \frac{d}{dt}\left[E_m\sin(\omega t+\alpha)\right]$$

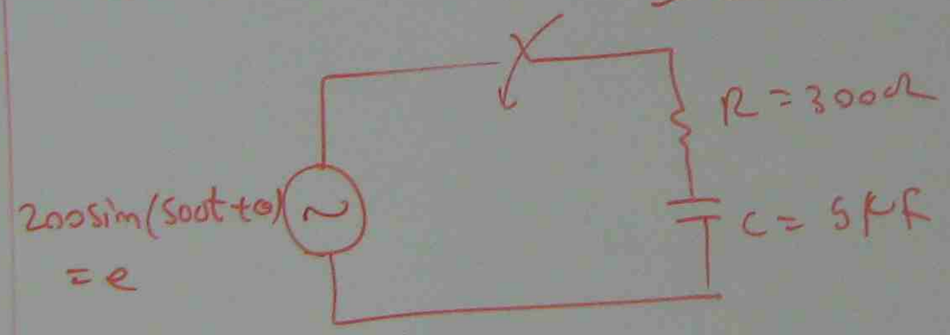
$$R\frac{di}{dt} + \frac{1}{c}\frac{d}{dt}\int i\,dt = E_m\frac{d}{dt}\sin(\omega t+\alpha)$$

$$R\frac{di}{dt} + \frac{1}{c}\times i = E_m\omega\cos(\omega t+\alpha)$$

THE GENERAL SOLUTION

$$I = Ae^{-t/\tau} + I_p \sin(\omega t + \theta - \phi)$$

pb AN E.M.F OF $e = 200 \sin(500t + \theta)$ VOLT IS APPLIED TO AN RC CIRCUIT WHERE $R = 300\ \Omega$ AND $C = 5\ \mu F$ WHEN θ IS 60°. DETERMINE THE EQUATION OF THE CIRCUIT IF THE INITIAL CHARGE ON THE CAPACITOR IS 250 MICRO COULOMBS.



$$i = Ae^{-t/\tau} + I_p \sin(\omega t + \theta - \phi)$$

$$\tau = R.C = 300 \times 5 \times 10^{-6} = 1.5 \times 10^{-3}$$

$$\theta = 60$$

$$Z = R - jX_c = 300 - j\frac{1}{2\pi fc}$$

$$= 300 - j\frac{1}{500 \times 5 \times}$$

$$= 300 - j400\ \Omega$$

$$= \sqrt{300^2 + 400^2}\ \angle$$

$$Z = 500\ \angle -53.1$$

$$\phi = -53.1$$

$$I_p = \frac{E}{Z} = \frac{200}{500} =$$

$$i = Ae^{-\frac{t}{1.5 \times 10^{-3}}} + 0.4 \sin$$

$$i = Ae^{-666.7t} + 0.4 \sin$$

$$t = 0 \rightarrow i(0) = Ae^{-666.}$$

$$i(0) = A + 0.4$$

$$E_m \sin(\omega t + \alpha)$$

$$\frac{d}{dt}\left[E_m \sin(\omega t + \alpha)\right]$$

$$= E_m \frac{d}{dt} \sin(\omega t + \alpha)$$

$$\omega \cos(\omega t + \alpha)$$

$$Z = R - jX_C = 300 - j\frac{1}{2\pi fc}$$

$$= 300 - j\frac{1}{500 \times 5\times10^{-6}}$$

$$= 300 - j400 \ \Omega$$

$$= \sqrt{300^2 + 400^2} \angle -\tan^{-1}\frac{400}{300}$$

$$Z = 500 \angle -53.1 \ \Omega$$

$$\phi = -53.1$$

$$I_p = \frac{E}{Z} = \frac{200}{500} = 0.4$$

$$i = A e^{-\frac{t}{1.5\times10^{-3}}} + 0.4 \sin(500t + 60 - (-53.1))$$

$$i = A e^{-666.7t} + 0.4 \sin(500t + 113.1) \ \text{Amp}$$

$$t = 0 \rightarrow i(0) = A e^{-666.7\times0} + 0.4 \sin(500\times0 + 113.1)$$

$$i(0) = A + 0.4 \sin 113.1 \quad \text{——} \ (1)$$

$$q = \int i\,dt = 250\ \mu C$$

INITIAL

$$iR + \frac{1}{C}\int i\,dt = e$$

$$iR + \frac{1}{C} \times q = e$$

$$i \times 300 + \frac{250\times10^{-6}}{5\times10^{-6}} = 200 \sin 60$$

$$i \times 300 + 50 = 200 \sin 60$$

$$i(0) = \frac{200\sin60 - 50}{300} = 0.4107$$

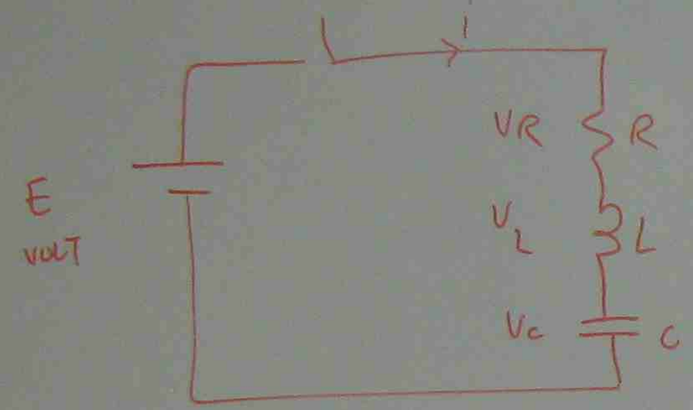
(1)

$$0.4107 = A + 0.4 \sin 113.1$$

$$A = 0.0433$$

$$i(t) = 0.0433 e^{-666.7t} + 0.4(500t + 113.1)$$

SOLVING THE SECOND ORDER DIFFERENTIAL EQUATION



$$V_R + V_L + V_c = E$$

$$iR + L\frac{di}{dt} + \frac{1}{c}\int i\,dt = E$$

$$\frac{d}{dt} iR + \frac{d}{dt} L\frac{di}{dt} + \frac{d}{dt}\frac{1}{c}\int i\,dt = \frac{d}{dt} E$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt} + \frac{1}{c} x\, i = 0$$

$$\frac{R}{L}\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{1}{Lc} i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc} i = 0$$

COMPARE WITH

$$a\frac{d^2i}{dt^2} + b\frac{di}{dt} + ci = 0$$

$$am^2 + bm + c = 0$$

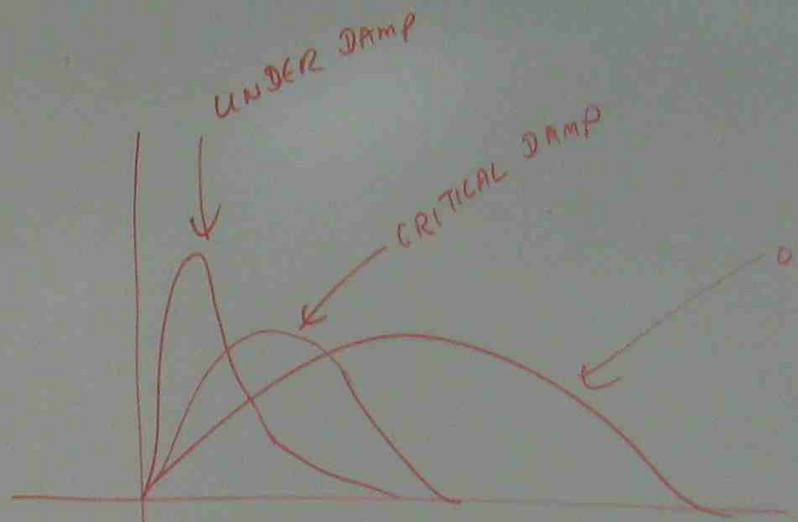
$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

LET

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$



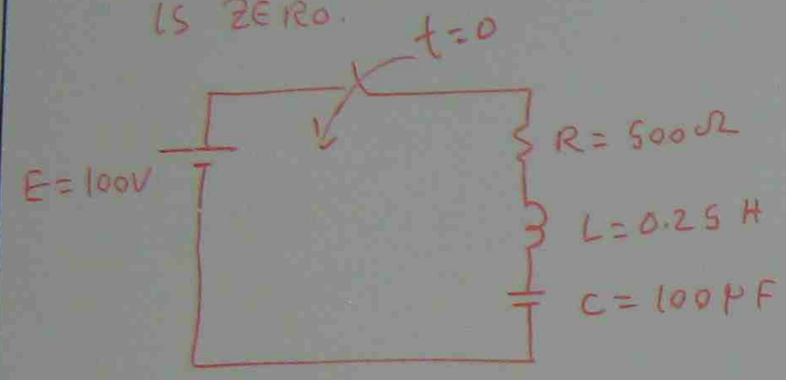
IF $\alpha > \omega_0 \Longrightarrow$ OVER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$

IF $\alpha = \omega_0 \Longrightarrow$ CRITICAL DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 + A_2 t\right)$

IF $\alpha < \omega_0 \Longrightarrow$ UNDER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 \sin \beta t + A_2 \cos \beta t\right)$

ph IN A GIVEN CIRCUIT, $E = 100V$ $R = 500\Omega$, $L = 0.25\ H$, $C = 100\mu F$.

DETERMINE THE EQUATION OF THE CURRENT IF THE INITIAL CHARGE ON CAPACITOR IS ZERO.



$$\alpha = \frac{R}{2L} = \frac{500}{2\times 0.25} = 1000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25\times 100\times 10^{-6}}} = 200$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha > \omega_0 \Rightarrow \text{OVER DAMP}$$

$$= \sqrt{1000^2 - 200^2}$$

$$= 979.8$$

$$i = e^{-\alpha t}\left[A_1 e^{\beta t} + A_2 e^{-\beta t}\right]$$

$$= A_1 e^{(-\alpha+\beta)t} + A_2 e^{(-\alpha-\beta)t}$$

$$= A_1 e^{(-1000+979.8)t} + A_2 e^{(-1000-979.8)t}$$

$$= A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

(1) $t = 0 \rightarrow i = 0$

$$0 = A_1 e^{-20.2\times 0} + A_2 e^{-1979.8\times 0}$$

$$A_1 + A_2 = 0 \quad \text{—(1)} \Rightarrow A_1 = -A_2$$

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

$$\frac{di}{dt} = A_1 \frac{d}{dt} e^{-20.2t} + A_2 \frac{d}{dt} e^{-1979.8t}$$

$$\frac{di}{dt} = -20.2 A_1 e^{-20.2t} + (-1979.8) A_2 e^{-1979.8t}$$

II

$L \frac{di}{dt}$ = INDUCTOR VOLTAGE

$t=0 \rightarrow L\frac{di}{dt} = E$

$\frac{di}{dt} = \frac{E}{L} = \frac{100}{0.25} = 400$

$\Rightarrow A_1 = -A_2$

$400 = -20.2 A_1 e^{0} - 1979.8 A_2 e^{0}$

$-20.2 A_1 - 1979.8 A_2 = 400$

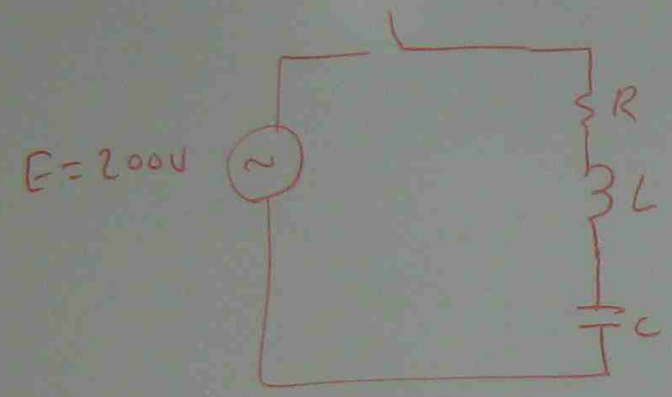
$-20.2(-A_2) - 1979.8 A_2 = 400$

$A_2 = -0.2041$

$\therefore A_1 = 0.02041$

$i = 0.02041 e^{-20.2t} - 0.204 e^{-1979.8t}$

EXERCISE



$E = 200V$, $R = 100\Omega$ $L = 0.5$ HENRY

$C = 200\mu f$

FIND THE EQUATION OF THE CURRENT

IF THE INITIAL CHARGE ON CAPACITOR IS

20 MILLICOULOMBS

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

COMPARE α & ω_0 SELECT THE EQUATION

CONDITION (1) $t = 0 \longrightarrow i = 0$

CONDITION (2)

$$q = 20\ mc$$

$$V_c = \frac{q}{C} = \frac{20 \times 10^{-3}}{200 \times 10^{-6}}$$

$$= 100V$$

$$V_c = \frac{1}{C}\int i\,dt$$

$$t = 0 \quad \frac{di}{dt} = \frac{E}{L} = \frac{100V}{0.5}$$

$$= 200$$

IF

IF

IF

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

LET

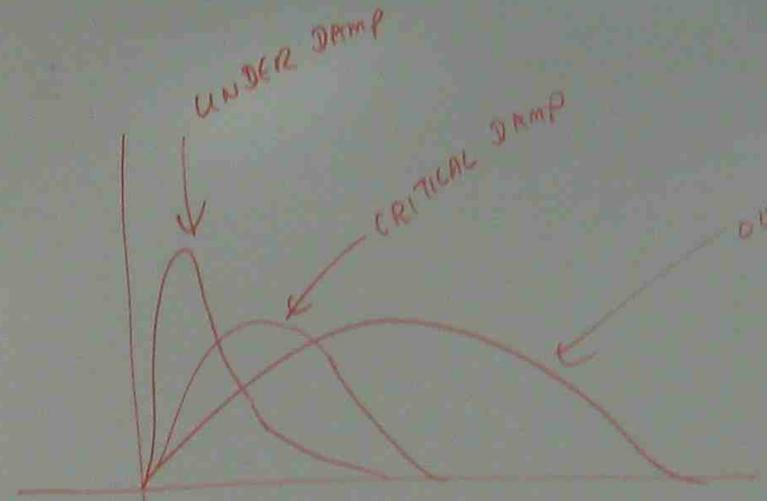
$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$

IF $\alpha > \omega_0 \Rightarrow$ OVER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$

IF $\alpha = \omega_0 \Rightarrow$ CRITICAL DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 + A_2 t\right)$

IF $\alpha < \omega_0 \Rightarrow$ UNDER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 \sin \beta t + A_2 \cos \beta t\right)$



REVISION

① $\int_{\pi}^{2\pi} (3\sin x + 10)\,dx$

$\int_{\pi}^{2\pi} 3\sin x\,dx + \int_{\pi}^{2\pi} 10\,dx$

$3\int_{\pi}^{2\pi} \sin x\,dx + 10\int_{\pi}^{2\pi} dx$

$3\times\left[-\cos x\right]_{\pi}^{2\pi} + 10\left[x\right]_{\pi}^{2\pi}$

$-3\left[\cos 2\pi - \cos\pi\right] + 10\left[2\pi - \pi\right]$

$-3\left[\cos 360 - \cos 180\right] + 10\times\pi$

$-3\left[1-(-1)\right] + 10\times 3.1416$

$-3\times 2 + 31.416$

25.416

② $\int (5x+7)^3 dx$

$d(5x+7) = d5x + d7$

$= 5dx$

$\therefore dx = \frac{d(5x+7)}{5}$

$\int (5x+7)^3 \frac{d(5x+7)}{5}$

$\frac{1}{5}\int (5x+7)^3 d(5x+7)$

$\frac{1}{5} \times \frac{(5x+7)^{3+1}}{3+1} + c$

$\frac{1}{5} \frac{(5x+7)^4}{4} + c = \frac{1}{20}(5x+7)^4 + c$

③ $\int \sin^2 2x \, dx$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 4x = 1 - 2\sin^2 2x$

$2\sin^2 2x = 1 - \cos 4x$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$\int \frac{1-\cos 4x}{2} dx$

$\frac{1}{2}\int [1 - \cos 4x] dx$

$\frac{1}{2}\left[\int dx - \int \cos 4x \, dx\right]$

$d4x = 4dx$

$\therefore dx = \frac{d4x}{4}$

$\frac{1}{2}\left[x - \int \cos 4x \times \frac{d4x}{4}\right]$

$\frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + c$

$\frac{x}{2} - \frac{\sin 4x}{8} + c$ ✱

④ $\int \sin 3x \cos 4x \, dx$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\sin 3x \cos 4x = \frac{1}{2}[\sin(3x+4x) + \sin(3x-4x)]$

$= \frac{1}{2}[\sin 7x + \sin(-x)]$

$= \frac{1}{2}[\sin 7x - \sin x]$

$\int \frac{1}{2}[\sin 7x - \sin x] \, dx$

$\frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} - (-\cos x)\right]$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$

$\frac{\cos x}{2} - \frac{\cos 7x}{14} + C$

⑤ $\int x\sqrt{2x+1} \, dx$

LET $\sqrt{2x+1} = u$

$2x+1 = u^2$

$2x = u^2 - 1$

$x = \frac{u^2-1}{2}$

$dx = d\left(\frac{u^2-1}{2}\right)$

$dx = \frac{du^2}{2} - d\cancel{\frac{1}{2}}^{0}$

$= \frac{2u\,du}{2}$

$dx = u\,du$

$\int$

$\frac{1}{2}[\int$

$\frac{1}{2}[$

$= \frac{1}{2}$

$=$

$$\int \frac{u^2-1}{2} \times u \times u \, du$$

$$\frac{1}{2}\left[\int (u^2-1)\, u^2 \, du\right]$$

$$\frac{1}{2}\left[\int (u^4-u^2)\, du\right]$$

$$= \frac{1}{2}\left[\int u^4 du - \int u^2 du\right]$$

$$= \frac{1}{2}\left[\frac{u^5}{5} - \frac{u^3}{3}\right] + c$$

$$= \frac{u^5}{10} - \frac{u^3}{6} + c$$

$$= \frac{(\sqrt{2x+1})^5}{10} - \frac{(\sqrt{2x+1})^3}{6} + c$$

$$\frac{(2x+1)^{5/2}}{10} - \frac{(2x+1)^{3/2}}{6} + c$$

---

⑥ $\int \frac{x}{\sqrt{7-x}}\, dx$

$u = \frac{1}{\sqrt{7-x}}$

$u = (7-x)^{-1/2}$

$u^2 = (7-x)^{-1}$

$u^2 = \frac{1}{7-x}$

$7-x = \frac{1}{u^2}$

$x = 7 - \frac{1}{u^2}$

$dx = d\left(7 - \frac{1}{u^2}\right)$

$dx = -d\,u^{-2}$

$= -(-2)\, u^{-2-1}\, du$

$= 2\, u^{-3}\, du$

$= \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times u \times \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times \frac{1}{u^2}\, du$

$\int \frac{14}{u^2} du - \frac{2}{u^4} du$

$14\int u^{-2} du - 2\int u^{-4} du$

$$\frac{(2x+1)^{5/2}}{10} - \frac{(2x+1)^{3/2}}{6} + c$$

---

⑥ $\int \frac{x}{\sqrt{7-x}}\,dx$

$u = \frac{1}{\sqrt{7-x}}$

$u = (7-x)^{-1/2}$

$u^2 = (7-x)^{-1}$

$u^2 = \frac{1}{7-x}$

$7-x = \frac{1}{u^2}$

$x = 7 - \frac{1}{u^2}$

$dx = d\left(7 - \frac{1}{u^2}\right)$

$dx = -d\,u^{-2}$

$= -(-2)\,u^{-2-1}\,du$

$dx = 2u^{-3}\,du$

$= \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times u \times \frac{2\,du}{u^3}$

$\int \left(14 - \frac{2}{u^2}\right) \times \frac{1}{u^2}\,du$

$\int \frac{14}{u^2}\,du - \frac{2}{u^4}\,du$

$14\int u^{-2}\,du - 2\int u^{-4}\,du$

$14 \times \frac{u^{-2+1}}{-2+1} - 2\frac{u^{-4+1}}{-4+1} + c$

$14\,\frac{u^{-1}}{-1} - \frac{2u^{-3}}{-3} + c$

$-\frac{14}{u} + \frac{2}{3u^3} + c$

$\frac{-14}{\frac{1}{\sqrt{7-x}}} + \frac{2}{3\times\left(\frac{1}{\sqrt{7-x}}\right)^3} + c$

$-14\sqrt{7-x} + \frac{2(7-x)^{3/2}}{3} + c$

$-14(7-x)^{-1/2} + \frac{2(7-x)^{3/2}}{3} + c$

(7) $\int \frac{e^{ax}}{e^{ax}+4}\,dx$

$u = e^{ax}+4$

$du = d e^{ax} + d4$

$du = e^{ax}\, d ax$

$du = e^{ax} \times a\, dx$

$\therefore e^{ax} = u-4$

$du = (u-4)\times a \times dx$

$dx = \frac{du}{(u-4)\times a}$

$\int \frac{u-4}{u} \times \frac{du}{(u-4)\times a}$

$\int \frac{du}{u \times a}$

$\frac{1}{a}\int \frac{du}{u}$

$\frac{1}{a}\int \frac{1}{u}\,du$

$\frac{1}{a} \ln u + c$

$\frac{1}{a} \ln(e^{ax}+4) + c$ //

(8) $\int$

$d \sin x$

$\int e^{\sin}$

$\int e$

e

(9) $\int_2^5 \frac{1}{x}\,dx = \left[\ln x\right]_2^5$

$= \ln 5 - \ln 2 = 1.609 - 0.693 = 0.915$

(10) $\int_1^2 e^{-3x}\,dx$

$d(-3x) = -3\,dx$

$dx = -\frac{1}{3}\,d(-3x)$

$\int_1^2 e^{-3x}\left(-\frac{1}{3}\,d(-3x)\right)$

$-\frac{1}{3}\int_1^2 e^{-3x}\,d(-3x)$

$-\frac{1}{3}\left[e^{-3x}\right]_1^2$

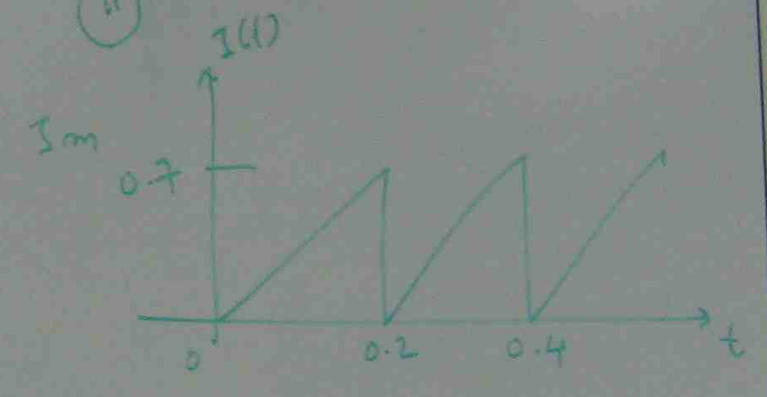
$-\frac{1}{3}\left[e^{-6} - e^{-3}\right]$

$-\frac{1}{3}\left[2.478\times10^{-3} - 0.049\right]$

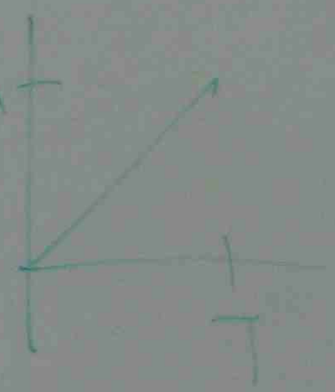
$-\frac{1}{3}\left[-0.046\right]$

$\therefore 0.015$

(11)



FIND I_{AVG} FOR GIVEN WAVE.



$f(t) = \frac{I_m}{T} \times t$

$$I_{AV} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$I_{AVE} = \frac{1}{T}\int_0^T \frac{I_m}{T}\,t\,dt$$

$$= \frac{I_m}{T^2}\int_0^T t\,dt$$

$$= \frac{I_m}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{I_m}{T^2} \times \frac{T^2}{2}$$

$$= \frac{I_m}{2}$$

$$= \frac{0.7}{2}$$

$$= 0.35$$

(12) A VOLTAGE WAVE FORM IS REPRESENTED BY THE EQUATION

$$V(t) = 20 - 20 \sin \omega t$$

DETERMINE THE FOLLOWINGS

(a) AVERAGE VALUE (b) RMS VALUE (c) FORM FACTOR

$$\omega t = \theta$$

$$V_{AVE} = \frac{1}{T}\int_0^T f(t)\,dt \quad (OR) \quad V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\,d\theta$$

$$v(t) = 20 - 20\sin\omega t \quad (OR) \quad V(\theta) = 20 - 20\sin\theta$$

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} (20 - 20\sin\theta)\,d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} 20\,d\theta - \int_0^{2\pi} 20\sin\theta\,d\theta\right]$$

$$= \frac{1}{2\pi}\left[20(\theta\right.$$

$$= \frac{1}{2\pi}\left[20\right.$$

$$= \frac{1}{2\pi}\left[40\right.$$

$$V_{AVG} = \frac{1}{2\pi} \times 40\pi$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T$$

$$V_{Rms} = \sqrt{\frac{1}{2}$$

$$= \frac{1}{2\pi}\left[20(\theta)_0^{2\pi} - 20\times -[\cos\theta]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[20\times 2\pi + 20\left[\cos 2\pi - \cos 0\right]\right]$$

$$= \frac{1}{2\pi}\left[40\pi + 20\left[1-1\right]\right]$$

$$V_{AVG} = \frac{1}{2\pi}\times 40\pi = 20 \text{ unit}$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T f(t)^2 dt} \quad (\text{or}) \quad V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v(\theta)^2 d\theta}$$

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(20-20\sin\theta)^2 d\theta}$$

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(400 - 800\sin\theta + 400\sin^2\theta)d\theta}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \longrightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} 400\,d\theta - \int_0^{2\pi} 800\sin\theta\,d\theta + 400\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[400\times 2\pi + 800(\cos\theta)_0^{2\pi} + \frac{400}{2}(\theta)_0^{2\pi} - \frac{1}{4}(\sin 2\theta)_0^{2\pi}\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(800\pi + 800(\cos 2\pi - \cos 0) + 200\times 2\pi - \frac{1}{4}(\sin 4\pi - \sin 0\right)\right.}$$

$$= \sqrt{\frac{1}{2\pi}\times(800\pi + 400\pi)}$$

$$V_{Rms} = \sqrt{600} = 24.49$$

(c)

$$\text{FORM FACTOR} = \frac{V_{Rms}}{V_{AVG}}$$

$$= \frac{24.49}{20}$$

$$= 1.225$$

$$\pi \quad 20\pi - [\cos\theta]_0^{2\pi} \quad]$$

$$\pi + 20\,[\cos 2\pi - \cos 0\,]$$

$$20\,[\,1 - 1\,]$$

$$= 0$$

20 unit

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (400 - 800\sin\theta + 400\sin^2\theta)\,d\theta}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \longrightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} 400\,d\theta - \int_0^{2\pi} 800\sin\theta\,d\theta + 400\int_0^{2\pi}\left(1-\frac{\cos 2\theta}{2}\right)d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[400\times 2\pi + 800(\cos\theta)_0^{2\pi} + \frac{400}{2}(\theta)_0^{2\pi} - \frac{400}{4}(\sin 2\theta)_0^{2\pi}\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[(800\pi + 800(\cos 2\pi - \cos 0) + 200\times 2\pi - \frac{400}{4}(\sin 4\pi - \sin 0)\right]}$$

($\cos 2\pi - \cos 0 \to 0$; $\sin 4\pi - \sin 0 \to 0$)

$$= \sqrt{\frac{1}{2\pi}\times(800\pi + 400\pi)}$$

$$V_{RMS} = \sqrt{600} = 24.49$$

(OR) $V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v(\theta)^2\,d\theta}$

$$\int_0^{2\pi} (20 - 20\sin\theta)^2\,d\theta$$

(c)

$$\text{FORM FACTOR} = \frac{V_{RMS}}{V_{AVG}}$$

$$= \frac{24.49}{20}$$

$$= 1.225$$

REVISION (STATISTICS)

(1) FIND THE MEAN OF THE FOLLOWING G F D

INTERVAL	FREQUENCY
0 → 9	1
10 → 19	2
20 → 29	4
30 → 39	8
40 → 49	4
50 → 59	1

(2) FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING SAMPLE OF GROUPED DATA

INTERVAL	FREQUENCY
10 — 14	4
15 — 19	7
20 — 24	5
25 — 29	3
30 — 34	1

$$\underset{\text{VARIANCE}}{s^2} = \frac{\Sigma f(x-\bar{x})^2}{n-1} = \frac{\Sigma f x^2 - n\bar{x}^2}{n-1}$$

$$\underset{\substack{\text{STANDARD}\\ \text{DEVIATION}}}{s} = \sqrt{\frac{\Sigma f x^2 - n\bar{x}^2}{n-1}}$$

(1)

10

20 →

30 →

40 →

50 →

THE VARIANCE AND STANDARD DEVIATION OF

NG SAMPLE OF GROUPED DATA

	FREQUENCY
	4
	7
	5
	3

$$\frac{\sum f(x-\bar{x})^2}{n-1} = \frac{\sum fx^2 - n\bar{x}^2}{n-1}$$

$$\sqrt{\frac{\sum fx^2 - n\bar{x}^2}{n-1}}$$

(1)

INTERVAL	MEAN = $\frac{\text{SMALLEST} + \text{HIGHEST}}{2}$ (x)		f	fx
$0 \rightarrow 9$	$\frac{0+9}{2}$	$= 4.5$	1	$4.5 \times 1 = 4.5$
$10 \rightarrow 19$	$\frac{10+19}{2}$	$= 14.5$	2	$14.5 \times 2 = 29$
$20 \rightarrow 29$	$\frac{20+29}{2}$	$= 24.5$	4	$24.5 \times 4 = 98$
$30 \rightarrow 39$	$\frac{30+39}{2}$	$= 34.5$	8	$34.5 \times 8 = 276$
$40 \rightarrow 49$	$\frac{40+49}{2}$	$= 44.5$	4	$44.5 \times 4 = 178$
$50 \rightarrow 59$	$\frac{50+59}{2}$	$= 54.5$	1	$54.5 \times 1 = 54.5$
			$\Sigma f = 20$	$\Sigma fx = 640$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{640}{20}$$

$$= 32$$

(2)

INTERVAL	
10 – 14	
15 → 19	
20 – 24	
25 – 29	
30 – 34	

$\bar{x} =$

VARIANCE =

STANDARD DEVIATION

AN = SMALLEST + HIGHEST / 2	f	fx
= 4.5	1	4.5×1 = 4.5
= 14.5	2	14.5×2 = 29
= 24.5	4	24.5×4 = 98
= 34.5	8	34.5×8 = 276
= 44.5	4	44.5×4 = 178
= 54.5	1	54.5×1 = 545
	$\Sigma f = 20$	$\Sigma fx = 640$

$$\frac{\Sigma x}{f} = \frac{640}{20}$$

$$= 32$$

(2)

INTERVAL	$\bar{x}$ = MID POINT	f	fx	fx^2
10 – 14	$\frac{10+14}{2} = 12$	4	12×4 = 48	$4 \times 12^2 = 576$
15 → 19	$\frac{15+19}{2} = 17$	7	17×7 = 119	$7 \times 17^2 = 2023$
20 – 24	$\frac{20+24}{2} = 22$	5	22×5 = 110	$5 \times 22^2 = 2420$
25 – 29	$\frac{25+29}{2} = 27$	3	27×3 = 81	$3 \times 27^2 = 2187$
30 – 34	$\frac{30+34}{2} = 32$	1	32×1 = 32	$1 \times 32^2 = 1024$
		$\Sigma f = 20$	$\Sigma fx = 390$	$\Sigma fx^2 = 8230$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{390}{20} = 19.5 \quad , n = \Sigma f = 20$$

$$\text{VARIANCE} = s^2 = \frac{\Sigma fx^2 - n(\bar{x})^2}{n-1} = \frac{8230 - 20 \times (19.5)^2}{20-1}$$

$$= \frac{8230 - 7605}{19} = 32.89$$

$$\text{STANDARD DEVIATION} = s = \sqrt{32.89} = 5.735$$

MATHS A+B REVISION

1. FACTORISE $8x^2 + 73x + 9$
2. $(x-7)(x+2) = 4x-1$
   FIND x
3. $\log_{10} \frac{k}{k-x} = t$, FIND x
4. PLOT GRAPH FOR $y = 2x^2 - 12x + 13$
5. THE FOLLOWING VALUES of x AND y ARE BELIEVED TO SATISFY THE EQUATION $y = Ax^2 + Bx$
   FIND A LINEAR EQUATION THAT SUITS THE INFORMATION AND EVALUATE A & B
6. CALCULATE THE AMOUNT THAT $5000 GROWS TO WHEN IT IS INVESTED FOR 3 WEEKS AT INTEREST RATE 13% PER YEAR INTEREST IS COMPOUNDED DAILY
7. PLOT $y = 7\sin 2\theta$

---

1. $8x^2 + 73x + 9$

   $8x$, 1 ; x , 9

   $72x + x = 73x$

   $(8x+1)(x+9)$

---

2. $(x-7)(x+2) = 4x-1$

   $x^2 - 7x + 2x - 14 = 4x - 1$

   $x^2 - 5x - 14 = 4x - 1$

   $x^2 - 5x - 14 - 4x + 1 = 0$

   $x^2 - 9x - 13 = 0$

   $Ax^2 + Bx + C = 0$

   $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

   $= \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}}{2 \times 1}$

$x = \frac{9 \pm \sqrt{}}{2}$

$= \frac{9 \pm \sqrt{}}{2}$

$= \frac{9 \pm 11.5}{2}$

$= \frac{9 + 11.53}{2}$

$= 10.265$

3. $\log_{10} \frac{k}{k-x}$

$\frac{k}{k-x}$

$\frac{k}{10^t}$

$x =$

$x =$

$x =$

$x+9$

1

9

$= 73x$

$x+9\ ]$

$- 4x - 1$

$2x - 14 = 4x - 1$

$-14 = 4x - 1$

$4 - 4x + 1 = 0$

$-13 = 0$

$+c = 0$

$\sqrt{B^2 - 4AC}$

$-A$

$1) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}$

2×1

$$x = \frac{9 \pm \sqrt{81+52}}{2}$$

$$= \frac{9 \pm \sqrt{133}}{2}$$

$$= \frac{9 \pm 11.53}{2}$$

$$= \frac{9+11.53}{2} \text{ (or) } \frac{9-11.53}{2}$$

$$= 10.265 \text{ (OR) } -1.265$$

③ $\log_{10} \frac{k}{k-x} = t$

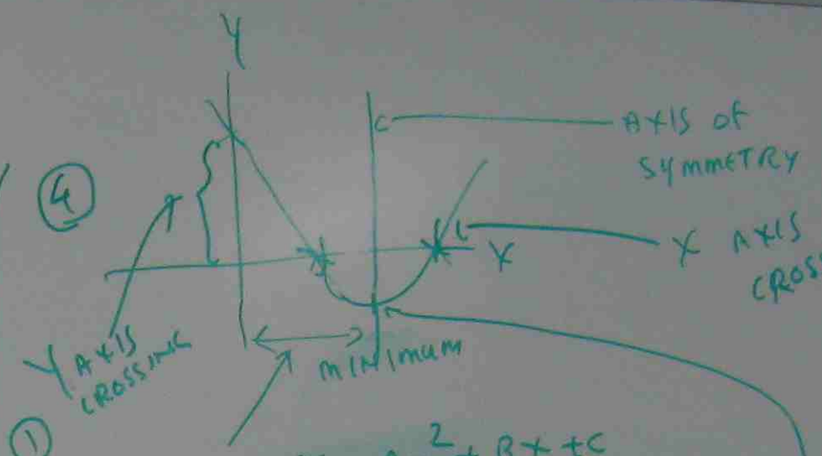
$$\frac{k}{k-x} = 10^t$$

$$\frac{k}{10^t} = k - x$$

$$x = k - \frac{k}{10^t}$$

$$x = k\left(1 - \frac{1}{10^t}\right)$$

$$x = k\left(\frac{10^t - 1}{10^t}\right)$$



④

① COMPARE WITH $Y = Ax^2 + Bx + C$

THEN $\frac{-B}{2A}$ = AXIS of SYMMETRY

② FIND Y VALUE AT AXIS OF SYMMETRY

③ FIND Y AXIS CROSSING BY PUTTING $x = 0$

④ FIND X AXIS CROSSING BY PUTTING $Y = 0$

$Y = 2x^2 - 12x + 13$

$Y = Ax^2 + Bx + C$

$A = 2,\ B = -12,\ C = 13$

① AXIS OF SY

② $Y = 2x$

Y AT AXIS

$= 2$

③ Y AX

$Y = 2$

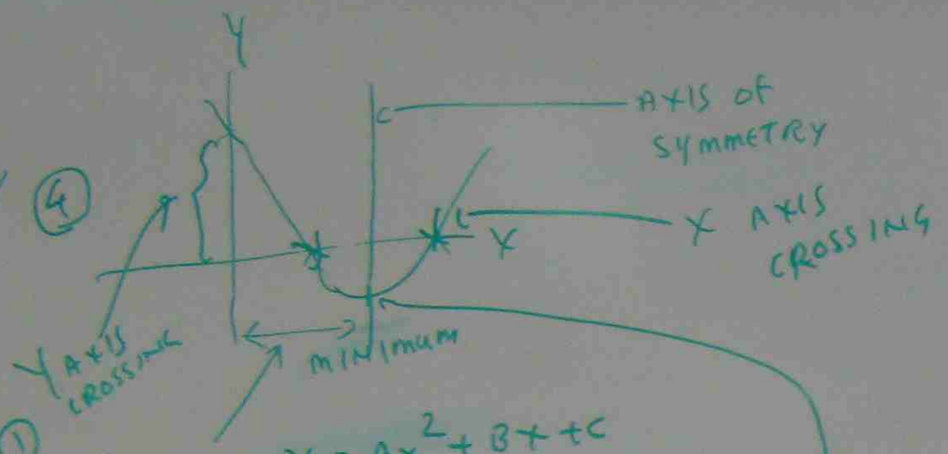
④ X A

$0 =$

$x =$

13

-5



④

① COMPARE WITH $Y = Ax^2 + Bx + C$

THEN $\frac{-B}{2A}$ = AXIS of SYMMETRY

② FIND Y VALUE AT AXIS of SYMMETRY

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④ FIND X AXIS CROSSING BY PUTTING $Y = 0$

$Y = 2x^2 - 12x + 13$

$Y = Ax^2 + Bx + C$

$A = 2,\ B = -12,\ C = 13$

① AXIS of SYMMETRY $= \frac{-B}{2A} = \frac{-(-12)}{2 \times 2} = \frac{12}{4} = 3$ ← X VALUE

② $Y = 2x^2 - 12x + 13$

Y AT AXIS of SYMMETRY

$= 2 \times 3^2 - 12 \times 3 + 13 = 2 \times 9 - 36 + 13 = -5$

③ Y AXIS CROSSING, $X = 0$

$Y = 2x^2 - 12x + 13 = 2 \times 0^2 - 12 \times 0 + 13 = 13$

④ X AXIS CROSSING $Y = 0$

$0 = 2x^2 - 12x + 13$

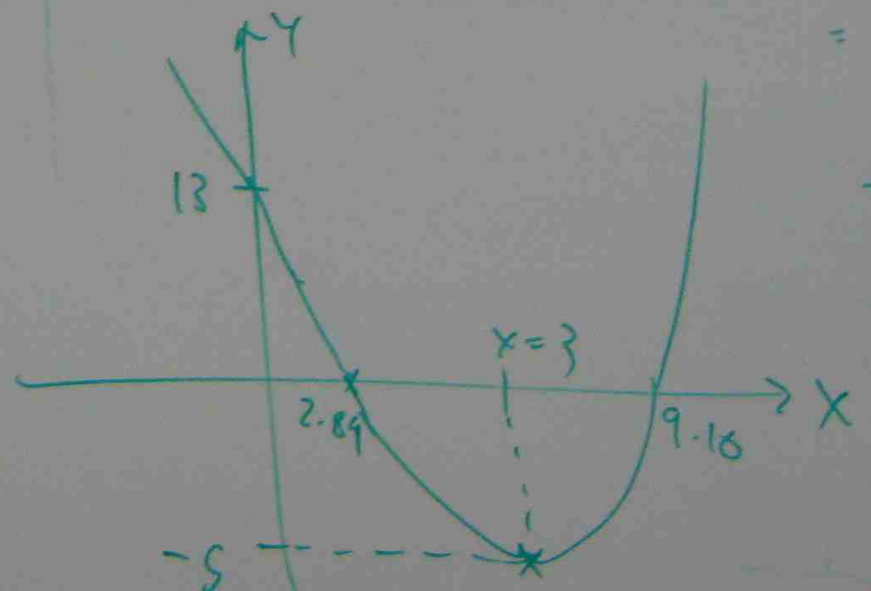
$Ax^2 + Bx + C = 0$

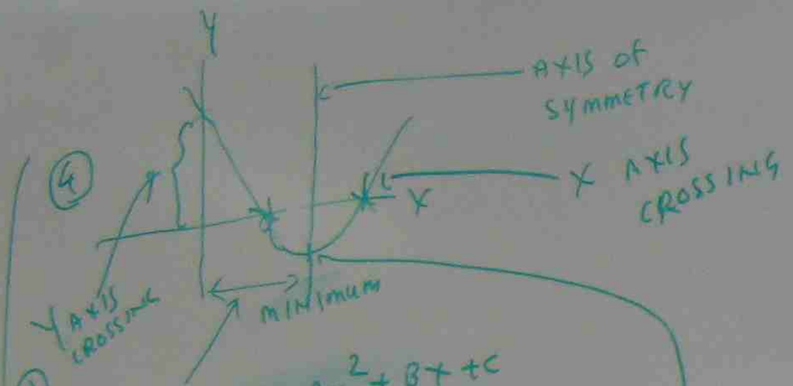
$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$

$= \frac{12 \pm \sqrt{144 - 104}}{4}$

$= \frac{12 \pm \sqrt{40}}{4} = \frac{12 \pm 6.32}{4}$

$= 9.16$ (or) 2.84





④

① COMPARE WITH $Y = Ax^2 + Bx + c$

THEN $\frac{-B}{2A}$ = AXIS of SYMMETRY

② FIND Y VALUE AT AXIS OF SYMMETRY

③ FIND Y AXIS CROSSING BY PUTTING $x = 0$

④ FIND X AXIS CROSSING BY PUTTING $Y = 0$

$Y = 2x^2 - 12x + 13$

$Y = Ax^2 + Bx + c$

$A = 2,\ B = -12,\ C = 13$

① AXIS of SYMMETRY $= \frac{-B}{2A} = \frac{-(-12)}{2 \times 2} = \frac{12}{4} = 3$ ← X VALUE

② $Y = 2x^2 - 12x + 13$

Y AT AXIS of SYMMETRY

$= 2 \times 3^2 - 12 \times 3 + 13 = 2 \times 9 - 36 + 13 = -5$

③ Y AXIS CROSSING, $X = 0$

$Y = 2x^2 - 12x + 13 = 2 \times 0^2 - 12 \times 0 + 13 = 13$

④ X AXIS CROSSING $Y = 0$

$0 = 2x^2 - 12x + 13$

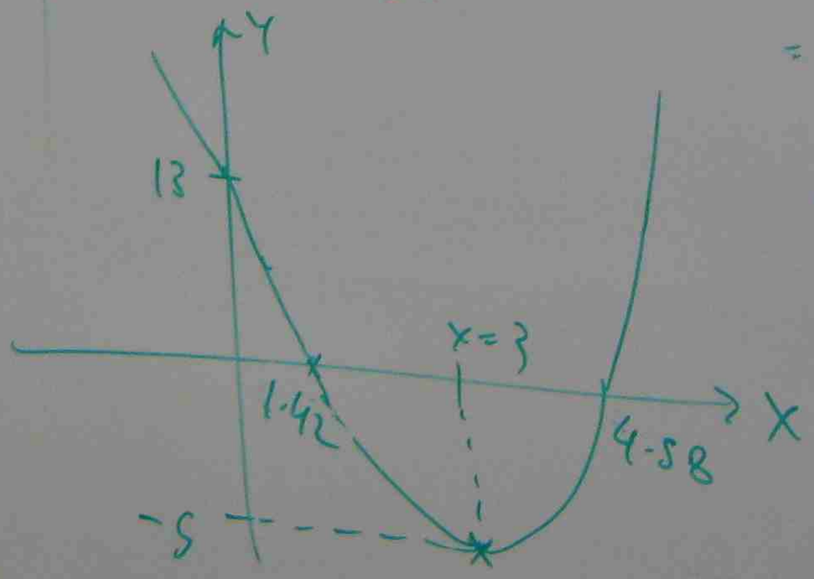
$Ax^2 + Bx + c = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$

$= \frac{12 \pm \sqrt{144 - 104}}{4}$

$= \frac{12 \pm \sqrt{40}}{4} = \frac{12 \pm 6.32}{4}$

$= 4.58$ (OR) 1.42



MATHS A+B REVISION

① FACTORISE $8x^2 + 73x + 9$

② $(x-7)(x+2) = 4x - 1$

FIND x

③ $\log_{10} \frac{k}{k-x} = t$, FIND x

④ PLOT GRAPH FOR $y = 2x^2 - 12x + 13$

⑤ THE FOLLOWING VALUES of x AND y ARE BELIEVED TO SATISFY THE EQUATION $y = Ax^2 + Bx$

$x =$	1	2	3	4	5
$y =$	5	16	34	57	84

FIND A LINEAR EQUATION THAT SUITS THE INFORMATION AND EVALUATE A & B

⑥ CALCULATE THE AMOUNT THAT $5000 GROWS TO WHEN IT IS INVESTED FOR 3 WEEKS AT INTEREST RATE 13% PER YEAR INTEREST IS COMPOUNDED DAILY

⑦ PLOT $y = 7\sin 2\theta$

① $8x^2 + 73$

$8x$

x

$72x + x$

$(8x+1)($

② $(x-7]$

$x^2 - 7x$

$x^2 - 5x$

$x^2 - 5x -$

$x^2 - 9x$

$Ax^2 + Bx$

$x = \frac{-B}{}$

$= -(-$

$=$

(5)

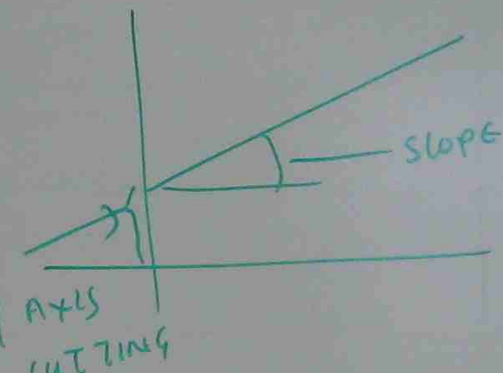
X	1	2	3	4	5
Y	5	16	34	57	84

X	1	2	3	4	5
$\frac{y}{x} = Y$	$\frac{5}{1}=5$	$\frac{16}{2}=8$	$\frac{34}{3}=11.3$	$\frac{57}{4}=14.3$	$\frac{84}{5}=16.8$

$$y = Ax^2 + Bx$$

$$\frac{y}{x} = Ax + B$$

$$Y = Ax + B$$



(6)

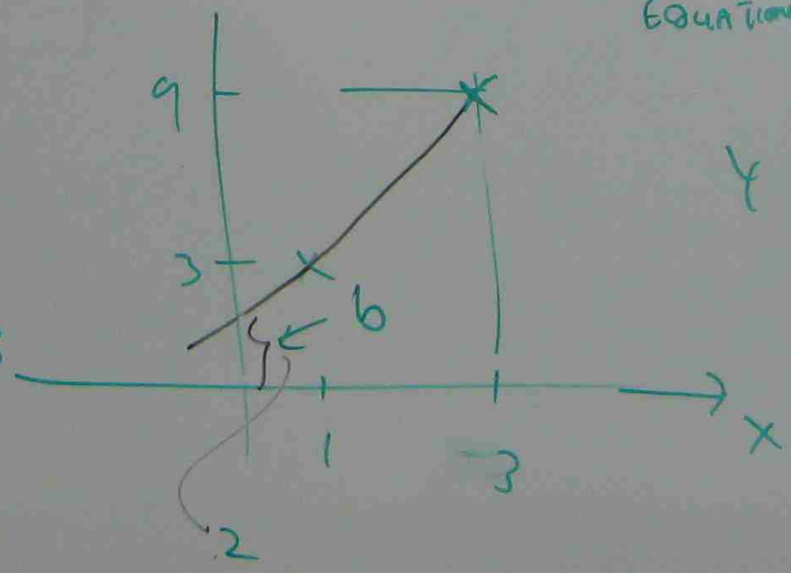
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-5}{2-1} = 3$$

Slope = 3

$x = 1 \rightarrow y = 3x = 3 \times 1 = 3$

$x = 3 \rightarrow y = 3 \times 3 = 9$



EQUATION = SLOPE × X + Y CUTTING POINT

$$Y = 3X + 2$$

$$\frac{y}{x} = 3x + 2$$

$$y = 3x^2 + 2x$$

(7)

- SLOPE

TION = SLOPE × X + Y CUTTING POINT

$= 3x + 2$

$\frac{y}{x} = 3x + 2$

$y = 3x^2 + 2x$

⑥

$A = \text{INVEST AMOUNT} \times \left(1 + \frac{\frac{\text{ANNUAL INTEREST RATE}}{365}}{100}\right)^{\text{No. of DAYS INVESTED}}$

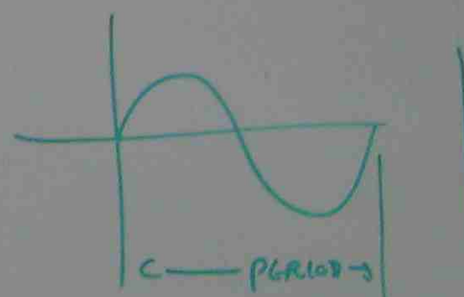
$= 5000 \times \left(1 + \frac{\frac{13}{365}}{100}\right)^{3\times 7}$

$= 5000 \left(1 + \frac{\frac{13}{365}}{100}\right)^{21} = \$5037.52$

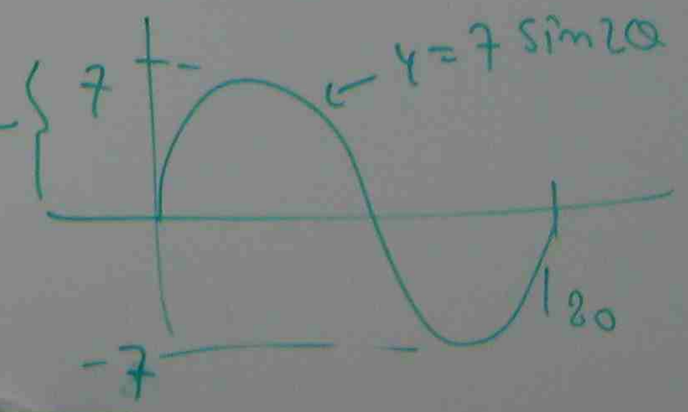
⑦ $Y = 7 \sin 2\theta$

HALF of AMPLITUDE

PERIOD = 360



$2\theta = 360 \rightarrow \theta = 180$



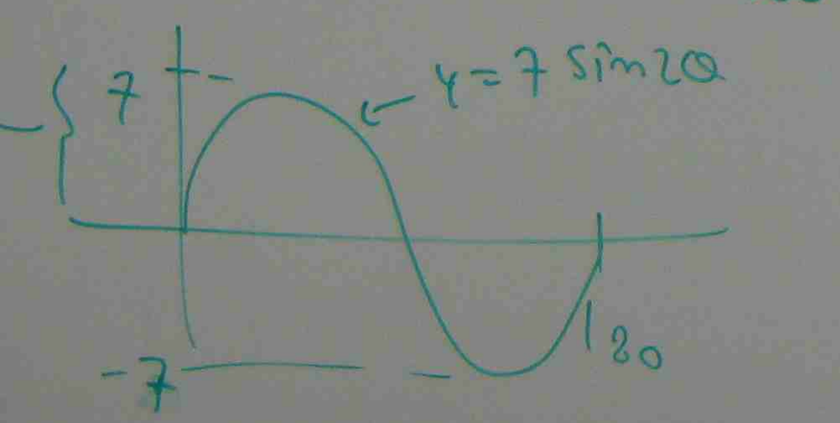
$$\left(1+\frac{\frac{\text{ANNUAL INTEREST RATE}}{365}}{100}\right)^{\text{NO. OF DAYS INVESTED}}$$

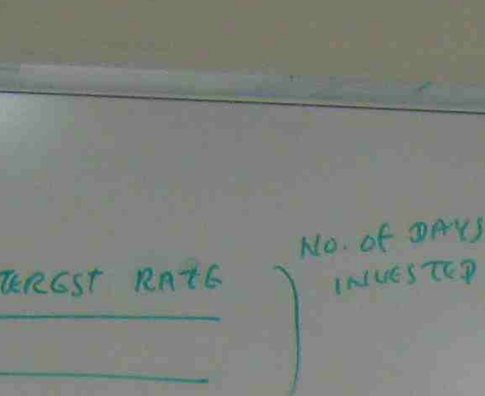
$$\left(1+\frac{\frac{13}{365}}{100}\right)^{3\times 7}$$

$$\left(\quad\right)^{21} = \$5037.53$$

PERIOD = 360

$\theta = 360 \rightarrow \theta = 180$





MATHS (B)

(1) SKETCH THE CURVE $5.37 \sin\left(136\times10^{3}t - 0.955\right)$ mA

(2) PROVE THAT $\dfrac{\sec^2 A - 1}{1+\tan^2 A} = \sin^2 A$

(3) FIND THE PHASE DIFFERENCE

$\sin(\theta - 30)$ & $\sin(\theta - 100)$

GRAPHICALLY

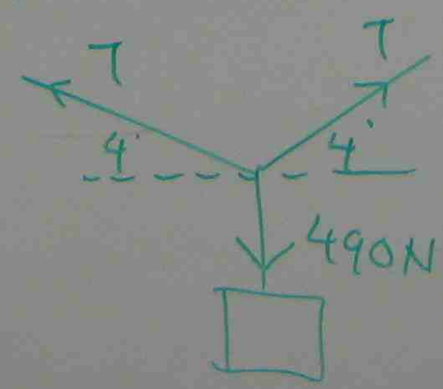
(4) IN $\triangle ABC$, $\angle B = 65$, $\angle C = 40$

$AB = 5.4$ m, FIND AC, BC, $\angle A$

(5) SIMPLIFY $\sin 5x \cos 4x - \cos 5x \sin 4x$

(6) FIND EXACT VALUE of $\sin 15$

(7)



FIND (i) TENSION IN CABLE

(ii) RATIO OF TENSION TO LOAD SUPPORTED

(1) AMPLITUDE, P

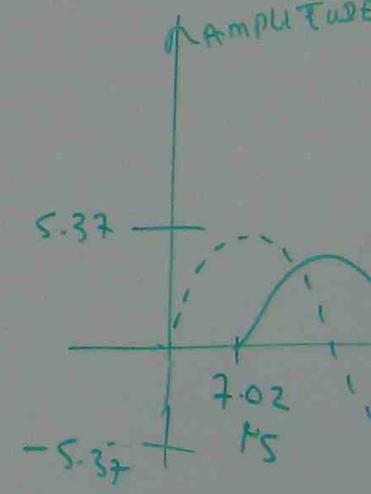
5.37 SI

↑

HALF of AMPLITUDE

PERI

AMPLITUDE



PHASE SHIFT

(955) mA

① AMPLITUDE, PERIOD, PHASE SHIFT ANGLE

$5.37 \sin\left(\underbrace{136 \times 10^3 t}_{\omega} - \underbrace{0.955}_{\phi}\right)$ mA

5.37 ↑ HALF of AMPLITUDE

ω ↑

ϕ ↑

$\text{PERIOD} = T = \frac{2\pi}{\omega}$

$= \frac{2 \times 3.1416}{136 \times 10^3}$

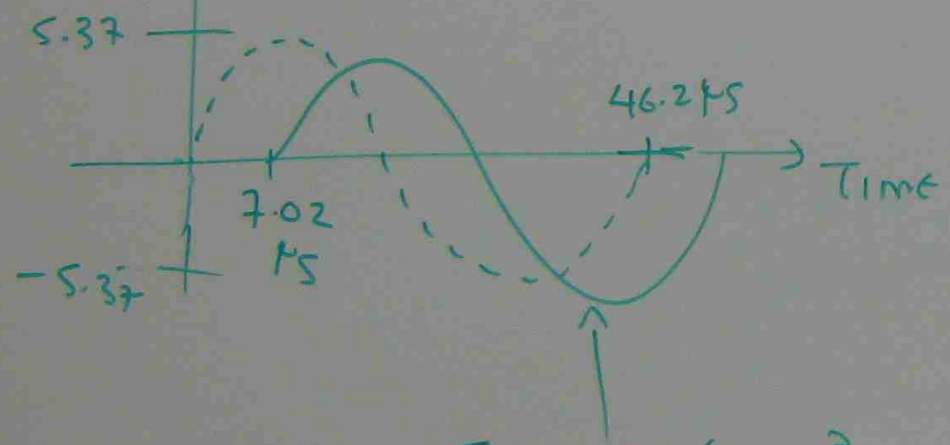
$= 46.2\,\mu s$

$\text{TIME} = \frac{\phi}{\omega}$

$t = \frac{0.955}{136 \times 10^3}$

$= 7.02\,\mu s$

↑ AMPLITUDE



$5.37 \sin\left(136 \times 10^3 t - 0.955\right)$ mA

4X

TO

ORTED

② LHS

$\frac{\sec^2 A - 1}{1 + \tan^2 A}$

$\sec A = \frac{1}{\cos A}$

$\tan A = \frac{\sin A}{\cos A}$

$\sin^2 A + \cos^2 A = 1$

$\frac{\frac{1}{\cos^2 A} - 1}{1 + \frac{\sin^2 A}{\cos^2 A}}$

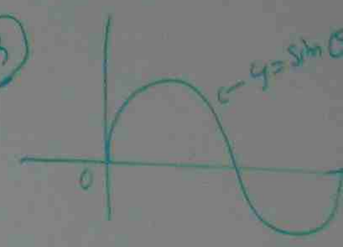
$\frac{\frac{1 - \cos^2 A}{\cos^2 A}}{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}} = \frac{1 - \cos^2 A}{1}$

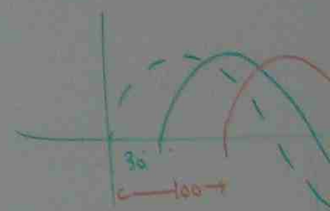
$= 1 - \cos^2 A$

$= \sin^2 A$

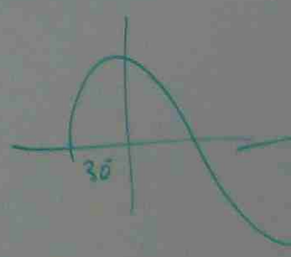
RHS

③





DIFFERENCE = 100



$\sin^2 A + \cos^2 A = 1$

$= 1$

$\frac{\sin^2 A}{\cos^2 A}$

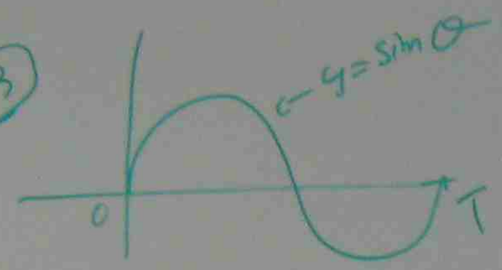
$\frac{\quad}{\cos^2 A + \sin^2 A} = \frac{1 - \cos^2 A}{1}$

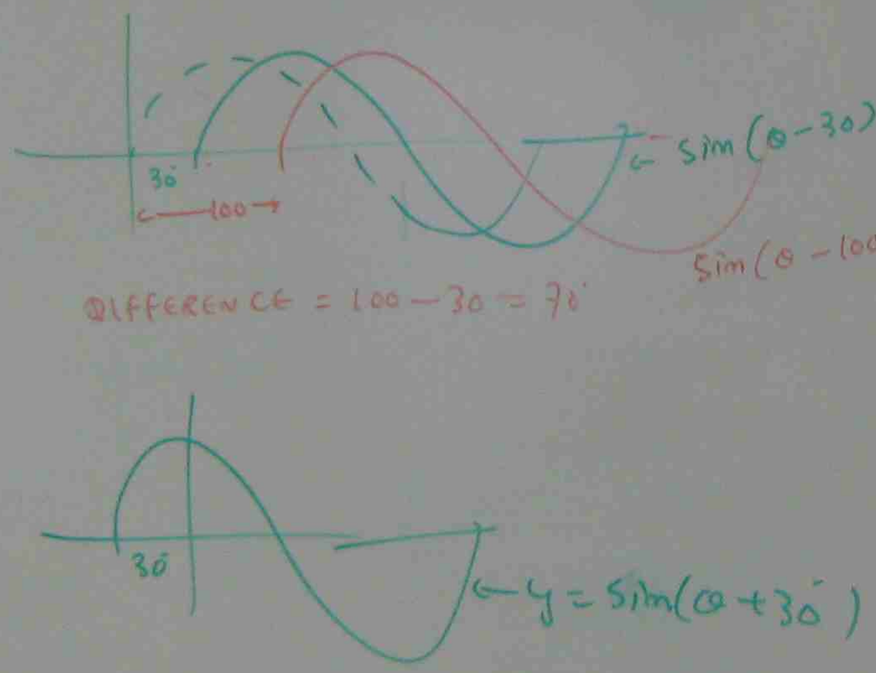
$= 1 - \cos^2 A$

$= \sin^2 A$

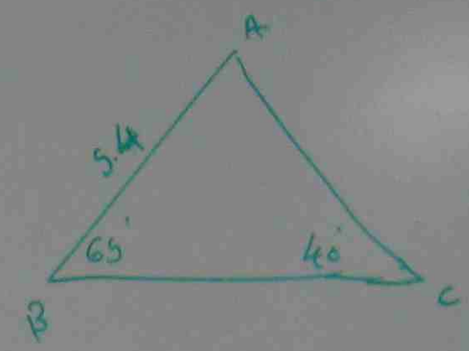
RHS

(3)





(4)



$\hat{A} = 180 - (\hat{B} + \hat{C})$

$= 180 - (65 + 40)$

$= 75°$

$\frac{AB}{\sin C} = \frac{AC}{\sin B}$

$\frac{5.4}{\sin 40} = \frac{AC}{\sin 65}$

$AC = \frac{5.4 \sin 65}{\sin 40}$

$= \frac{5.4 \times 0.906}{0.642}$

$= 7.62$

$\frac{BC}{\sin A} = \frac{A}{\sin}$

$\frac{BC}{\sin 75} = \frac{5}{\sin}$

$BC = \frac{5.4}{\sin}$

$= 5.4 *$

$= 8.$

(5) $\sin(A+B)$

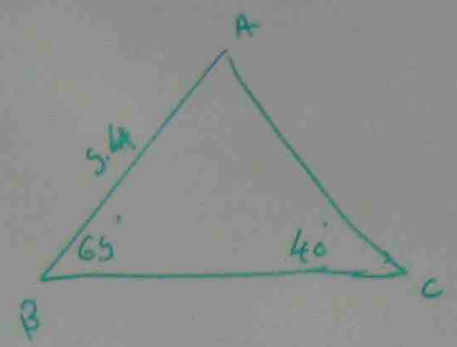
$\sin(A \rightarrow$

$\sin 5 \times \cos$

$= \sin($

$= \sin$

④



$\hat{A} = 180 - (\hat{B} + \hat{C})$

$= 180 - (65 + 40)$

$= 180 - 105 = 75°$

$\frac{AB}{\sin C} = \frac{AC}{\sin B}$

$\frac{5.4}{\sin 40} = \frac{AC}{\sin 65}$

$AC = \frac{5.4 \sin 65}{\sin 40}$

$= \frac{5.4 \times 0.906}{0.642}$

$= 7.62$

$\frac{BC}{\sin A} = \frac{AB}{\sin C}$

$\frac{BC}{\sin 75} = \frac{5.4}{\sin 40}$

$BC = \frac{5.4 \sin 75}{\sin 40}$

$= \frac{5.4 \times 0.965}{0.642}$

$= 8.11$

⑤ $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin 5x \cos 4x - \cos 5x \sin 4x$

$= \sin(5x - 4x)$

$= \sin x$

⑥ $\sin 15° = ?$

$\sin 15° = \sin(45 - 30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

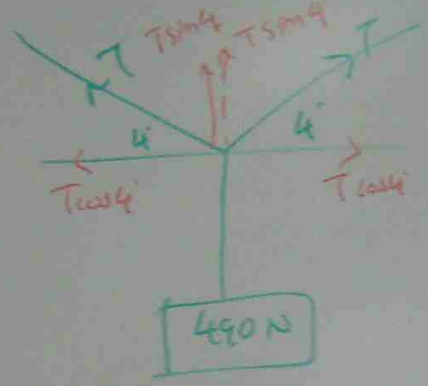
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{2 \times 2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

(7)



$$2T\sin 4 = 490$$

$$T = \frac{490}{2\sin 4}$$

$$= \frac{490}{2\times 0.069}$$

$$= 3550$$

$$\frac{T}{W} = \frac{3550}{490} = 7.24:1$$