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ANTI-JAMMING TECHNIQUES FOR GPS RECEIVERS

Villanova University

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The fundamental objective of this research project is to improve the GPS receiver performance when subjected to strong nonstationary interference. The techniques applied and proposed to achieve this objective rely on the estimation of both time-frequency and spatial signatures of the jammers. The report consists of four chapters addressing important problems in nonstationary interference mitigation in GPS receivers. The first two chapters deal with a single antenna receiver, whereas the last two chapters consider the presence of multi-antenna array. In all four chapters, the nonstationary interference is cast as an FM signal and it is mitigated using its temporal and spatial characteristics through subspace projection methods. The first three chapters are GPS specific and utilize the GPS signal structure and its deterministic nature. Chapter 4 applies to the general problem of suppressing instantaneous narrowband signals in broadband communication platforms.

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Anti-Jamming Techniques for GPS Receivers

Executive Summary

This is the final report for the Air Force Research Lab (AFRL) contract no. F30602-99-2-0504. It provides research results obtained over the period of April 1999 to January 2001.

The contributors to the research over the span of the contract are Professor Moeness Amin (PI), Dr. Yimin Zhang (Postdoctoral Fellow), and Mr. Liang Zhao (Graduate Student) from Villanova University. Dr. Alan Lindsey (Project Manager) from AFRL has worked closely with the research team at Villanova University and has provided valuable insights into several issues vital to progress and advances in research. Both Dr. Zhang and Mr. Zhao continue to work on GPS signal processing for ant-jamming. They, along with Prof. Amin, are supported by a new contract from the AFRL.

The fundamental objective of this research project is to improve the GPS receiver performance when subjected to strong nonstationary interference. The techniques applied and proposed to achieve this objective rely on the estimation of both time-frequency and spatial signatures of the jammers.

The report consists of four chapters addressing important problems in nonstationary interference mitigation in GPS receivers. Each chapter has its own abstract, introduction, equation numbers, figure numbers and captions, appendices, conclusions, and references. The first two chapters deal with a single antenna receiver, whereas the last two chapters consider the presence of multi-antenna array. In all four chapters, the nonstationary interference is cast as an FM signal and it is mitigated using its temporal and spatial characteristics through subspace projection methods. The first three chapters are GPS specific and utilize the GPS signal structure and its deterministic nature. Chapter 4 applies to the general problem of suppressing instantaneous narrowband signals in broadband communication platforms.

Subspace projection techniques are applied in Chapter 1 as a pre-correlation signal processing method for FM interference suppressions in GPS receivers. The FM jammers are instantaneous narrowband and have clear time-frequency (t-f) signatures that are distinct from the GPS C/A spread spectrum code. In the proposed technique, the instantaneous frequency (IF) of the jammer is estimated and used to construct a rotated signal space in which the jammer occupies one dimension. The anti-jamming system is implemented by projecting the received data sequence onto the jammer-free subspace. Chapter 1 focuses on the characteristics of the GPS C/A code and derives the signal to interference and noise ratio (SINR) of the GPS receiver, implementing the subspace projection techniques.

Frequency modulated signals in the frequency band 1.217-1.238 GHz and 1.565-1.586 GHz present a source of interference to the GPS, which should be properly mitigated. The problem of mitigating periodic interferers in GPS receivers using subspace projection techniques is addressed in chapter 2. In this Chapter, the signal-to-interference-and-noise ratio (SINR) of the GPS receiver implementing subspace projection techniques for suppression of FM jammers is derived. The general case in which the jammer may have equal or different cycles than the coarse acquisition (C/A) code of the GPS signals is considered. It is shown that the weak correlations between the FM interference and the gold codes allow effective interference cancellation without significant loss of the desired signal

Subspace array processing for the suppression of FM jammers in GPS receivers is introduced in Chapter 3. In this Chapter, subspace projection array processing techniques are applied for suppression of frequency modulated (FM) jammers in GPS receivers. In the proposed technique, the instantaneous frequency (IF) of the jammer is estimated and used to construct the jammer subspace. With a multi-sensor receiver, both spatial and time-frequency signatures of signal arrivals are used for effective interference suppression. Chapter 3 considers the deterministic nature of the GPS C/A code. The receiver SINR is derived and shown to offer improved performance in strong interference environments.

Combined spatial and time-frequency signatures of signal arrivals at a multi-sensor array are used in Chapter 4 for nonstationary interference suppression in direct-sequence spread-spectrum (DS/SS) communications. With random PN spreading code and deterministic nonstationary interferers, the use of antenna arrays offers increased DS/SS signal dimensionality relative to the interferers. Interference mitigation through spatio-temporal subspace projection technique leads to reduced DS/SS signal distortion and improved performance over the case of a single antenna receiver. The angular separation between the interference and desired signals is shown to play a fundamental role in trading off the contribution of the spatial and time-frequency signatures to the interference mitigation process. The expressions of the receiver SINR implementing subspace projections are derived and numerical results are provided.

Following this executive summary are the lists of papers submitted/published based on the above contributions. The four chapters included in this report properly integrate and extensively cover the material presented in all paper submissions under this contract.

List of Journal Publications

[1] Yimin Zhang, and M. G. Amin, "Array Processing for Nonstationary Interference Suppression in DS/SS Communications Using Subspace Projection Techniques," submitted to *IEEE Trans. Signal Processing*, Sept. 2000. Revised March 2001.

List of Conference Publications

[1] L. Zhao, M. G. Amin, and A. R. Lindsey, "Subspace projection techniques for anti-FM jamming GPS receivers," *Proceedings of the 10-th IEEE Workshop on Statistical Signal and Array Processing*, pp. 529-533, Aug. 2000.

[2] L. Zhao, M. G. Amin, and A. R. Lindsey, "Subspace Array Processing for the Suppression of FM Jamming in GPS Receivers," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, Monterey, CA, Oct. 2000.

[3] L. Zhao, M. G. Amin, and A. R. Lindsey, "Mitigation of Periodic Interferers in GPS Receivers Using Subspace Projection Techniques," Submitted to *ICASSP 2001*, Dec. 2000.

List of Conference Presentations

[1] L. Zhao, M. G. Amin, and A. R. Lindsey, "Subspace projection techniques for anti-FM jamming GPS receivers," *the 10-th IEEE Workshop on Statistical Signal and Array Processing*, pp. 529-533, Aug. 2000. Presented by L. Zhao.

[2] L. Zhao, M. G. Amin, and A. R. Lindsey, "Subspace Array Processing for the Suppression of FM Jamming in GPS Receivers," *the Asilomar Conference on Signals, Systems, and Computers*, Monterey, CA, Oct. 2000. Presented by A. R. Lindsey.

Chapter 1

Subspace Projection Techniques for Anti-FM Jamming GPS Receivers

Abstract

This report applies subspace projection techniques as a pre-correlation signal processing method for the FM interference suppressions in GPS receivers. The FM jammers are instantaneous narrowband and have clear time-frequency (t-f) signatures that are distinct from the GPS C/A spread spectrum code. In the proposed technique, the instantaneous frequency (IF) of the jammer is estimated and used to construct a rotated signal space in which the jammer occupies one dimension. The anti-jamming system is implemented by projecting the received sequence onto the jammer-free subspace. This report focuses on the characteristics of the GPS C/A code and derives the signal to interference and noise ratio (SINR) of the GPS receivers implementing the subspace projection techniques.

I. Introduction

The Global Positioning System (GPS) is a satellite-based, worldwide, all-weather navigation and timing system [1]. The ever-increasing reliance on GPS for navigation and guidance has created a growing awareness of the need for adequate protection against both unintentional and intentional interference. Jamming is a procedure that attempts to block reception of the desired signal by the intended receiver. In general terms, it is high power signal that occupies the same frequency as the desired signal, making reception by the intended receiver difficult or impossible. Designers of military as well as commercial communication systems have, through the years, developed numerous anti-jamming techniques to counter these threats. As these techniques become effective for interference removal and mitigation, jammers themselves have become smarter and more sophisticated, and generate signals, which are difficult to combat.

The GPS system employs BPSK-modulated direct sequence spread spectrum (DSSS) signals. The DSSS systems are implicitly able to provide a certain degree of protection against intentional or non-intentional jammers. However, in many cases, the jammer may be much stronger than the GPS signal, and the spreading gain might be insufficient to decode the useful data reliably [2]. There are several methods that have been proposed for interference suppression in DSSS communications [3], [4], [5]. The recent development of the bilinear time-frequency distributions (TFDs) for improved signal power localization in the time-frequency plane has motivated several new effective approaches, based on instantaneous frequency (IF) estimation, for non-stationary interference excisions [6]. One of the important IF-based interference rejection techniques uses the jammer IF to construct a time-varying excision notch filter that effectively removes the interference [7]. However, this notch filtering excision technique causes significant distortions to the desired signal, leading to undesired receiver performance.

Recently, subspace projection techniques, which are also based on IF estimation, have been devised for non-stationary FM interference excision in DSSS communications [8]. The techniques assume clear jammer time-frequency signatures and rely on the distinct differences in the localization properties between the jammer and the spread spectrum signals. The jammer instantaneous frequency, whether provided by the time-frequency distributions or any other IF estimator, is used to form an interference subspace. Projection can then be performed to excise the jammer from the incoming signal prior to correlation with the receiver PN sequence. The result is improved receiver SINR and reduced BERs.

In this report, we apply the subspace projection techniques as a pre-correlation signal processing method to the FM interference suppression in GPS receivers. The GPS receiver and signal structure impose new constraints on the problem since the spreading code from each satellite is known and periodic within one navigation data symbol. This structure and the signal model are reviewed in Section 2. In Section 3, we depict the received GPS signal properties in time-frequency domain. The SINR of the GPS receiver implementing the subspace projection techniques is derived in Section 4, which shows improved performance in strong interference environments.

II. Signal Model

GPS employs BPSK-modulated DSSS signals. The navigation data is transmitted at a symbol rate of 50 bps. It is spread by a coarse acquisition (C/A) code and a precision (P) code. The C/A code is a Gold sequence with a chip rate of 1.023 MHz and a period of 1023 chips, i.e. its period is 1ms, and there are 20 periods within one data symbol. The P code is a pseudorandom code at the rate of 10.23 MHz and with a period of 1 week. These two spreading codes are multiplexed in quadrature phases. Figure 1 shows the signal structure. The carrier L1 is modulated by both C/A code and P code, whereas the carrier L2 is only modulated by P code. In this report, we will mainly address the problem of anti-jamming for the C/A code, for which the peak power spectral density exceeds that of the P code by about 13 dB [1]. The transmitted GPS signal is also very weak with Jammer-to-Signal Ratio (JSR) often larger than 40 dB and Signal-to-Noise Ratio (SNR) in the range -14 to -20 dB [2], [9]. Due to the high JSR, the FM jammer often has a clear signature in the time-frequency domain as shown in Section 3. As the P code is very weak compared to the C/A code, noise and jammer, we can ignore its presence in our analysis.

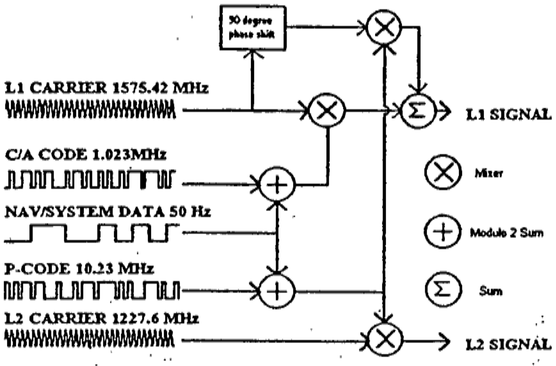

Fig. 1. The GPS signal structure.

The BPSK-modulated DSSS signal may be expressed as

$$s(t) = \sum_i I_i \; b_i(t - iT_b) \qquad I_i \in \{-1, 1\} \forall i \tag{1}$$

here I_i represents the binary information sequence and T_b is the bit interval, which is 20ms in the case of GPS system. The i^{th} binary information bit, $b_i(t)$ is further decomposed as a superposition of L spreading codes, $p(n)$, pulse shaped by a unit-energy function, $q(t)$, of duration of τ_c, which is 1/1023 ms in the case of C/A code. Accordingly,

$$b_i(t) = \sum_{n=1}^{L} p(n) \; q(t - n\tau_c) \tag{2}$$

The signal for one data bit at the receiver, after demodulation, and sampling at chip rate, becomes

$$x(n) = p(n) + w(n) + j(n) \qquad\qquad 1 \leq n \leq L \tag{3}$$

where $p(n)$ is the chip sequence, $w(n)$ is the white noise, and $j(n)$ is the interfering signal. The above equation can be written in the vector form

$$\mathbf{x} = \mathbf{p} + \mathbf{w} + \mathbf{j} \tag{4}$$

where

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} x(1) & x(2) & x(3) & \cdots & x(L) \end{bmatrix}^T, \\ \mathbf{p} &= \begin{bmatrix} p(1) & p(2) & p(3) & \cdots & p(L) \end{bmatrix}^T \\ \mathbf{w} &= \begin{bmatrix} w(1) & w(2) & w(3) & \cdots & w(L) \end{bmatrix}^T, \\ \mathbf{j} &= \begin{bmatrix} j(1) & j(2) & j(3) & \cdots & j(L) \end{bmatrix}^T \end{aligned}$$

All vectors are of dimension $L \times 1$, and 'T' denotes vector or matrix transposition. It should be noted that the P vector is real, whereas all other vectors in the above equation have complex enteries.

III. Periodic Signal Plus Jammer in the Time-Frequency Domain

For GPS C/A code, the PN sequence is periodic. The PN code of length 1023 repeats itself 20 times within one symbol of the 50 bps navigation data. Consequently, it is no longer of a continuous spectrum in the frequency domain, but rather of spectral lines. The case is the same for periodic jammers. Figure 2 and Figure 3 show the effect of periodicity of the signal and the jammer on their respective power distribution over time and frequency, using Wigner-Ville distribution. In both figures, a PN sequence of length 32 samples that repeats 8 times is used. A non-periodic chirp jammer of a 50dB JSR (jammer-to-signal ratio) is added in Figure 2. A periodic chirp jammer of 50 dB JSR with the same period as the C/A code is included in Figure 3. We note that the chosen value of 50dB JSR has a practical significance. The spread spectrum systems in a typical GPS C/A code receiver can tolerate a narrowband interference of approximately 40 dB

JSR without interference mitigation processing. However, field tests show that jammer strength often exceeds that number due to the weakness of the signal. SNR in both figures are -20dB, which is also close to its practical value [2], [9]. Due to high JSR, the jammer is dominant in both figures. From Figure 3, it is clear that the periodicity of the jammer brings more difficulty to IF estimation than the non-periodic jammers. This problem can be solved by applying a short data window when using Wigner-Ville distribution. Note that the window length should be less than the jammer period. Figure 4 shows the result of applying a window of length 31 to the same data used in Fig. 3. It is evident from the Fig. 4 that the horizontal discrete harmonic lines have disappeared.

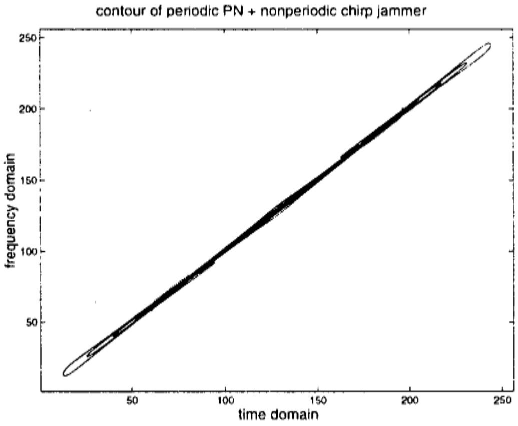

Fig. 2. Periodic signal corrupted by a non-periodic jammer in time-frequency domain

IV. GPS Anti-Jamming Using Projection Techniques

The concept of subspace projection for instantaneously narrowband jammer suppression is to remove the jammer components from the received data by projecting it onto the subspace that is orthogonal to the jammer subspace, as illustrated in Fig. 5.

Once the instantaneous frequency (IF) of the non-stationary jammer is estimated from the time-frequency domain, or by using any other IF estimator [10], [11], [12], [13], the interference signal vector **j** in (4) can be constructed, up to ambiguity in phase and possibly

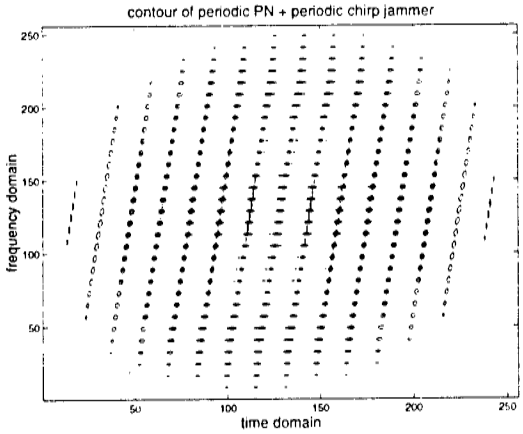

Fig. 3. Periodic signal corrupted by a periodic jammer in time-frequency domain

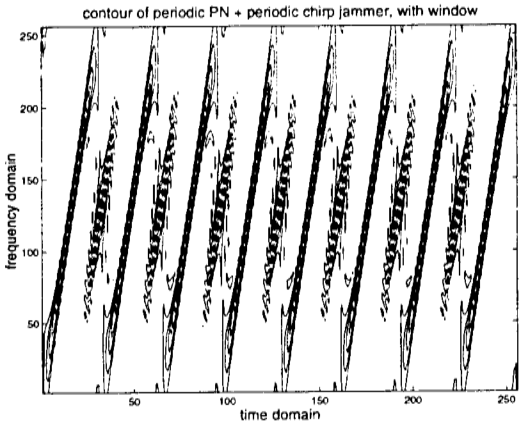

Fig. 4. Periodic signal corrupted by a periodic jammer in time-frequency domain (with window)

in amplitude. In the proposed interference excision approach, the data vector is partitioned into Q blocks, each of length P, i.e. L=PQ. For the GPS C/A code, Q=20, P=1023, and all Q blocks are identical, i.e., the signal PN sequence is periodic. Block-processing provides the flexibility to discard the portions of the data bit, over which there are significant errors in the IF estimates. The orthogonal projection method makes use of the fact that, in each block, the jammer has a one-dimensional subspace $\mathcal{J}$ in the P-dimensional space $\mathcal{V}$, which is spanned by the received data vector. The interference can be removed from each block by projecting the received data on the corresponding orthogonal subspace $\mathcal{G}$ of

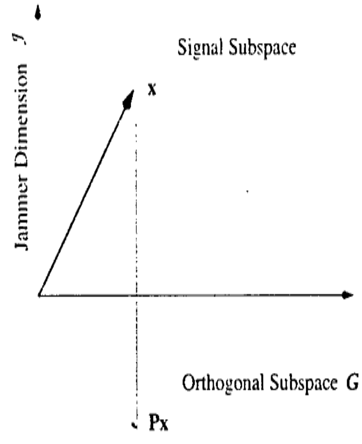

Fig. 5. Jammer excision by subspace projection

interference subspace $\mathcal{J}$. The subspace $\mathcal{J}$ is estimated using the IF information. The projection matrix for the k^{th} block is given by

$$\mathbf{V}_k = \mathbf{I} - \mathbf{u}_k \ \mathbf{u}_k^H \tag{5}$$

The vector $\mathbf{u}_k$ is the unit norm basis vector in the direction of the interference vector of the k^{th} block, and 'H' denotes vector or matrix Hermitian. Since the FM jammer signals are uniquely characterized by their IFs, the i^{th} FM jammer in the k^{th} block can be expressed as

$$u_k(i) = \frac{1}{\sqrt{P}} exp[j\phi_k(i)] \tag{6}$$

The result of the projection over the k^{th} data block is

$$\bar{\mathbf{x}}_k = \mathbf{V}_k \ \mathbf{x}_k \tag{7}$$

where $\mathbf{x}_k$ is the input data vector. Using the three different components that make up the input vector in (4), the output of the projection filter $\mathbf{V}_k$ can be written as

$$\bar{\mathbf{x}}_k = \mathbf{V}_k \ \left[\mathbf{p}_k + \mathbf{w}_k + \mathbf{j}_k\right] \tag{8}$$

The noise is assumed to be complex white Gaussian with zero-mean,

$$E\left[w(n)\right] = 0, \ E\left[w(n)^* w(n+l)\right] = \sigma^2 \delta(l), \forall l \tag{9}$$

Since we assume total interference excision through the projection operation, then

$$\mathbf{V}_k \mathbf{j}_k = \mathbf{0}, \qquad \bar{\mathbf{x}}_k = \mathbf{V}_k\ \mathbf{p}_k + \mathbf{V}_k\ \mathbf{w}_k \tag{10}$$

The decision variable y_r is the real part of y that is obtained by correlating the filter output $\bar{\mathbf{x}}_k$ with the corresponding kth block of the receiver PN sequence and summing the results over the K blocks. That is,

$$y = \sum_{k=0}^{K-1} \bar{\mathbf{x}}_k^H\ \mathbf{p}_k \tag{11}$$

Since the PN code is periodic, we can strip off the subscript k in p_k. The above variable can be written in terms of the constituent signals as

$$y = \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} + \sum_{k=0}^{Q-1} \mathbf{w}^H \mathbf{V}_k \mathbf{p} \triangleq y_1 + y_2 \tag{12}$$

where y_1 and y_2 are the contributions of the PN and noise sequences to the decision variable, respectively. In [8], y_1 is considered as a random variable. However, in GPS system, due to the fact that each satellite is assigned a fixed Gold code [1], and that the Gold code is the same for every navigation data symbol, y_1 can no longer be treated as a random variable, but rather a deterministic value. This is a key difference between the GPS system and other spread spectrum systems. The value of y_1 is given by

$$\begin{aligned} y_1 &= \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} \\ &= \sum_{k=0}^{Q-1} \mathbf{p}^T (\mathbf{I} - \mathbf{u}_k \mathbf{u}_k^H) \mathbf{p} \\ &= \sum_{k=0}^{Q-1} (\mathbf{p}^T \mathbf{p} - \mathbf{p}^T \mathbf{u}_k \mathbf{u}_k^H \mathbf{p}) \\ &= QP - \sum_{k=0}^{Q-1} (\mathbf{p}^T \mathbf{u}_k \mathbf{u}_k^H \mathbf{p}) \end{aligned} \tag{13}$$

Define

$$\beta_k = \frac{\mathbf{p}^T \mathbf{u}_k}{\sqrt{P}} \tag{14}$$

as the correlation coefficient between the PN sequence vector $\mathbf{p}$ and the jammer vector $\mathbf{u}$. β_k reflects the the component of the signal that is in the jammer subspace, and represents the degree of resemblance between the signal sequence and the jammer sequence. Since the signal is a PN sequence, and the jammer is a non-stationary FM signal, the correlation coefficient is typically very small. With the above definition, y_1 can be expressed as

$$y_1 = P(Q - \sum_{k=0}^{Q-1} |\beta_k|^2) \tag{15}$$

From (15), it is clear that y_1 is a real value, which is the result of the fact that the projection matrix V is Hermitian. With the assumptions in (9), y_2 is complex white Gaussian with zero-mean. Therefore,

$$\begin{aligned}
\sigma_{y_2}^2 &= E\left[|y_2|^2\right] \\
&= E\left[(\sum_{k=0}^{Q-1} \mathbf{w}^H \mathbf{V}_k \mathbf{p})^H (\sum_{l=0}^{Q-1} \mathbf{w}^H \mathbf{V}_l \mathbf{p})\right] \\
&= \sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k E\left[\mathbf{w}_k \mathbf{w}_l^H\right] \mathbf{V}_l \mathbf{p} \\
&= \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k E\left[\mathbf{w}_k \mathbf{w}_k^H\right] \mathbf{V}_k \mathbf{p} \\
&= \sigma^2 \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{V}_k \mathbf{p} \\
&= \sigma^2 \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_k \mathbf{p} = \sigma^2 y_1
\end{aligned} \tag{16}$$

the above equations make use of the noise assumptions in (9) and the properties of the projection matrix. The decision variable y_r is the real part of y. Consequently, y_r is given by

$$y_r = y_1 + Re\{y_2\} \tag{17}$$

where $Re\{y_2\}$ denotes the real part of y_2. $Re\{y_2\}$ is real white Gaussian with zero-mean and variance $\frac{1}{2}\sigma_{y_2}^2$. Therefore, the SINR is

$$SINR = \frac{y_1^2}{var\{Re\{y_2\}\}}$$

$$\begin{aligned} &= \frac{y_1^2}{\frac{1}{2}\sigma_{y_2}^2} = \frac{2y_1}{\sigma^2} \\ &= \frac{2P(Q - \sum_{k=0}^{Q-1} |\beta_k|^2)}{\sigma^2} \end{aligned} \tag{18}$$

In the absence of jammers, no excision is necessary, and the SINR(SNR) of the receiver output will become $2PQ/\sigma^2$, which represents the upper bound for the anti-jamming performance. Clearly, $\frac{2P\sum_{k=0}^{Q-1}|\beta_k|^2}{\sigma^2}$ is the reduction in the receiver performance caused by the proposed jammer suppression techniques. It reflects the energy of the power of the signal component that is in the jammer subspace. If the jammer and spread spectrum signals are orthogonal, i.e., their correlation coefficient $|\beta| = 0$, then interference suppression is achieved with no loss in performance. However, as stated above, in the general case, β_k is often very small, so the projection technique can excise FM jammers effectively with only very insignificant signal loss. The lower bound of SINR is zero and corresponds to $|\beta| = 1$. This case requires the jammer to assume the C/A code, i.e., identical and synchronous with actual one. Figure 6 depicts the theoretical SINR in (18), its upper bound, and estimated values using computer simulation. The SNR assumes five different values [-25, -20, -15, -10,-5] dB. In this figure, the signal is the Gold code of satellite SV#1, and the jammer is a periodic chirp FM signal with frequency 0–0.5 and has the same period as the C/A code. For this case, the correlation coefficient β is very small, $|\beta| = 0.0387$. JSR used in the computer simulation is set to 50dB. Due to the large computation involved, we have used 1000 realizations for each SNR value. Figure 6 demonstrates that the theoretical value of SINRs is almost the same as the upper bound and both are very close to the simulation result. In the simulation as well as in the derivation of equation (18), we have assumed exact knowledge of the jammer IF. Inaccuracies in the IF estimation will have an effect on the receiver performance [8].

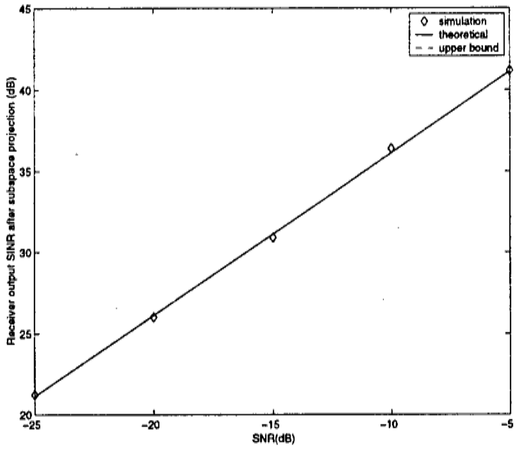

Fig. 6. Receiver SINR vs SNR.

V. Conclusions

GPS receivers are vulnerable to strong interferences. In this report, subspace projection techniques are adapted for the anti-FM jamming GPS receiver. These techniques are based on IF estimation of the jammer signal, which can be easily achieved, providing that the C/A code and the jammer have distinct time-frequency signatures. The IF information is used to construct the FM interference subspace which, because of signal nonstationarities, is otherwise difficult to obtain. Due to the characteristic of the GPS spread spectrum signal structure and the fact that the C/A codes are fixed for the different satellites and known to all, the analysis of the receiver SINR becomes different from common spread spectrum systems. The theoretical and simulation results suggest that the subspace projection techniques can effectively excise FM jammers for GPS receivers with insignificant loss in the spreading gain.

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Chapter 2

Mitigation of Periodic Interferers in GPS Receivers Using Subspace Projection Techniques

Abstract

Frequency modulated signals in the frequency band 1.217-1.238 GHz and 1.565-1.586 GHz present a source of interference to the GPS, which should be properly mitigated. In this report, we derive the Signal-to-Interference-and-Noise Ratio (SINR) of the GPS receiver implementing subspace projection techniques for suppression of FM jammers. We consider the general case in which the jammer may have equal or different cycles than the coarse acquisition (C/A) code of the GPS signals. It is shown that the weak correlations between the FM interference and the Gold codes allow effective interference cancellation without significant loss of the desired signal

I. Introduction

Subspace projection techniques based on time-frequency distributions have been employed for suppression of non-stationary FM interference in broadband communication platforms [1]-[4]. Most recently, they have been applied to Global Positioning System (GPS) with single [5] and multi-sensor [6] receivers. These techniques assume clear jammer time-frequency signatures and rely on the distinct differences in the localization properties between the interference FM waveforms and the coarse acquisition (C/A) Gold codes of the GPS signals. The FM jammer instantaneous frequency (IF), whether provided by the time-frequency distributions or any other IF estimator [7][8], is used to define the temporal signature of the interference, which is in turn used to construct the interference subspace. The respective projection matrix is used to excise the jammer power in the incoming signal prior to correlation with the receiver C/A codes. The result is improved receiver signal-to-interference-plus-noise ratio (SINR) and reduced BERs.

In this report, we generalize the results in Chapter 1 and reference [5] by considering jammer signals with different periodic structures from that of the GPS C/A codes. In the underlying problem, we deal with the case in which multiplicities of the jammer period span a finite number of the GPS information symbols. Therefore, unlike the previous work that assumes periodic synchronization between the interference and the desired signal, the generalization herein allows different portions of the jammer signal to infringe on different symbols of the GPS C/A code. It is shown, however, that due to the weak correlation between the FM waveforms and the Gold codes, the GPS receiver implementing subspace projections is robust to FM jammer periodic patterns and repetition cycles, achieving full interference suppression with no significant performance degradation from that obtained in the optimum interference free environment.

II. SINR Analysis for Periodic Jammers

The derivation in Chapter 1 implicitly assumes that the jammer period, T_j, is equal to the symbol length, T_g, of the GPS signal, i.e., $T_j=T_g=PQ$. This assumption limits the *SINR* result

$$SINR = 2P\left(Q - \sum_{k=0}^{Q-1} |\beta_k|^2 \right) \Big/ \sigma^2 \tag{1}$$

to the special case where the jammer and GPS signals have the same cycle. Considering the more general case, the jammer is presumed to be a periodic signal with $T_j = \frac{N}{M} PQ$, where N and M are integers and relatively prime. Recall that P=1023 is the period of the GPS spreading code, and Q=20 is the number of times the code repeats itself over one symbol. From the above definition, M jammer periods extend over N symbols of the GPS signal, and, as such, different segments of the jammer signature will infringe on different GPS symbols. Therefore, in the analysis herein, we add the subscript i to associate the receiver variables with the k^{th} symbol. It is straightforward to show that the correlation output at the receiver yields

$$y_i = \sum_{k=0}^{Q-1} \mathbf{p}^T \mathbf{V}_{ik} \mathbf{p} + \sum_{k=0}^{Q-1} \mathbf{w}^T(n) \mathbf{V}_{ik} \mathbf{p} \overset{\Delta}{=} y_{i1} + y_{i2} \tag{2}$$

The decision variable is the real part of y_i. It can be shown that

$$y_{i1} = P(Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2) \tag{3}$$

On the other hand, the correlation output due to the noise, y_{i2}, is a complex Gaussian zero-mean random variable, and its variance can be readily obtained as

$$\sigma^2_{y_{i2}} = \sigma^2 P(Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2) \tag{4}$$

It is noted that since there are N symbols for every M jammer periods, and $M \neq N$, both variables y_{i1} and y_{i2} assume different values over N consecutive GPS symbols. The jammer can then be cast as symbol-dependent, assuming N distinct waveforms. In this case, one simple measure of the receiver performance is to average the SINR over N consecutive symbols, i.e.,

$$\begin{aligned} SINR_{av} &= E[SINR_i] \\ &= \sum_{i=1}^{N} P_r(SINR_i \mid J_i) P_r(J_i) \\ &= \frac{1}{N} \sum_{i=1}^{N} SINR_i \end{aligned} \tag{5}$$

where $\mathrm{SINR_i}$ and $\mathrm{SINR_{av}}$ denote, respectively, the receiver signal-to-interference-and-noise ratio over the i^{th} symbol and the average receiver SINR. In the above equation, $SINR_i$ is treated as a discrete random variable that takes N possible values with equal probability. $J_i(i = 1, \ldots, N)$ are the segments of the jammer signal over N consecutive symbols. In equation (5) $P_r(x)$ denotes the probability of the event x, and $P_r(J_i)=1/N$. The $SINR_i$ is

$$SINR_i = \frac{2P(Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2)}{\sigma^2}. \tag{6}$$

Accordingly,

$$SINR_{av} = \frac{2P(Q - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=0}^{Q-1} |\beta_{ik}|^2)}{\sigma^2}. \tag{7}$$

The above expression, although simple to calculate, smoothes out high and low SINR values. In this regard, the average value in (7) does not properly penalize poor or reward good receiver performance. Further, it is difficult to establish a relationship between the receiver SINR_{av} and its BER. Most importantly, expression (7) does not account for the self-noise term that reflects the level of signal distortion produced by the induced correlation of the code chips as a result of the excision process. Hence, a more proper way to measure the receiver performance is to deal with y_l as a random variable. In this case the average receiver signal to interference plus noise ratio is referred to as $\overline{SINR}$ to distinguish it from equation (1). We assume that symbol "1" is transmitted and contaminated by one of N possible jammer signals occurring with the same probability. In this case, the mean value and the variance of the correlator output due to the GPS signal can be derived as

$$\begin{aligned} E[y_1] &= \sum_{i=1}^{N} E[y_1 \mid J_i] P_r(J_i) = \frac{1}{N} \sum_{i=1}^{N} E[y_1 \mid J_i] \\ &= \frac{\sum_{i=1}^{N} [P(Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2)]}{N} = PQ(1 - \frac{1}{NQ} \sum_{i=1}^{N} \sum_{k=0}^{Q-1} |\beta_{ik}|^2) \end{aligned} \tag{8}$$

$$\begin{aligned} \sigma_{y_1}^2 &= E[y_1^2] - E^2[y_1] \\ &= \frac{1}{N} \sum_{i=1}^{N} P^2 (Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2)^2 - P^2 Q^2 (1 - \frac{1}{NQ} \sum_{i=1}^{N} \sum_{k=0}^{Q-1} |\beta_{ik}|^2)^2 \\ &= \frac{P^2}{N} \sum_{i=1}^{N} (\sum_{k=0}^{Q-1} |\beta_{ik}|^2)^2 - \frac{P^2}{N^2} (\sum_{i=1}^{N} \sum_{k=0}^{Q-1} |\beta_{ik}|^2)^2 \end{aligned} \tag{9}$$

Similarly, the average noise power is

$$
\begin{aligned}
\sigma_{y_2}^2 &= E[y_2^2] = \sum_{i=1}^{N} E[y_2^2 \mid J_i] P_r(J_i) \\
&= \frac{1}{N}\sum_{i=1}^{N} E[y_2^2 \mid J_i] \\
&= \frac{\sum_{i=1}^{N} \sigma^2 P(Q - \sum_{k=0}^{Q-1} |\beta_{ik}|^2)}{N} \\
&= \sigma^2 PQ(1 - \frac{1}{NQ}\sum_{i=1}^{N}\sum_{k=0}^{Q-1}|\beta_{ik}|^2)
\end{aligned}
\tag{10}
$$

From the above and (1) the average SINR is given by,

$$
\begin{aligned}
\overline{SINR} &= \frac{E^2[y_1]}{\sigma_{y_1}^2 + \frac{1}{2}\sigma_{y_2}^2} \\
&= \frac{P^2Q^2(1 - \frac{1}{NQ}\sum_{i=1}^{N}\sum_{k=0}^{Q-1}|\beta_{ik}|^2)^2}{\frac{P}{N}\sum_{i=1}^{N}(\sum_{k=0}^{Q-1}|\beta_{ik}|^2)^2 - \frac{P}{N^2}(\sum_{i=1}^{N}\sum_{k=0}^{Q-1}|\beta_{ik}|^2)^2 + \frac{1}{2}\sigma^2 Q(1 - \frac{1}{NQ}\sum_{i=1}^{N}\sum_{k=0}^{Q-1}|\beta_{ik}|^2)}
\end{aligned}
\tag{11}
$$

This expression represents the SINR of the receiver implementing subspace projections to remove a periodic jammer and is also valid for the case in which the jammer assumes N possible waveforms. In the case that the jammer has the same period as the GPS data symbol, then N=1 and the above equation simplifies to single antenna case. Comparing (11) to (7) and SINR expression in Chapter 1, it is clear that (11) includes the self-noise component $\sigma_{y_1}^2$ that arises due to the differences in the distortion effects of interference excision on the GPS signal over the N symbols. In the absence of jamming, no excision is necessary, and the SINR of the receiver output becomes $2PQ/\sigma^2$, which represents the upper bound of the interference suppression performance. Moreover, if the jammer and the spreading codes are orthogonal, i.e., $\beta_{ik} = 0$, the interference suppression is also achieved with no loss in optimum receiver performance.

It is noted, however, that the values of the cross-correlation coefficient, $|\beta_{ik}|$, between the PN sequence signal and the non-stationary FM jammer are typically very small. This allows the proposed projection technique to excise FM jammers effectively with insignificant signal loss. Computer simulations show that $|\beta_{ik}|$ ranges from 0 to 0.14. With these values, the self-noise $\sigma^2_{y_1}$ is negligible compared to the Gaussian noise for the low SNR conditions that often prevail in GPS environment. In this case, equation (11) can be simplified to the following

$$\overline{SINR} \approx \frac{E^2[y_1]}{\frac{1}{2}\sigma^2_{y_2}} = \frac{2PQ\left(1-\frac{1}{NQ}\sum_{i=1}^{N}\sum_{k=0}^{Q-1}|\beta_k|^2\right)}{\sigma^2} \tag{12}$$

which is similar to the SINR expression in Chapter 1 and has the same form as (7). Therefore, $SINR_{av}$ and $\overline{SINR}$ approximately yield the same performance measure.

III. Simulation Results

Fig. 1 plots the receiver SINR vs SNR according to (11) for the two cases of N=1 and N/M=5/3. In both cases, the normalized start and end frequencies of the chirp jammer are 0 and 0.5, respectively. The SNR values range from –25dB to –5dB, and the GPS signal is the Gold code of satellite SV#1. It is clear that the period of the jammer has little effects on the result of interference suppression performance, as both SINR curves are very close to the upper bound. From Fig. 1, we can also observe that the SINR change linearly with the input SNR, which can be easily recognized from (12). Fig. 2 shows the $|\beta_{ik}|$ values for the underlying example. It is evident from this figure that there is no clear variation patterns of the cross-correlation coefficients. The range values of $|\beta_{ik}|$ do not