UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING

Electronics Engineering Section

Third Year

AC Machine and Power Electronics

2016-2017

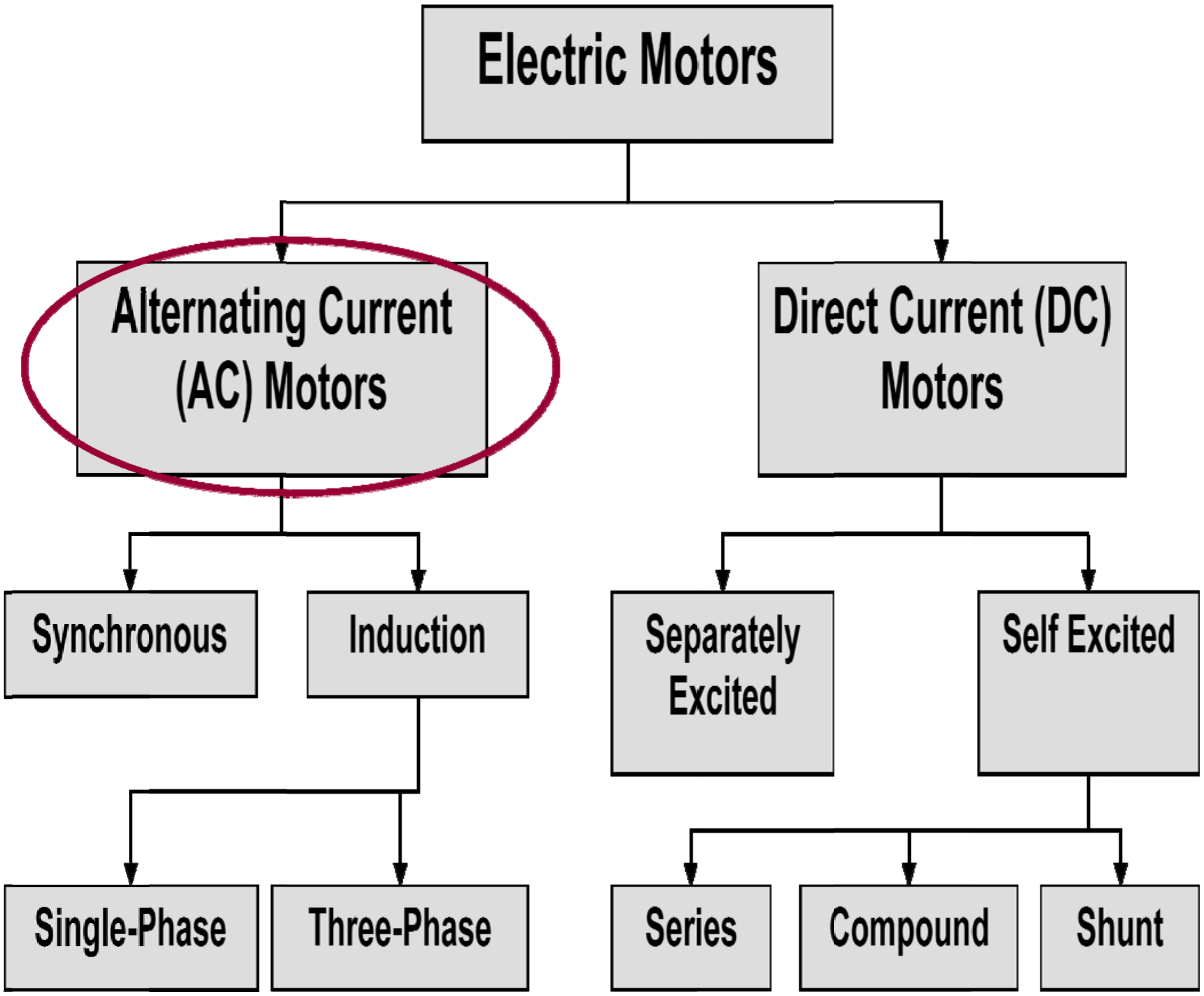
Module-I: AC Machines:

- *Three-phase Induction Motors*
- *Single-phase Induction Motors*
- *Special Motors*

Textbooks:

1. *H.C. Gerhard Henneberger, Electrical Machines.*
2. *M. Rashid, Power electronic handbook.*

Classification of Motors



Induction Motors

1. Introduction

The induction motor derives its name from the fact that ac voltages are induced in the rotor circuit by the rotating magnetic field of the stator. In many ways, induction in this motor is similar to the induction between the primary and secondary windings of a transformer.

Induction machines are the most widely used of all electric motors. They offer the following attractive features:

- ○ Generally easy to build and cheaper than corresponding dc or synchronous motors
- ○ Rugged and require little maintenance
- ○ Offer reasonable asynchronous performance
- ○ Generally satisfactory efficiency
- ○ Range in size from few Watts to several MW

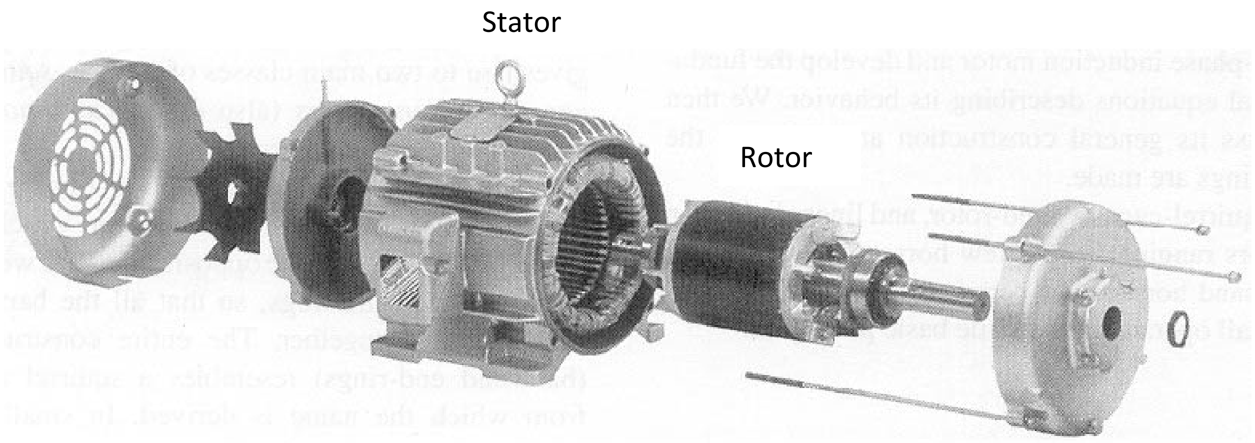
Some disadvantages of induction motors are:

- ○ Speeds not as easily controlled as dc motors
- ○ Draw large starting currents, typically 6-8 x their full load values
- ○ operate with a poor lagging power factor when lightly loaded

2. Construction

A three phase induction motor consists essentially of two parts:

1. Stator
2. Rotor



2.1 Stator

- The stator is the stationary part is built up of high – grade alloy steel laminations to reduce the eddy current losses.
- The laminations are slotted on the inner periphery and are insulated from each other.
- These laminations are supported in a stator frame of cast iron or fabricated steel plate. The insulated stator conductors are placed in these slots. The stator conductors are connected to form a three- phase winding. The phase winding may be either star or delta connected.

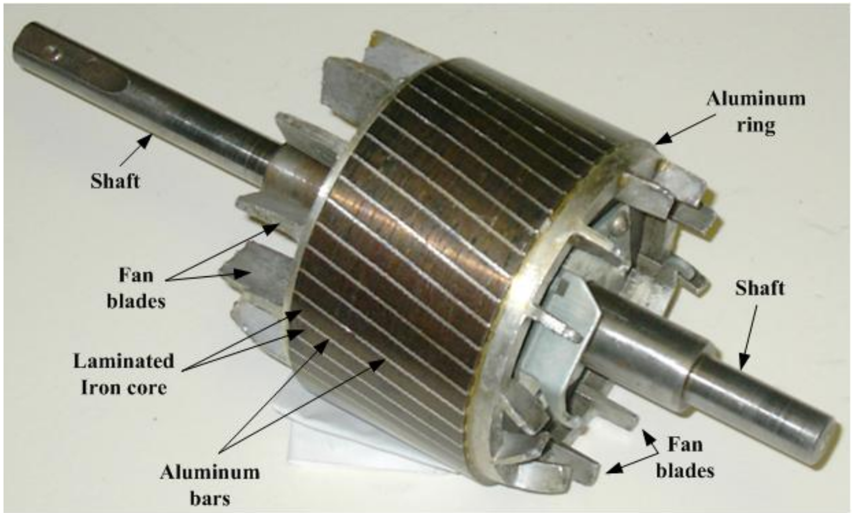
2.2 Rotor

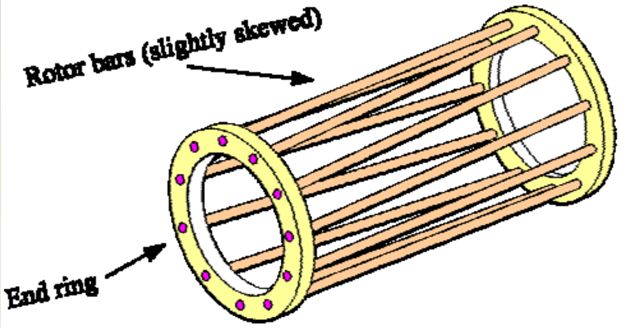
There are two-types of induction motor rotors:

1. Squirrel-cage rotor or simply cage rotor (most common)
2. Wound-rotor or slip -ring.

2.2.1 Squirrel-cage

It consists of a cylindrical laminated core with slots nearly parallel to the shaft axis, or skewed. Each slot contains an uninsulated bar conductor of aluminum or copper. At each end of the rotor, the rotor bar conductors are short- circuited by heavy end rings.





The skewing of cage rotor conductors offers the following advantages:

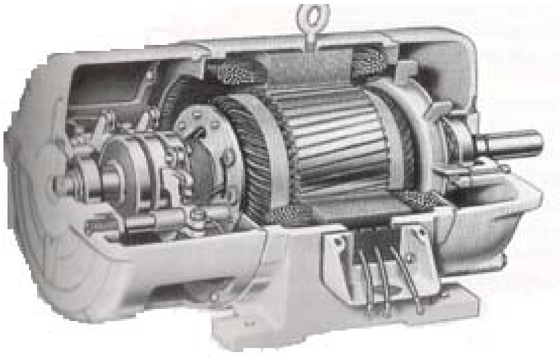
1. More uniform torque is produced and the noise is reduced during operation.

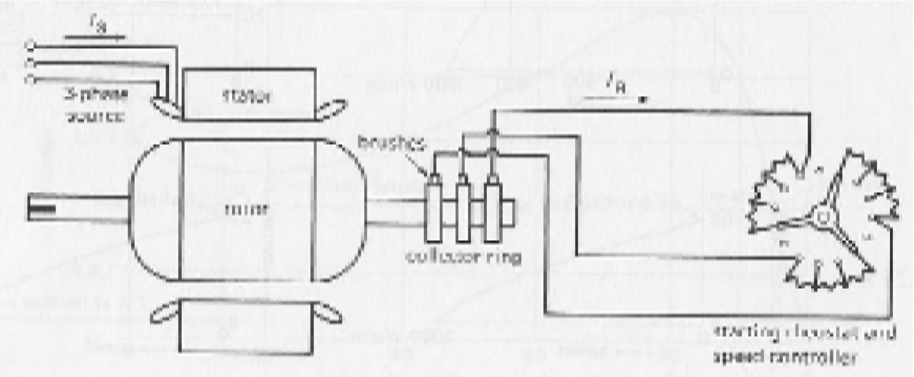
2. The locking tendency of the rotor is reduced. During locking the rotor and stator teeth attract each other due to magnetic action.

2.2.2 <u>Wound-rotor (slip –ring)</u>

A wound rotor has a three phase winding, similar to the stator winding. The rotor winding terminals are connected to three slip Rings. The slip rings are mounted on the shaft with brushes resting on them. The slips/ brushes allow external resistors to be connected in series with the winding.

The external resistors are mainly used during start-up, under normal running conditions the windings short circuited externally.





- Cutaway of a wound-rotor induction motor

3 Basic principle of operation

When the three phase stator winding, are fed by a three phase supply then, a magnetic flux of constant magnitude but rotating at synchronous speed is set up.

The flux passes through the air gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the conductors. The frequency of the induced e.m.f. is the same as the supply frequency. Due to closed paths, the rotor currents circulate. These rotor currents interact with the magnetic field (produced by the stator) in the air gap and develop a torque which rotates the rotor in the same direction.

The cause which produced the current in the rotor conductors is the relative speed between revolving flux of the stator and the stationary rotor conductors. Thus to reduce the relative speed, the rotor starts moving in the same direction as that of the flux and tries to catch the revolving flux.

4 Slip of Induction Motor

In practice, the motor never succeeds in ‘catching up’ with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f, no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor. The difference between the synchronous speed Ns and the rotor speed N can be expressed as a percentage of synchronous speed, known as the slip.

$$S = \frac{N_S - N}{N_S} * 100$$

s: slip

N_S: synchronous speed (r.p.m)

N: rotor speed (r.p.m)

Sometimes, N_s – N is called slip speed.

5 Frequency of rotor voltage and current

The frequency in the rotor winding is variable and depends on the difference between the synchronous speed and the rotor speed. Hence the rotor frequency depends upon the slip. The rotor frequency is given by

$$f_r = \frac{p(N_s - N)}{120}$$

$$\frac{f_r}{f} = \frac{N_s - N}{N_s}$$

but $\frac{N_s - N}{N_s} = s$

$$\therefore f_r = sf$$

Example (1):

An induction motor having 8 – poles run on 50 Hz supply. If it operate at full-load at 720 r.p.m. calculate the slip?

Sulotion:

$$N_s = \frac{120f}{P} = \frac{120 * 50}{8} = 750\, r.p.m.$$

$$slip\ speed = N_s - N = 750 - 720 = 30\, r.p.m.$$

$$\therefore slip = \frac{N_s - N}{N_s} = \frac{750 - 720}{750} * 100 = 4\%$$

Example (2):

The frequency of e.m.f. in the stator of 4 pole induction motor is 50 Hz and that in the rotor is 1.5 Hz. What is the slip, and at what speed is the motor running?

Solution:

$$f_r = s f_s$$

$$s = \frac{f_r}{f_s} = \frac{1.5}{50} = 0.03\, p.u. = 3\%$$

$$N_s = \frac{f_s 120}{P} = \frac{120 * 50}{4} = 1500\, r.p.m.$$

Speed of motor

$$N = (1 - s)N_s = (1 - 0.03)1500 = 1455\, r.p.m.$$

Example (3):

A 6 – pole alternator running at 1000 r.p.m. supplies an 8 – pole induction motor. What is the actual speed of the motor if the slip is 2.5%

Solution:

synchronous speed of alternator

$$N_s = 1000$$

$$\therefore f = \frac{PN_s}{120} = \frac{6 * 1000}{120} = 50 \ c/s \ (supply \ frequqncy \ of \ moto$$

$$\therefore N_s \ of \ motor$$

$$N_s = \frac{120f}{P} = \frac{120 * 50}{8} = 750 \, r.p.m.$$

$$N = (1 - s)N_s = (1 - 0.025) * 750 = 731.25 \, r.p.m.$$

6. Equivalent Circuit of An Induction Motor

The equivalent circuit of induction motor can be derived from knowledge of transformers and from what already know about variation of rotor frequency with speed in induction motors.

Stator circuit model

The stator model of the induction motor is shown in figure (1)

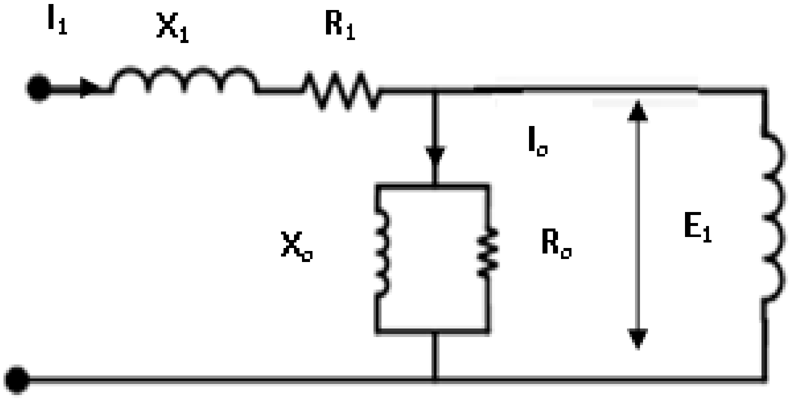


Fig. (1) Stator model of an I. M.

As shown there is certain resistance and self inductance in stator windings which must be represented in equivalent circuit of machine. Stator resistance called R_1 and stator leakage reactance called X_1. These appear right at input to machine model.

The core losses (hysteresis and eddy current losses) in an induction motor can be represented by a shunt resistance and because the stator and rotor are magnetically coupled, a magnetic reactance, just as in a transformer, shunted to the shunt resistance.

The total magnetizing current I_o is considerably larger in the case of the induction motor as compared to a transformer. This is due to the higher reluctance caused by the air gap of the induction motor. The magnetizing reactance X_o in an induction motor will have a much smaller value. In a transformer, I_o is about 2 to 5% of the rated current while in an induction motor it is approximately 25 to 40% of the rated current depending upon the size of motor.

Rotor circuit model

In an induction motor, when a 3 phase supply is applied to the stator windings, a voltage is induced in the rotor windings of the machine. The greater the relative motion of the rotor and the stator magnetic fields, the greater the resulting rotor voltage. If the induced rotor voltage at standstill condition, (s=1), is E_{20}

$$E_{20} = 4.44 \mathrm{k} \emptyset_m \mathrm{f}_s N_2 \qquad \ldots(1)$$

Where, N_2:no.of rotor winding turns per phase

$\emptyset_m$: flux per pole produced by stator m.m.f.

Then the induced voltage of an I.M. running at speed of N r.p.m., we have

$$E_{2s} = 4.44 k \emptyset_m S f_s N_2$$

$$\therefore E_{2s} = S E_{20} \qquad \ldots(2)$$

The rotor resistance R_2 is a constant (except for the skin effect), it is independent of slip. While the reactance of the induction motor rotor depends upon the inductance of the rotor and the frequency of the voltage and current in the rotor.

If L_2 is inductance of rotor, the reactance is given by

$$X_2 = 2\pi f_2 L_2$$

$$but \qquad f_2 = s f_s$$

$$X_2 = 2\pi s f_s L_2$$

$$\therefore X_2 = sX_{20}$$

where X_{20} is the standstill reactance of rotor.

The rotor impedance is given by

$$Z_{2s} = R_2 + jSX_{20} \qquad \ldots(3)$$

The rotor circuit is shown in figure (2).

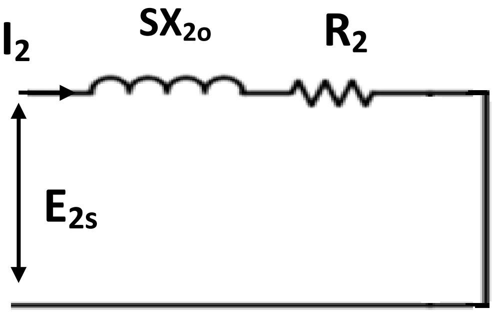


Fig.(2) Rotor model of induction motor

The rotor current per phase may be expressed as

$$I_{2s} = \frac{E_{2s}}{Z_{2s}}$$

$$I_{2s} = \frac{SE_{20}}{R_2 + jSX_{20}} \qquad \ldots(4)$$

By dividing both the numerator and the denominator of equation (4) by the slip s, we get

$$I_{2s} = \frac{E_{20}}{\frac{R_2}{S} + jX_{20}} \qquad \dots (5)$$

The circuit interpretation of equation (5) is shown in figure (3)

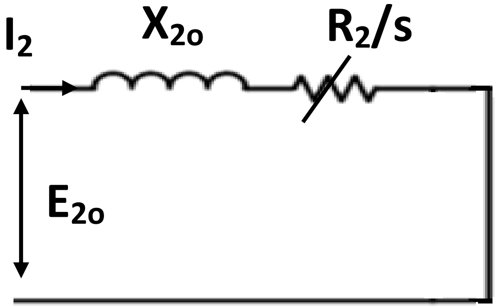


Fig. (3) Rotor circuit model.

The significance of equation (5) should be understood clearly. This equation describes the secondary circuit of a fictitious transformer, one with a constant voltage ratio and with same frequency of both sides.

This fictitious stationary rotor carries the same current as the actual rotating rotor, and thus produces the same m.m.f. waves. This concept of fictitious stationary rotor makes it possible to transfer the secondary (rotor) impedance to the stator side.

In order to obtain the complete per phase equivalent circuit for an I.M., it is necessary to refer the rotor part of model over to the stator circuit's frequency and voltage level.

The voltage, currents and impedances on the rotor can by transferred to the stator side by means of turns ratio a of I.M.

$$\grave{E}_{20} = a\,E_{20} = E_1$$

$$\grave{I}_2 = \frac{I_2}{a}$$

$$\grave{Z}_{20} = a^2 Z_{2o}$$

$$\grave{R}_2 = a^2 R_2$$

$$\grave{X}_{20} = a^2 X_{20}$$

The complete equivalent circuit of the I.M. is shown in the figure (4).

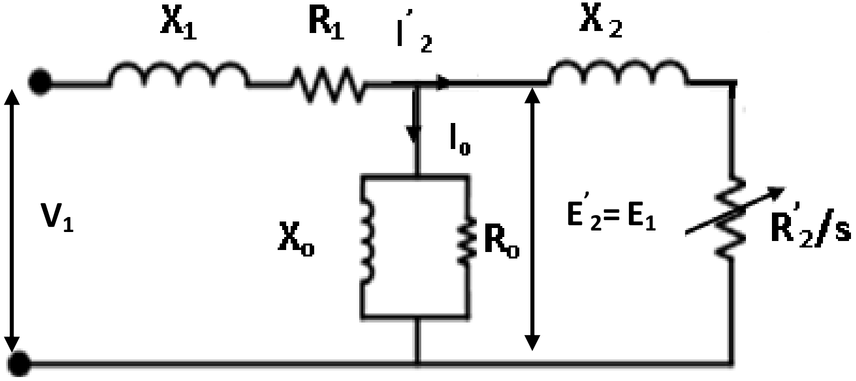


Fig. (4) The complete equivalent circuit of the I.M. referred to stator

If the voltage drop across R_1 and X_1 is small, $V_1 \cong E_1$, the shunt impedance branches R_o and X_o can by shifted to the input terminals as shown in figure (5).

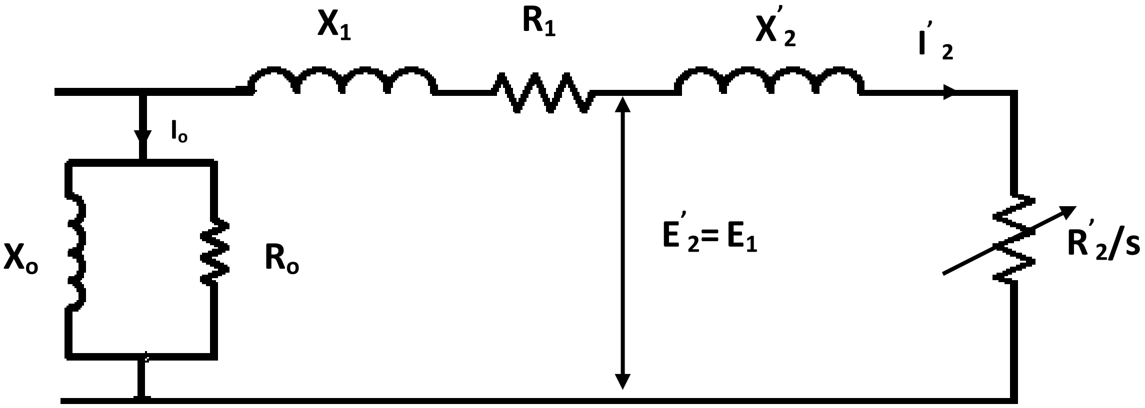


Fig. (5) Approximate equivalent circuit of an I.M.

In the circuit model shown in figure (5) the resistance $\frac{\grave{R}_2}{s}$ consumes the total rotor input (air gap power). Therefore the air gap power is given by

$$P_g = 3\grave{I}_2^2 \frac{\grave{R}_2}{s} \qquad \ldots(6)$$

The actual resistive losses (copper losses) in the rotor circuit are given by

$$P_c = 3\grave{I}_2^2 R_2 \qquad \ldots(7)$$

Developed mechanical power

$$P_{md} = P_g - P_c$$

$$= 3\grave{I}_2^2 \grave{R}_2 (\tfrac{1}{S} - 1) \qquad \ldots(8)$$

but $\quad P_g = P_{md} + P_c = 3\grave{I}_2^2[\grave{R}_2 + \grave{R}_2(\frac{1-S}{S})] \qquad \ldots(9)$

and $\quad \grave{R}_2 + \grave{R}_2\left(\frac{1-S}{S}\right) = \grave{R}_2 + \frac{\grave{R}_2}{S} - \grave{R}_2 = \frac{\grave{R}_2}{S}$

From the above equation, it is seen that $\frac{\grave{R}_2}{S}$ may be divided into two components:

$\grave{R}_2$, representing the rotor copper loss per phase,

and $\grave{R}_2(\frac{1-S}{S})$, representing the developed mechanical power.

This expression is useful in analysis because it allows any mechanical load to by represented in the equivalent circuit by resistor. So, the equivalent circuit can by drown as shown in figure (6).

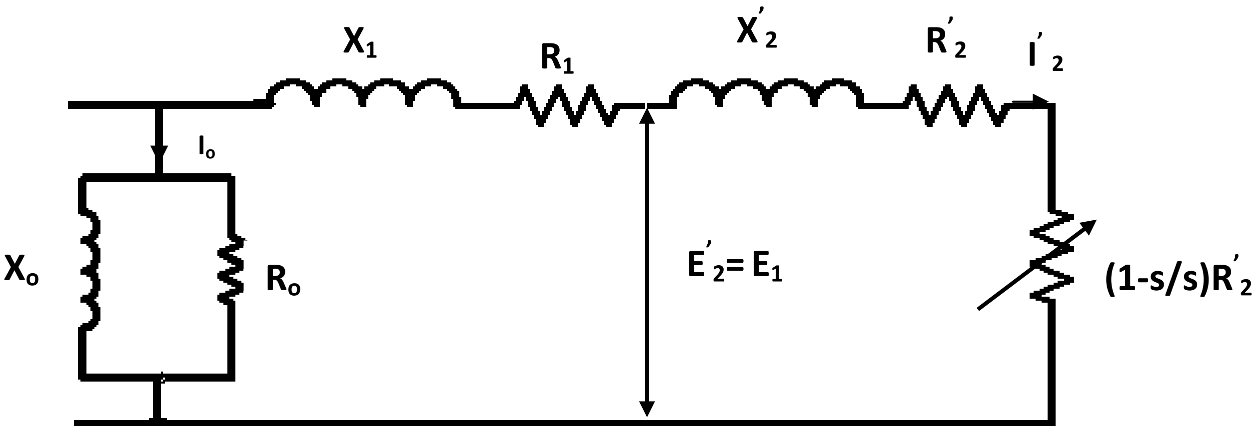


Fig. (6)Approximate equivalent circuit.

This approximate equivalent circuit model has become the standard for all performance calculation of an I.M. Referring to fig.(6), the following equation can by written down for one phase at any given slips:

Impedance beyond AB is

$$Z_{AB} = \left(R_1 + \frac{\grave{R}_2}{S}\right) + j(X_1 + \grave{X}_2) \qquad \ldots(10)$$

$$\grave{I}_2 = \frac{V_1}{Z_{AB}} = \frac{V_1}{\left(R_1+\frac{\grave{R}_2}{S}\right)+j(X_1+\grave{X}_2)} \qquad \ldots(11)$$

Power factor

$$\cos\varphi_2 = \frac{R_1+{}^{\grave{R}_2}/_{S}}{|Z_{AB}|} \qquad \ldots(12)$$

no load current

$$I_o = I_w + jI_u = V_1[\frac{1}{R_o} - j\,\frac{1}{X_o}] \qquad \ldots(13)$$

Air gap power per phase

$$P_g = V_1\,\grave{I}_2\cos\varphi_2 = \acute{I}_2^2\,\frac{\acute{R_2}}{S} = \frac{V_1^2({}^{\acute{R}_2}/_{S})}{(R_1+{}^{\acute{R}_2}/_{S})^2+(X_1+\acute{X}_2)^2} \qquad \ldots(14)$$

Developed torque

$$T_d = \frac{P_g}{\omega_s} = \frac{V_1^2({}^{\acute{R}_2}/_{S})}{\omega_s[(R_1+{}^{\acute{R}_2}/_{S})^2+(X_1+\acute{X}_2)^2]} \qquad \ldots(15)$$

Example (4):

A 6 pole, 50 Hz, 3 phase induction motor running on full load develops a useful torque of 150 N.m at a rotor frequency of 1.5Hz. Calculate the shaft power output, if the mechanical torque lost in friction be 10 N.m, determine (a) rotor copper loss, (b) the input to the motor, and (c) the efficiency. The total stator loss is 700 watt.

Solution:

$$N_S = \frac{120 f_s}{P} = \frac{120 * 50}{6} = 1000\, r.p.m.$$

$$S = \frac{f_r}{f_s} = \frac{1.5}{50} = 0.03\ or\ 3\%$$

$$N_r = (1 - S)N_s = (1 - 0.03) * 1000 = 970\, r.p.m.$$

$$\omega_r = 2\pi N_r = \frac{2\pi 970}{60} = 101.58\ \text{rad/s}$$

Shaft power output, $P_o = T_o \omega_r$

$$= 150 * 101.58 = 15.236\ kw$$

Mechanical power developed

$$P_{md} = (150 + 10) * 101.58 = 16.252\ kw$$

(a) Rotor copper loss $\mathrm{P_{rc}} = \left(\frac{\mathrm{S}}{1-\mathrm{S}}\right)\mathrm{P_{md}}$

$$= \frac{0.03}{1 - 0.03} * 16.252 = 0.5026\ kw$$

(b) Input to motor $\mathrm{P_i} = \mathrm{P_{md}} + \mathrm{P_{rc}} + \mathrm{P_{sc}}$

$$= 16.252 + 0.5026 + 0.7 = 17.4546\ kw$$

(c) $efficiency = \frac{P_o}{P_{in}} = \frac{15.236}{17.4546} = 87.29\%$

Example (5): a 500V, 6-pole, 50Hz, 3-phase induction motor develops 20kw inclusive of mechanical losses when running at 995 r.p.m., the p.f. being 0.87. Calculate (a) the slip, (b) the rotor I^2R loss, (c) the total input if the stator loss is 1500 w, (d) line current, (e) the rotor current frequency.

Solution:

$$\mathrm{N_S} = \frac{120\mathrm{f_s}}{\mathrm{P}} = \frac{120 * 50}{60} = 1000\ \mathrm{r.p.m.}$$

$$s = \frac{N_S - N_r}{N_S} = \frac{1000 - 995}{1000} = 0.005\ pu$$

Rotor on loss = slip * rotor power input

=s (mech. power developed +rotor Cu loss)

$$P_{rc} = s(P_m + P_{rc})$$

$$P_{rc}(1 - s) = sP_m$$

$$P_{rc} = \frac{sP_m}{(1 - s)} = \frac{0.005}{1 - 0.005} * 20 * 1000 = 100.5 \text{ W}$$

Total input to stator =rotor power input +stator loss

$$Rotor\ input = \frac{1}{s} *\ rotor\ Cu\ loss$$

$$= \frac{1}{0.005} * 100.5 = 20100\ W = 20.1\ KW$$

$$Hence\ total\ input = 20.1 + 1.5 = 21.6KW$$

$$Line\ current = \frac{21600}{\sqrt{3} * 500 * 0.87} = 28.7A$$

$$f_r = sf_s = 0.005 * 50 = 0.25\ Hz$$

7. Torque of an Induction Motor

Similar to a d.c motor the torque of an I.M. is also proportional to the product of stator flux per pole, the rotor current and power factor of the rotor, thus the torque is

$$T \propto \emptyset I_2 \cos\varphi_2$$

or

$$T = K\emptyset I_2 \cos\varphi_2 \qquad \ldots(16)$$

where:

K: constant

$\emptyset$: flux per pole

Since rotor e.m.f. per phase, $\mathrm{E_2}$ is proportion to the stator flux (at rotor stand still)

$$\therefore T = K_1 E_2 I_2 \cos\varphi_2$$

and

$$\mathrm{I_2} = \frac{\mathrm{sE_{20}}}{\sqrt{\mathrm{R_{20}^2} + (\mathrm{sx_{2o}})^2}}$$

$$\cos \varphi_2 = \frac{\mathrm{R_2}}{\sqrt{\mathrm{R_2^2} + (\mathrm{sx_{2o}})^2}}$$

Therefore

$$T = K_1 E_{20} \frac{\mathrm{sE_{20}}}{\sqrt{\mathrm{R_{20}^2} + (\mathrm{sx_{2o}})^2}} * \frac{\mathrm{R_2}}{\sqrt{\mathrm{R_2^2} + (\mathrm{sx_{2o}})^2}}$$

$$\therefore T = \frac{K_1 s R_2 E_{2o}^2}{R_2^2 + s^2 x_{2o}^2} \qquad \ldots(17)$$

Where, K$_1$ is constant, its value can be equal to $\frac{3}{2\pi \mathrm{N_S}}$.

Hence, expression for torque becomes

$$\therefore T = \frac{3}{2\pi N_S} . \frac{s R_2 E_{2o}^2}{R_2^2 + s^2 x_{2o}^2} \qquad \ldots(18)$$

Starting Torque

At start, s=1, therefore, stating torque may be obtained by putting s=1 in equation (18)

$$T_S = \frac{3}{2\pi N_S} . \frac{R_2 E_{2o}^2}{R_2^2 + x_{2o}^2} \qquad \ldots(19)$$

8. Condition for Maximum Torque

The torque of a rotor under running condition, (eq. 17), can be written as

$$T = \frac{K \frac{R_2}{s} E_{2o}^2}{(\frac{R_2}{s})^2 + x_{2o}^2} \qquad \ldots(20)$$

The condition for maximum torque may be obtained by differentiating equation (20) with respect to ($\frac{\mathrm{R_2}}{\mathrm{s}}$) and then putting it equal to zero.

$$\therefore \frac{dT}{d(\frac{R_2}{s})} = KE_{2o}^2 \left[\frac{1}{\left(\frac{R_2}{s}\right)^2 + x_{2o}^2} - \frac{\left(\frac{R_2}{s}\right)\left(2\frac{R_2}{s}\right)}{\left[\left(\frac{R_2}{s}\right)^2 + x_{2o}^2\right]^2} \right] = 0$$

$$\therefore \left(\frac{R_2}{s}\right)^2 + x_2^2 = 2\left(\frac{R_2}{s}\right)^2$$

$$R_2 = sX_{2o}$$

Slip corresponding to maximum torque is

$$\mathrm{s_m} = \frac{\mathrm{R_2}}{\mathrm{X_2}} \qquad \dots(21)$$

This slip is sometime written as S_b and the maximum torque as T_b.

Putting $\mathrm{R_2} = \mathrm{sX_{2o}}$ in the eq. (20) for the torque, we get

$$T_{max} = \frac{KE_{2o}^2}{2X_{2o}} \qquad \dots(22)$$

For obtaining maximum torque at starting (s=1), rotor resistance must equal to rotor reactance.

$$R_2 = x_{2o}$$

9. Torque – Slip and Torque Speed Characteristics

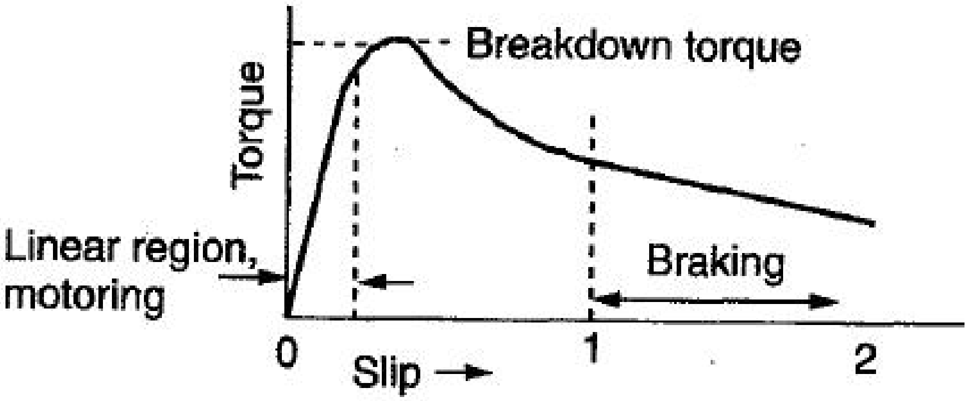
The torque equation can be written as

$$T = \frac{KsR_2E_{20}^2}{R_2^2 + (sX_{20})^2}$$

It is seen that if R_2 and X_{20} are kept constant, the torque T depends upon the slip s. The torque – slip characteristic curve can be divided roughly into three regions:

- Low – Slip region
- Medium – Slip region
- High – Slip region

The torque – slip curve is given below



Torque – slip curve.

Low – Slip region

At synchronous speed s=0, therefore, the torque is zero. When the speed is very near to synchronous speed, the slip is very low and $(SX_{20})^2$ is negligible in comparison with R_2. Therefore,

$$T = \frac{K_1 s}{R_2}$$

If R_2 is constant, the equation become as

$$T = K_2 s \qquad \dots (*)$$

where $K_2 = \frac{K_1}{R_2}$

Relation (*) shows that the torque is proportional to the slip. Hence, when the slip is small (which is the normal working region of the motor), the torque – slip curve is straight line.

Medium – Slip region

As slip increases (that is, as the speed decreases with the increase with the load), the term $(sX_{20})^2$ becomes large, so that R_2^2 may be neglected in comparison with $(sX_{20})^2$ and

$$T = \frac{K_3 R_2}{s X_{20}^2}$$

Thus, the torque is inversely proportional to slip towards standstill conditions. The torque slip characteristic is represented by a rectangular hyperbola. For intermediate value of the slip, the graph changes from one form to another. In doing so, it passes through the point of maximum torque when $R_2 = sX_{20}$. The maximum torque developed in an induction motor is called pull – out torque or breaking torque.

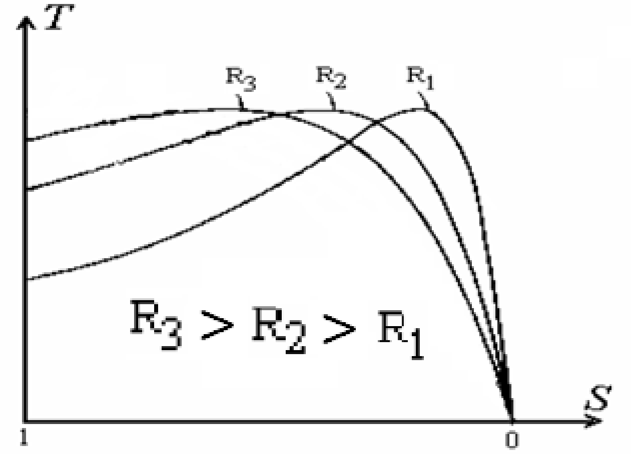
High – slip region

When the slip is further increased (the motor speed is further decreased) with further increase in motor load, then R_2 becomes negligible as compared to (SX_{20}). Therefore, for large value of slip

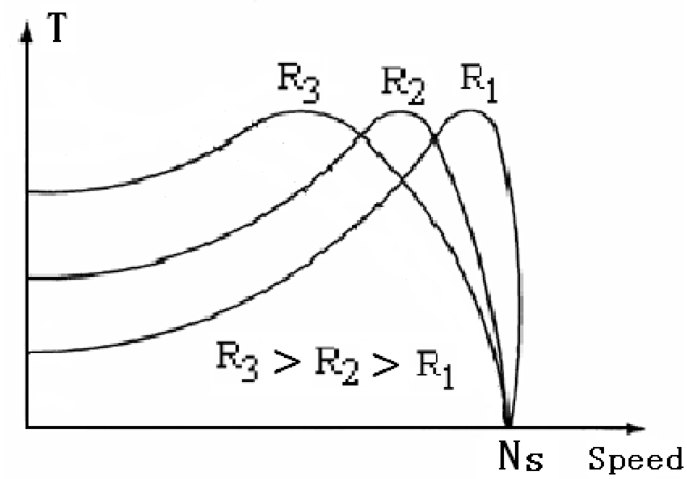
$$T \propto \frac{s}{(SX_{20})^2} \quad or \quad T \propto \frac{1}{s}$$ (if standstill reactance is constant)

It will be seen that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. At this stage, the overload protection must immediately disconnect the motor from the supply to prevent damage due to overheating.In fact the stable operation of the motor lies between the values of s=0 and that corresponding to maximum torque.

The torque – slip and torque – speed curves are given below respectively, with R_2 as third parameter. It is seen that although maximum torque does not depend on R_2 , yet the exact location of T_{max} is dependent on it. Greater the R_2, the greater is the value of slip at which the maximum torque occurs.



torque – slip curve



torque – speed curve

Example 6

The rotor resistance and standstill reactance pre phase of a 3 – phase slip – ring induction motor are 0.05Ω and 0.1Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to give maximum torque at starting?

Solution

Let external resistance per phase added to the rotor circuit be r ohms.

Rotor resistance per phase, $R_2 = (0.05 + r)$

The starting torque should be maximum when

$$R_2 = X_{20}$$

$$0.05 + r = 0.1$$

$$\therefore\ r = 0.05\Omega$$

Example 7: A 3-phase induction motor with rotor resistance per phase equal to the standstill rotor reactance has a starting torque of 25 Nm. For negligible stator impedance and no – load current, determine the starting torque in case the rotor circuit resistance per phase is (a) doubled, (b) halved.

Solution

$$R_2 = X_{20}$$

$$T_s = \frac{kR_2}{R_2^2 + X_{20}^2}$$

$$25 = \frac{kR_2}{R_2^2 + X_{20}^2}$$

$$\therefore\ k = 50R_2$$

(a) New rotor resistance $= 2R_2$

$$\therefore\ T_s = \frac{k(2R_2)}{(2R_2)^2 + R_2^2} = \frac{50R_2 * 2R_2}{5R_2^2} = 20\ Nm$$

(b) Rotor resistance $= \frac{1}{2}R_2$

$$\therefore\ T_s = \frac{k(\frac{1}{2}R_2)}{(\frac{1}{2}R_2)^2 + R_2^2} = \frac{50R_2 * \frac{1}{2}R_2}{\frac{R_2^2}{4} + R_2^2} = 20\ Nm$$

Example 8: A 6-pole, 3-phase, 50 Hz induction motor develops a maximum torque of 30Nm at 960 r.p.m. Determine the torque exerted by the motor at 5% slip. The rotor resistance per phase is 0.6Ω.

Solution

$$N_s = \frac{120f}{P} = \frac{120*50}{6} = 1000\, r.p.m.$$

Speed at maximum torque, $N_m = 960\, r.p.m.$

Slip at maximum speed

$$s_M = \frac{N_s - N_m}{N_s} = \frac{1000 - 960}{1000} = 0.04$$

Also $$\mathrm{s_M} = \frac{\mathrm{R_2}}{\mathrm{X_{2o}}}$$

$$\mathrm{X_{2o}} = \frac{\mathrm{R_2}}{\mathrm{s_M}} = \frac{0.6}{0.04} = 15\Omega$$

If T_{fs} is the torque at slip s

$$\frac{\mathrm{T_{fs}}}{\mathrm{T_M}} = \frac{2\mathrm{s}\mathrm{s_M}}{\mathrm{s}^2 + \mathrm{s_M^2}}$$ Here s=0.05 , $T_M = 30$ Nm

$$\mathrm{T_{fs}} = \frac{2*0.05*0.04}{(0.05)^2 + (0.04)^2} * 30 = 29.27\mathrm{Nm}$$

10. Starting Method for Induction Motors

A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation.

Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;

1) Reduce heavy starting currents and prevent motor from overheating.

2) Provide overload and no-voltage protection.

There are many methods in use to start 3-phase induction motors. Some of the common methods are;

- Direct On-Line Starter (DOL)
- Star-Delta Starter
- Auto Transformer Starter

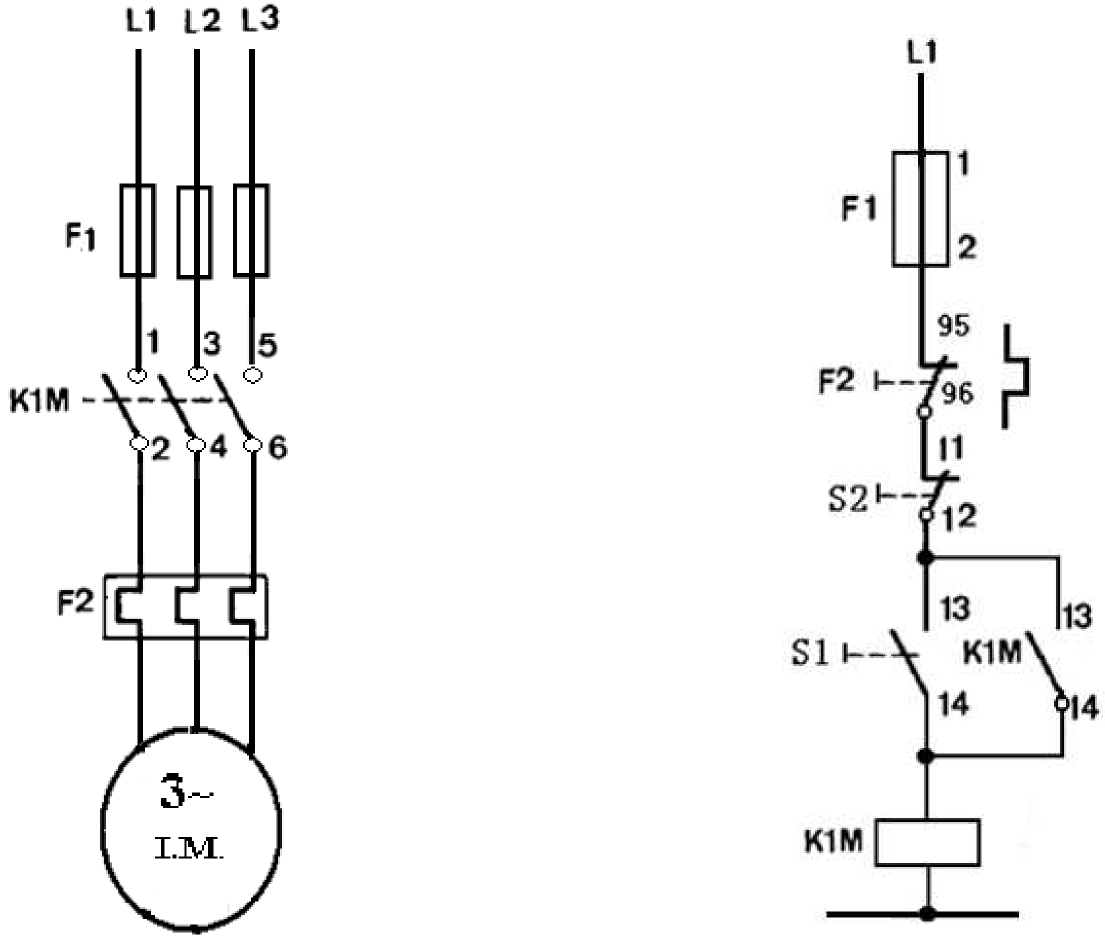
- Rotor Impedance Starter
- Power Electronics Starter

Direct On-Line Starter (DOL)

The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors. It directly connects the contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current. The starting torque is likely to be 0.75 to 2 times the full load torque. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW

There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor.The power and control circuits of induction motor with DOL starter are shown in figure(1).

* K1M Main contactor



Power Circuit

Control Circuit

Fig.(1): power and control circuits of I.M. with DOL starter

The DOL starter consists of a coil operated contactor K1M controlled by start and stop push buttons. On pressing the start push button S1, the contactor coil K1M is energized from line L1. The three mains contacts (1-2), (3-4), and (5-6) in fig. (1) are closed. The motor is thus connected to the supply. When the stop push button S2 is pressed, the supply through the contactor K1M is disconnected. Since the K1M is de-energized, the main contacts (1-2), (3-4), and (5-6) are opened. The supply to motor is disconnected and the motor stops.

Star-Delta Starter

The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters. This method used reduced supply voltage in starting. Figure(2) shows the connection of a 3phase induction motor with a star – delta starter.

The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration.

By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta. At the time of starting when the stator windings are start connected, each stator phase gets voltage $V_L/\sqrt{3}$, where V_L is the line voltage. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star- delta starting reduced the starting torque to one – third that obtainable by direct delta starting.

- K2M Main Contactor
- K3M Delta Contactor
- K1M Star Contactor
- F1 Thermal Overload Relay

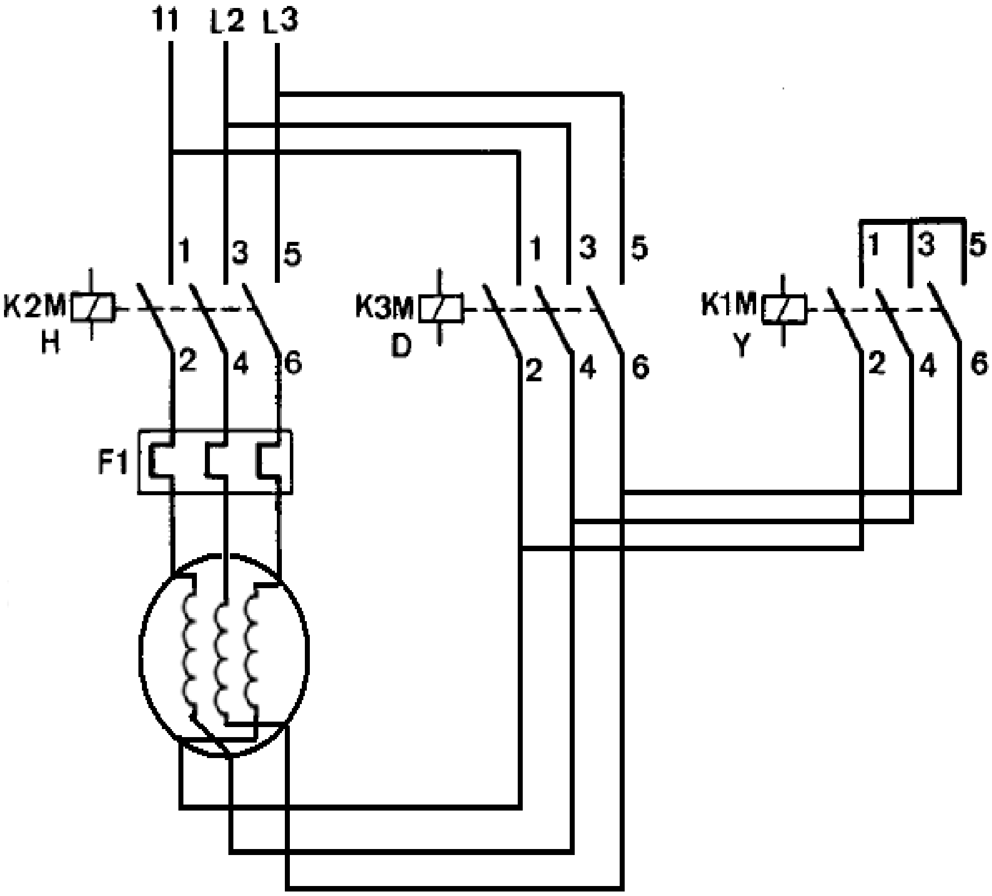


Fig.(2) Induction Motor with Star Delta Starter

Auto Transformer Starter

The operation principle of auto transformer method is similar to the star delta starter method. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage.

The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer. When the star delta method is considered, voltage can be adjusted only by factor of $1/\sqrt{3}$.

Figure (3) shows the connection of a 3phase induction motor with auto transformer starter.

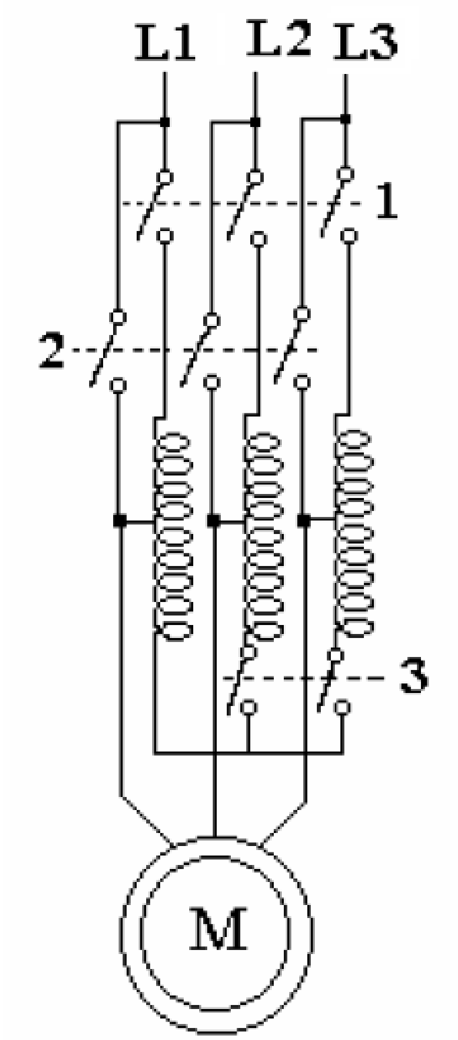


Fig.(3) shows I.M with auto transformer starter.

Rotor Impedance Starter

This method allows external resistance to be connected to the rotor through slip rings and brushes. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero.

The rotor impedance starting mechanism is usually very bulky and expensive when compared with other methods. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them. The starting frequency is also limited in this method. However, the rotor impedance method allows the motor to be started while on load. Figure (4) shows the connection of a 3phase induction motor with rotor resistance starter.

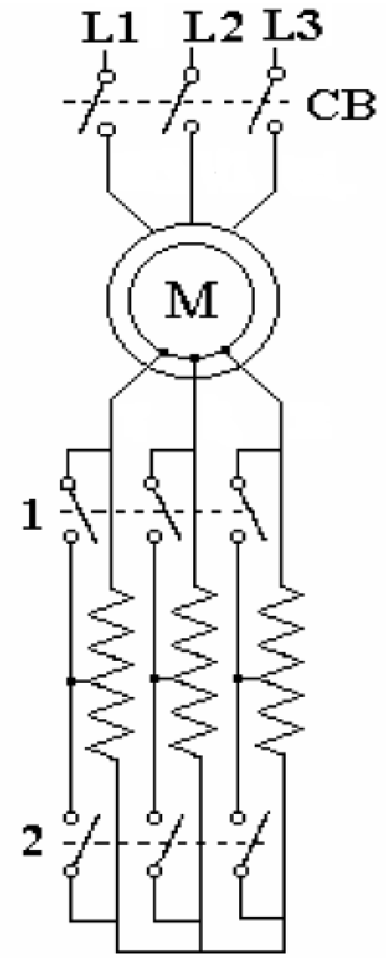


Fig. (4) Shows the I.M. with rotor resistance starter.

Example (9):

It is desired to install a 3-phase cage induction motor restricting the maximum line current drawn from a 400 V 3-phase supply to 120 A. if the starting current is 6 times full load current, what is the maximum permissible full load kVA of the motor when

i. It is directly connected to the mains
ii. It is connected through an auto-transformer with a tapping of 60%
iii. It is designed for used with star-delta starter.

Solution:

i. Direct-on-line starting

Maximum line current, $\mathrm{I_L = 120A}$

Starting current $\mathrm{I_{st} = 6 \times full\ load\ current = 6I_{fl}}$

Since the maximum line current drawn from the supply is 120A

$$6\mathrm{I_{ft}} = 120, \qquad \mathrm{I_f} = \frac{120}{6} = 20\mathrm{A}$$

Maximum permissible rating of the motor

$$= \sqrt{3}\mathrm{V_L I_{ft}} = \sqrt{3} \times 20 \times 400 = 13856\ \mathrm{VA} = 13.856\ \mathrm{k\,VA}$$

ii. Auto-transformer starting

$$\mathrm{I_{st}} = \mathrm{x^2 I_{sc}} = \mathrm{x^2(6I_{ft})}$$

$$120 = (0.6)^2(6\mathrm{I_{ft}})$$

$$\mathrm{I_{ft}} = \frac{120}{6 \times (0.6)^2} = 55.55\mathrm{A}$$

Maximum permissible rating of the motor

$$= \sqrt{3}\mathrm{V_L I_{ft}} = \sqrt{3} \times 400 \times 55.55 = 38.49\ \mathrm{k\,VA}$$

iii. Star-delta starting

$$\mathrm{I_{st}} = \frac{1}{3}(6\mathrm{I_{ft}})$$

$$120 = 2I_{ft}, \qquad I_{ft} = 60A$$

Maximum permissible kVA rating of the motor

$$= \sqrt{3}\mathrm{V_L I_{ft}} = \sqrt{3} \times 400 \times 60 = 41.56\ \mathrm{k\,VA}$$

11. SPEED CONTROL OF INDUCTION MOTORS

The speed of an induction motor is given as

N = 120f/p (1-S).

So obviously the speed of an induction motor can be controlled by varying any of three factors namely supply frequency f, number of pole P or slip S.

The main methods employed for speed control of induction motors are as follows:

1. Pole changing
2. Stator voltage control
3. Supply frequency control
4. Rotor resistance control
5. Slip energy recovery.

The basic principles of these methods are described below

Pole changing

The number of stator poles can be change by

- Multiple stator windings
- Method of consequent poles
- Pole amplitude modulation (PWM)

The methods of speed control by pole changing are suitable for cage motors only because the cage rotor automatically develops number of poles equal to the poles of stator winding.

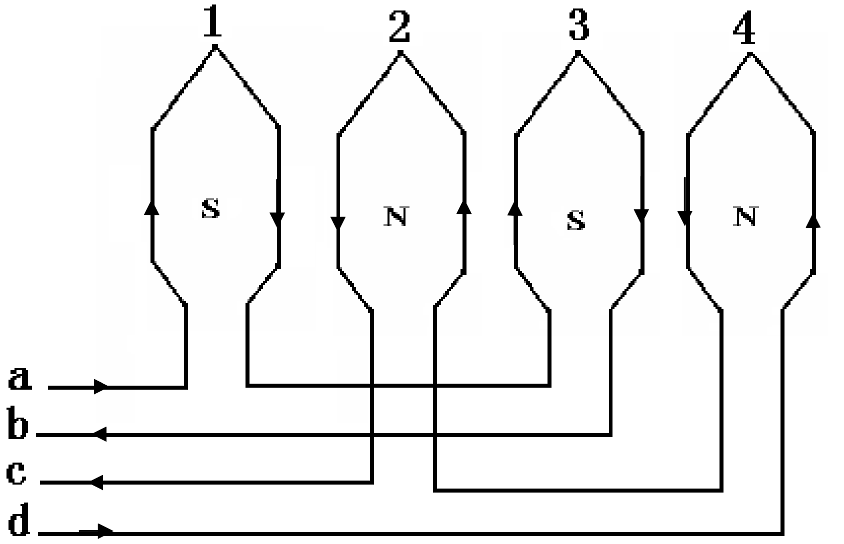
1. Multiple stator windings

In this method the stator is provided with two separate windings which are wound for two different pole numbers. One winding is energized at a time. Suppose that a motor has two windings for 6 and 4 poles. For 50 Hz supply the synchronous speed will be 1000 and 1500 rpm respectively. If the full load slip is 5% in each case, the operating speeds will be 950 rpm and 1425 rpm respectively. This method is less efficient and more costly, and therefore, used only when absolutely necessary.

2.Method of consequent poles

In this method a single stator winding is divided into few coil groups. The terminals of all these groups are brought out. The number of poles can be changed with only simple changes in coil connections. In practice, the stator winding is divided only in two coil groups. The number of poles can be changed in the ratio of 2:1.

Fig.(1) shows one phase of a stator winding consisting of 4 coils divided into two groups a – b and c – d. Group a – b consists of odd numbered coils(1,3) and connected in series. Group c – d has even numbered coils (2, 4) connected in series. The terminals a,b,c,d are taken out as shown.



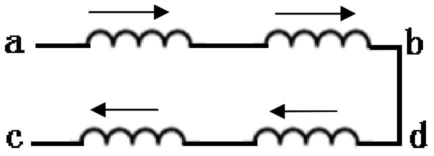


fig. (1-b)

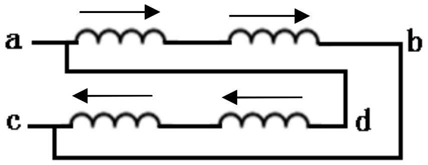


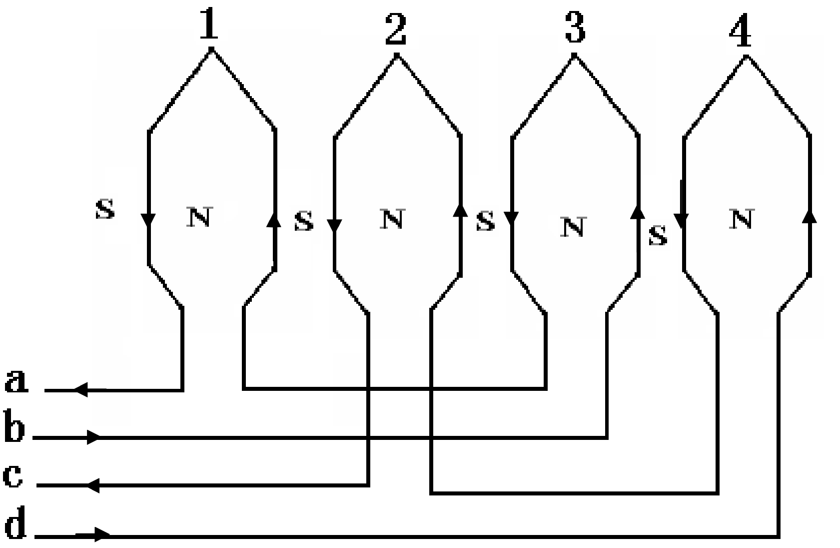
fig. (1-c)

Fig. (1) Stator phase connections for 4 poles

The coils can be made to carry current in given directions by connecting coil groups either in series or parallel shown in fig. (1-b) and fig.(1-c) respectively.

With this connection, there will be a total of 4 poles giving a synchronous speed of 1500 rpm for 50 Hz system. If the current through the coils of group a – b is reversed (fig.2), then all coils will produce north (N) poles.

In order to complete the magnetic path, the flux of the pole groups must pass through the spaces between the groups, thus inducing magnetic poles of opposite polarity (S poles) in the inter – pole spaces.



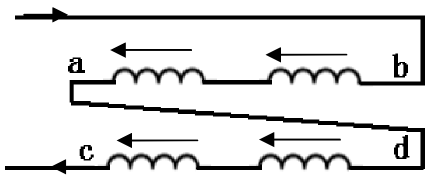


fig. (2-b) series connection

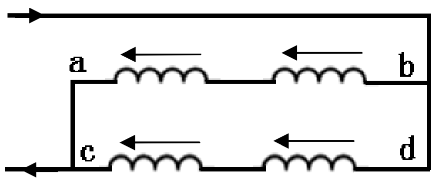


fig. (2-c) parallel connection

Fig. (2) Stator phase connections for 8 poles

. Stator Voltage Control

The torque developed by an induction motor is proportional to the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig.(3). These curves show that the slip at maximum torque s_m remains same, while the value of stall torque comes down with decrease in applied voltage.

Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed $T \propto \omega^2$.

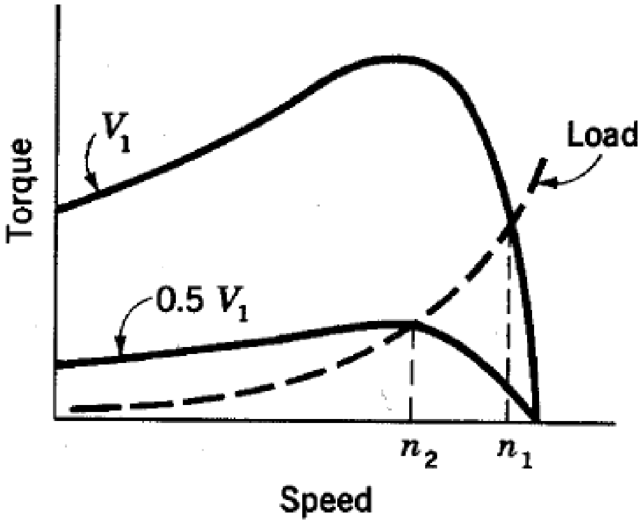


Fig. (3): Torque - speed curves for various terminal voltages

Supply Frequency Control

The synchronous speed of an induction motor is given by

$$N_S = \frac{120 f_s}{P}$$

The synchronous speed and, therefore, the speed of motor can be controlled by varying the supply frequency.

The emf induced in the stator of an induction motor is given by

$$E = 4.44 \mathrm{k} \emptyset_m \mathrm{f}_s N_1$$

Therefore, if the supply frequency is change, E_1 will also change to maintain the same air gap flux. If the stator voltage drop is neglected the terminal voltage V_1 is equal to E_1 . in order to avoid saturation and to minimize losses, motor is operated at rated air gap flux by varying terminal voltage with frequency so as to maintain $(^{\mathrm{V}}/_{\mathrm{f}})$ ratio constant at rated value.

This type of control is known as constant volt in per hertz. Thus, the speed control of an induction motor using variable frequency supply requires a variable voltage power source.

Rotor Resistance Control

In wound rotor induction motor, it is possible to change the shape of the torque – speed curve by inserting extra resistance into rotor circuit of the machine. The resulting torque – speed characteristic curves are shown in fig.(4).

This method of speed control is very simple. It is possible to have a large starting torque and low starting current at small value of slip.

The major disadvantage of this method is that the efficiency is low due to additional losses in resistors connected in the rotor circuit. Because of convenience and simplicity, it is often employed when speed is to be reduced for a short period only (cranes).

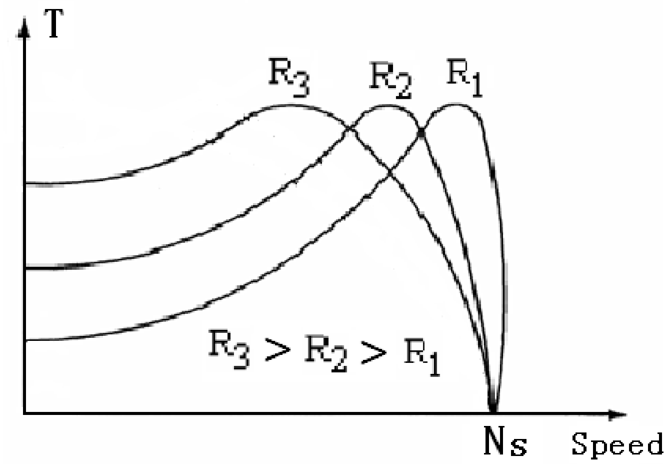


Fig.(4) Torque – speed curve for rotor resistance variation

Single Phase Motors

As the name suggests, these motors are used on single – phase supply. Single phase motors are the most common type of electric motors, which finds wide domestic, commercial and industrial applications. Single phase motors are small size motors of fraction – kilowatt ratings. Domestic applications like fans, hair driers, washing machines, mixers, refrigerators, food processors and other kitchen equipment employ these motors. These motors also find applications in air – conditioning fans, blower’s office machinery etc.

Single phase motors may be classified into the following basic types:

1. Single phase induction motors
2. AC. Series motor (universal motor)
3. Repulsion motors
4. Synchronous motor

Single Phase Induction Motor

A single phase induction motor is very similar to 3 – phase squirrel cage induction motor. It has a squirrel – cage rotor identical to a 3 - phase squirrel cage motor and a single – phase winding on the stator. Unlike 3 – phase induction motor, a single phase induction motor is not self starting but requires some starting means.

Figure (1) shows 1 – phase induction motor having squirrel cage rotor and single phase distributed stator winding.

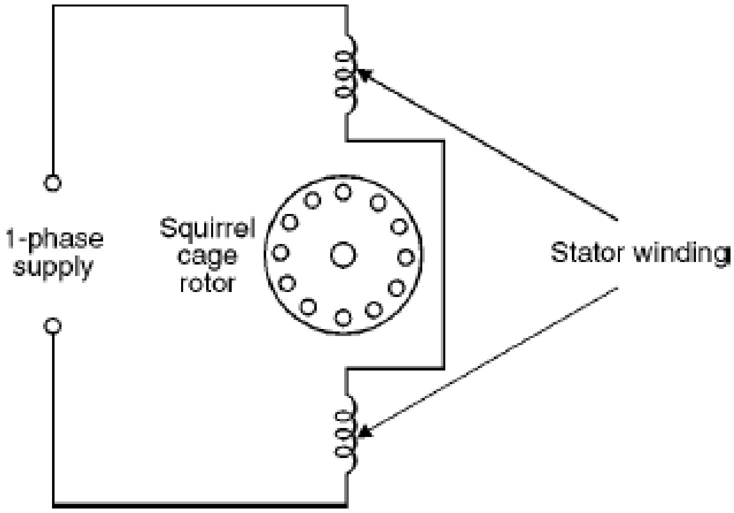


Fig. (1) Single – phase induction motor

If the stator winding is connected to single – phase a.c. supply, the stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel cage rotor. However, if the rotor is started by auxiliary means, the motor will quickly attain the final speed. The behavior of single – phase induction motor can be explained on the basic of double – field revolving theory.

Double – Field Revolving Theory

The pulsating field produced in single phase AC motor is resolved into two components of half the magnitude and rotating in opposite directions at the same synchronous speed.

Let $\emptyset_m$ be the pulsating field which has two components each of magnitude ${}^{\emptyset_m}/_{2}$. Both are rotating at the same angular speed ω rad/sec but in opposite direction as shown in the Figure (2-a). The resultant of the two fields is $\emptyset_m \cos\theta$. Thus the resultant field varies according to cosine of the angle θ. The wave shape of the resultant field is shown in Figure (2-b).

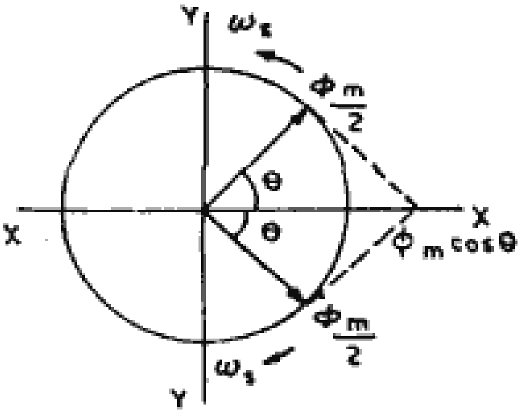


Fig. (2-a)

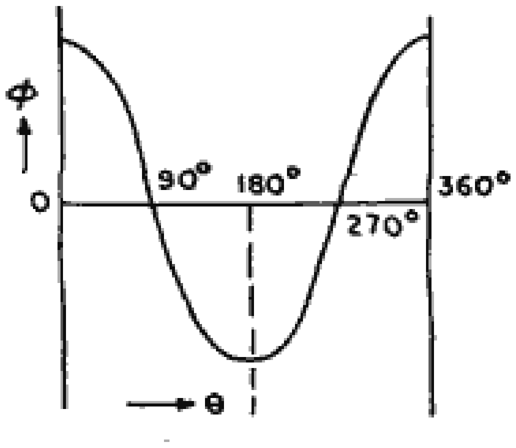


Fig. (2-b)

Thus the alternating flux produced by stator winding can be presented as the sum of two rotating fluxes $\emptyset_1$ and $\emptyset_2$ each equal to one half of the maximum value of alternating flux and each rotating at synchronous speed in opposite directions. Let the flux $\emptyset_1$ (forward) rotate in anticlockwise direction and flux $\emptyset_2$ (backward) in clockwise direction. The flux $\emptyset_1$ will result in the production of torque T_1 in the anticlockwise direction and flux $\emptyset_2$ will result in the production of torque T_2 in the clockwise direction. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single – phase induction motor is not self – starting. This fact is illustrated in figure(3).

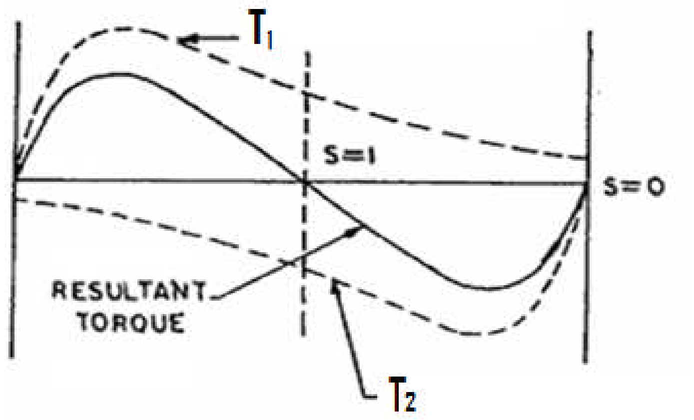


Fig. (3) Torque – slip characteristic of 1- phase induction motor

Rotor Running

Assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux $\emptyset_f$ and that in the other direction is the backward rotating flux $\emptyset_b$. The slip w.r.t. the forward flux will be

$$s_f = \frac{N_s - N}{N_s}$$

Where N_s = synchronous speed

N = speed of rotor in the direction of forward flux

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t the backward flux will be

$$s_b = \frac{N_s - (-N)}{N_s} = \frac{N_s + N}{N_s} = \frac{2N_s - N_s + N}{N_s}$$

$$= \frac{2N_s}{N_s} - \frac{(N_s - N)}{N_s} = 2 - s$$

$$\therefore \qquad s_b = 2 - s$$

Thus for forward rotating flux, slip is s (less than unity) and for backward rotating flux, the slip is 2-s (greater than unity) since for usual rotor resistance/reactance ratios, the torque at slips of less than unity are greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

Starting of Single Phase Induction Motors

The single phases induction motors are classified based on the method of starting method and in fact are known by the same name descriptive of the method.

1. Split – phase Induction Motor

The stator of a split – phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90 electric degrees as shown in figure (4-a).

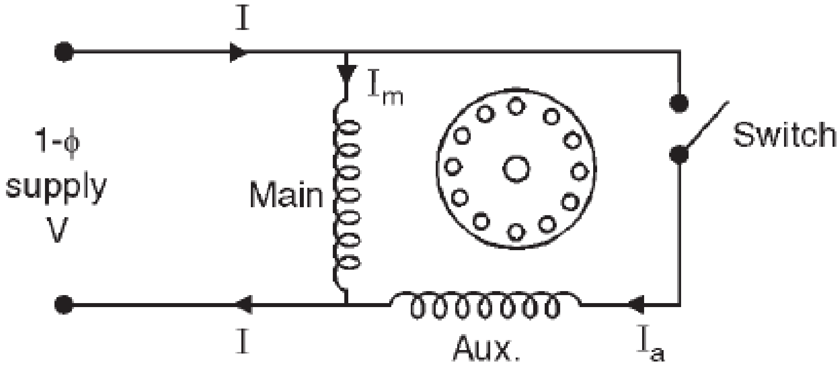


Fig.(4-a) split phase I.M.

The auxiliary winding is made of thin wire so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. When the two stator windings are energized from a single – phase supply, the current I_m and I_a in the main winding and auxiliary winding lag behind the supply voltage V, and I_a leading the current I_m as shown in figure (4-b).

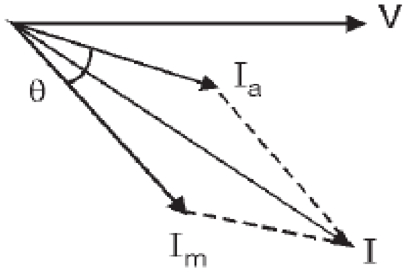
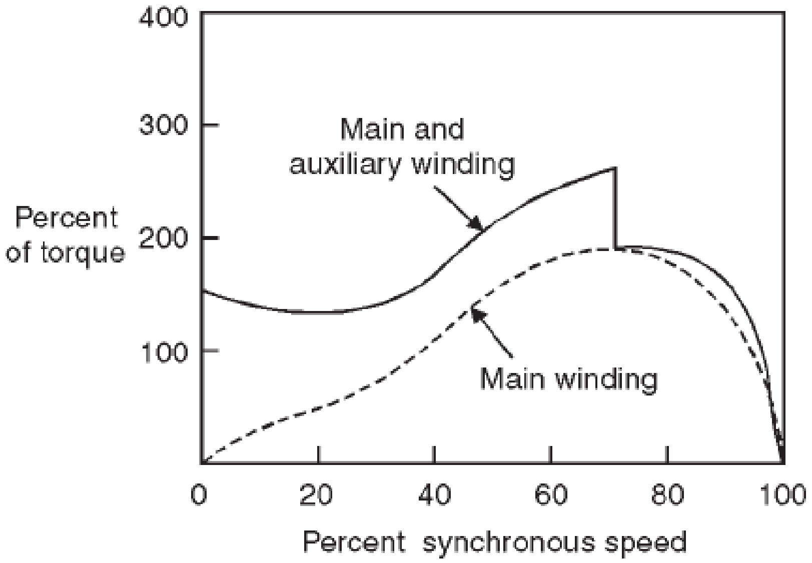


Fig.(4-b) Phasor diagram at starting

This means the current through auxiliary winding reaches maximum value first and the mmf or flux due to I_a lies along the axis of the auxiliary winding and after some time the current I_m reaches maximum value and the mmf due to I_m lies along the main winding axis. Thus the motor becomes a 2 – phase unbalanced motor. Because of these two fields a starting torque is developed and the motor becomes a self starting motor. After the motor starts, the auxiliary winding is disconnected usually by means of centrifugal switch that operates at about 75% of synchronous speed. Finally the motor runs because the main winding. Since this being single phase some level of humming noise is always associated with the motor during running. The power rating of such motors generally lies between 60- 250W.

The typical torque – speed characteristic is shown in fig (4-c).



Characteristics

- Due to their low cost, split – phase induction motors are most popular single – phase motors in the market
- Since the starting winding is made of thin wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built – in thermal relay. This motor is, therefore, suitable where starting periods are not frequent.

2. Capacitor – Start Motor

Capacitors are used to improve the starting and running performance of the single phase inductions motors.

The capacitor – start motor is identical to a split – phase motor except that the starting winding has as many turns as the main winding. Moreover, a capacitor C is connected in series with the starting winding as shown in figure (5-a).

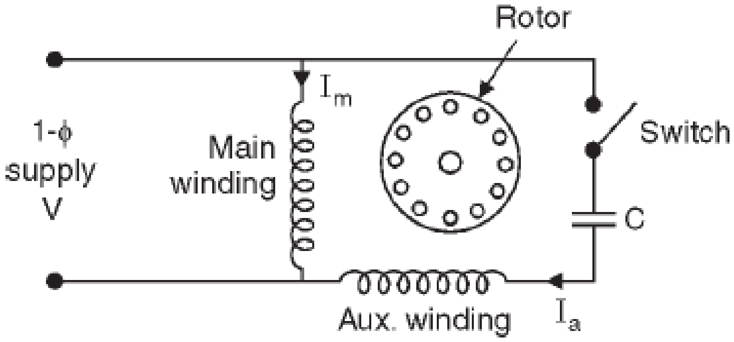
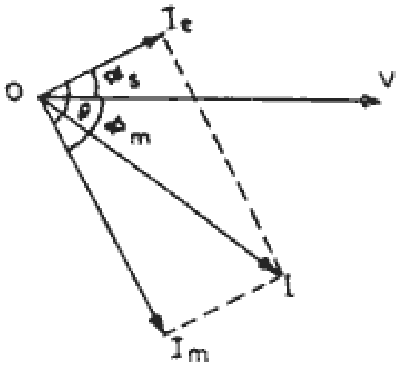
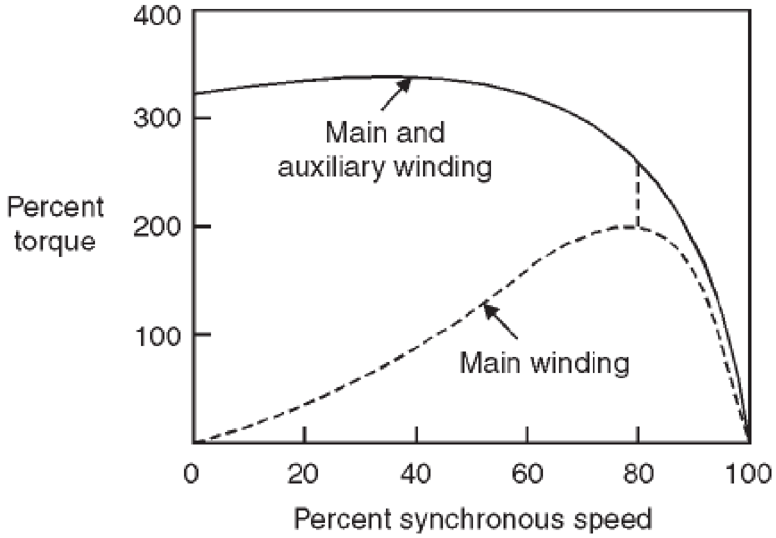


Fig.(5-a) Capacitor Start Motor

The value of capacitor is so chosen that I_a leads I_m by about 90^o (Fig.5-b) so that the starting torque is maximum for certain values of I_a and I_m. Again, the starting winding is opened by the centrifugal switch when the motor attains about 75% of synchronous speed. The motor then operates as a single – phase induction motor and continues to accelerate till it reaches the normal speed.



The typical torque – speed characteristic is shown in fig (5-c).



Characteristics

- Although starting characteristics of a capacitor – start motor are better than those of a split – phase motor, both machines possess the same running characteristics because the main windings are identical.
- The phase angle between the two currents is about 90 compared to about 25° in a split – phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split – phase motor. Therefore, the starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.

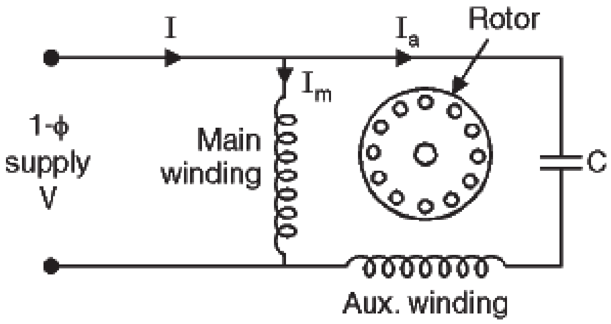
- Capacitor – start motors are used where high starting torque is required and where high starting period may be long e.g. to drive:
  a) Compressors b) large fans c) pumps d)high inertia loads

  The power rating of such motors lies between 120W and 0.75 kW.

3. Permanent – Split Capacitor Motor

In this motor, as shown in fig.(6-a), the capacitor that is connected in series with the auxiliary winding is not cut out after starting and is left in the circuit all the time. This simplifies the construction and decreases the cost because the centrifugal switch is not needed. The power factor, torque pulsation, and efficiency are also improved because the motor runs as a two – phase motor. The motor will run more quietly.

The capacitor value is of the order of 20 – 50 μF and because it operates continuously, it is an ac paper oil type. The capacitor is compromise between the best starting and running value and therefore starting torque is sacrificed. The typical torque – speed characteristic is shown in fig (6-b).



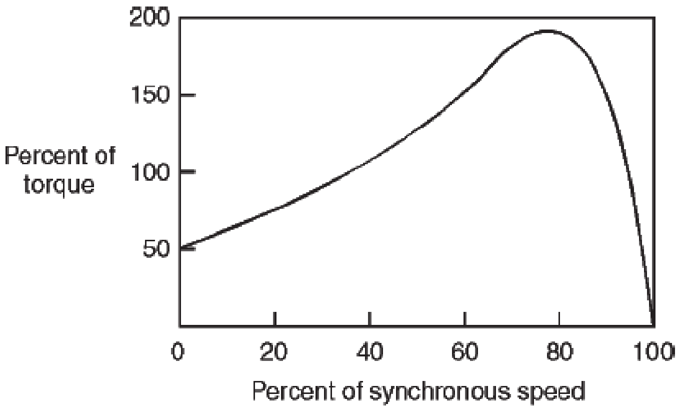


Fig.(6-a) Permanent – Split Capacitor Motor Fig.(6-b) torque – speed characteristic

Characteristic

- These motor are used where the required starting torque is low such as air – moving equipment i.e. fans, blowers and voltage regulators and also oil burners where quite operation is particularly desirable.

4. Capacitor - Start Capacitor - Run

Two capacitor, one for starting and one for running, can be used, as shown in fig.(7-a).

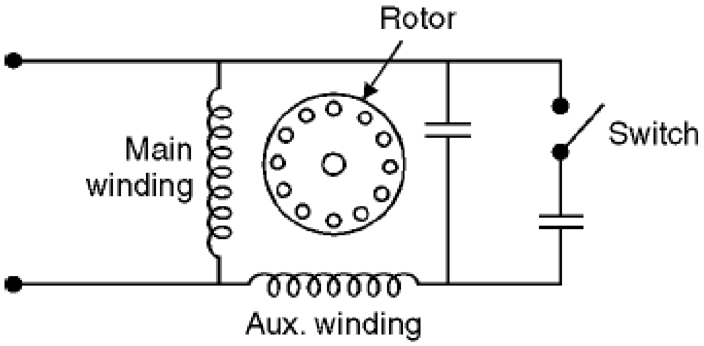


Fig. (7-a) Capacitor - Start Capacitor – Run motor

Theoretically, optimum starting and running performance can be achieved by having two capacitors. The starting capacitor is larger in value and is of the ac electrolytic type. The running capacitor permanently connected in series with the starting winding, is of smaller value and is of the paper oil type. Typical values of these capacitors for a 0.5 hp are C_s =300µF, C_r =40µF. The typical torque – speed characteristic is shown in fig. (7- b).

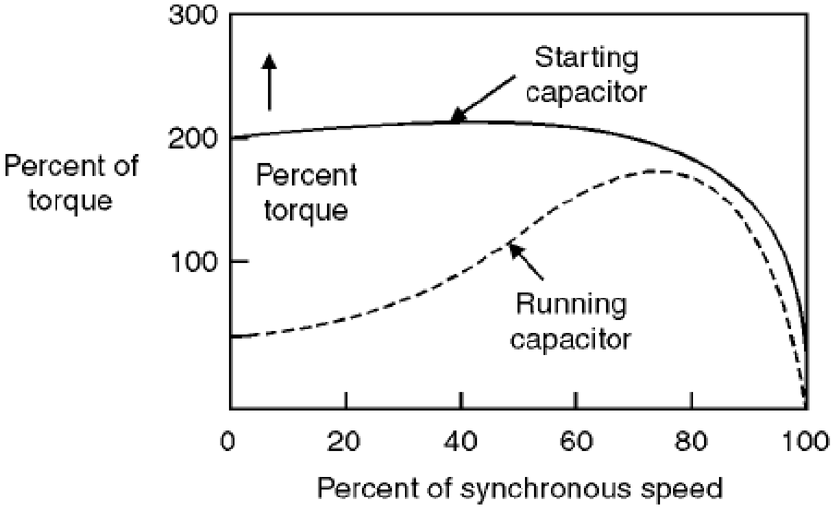


Fig.(7-b) torque – speed characteristic

Characteristic

- Ability to start heavy loads
- Extremely quiet operation
- Higher efficiency and power factor
- Ability to develop 25 per cent overload capacity. Hence, such motors are ideally suited where load requirements are severe as in the case of compressors and conveyors ect.

5.Shaded Pole Induction Motor

These motors have a salient pole construction. A shaded band consisting of a short – circuited copper turn, known as a shading coil, is used on one portion of each pole, as shown in fig(8-a)

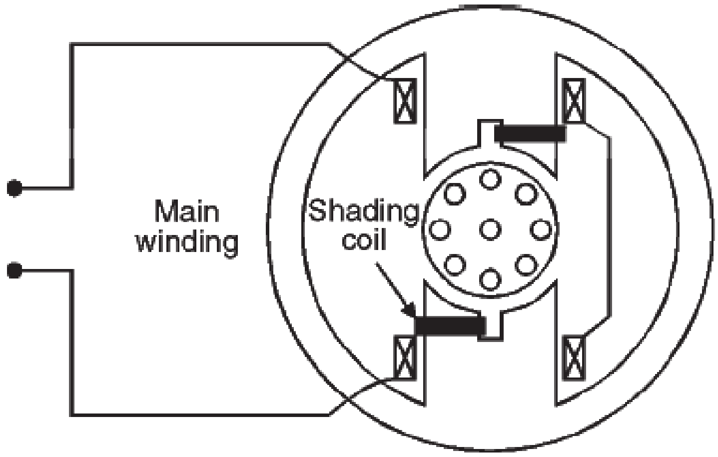


Fig (8-a) Shaded Pole Induction Motor

When alternating current flow in the field winding, an alternating flux is produced in the field core. A portion of this flux links with the shading coil, which behaves as short – circuited secondary of a transformer. A voltage is induced in the shading coil, and this voltage circulates a current in it. The induced current produces a flux called the induced flux which opposes the main core flux. The shading coil, thus, causes the flux in the shaded portion to lag behind the flux in the unshaded portion of the pole. At the same time, the main flux and the shaded pole flux are displaced in space. This displacement is less than 90°. Since there is time and space displacement between the two fluxes, the conditions for setting up a rotating magnetic field are produced. Under the action of the rotating flux a starting torque is developed on the cage rotor. The direction of this rotating field (flux) is from the unshaded to the shaded portion of the pole.

The typical torque-speed characteristic is shown in fig. (8-b).

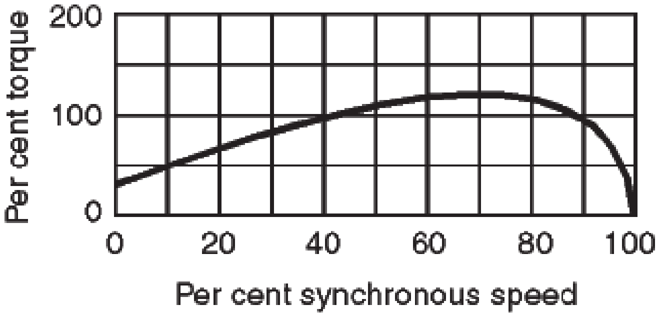


Fig.(8-b) torque – speed characteristic

Characteristic

- The salient features of this motor are extremely simple construction and absence of centrifugal switch
- Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g. to drive: Small fans b) toys c) hair driers. The power rating of such motors is up to about 30 W.

Equivalent Circuit of Single – Phase Induction Motor

When the stator of single phase induction motor is connected to single – phase supply, the stator current produces a pulsating flux. According to the double – revolving field theory, the pulsating air – gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor. Since the magnitude of each rotating flux is one – half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single – phase induction motor may be considered as consisting of two motors having a common stator winding and two imaginary rotors, which rotate in opposite directions. The standstill impedance of each rotor referred to the main stator winding is $(\frac{\acute{R}_2}{2} + j\,\frac{\acute{X}_2}{2})$.

The equivalent circuit of single – phase induction motor at standstill is shown in fig.(9).

R_{1m} = resistance of stator winding

X_{1m} = leakage reactance of stator winding

X_M = total magnetizing reactance

$\acute{R}_2$= resistance of rotor referred to the stator

$\acute{X}_2$= leakage reactance of rotor referred to the stator

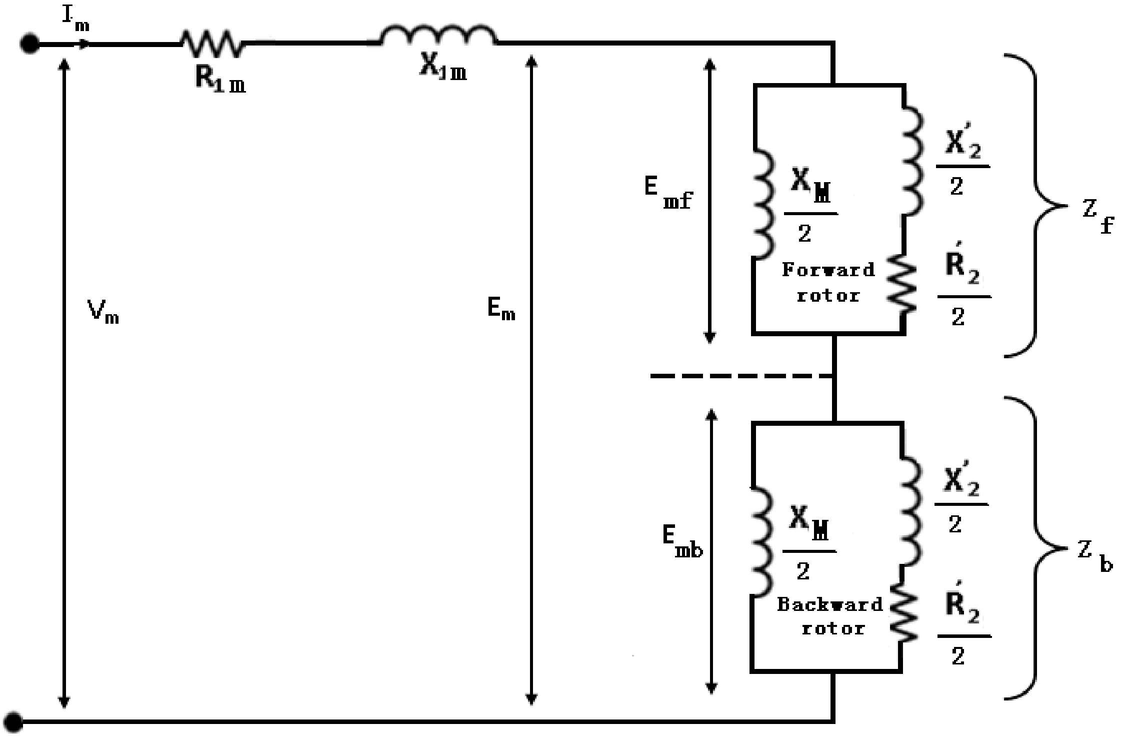


Fig.(9)

In this diagram, the portion of the equivalent circuit representing the effects of air gap flux is split into two portions. The first portion shows the effect of forward rotating flux, and the second portion shows the effect of backward rotating flux.

The forward flux induces a voltage E_{mf} in the main stator winding. The backward rotating flux induces a voltage E_{mb} in the main stator winding. The resultant induced voltage in the main stator winding is E_m , where

$$\mathbf{E_m = E_{mf} + E_{mb}}$$

At standstill, $\mathbf{E_{mf} = E_{mb}}$

Now suppose that the motor is started with the help of an auxiliary winding. The auxiliary winding is switched out after the motor gains it normal speed.

The effective rotor resistance of an induction motor depends on the slip of the rotor. The slip of the rotor with respect to the forward rotating flux is S. The slip the rotor with respect to the backward rotating flux is (2-S).

When the forward and backward slips are taken in account, the result is the equivalent circuit shown in fig.(10) which represent the motor running on the main winding alone.

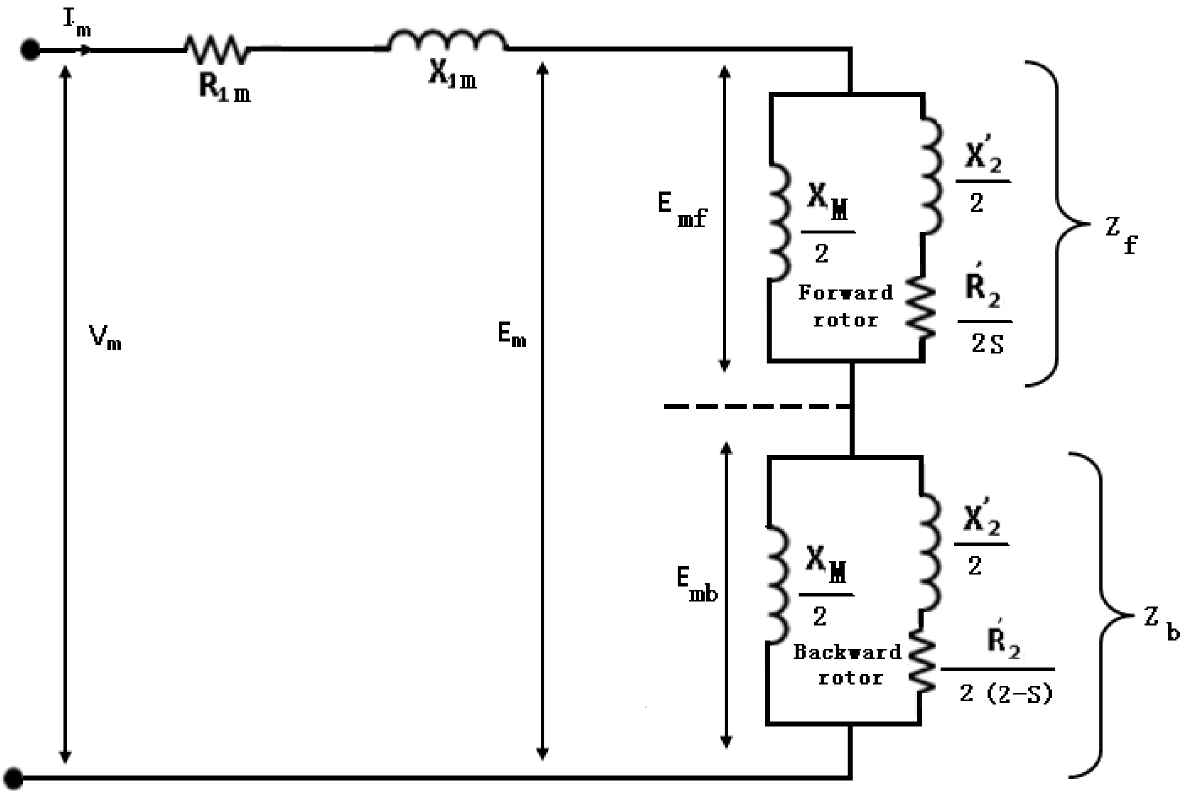


Fig.(10)

The rotor impedance representing the effect of forward field referred to the stator winding m is given by an impedance $(\frac{\acute{R}_2}{2S} + j\,\frac{\acute{X}_2}{2})$ in parallel with $j\,\frac{X_M}{2}$.

$$\therefore \qquad Z_f = R_f + jX_f = (\frac{\acute{R}_2}{2S} + j\,\frac{\acute{X}_2}{2}) \parallel (j\,\frac{X_M}{2})$$

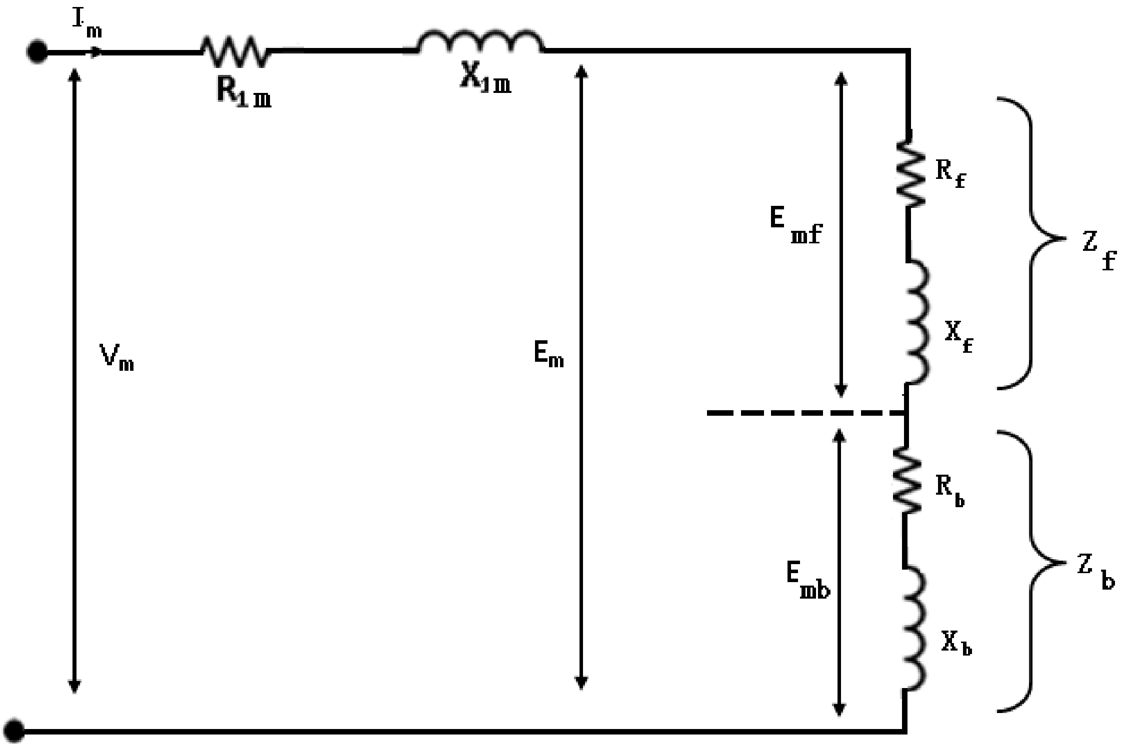
$$Z_f = \frac{(\frac{\acute{R}_2}{2S} + j\,\frac{\acute{X}_2}{2})(j\,\frac{X_M}{2})}{\frac{\acute{R}_2}{2S} + j\,\frac{\acute{X}_2}{2} + j\,\frac{X_M}{2}}$$

Similarly, the rotor impedance representing the effect of backward field referred to the stator winding m is given by an impedance $(\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2})$ in parallel with $j\frac{X_M}{2}$.

$$\therefore \qquad Z_b = R_b + jX_b = (\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2}) \parallel (j\frac{X_M}{2})$$

$$Z_b = \frac{(\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2})(j\frac{X_M}{2})}{\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

The simplified equivalent circuit of single – phase induction motor with only main winding energized is shown in fig.(11).



The current in the stator winding is

$$I_m = \frac{V_m}{Z_{1m} + Z_f + Z_b}$$

The torque of the backward field is in opposite direction to that of the forward field, and therefore the total air – gap power in a single phase induction motor is

$$P_g = P_{gf} - P_{gb}$$

Where P_{gf} = air – gap power for forward field

$$P_{gf} = I_m^2 R_f$$

Where P_{gb} = air – gap power for backward field

$$P_{gb} = I_m^2 R_b$$

∴

$$P_g = I_m^2 R_f - I_m^2 R_b = I_m^2 (R_f - R_b)$$

The torque produced by the forward field

$$T_f = \frac{1}{\omega_s} P_{gf} = \frac{P_{gf}}{2\pi n_s}$$

The torque produced by the backward field

$$T_b = \frac{1}{\omega_s}\, P_{gb} = \frac{P_{gb}}{2\pi n_s}$$

The resultant electromagnetic or induced torque T_{int} is the difference between the torque T_f and T_b :

$$T_{int} = T_f - T_b$$

As in the case of the 3 - phase I.M., the induced torque is equal to the air gap power divided by synchronous angular velocity.

$$T_{int} = \frac{P_g}{\omega_s} = \frac{1}{\omega_s}\left(P_{gf} - P_{gb}\right) = \frac{I_m^2}{\omega_s}(R_f - R_b)$$

The total copper loss is the sum of rotor copper loss due to the forward field and the rotor copper loss due to the backward field.

$$P_{cr} = P_{crf} + P_{crb}$$

And rotor copper loss in a 3 – phase induction motor

P_{cr}=slip * air gap power

$$P_{cr} = sP_{gf} + (2 - s)P_{gb}$$

The power converted from electrical to mechanical form in a single phase induction motor is given by

$$P_{mech} = P_{conv} = \omega T_{ind}$$

$$P_{mech} = (1-s)\omega_s T_{ind}$$

$$= (1-s)P_g = (1-s)(P_{gf} - P_{gb})$$

Or

$$P_{mech} = I_m^2\left(R_f - R_b\right)(1-s)$$

Shaft output power

$$P_{out} = P_{mech} - core\ loss - mechanical\ losses - stray\ losses$$

$$P_{out} = P_{mech} - P_{rot}$$

Where $P_{rot} = rotational\ losses$

Example

A 230 V, 50 Hz, 4 – pole single phase induction motor has the following equivalent circuit impedances:

$$R_{1m} = 2.2\Omega, \qquad \acute{R}_2 = 4.5\Omega$$

$$X_{1m} = 3.1\Omega, \quad \acute{X}_2 = 2.6\Omega, \quad \mathrm{X_M} = 80\Omega.$$

Friction, windage and core loss = 40 W

For a slip of 0.03pu, calculation (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency.

Solution. Form the given data

$$\frac{\grave{R}_2}{2s} = \frac{4.5}{2 \times 0.03} = 75\Omega$$

$$\frac{\grave{R}_2}{2(2-s)} = \frac{4.5}{2(2-0.03)} = 1.142\Omega$$

$$\frac{1}{2}\grave{X}_2 = \frac{1}{2} \times 2.6 = 1.3\Omega$$

$$\frac{1}{2}X_M = \frac{1}{2} \times 80 = 40\Omega$$

For the forward field circuit

$$Z_f = R_f + jX_f = \frac{\left(\frac{\acute{R}_2}{2s} + j\frac{\acute{X}_2}{2}\right)\left(j\frac{X_M}{2}\right)}{\frac{\acute{R}_2}{2s} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

$$= \frac{(75 + j1.3)(j40)}{75 + j1.3 + j40} = \frac{(75.011\angle 0.993°)(40\angle 90°)}{85.619\angle 28.84°}$$

$$= 35.04\angle 62.15°\Omega = 16.37 + j30.98\Omega$$

For the backward field

$$Z_b = R_b + jX_b = \frac{\left(\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2}\right)\left(j\frac{X_M}{2}\right)}{\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

$$= \frac{(1.142 + j1.3)(j40)}{1.142 + j1.3 + j40} = \frac{(1.73\angle 48.7°)(40\angle 90°)}{41.316\angle 88.4°}$$

$$= 1.675\angle 50.3° = 1.07 + j1.29\Omega$$

$$Z_{1m} = R_{1m} + jX_{1m} = 2.2 + j3.1$$

The total series impedance

$$Z_e = Z_{1m} + Z_f + Z_b$$

$$= 2.2 + j3.1 + 16.37 + j30.98 + 1.07 + j1.29$$

$$= 19.64 + j35.37 = 40.457\angle 60.96°\Omega$$

(a) Input current

$$I_m = \frac{V_m}{Z_e} = \frac{230\angle 0°}{40.457\angle 60.96°} = 5.685\angle - 60.96° A.$$

(b) Power factor = $\cos(-60.95°) = 0.4856\ lagging.$

(c) Developed power

$$P_{conv} = P_d = I_m^2\left(R_f - R_b\right)(1 - s)$$

$$= (5.685)^2(16.37 - 1.07)(1 - 0.03) = 479.65\ W$$

(d) Output power = $\mathrm{P_d} - \mathrm{P_{rot}} = 479.65 - 40 = 439.65\ \mathrm{W}$

Input power $= VI_m \cos \emptyset = 230 \times 5.685 \times 0.4856 = 634.9\ W$

(e) Efficiency $= \frac{output}{input} = \frac{439.65}{634.9} = 0.692\ pu.$

Determination of Equivalent Circuit Parameters

The parameter of the equivalent circuit of single – phase induction motor can be determined from the blocked – rotor and no – load tests. These tests are performed with auxiliary winding kept open, except for the capacitor – run motor.

Blocked – rotor test

In this test the rotor is at rest (blocked). A low voltage is applied to the stator so that rated current flows in the main winding. The voltage (V_{scr}), current(I_{scr}) and power input (P_{scr}) are measured. With the rotor blocked, s =1 the impedance $\frac{X_M}{2}$ in the equivalent circuit is so large compared with $(\frac{\acute{R}_2}{2}+j\frac{\acute{X}_2}{2})$ that it may be neglected from the equivalent circuit. Therefore the equivalent circuit at s=1 is shown in fig.(12).

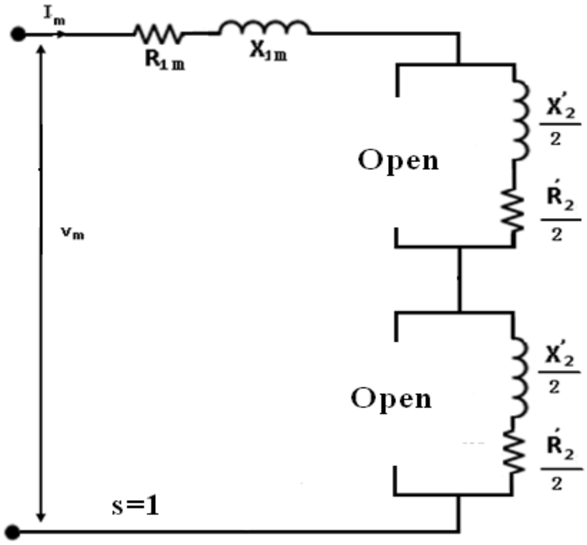


Fig.(12) simplified equivalent circuit of single phase I.M. with locked rotor

$$Z_e = \frac{V_{scr}}{I_{scr}}$$

From fig.(12), the equivalent series resistance R_e of the motor is

$$R_e = R_{1m} + \frac{\acute{R}_2}{2} + \frac{\acute{R}_2}{2} = R_{1m} + \acute{R}_2 = \frac{P_{scr}}{I_{scr}^2}$$

Since the resistance of the main stator winding R_{1m} is already measured, the effective rotor resistance at line frequency is given by

$$\acute{R}_2 = R_e - R_{1m} = \frac{P_{scr}}{I_{scr}^2} - R_{1m}$$

From fig.(12), the equivalent reactance $X_e$is given by

$$X_e = X_{1m} + \frac{\acute{X}_2}{2} + \frac{X_2{}'}{2} = X_{1m} + \acute{X}_2$$

Since the leakage reactance X_{1m} and $\acute{X}_2$cannot be separated out we make a simplifying assumption that $X_{1m} = \acute{X}_2$.

$$\therefore \qquad X_{1m} = \acute{X}_2 = \frac{1}{2} X_e = \frac{1}{2}\sqrt{Z_e^2 - R_e^2}$$

Thus, from blocked - rotor test, the parameters $\acute{R}_2, \acute{X}_{1m}$, $\acute{X}_2$ can be found if R_{1m} is known.

No - load test

The motor is run without load at rated voltage and rated frequency. The voltage(V_o), current(I_o) and input power (P_o) are measured. At no load, the slip s is very small close to zero and $\frac{\acute{R}_2}{2s}$ is very large as compared to $\frac{X_M}{2}$.

The resistance $\frac{\acute{R}_2}{2(2-S)} \cong \frac{\acute{R}_2}{4}$ associated with the backward rotating field is so small as compared to $\frac{X_M}{2}$, that the backward magnetizing current is negligible. Therefore, under no load conditions, the equivalent circuit becomes as shown in fig.(13).

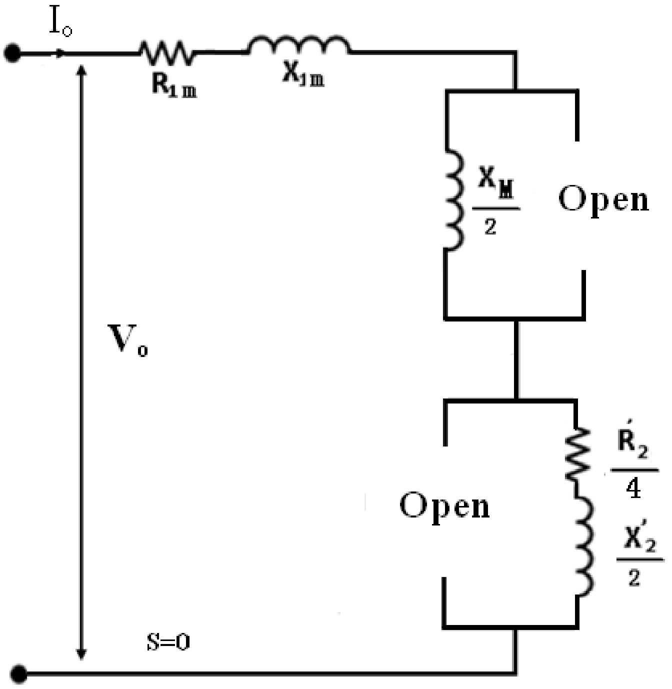


Fig.(13) simplified equivalent circuit of single phase I.M. at no load

From the fig (13), the equivalent reactance at no load is given by

$$X_o = X_{1m} + \frac{X_M}{2} + \frac{\acute{X}_2}{2}$$

Since X_{1m} and $\acute{X}_2$ are already known from the blocked rotor test, the magnetizing reactance X_M can be calculated from above equation.

And $$X_o = Z_o \sin \emptyset_o = Z_o \sqrt{1 - \cos^2 \emptyset_o}$$

$$\cos \emptyset_o = \frac{P_o}{V_0 I_o}$$

$$Z_o = \frac{V_0}{I_0}$$

Example

A 220 V, single – phase induction motor gave the following test results:

Blocked – rotor test : 120V, 9.6A, 460W

No – load test : 220V, 4.6A, 125W

The stator winding resistance is 1.5Ω, and during the blocked – rotor test, the starting winding is open. Determine the equivalent circuit parameters. Also, find the core, fraction and windage losses.

Solution

Blocked – rotor test

V_{scr}=120V, $I_{scr} = 9.6$A , P_{scr}=460W

$$Z_e = \frac{V_{scr}}{I_{scr}} = \frac{120}{9.6} = 12.5\Omega$$

$$R_e = \frac{P_{scr}}{I_{scr}^2} = \frac{460}{(9.6)^2} = 4.99\Omega$$

$$X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{(12.5)^2 - (4.99)^2} = 11.46\Omega$$

$$X_{1m} = \acute{X}_2 = \frac{1}{2} X_e = \frac{1}{2} * 11.46 = 5.73\Omega$$

$$R_{1m} = 1.5\Omega$$

$$R_e = R_{1m} + \acute{R}_2$$

$$\acute{R}_2 = R_e - R_{1m} = 4.99 - 1.5 = 3.49\Omega$$

No – load test: V_o=220V, $I_o = 4.6$A , P_o=125W

$$\cos\emptyset_o = \frac{P_o}{V_0 I_o} = \frac{125}{220 * 4.6} = 0.1235$$

$$\therefore \quad \sin\emptyset_o = 0.9923$$

$$Z_o = \frac{V_0}{I_0} = \frac{220}{4.6} = 47.83\Omega$$

$$\therefore \quad X_o = Z_o \sin\emptyset_o = 47.83 * 0.9923 = 47.46\Omega$$

Core, fraction and windage losses

=power input to motor at no load – no load copper loss

$$= P_o - I_o^2 \left(R_{1m} + \frac{\acute{R}_2}{4} \right)$$

$$= 125 - (4.6)^2 \left(1.5 + \frac{3.49}{4} \right) = 74.8W$$

Single-Phase Series Motor (Universal)

The single-phase series motor is a commutator-type motor. If the polarity of the line terminals of a dc series motor is reversed, the motor will continue to run in the same direction. Thus, it might be expected that a dc series motor would operate on alternating current also.

The direction of the torque developed in a dc series motor is determined by both filed polarity and the direction of current through the armature($T \propto \Phi i_a$). Let a dc series motor be connected across a single-phase ac supply. Since the same current flows through the field winding and the armature, it follows that ac reversals from positive to negative, or from negative to positive, will simultaneously affect both the field flux polarity and the current direction through the armature. This means that the direction of the developed torque will remain positive, and rotation will continue in the same direction. Thus, a series motor can run both on dc and ac.

Motor that can be used with a single-phase ac source as well as a dc source of supply voltages are called **universal motor**. However, a series motor which is specifically designed for dc operation suffers from the following drawbacks when it is used on single-phase ac supply:

1. Its efficiency is low due to hysteresis and eddy-current losses.

2. The power factor is low due to the large reactance of the field and the armature winding.
3. The sparking at the brushes is excessive.

In order to overcome these difficulties, the following modifications are made in a d.c. series motor that is to operate satisfactorily on alternating current:

- The field core is constructed of a material having low hysteresis loss. It is laminated to reduce eddy-current loss.
- The field winding is provided with small number of turns. The field-pole areas is increased so that the flux density is reduced. This reduces the iron loss and the reactive voltage drop.
- The number of armature conductors is increased in order to get the required torque with the low flux.
- In order to reduce the effect of armature reaction, thereby improving commutation and reducing armature reactance, a compensating winding is used.

  The compensating winding is put in the stator slots. The axis of the compensating winding is 90° (electrical) with the main field axis. It may be connected in series with both the armature and field as shown in fig.1 In such a case the motor is conductively compensated.

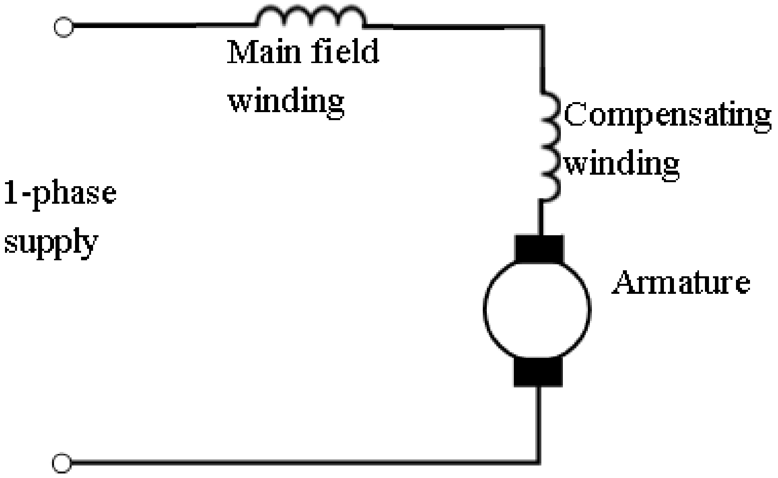


Figure. 2

The compensating winding may be short circuited on itself, in which case the motor is said to be inductively compensated (fig.2)

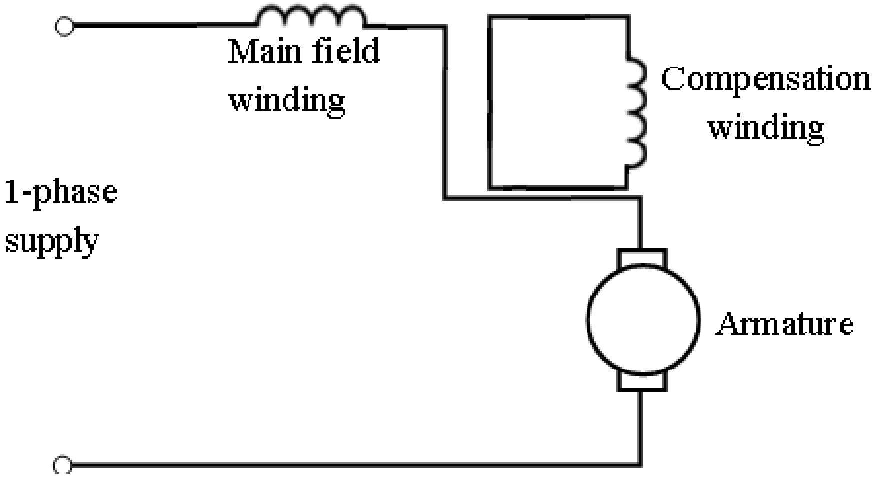


Fig. 2. Series motor with inductively compensated winding

- The armature of universal motor is of the same construction as ordinary series motor. In order to minimize commutation problems, high resistance brushes with increased brush area are used.

The universal motor is simply, and cheap. It is used usually for rating not greater than 750 W.

The characteristics of universal motor are very much similar to those of d.c. series motors, but the series motor develops less torque when operating from an a.c. supply than when working from an equivalent d.c. supply. The direction of rotation can be changed by interchanging connections to the field with respect to the armature as in d.c. series motor.

Speed control of universal motors is best obtained by solid-state devices. Since the speed of these is not limited by the supply frequency and may be as high as 20,000 r.p.m. (greater than the maximum synchronous speed of 3000 r.p.m. at 50 Hz), they are most suitable for applications requiring high speeds.

Applications

There are numerous applications where universal motors are used, such as portable drills, hair dryers, grinders, table-fans, blowers, polishers, kitchen appliances ets. They are also used for many other purposes where speed control and high values of speed are necessary. Universal motors of a given horse power rating are significantly smaller than other kinds of a.c. motors operating at the same frequency.

Phasor Diagram of a.c. Series Motor

The schematic diagram and phasor diagram for the conductively coupled single-phase ac series motor are shown in fig. 3

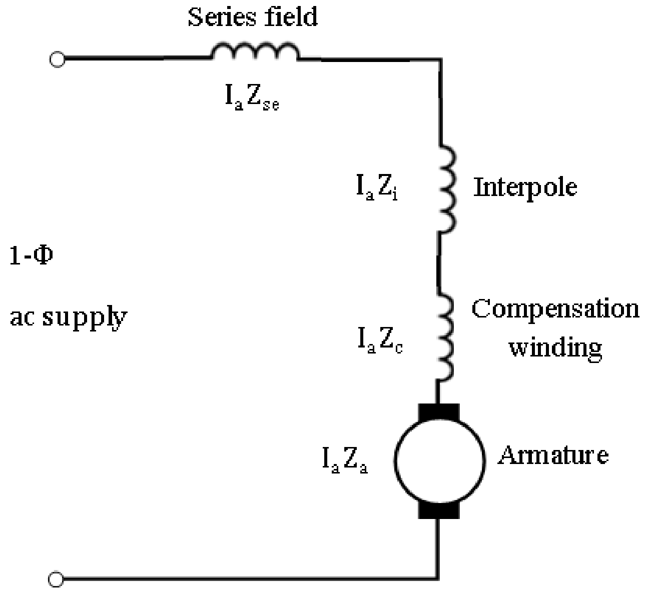


Fig. (3-a). Schematic diagram of conductively coupled ac series motor.

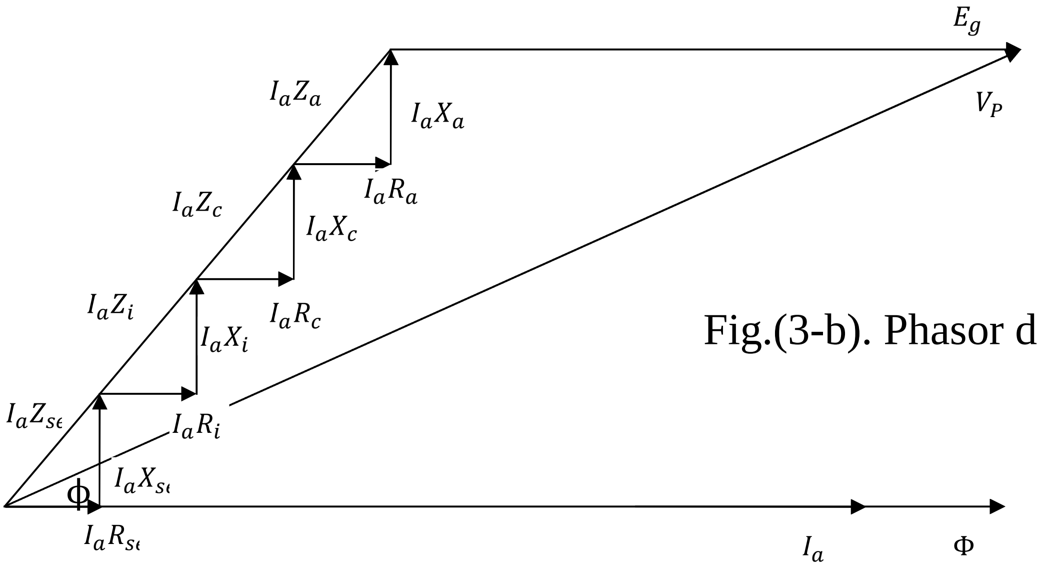


Fig.(3-b). Phasor diagram

The resistance drops $I_aR_{se},, I_aR_i, I_aR_c$ and I_aR_a due to resistances of series field, interpole winding, compensating winding and of armature respectively are in phase with armature current I_a. The reactance drops $I_aX_{se}, I_aX_i, I_aX_c$ and I_aX_a due to reactance of series field, interpole winding compensating winding and of armature respectively lead current I_a by 90°. The generated armature counter emf is E_g. the terminal phase voltage V_p is equal to the phasor sum of E_g and all the impedance drops in series.

$$V_p = E_g + I_aZ_{se} + I_aZ_i + I_aZ_c + I_aZ_a$$

The power factor angle between V_p and I_a is φ.

Example.

A universal series motor has a resistance of 30Ω and an inductance of 0.5 H. when connected to a 250 V dc supply and loaded to take 0.8 A it runs at 2000 rpm. Determine the speed, torque and power factor, when connected to a 250 V, 50 Hz ac supply and loaded to take the same current.

Solution Operation of motor on dc

$$\mathrm{E_{bdc} = V - I_aR_a = 250 - 0.8 \times 30 = 226\,V}$$

$$\mathrm{N_{dc} = 2000\ r.p.m}$$

Operation motor on ac

$$\mathrm{X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157\,\Omega}$$

From the phasor diagram shown in fig. 4,

$$\mathrm{AF^2 = AG^2 + GF^2}$$

$$\mathrm{V^2 = (AB + BG)^2 + GF^2 = (AB + DF)^2 + GF^2}$$

$$\mathrm{= (I_aR_a + E_{bac})^2 + (I_aX_L)^2}$$

$$\mathrm{E_{bac} + I_aR = \sqrt{V^2 - (I_aX_L)^2}}$$

$$\mathrm{E_{bac} = -0.8 \times 30 + \sqrt{(250)^2 - (0.8 \times 157)^2}}$$

$$= -24 + 216.12 = 192.12\,V$$

Since the currents in dc and ac operation are equal, the flux will also be equal $(\Phi_{ac} = \Phi_{dc})$

$$\frac{E_{bdc}}{E_{bac}}=\frac{KN_{dc}\Phi_{dc}}{KN_{ac}\Phi_{ac}}=\frac{N_{dc}}{N_{ac}}$$

$$\mathrm{N_{ac}}=\mathrm{N_{dc}}\frac{\mathrm{E_{bac}}}{\mathrm{E_{bdc}}}=2000\times\frac{192.12}{226}=1700\text{ rpm}$$

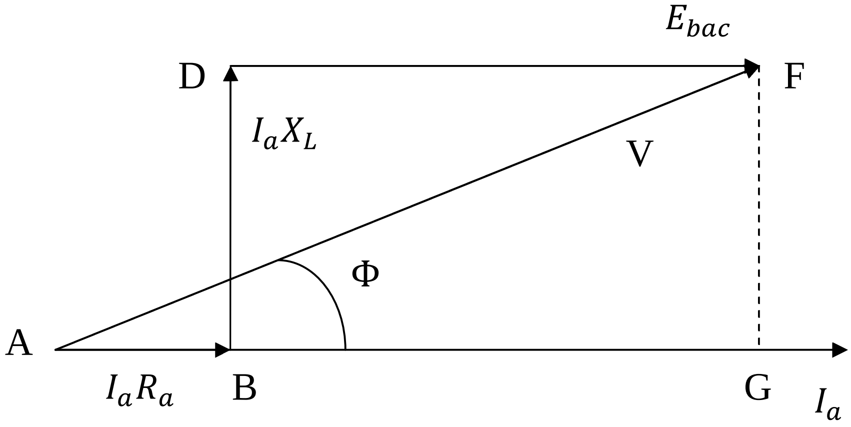


Fig.4

Power factor, $\cos\phi=\frac{AG}{AF}=\frac{E_{bac}+I_aR_a}{V}$

$$=\frac{192.12+0.8\times 30}{250}=0.8645\ (lagging)$$

Mechanical power developed

$$\mathrm{P_{mech}}=\mathrm{E_{bac}I_a}=192.12\times 0.8=153.7\text{ W}$$

Torque developed

$$\tau=\frac{P_{mech}}{\omega_m}=\frac{P_{mech}}{2\pi n_{ac}}$$

$$=\frac{153.7}{2\pi\times(1700/60)}=0.8633\ Nm$$

Stepper (Or Stepping) Motors

The stepper or stepping motor has a rotor movement in discrete steps. The angular rotation is determined by the number of pulses fed into the control circuit. Each input pulse initiates the drive circuit which produces one step of angular movement. Hence, the device may be considered as a digital-to-analogue converter.

There are three most popular types of rotor arrangements:

1. Variable reluctance (VR) type
2. Permanent magnet (PM) type
3. Hybrid type, a combination of VR and PM.

Step Angle

The angle by which the rotor of a stepper motor moves when one pulse is applied to the stator (input) is called **step angle.** This is expressed in degrees. The resolution of positioning of stepper motor is decided by the step angle. Smaller the step angle the higher is the resolution of positioning of the motor. The **step number or resolution** of a motor is the number of steps it makes in one revolution of the rotor.

$$Resolution = \frac{number\ of\ steps}{number\ of\ revloutions\ of\ the\ rotor}\ .$$

Higher the resolution, grater is the accuracy of positioning of objects by the motor. Stepper motor are realizable for very small step angles. Some precision motors can make 1000 steps in one revolution with a step angle of 0.36°. A standard motor will have a step angle of 1.8° with 200 steps per revolution.

Variable Reluctance (VR) Stepper Motor

A variable reluctance (VR) stepper motor can be of single-stack type or the multi-stack type.

i- Single-Stack Variable Reluctance Motor

A Variable reluctance stepper motor has salient-pole (or tooth) stator. The stator has concentrated winding places over the stator poles (teeth). The number of phases of the stator depends upon the connection of stator poles. Usually three or four phases winding are used. The rotor is slotted structure made from ferromagnetic material and carries no winding. Both the stator and rotor are made up of high quality magnetic materials having very high permeability so that the exciting current required is very small. When the stator phases are excited in a proper sequence from dc source with the help of semiconductor switch, a magnetic field is produced. The ferromagnetic rotor occupies the position which presents minimum reluctance to the stator field. That is, the rotor axis aligns itself to the stator field axis.

Elementary operation of variable reluctance motor can be explained through the diagram of fig. 1.

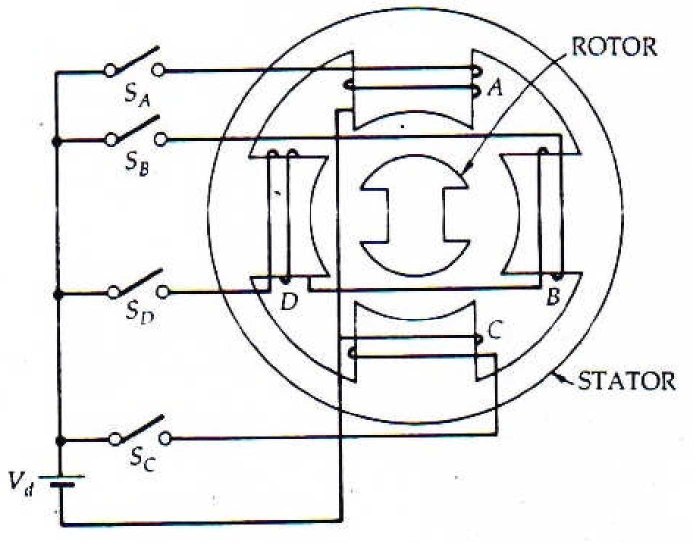


Fig.(1)four phase 4/2 VR stepper motor

It is a four-phase, 4/2-pole (4 poles in the stator and 2 in the rotor), single-stack, variable reluctance stepper motor. Four phases are A, B, C and D are connected to dc source with the help of semiconductor switches S_A, S_B, S_C and S_D respectively. The phase winding of the stator are energised in the sequence A, B, C, D, A. When winding A is excited, the rotor aligns with the axis of phase A. The rotor is stable in this position and cannot move until phase A is de-energised. Next, phase B is excited and A is disconnected. The rotor moves through 90°in clockwise direction to align with the resultant air gap field which now lies along the axis of phase B. Further, phase C is excited and B is disconnected, the rotor moves through a further step of 90° in the clockwise direction. In this position, the rotor aligns with the resultant air gap field which now lies along the axis of phase C. Thus, as the phases are excited in the sequence A, B, C, D, A the rotor moves through a step of 90 at each transition in clockwise direction. The rotor completes one revolution through four steps. The direction of rotation can be reversed by reversing the sequence of switching the winding, that is A, D, C, B, A. it is seen that the direction of rotation depends only on the sequence of switching the phases and is independent of the direction of currents through the phases.

The magnitude of step angle for any VR and PM stepper motor is given by

$$\alpha = \frac{360}{m_s N_r}$$

Where $\alpha = step\ angle$

$m_s = number\ of\ stator\ phases\ or\ stacks$

$N_r = number\ of\ rotor\ teeth\ (or\ rotor\ poles)$

The step angle is also expressed as

$$\alpha = \frac{N_s - N_r}{N_s N_r} \times 360°$$

Where $N_s = stator\ poles\ (teeth)$

The step angle can be reduced from 90 to 45 by exciting phases in the sequence A, A+B , B , B+ C , C, C+ D , D, D + A, A. Here (A+B) means that phase winding A and B are excited together and the resultant stator field will be midway between the poles carrying phase winding A and B. That is, the resultant field axis makes an angle of 45 with the axis of pole A in the clockwise direction. Therefore when phase A is excited, the rotor aligns with the axis of phase A. when phases A and B are excited together, the rotor moves by 45 in the clockwise direction. Thus, It is seen if the windings are excited in the sequence A, A+ B, B, B+ C, C, C+ D, D, D+A, A the rotor rotates in steps of 45 in the clockwise direction. The rotor can be rotated in steps of 45 in the anticlockwise direction by exciting the phase in the sequence A, A + D, D, D + C, C, C + B, B, B + A, A. this method of gradually shifting excitation from one phase to another (for example, from A to B with an intermediate step of A+ B) is known as micro stepping. It is used to realize smaller steps. Lower values of step angle can be obtained by using a stepping motor with more number of poles on stator and teeth on rotor. Consider a four-phase, 8/6 pole, single stack variable reluctance motor shown in fig 2.

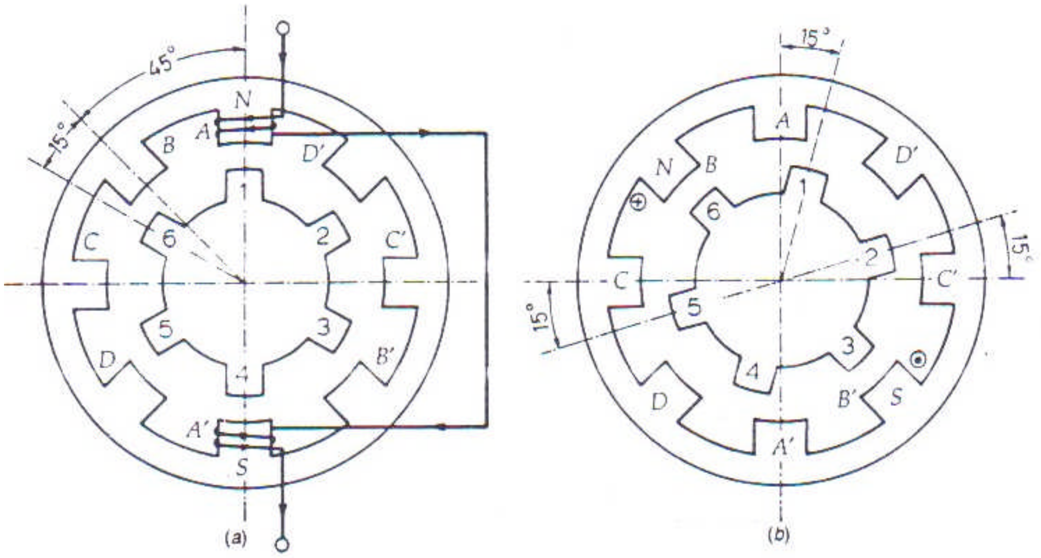


Fig. (2): four – phase, 8/6 VR stepper motor.

The coils wound around diametrically opposite poles are connected in series and four circuits (phase) are found. These phases are energised from a dc source through electronic switching device. The rotor has six poles (teeth). For the sake of simplicity, only phase A winding is shown in fig (2-a). when phase A (coil A-A`) is excited, rotor teeth numbered 1 and 4 aligned along the axis of phase A winding. Next phase winding A is de-energised and phase winding B is excited. Rotor teeth numbered 3 and 6 get aligned along the axis of phase B and the rotor moves through a step angle of 15 in the clockwise direction. Further clockwise rotation of 15 is obtained by de-energising phase winding B and exciting phase winding C. with the sequence A, B, C, D, A, four steps of rotation are completed and the rotor moves through 60 in clockwise direction. For one complete revolution of the rotor, 24 steps are required. For anticlockwise rotation of rotor through each step of 15, the phase windings are excited in reverse sequence of A, D, C, B, A.

ii- Multi-Stack Variable Reluctance Stepper Motor

A multi-stack (or m-stack) variable reluctance stepper motor can be considered to be made up of m identical single-stack variable reluctance motors with their rotors mounted on a single shaft. The stators and rotors have the same number of poles (or teeth) and, therefore, same poles pitch. For m-stack motor, the stator poles in all m stacks are aligned, but the rotor poles are displaced by 1/m of the pole pitch angle from one another. All the stator pole windings in given stack are excited simultaneously and, therefore, the stator winding of each stack forms one phase. Thus, the motor has the same number of phases as the number of stacks.

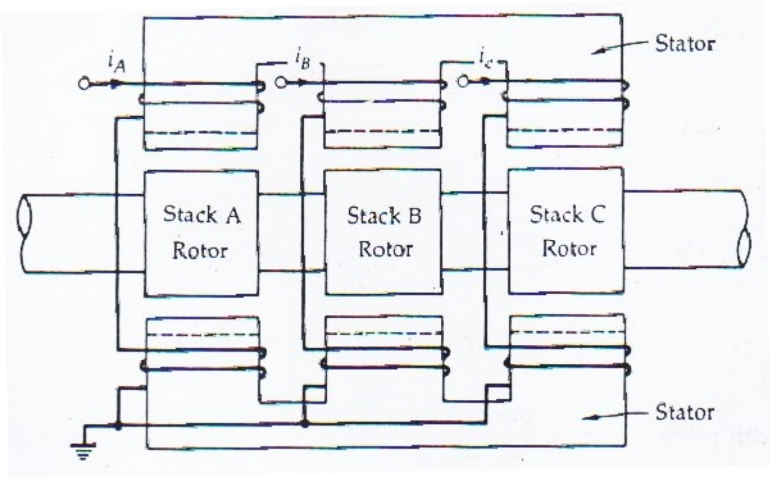


Fig.3 shows the cross-section of a three-stack (three-phase) motor parallel to the shift. In each stack, stators and rotors have 12 poles. For a 12-pole rotor, the pole pitch is 30, and therefore the rotor poles are displaced from each other by one-third of the pole pitch or 10°. The stator teeth in each stack are aligned. When the phase winding A is excited rotor teeth of stack A are aligned with the stator teeth as shown in fig. (4-a).

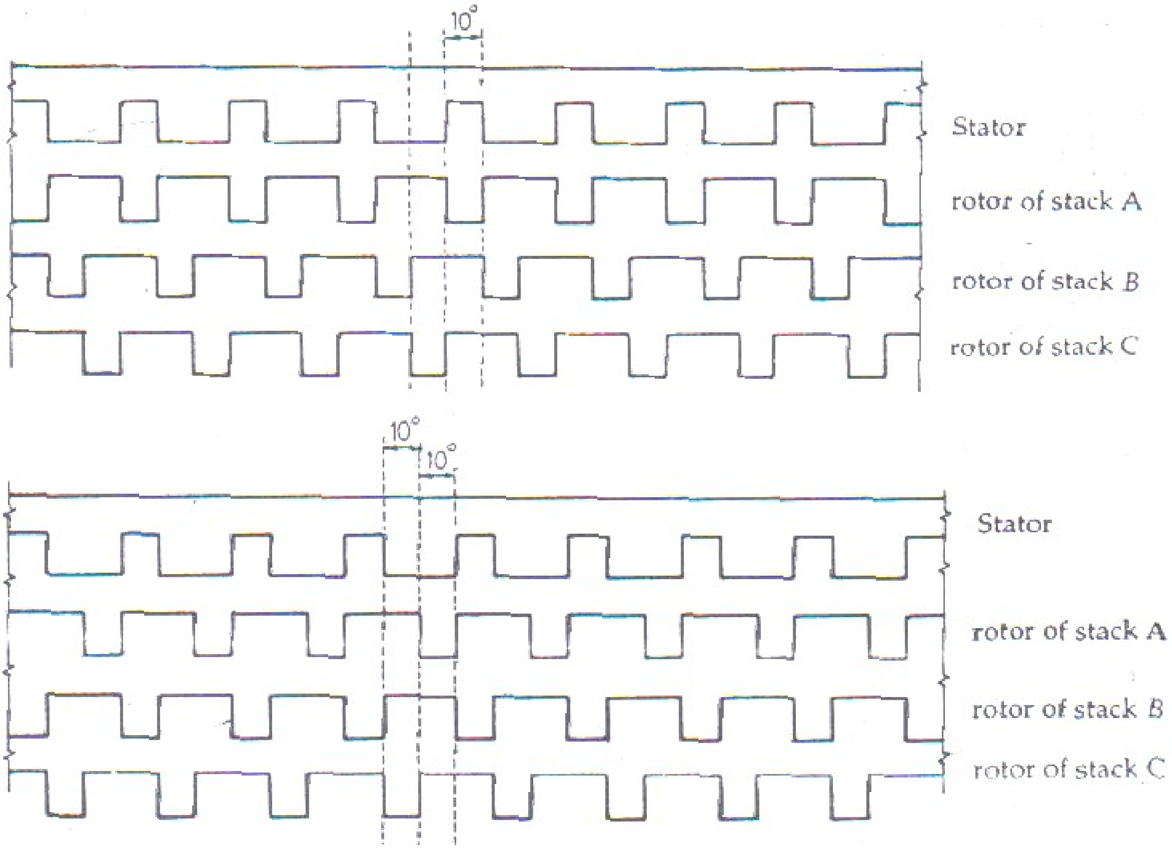


Fig.(4) position of stator and rotor teeth in 3 – stack VR motor
(a)phase A is excited (b) phase B is excited

When phase A is de-energized and phase B is excited, rotor teeth of stack B are aligned with stator teeth. This new alignment is made by the rotor movement of 10° in the anticlockwise direction. Thus the motor moves one step (equal to $\frac{1}{3}$ pole pitch) due to change of excitation from stack A to B (fig.4-b).

Next phase B is de-energized and phase C is excited. The rotor moves by another step of one-their of pole pitch in the anticlockwise direction. Another change of excitation from stack C to stack A will once more align the stator and rotor teeth in stack A. however, during this process $(A \to B \to C \to A)$ the rotor has move one rotor tooth pitch.

Let N_r be the number of rotor teeth and m the number of stacks or phases.

Then

$$Tooth\,pitch \qquad \tau_p = \frac{360°}{N_r}$$

$$Step\ angle \qquad = \frac{360°}{mN_r}$$

$$In\ our\ case, \qquad \tau_p = \frac{360°}{12} = 30°$$

$$Step\ angle \qquad = \frac{360°}{3 \times 12} = 10°$$

Multi-stack variable reluctance stepper motors are widely used to obtain smaller step size, typically in the range of 2 to 15 degrees.

The variable reluctance motors, both single and multi-stack type, have high torque to inertia ratio. The reduced inertia enables the VR motor to accelerate the load faster.

Permanent Magnet (PM) Stepper Motor

Permanent-magnet (PM) stepper motor has a stator construction similar to that of the single-stack variable reluctance motor. The rotor is cylindrical and consists of permanent-magnet poles made of high retentivity steel. Figure 5 shows a 4/2-pole PM stepper motor. The concentrated windings on diametrically opposite poles are connected in series to form 2-phase winding on the stator.

The rotor poles align with the stator teeth depending on the excitation of the winding. The two coils A-A` connected in series form phase A winding. Similarly, the two coils BB` connected in series form phase B winding.

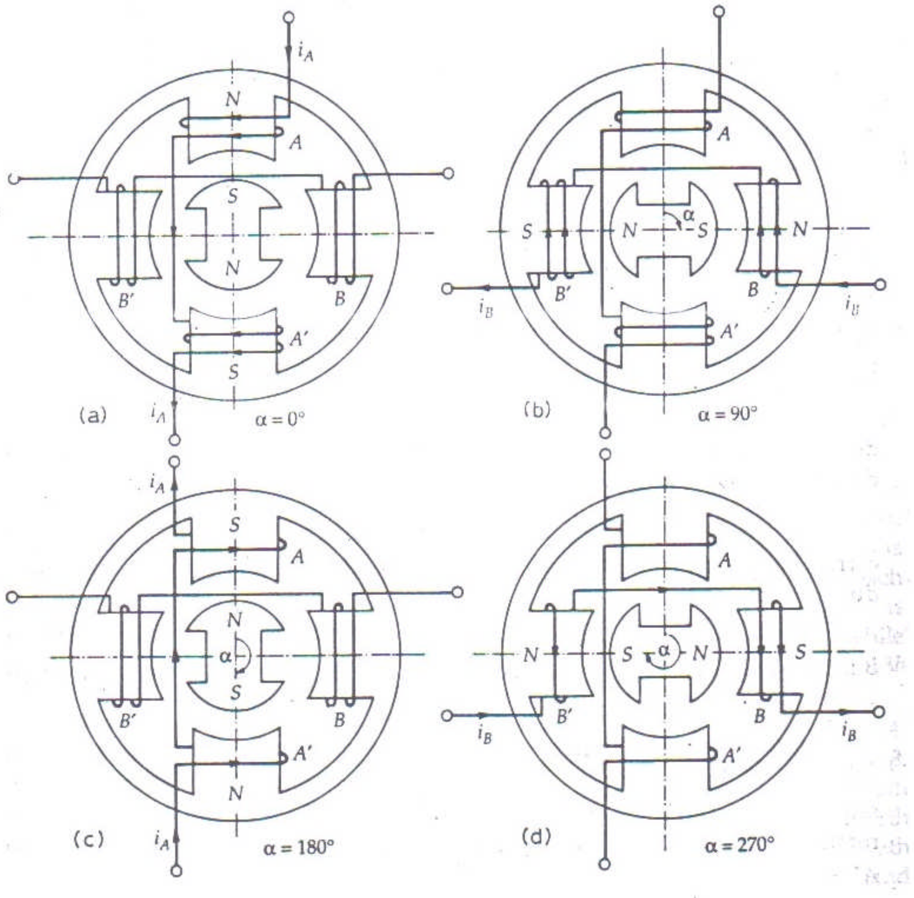


Fig .5 2-phase 4/2-pole PM stepper motor

Fig.5-a shows the condition when phase A winding is excited with current i_A^+. here south pole of rotor is attracted by the stator phase A pole so that the magnetic axes of the stator and rotor coincide and $\alpha = 0°$.

In fig.5-b, phase A winding does not carry any current and the phase B winding is excited by i_B^+. stator produced poles now attract the rotor pole and the rotor moves by a step of 90 in the clockwise direction, that is, $\alpha = 90$

In fig.5-c, phase A winding is excited by i_A^- and phase B winding is de-energized. The rotor moves through a further step of 90 in clockwise direction so that $\alpha = 180$.

In fig.5-d, phase B winding is excited by i_B^- and phase A winding carries no current. The rotor again moves further by a step of 90 in clockwise direction so that $\alpha = 270$.

For further 90 clockwise rotation of rotor so that $\alpha = 360$, phase winding B is de-energized and phase A winding is excited by current i_A^+ . Thus, four steps complete one revolution of the rotor.

It is seen that in a permanent-magnet stepper motor, the direction of rotation depends on polarity of phase current. For clockwise rotor movement the sequence of exciting the stator phase windings is A^+, B^+, A^-, B^-, A^+. for anticlockwise rotation the sequence of switching the phase windings should be reversed to A^+, B^-, A^-, B^+, A^+.

It is difficult to make small PM rotor with large numbers of poles and , therefore stepper motor of this type are restricted to larger step size in the range of 30 to 90. However disc type PM stepper motors are available to give small step size and low inertia.

Permanent magnet stepper motors have higher inertia and , therefore lower acceleration than VR stepper motors. The maximum step rate for PM stepper motors is 300 pulses per second, whereas it can be as high as 1200 pulses per second for VR stepper motors. The PM stepper motors produces more toque per ampere stator current than VR motor.

Detent Torque or Restraining Torque

The residual magnetism in the permanent magnet material produced a detent torque on the rotor when the stator coils are not energized. This torque prevent the motor from drifting when the machine supply is turned off.

In case the motor is unexcited, the permanent magnet and hybrid stepping motor are able to develop a detent torque restricting the rotor rotation. The detent torque is defined as the maximum load torque that can be applied to the shaft of unexcited motor without causing continuous rotation.

Hybrid Stepper Motor

A hybrid stepper motor combines the features reluctance and permanent magnet stepper motors.

The main advantages of hybrid stepper motors compared with variable reluctance stepper motors are as follows:

1. Small step length.
2. Greater torque per unit volume.
3. Provides detent torque with windings de-energized.
4. Less tendency to resonate.
5. High efficiency at lower speed and lower stepping rates.

Disadvantages of hybrid stepper motors

1. High inertia and weight due to presence of rotor magnet.
2. Performance affected by change in magnetic strength.
3. More costly than variable reluctance stepper motor.

Example (1)

Calculate the stepping angle for a 3- stack, 16 – teeth variable reluctance motor.

Solution : stepping angle $\propto = \frac{360}{m_s \times N_r}$

Where $m_s = number\ of\ stator\ phases\ or\ stacks$

$$N_r = number\ of\ rotor\ teeth\ (rotor\ poles)$$

$$\therefore \propto = \frac{360}{3 \times 16} = 7.5° \ per\ step.$$

Example (2)

Calculate the stepping angle for a 3- phase , 24 – pole permanent magnet stepper motor.

Solution :

stepping angle $\propto = \frac{360}{m_s \times P_r} = \frac{360}{3 \times 24} = 5° \, per \ step.$